Derivative Investments

CFA二级培训项目

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Topic Weightings in CFA Level II

Session NO.	Content	Weightings
Study Session 1-2	Ethics & Professional Standards	10-15
Study Session 3	Quantitative Methods	5-10
Study Session 4	Economic Analysis	5-10
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Study Session 7-8	Corporate Finance	5-15
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Study Session 12-13	Fixed Income Analysis	10-20
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Framework

Derivative Investments

SS14 Derivative Instruments ——Valuation and Strategies

- R40 Pricing and Valuation of Forward Commitments
- R41 Valuation of Contingent Claims
- R42 Derivatives Strategies

Reading 40

Pricing and Valuation of Forward Commitments

Framework

1. Forward
   - Principle of Arbitrage-free Pricing
   - Equity Forward and Futures Contracts
   - Interest Rate Forward and Futures Contracts(FRA)
   - Fixed-Income Forward and Futures Contracts
   - Currency Forward Contracts
2. T-bond Futures
3. Swap
   - Interest Rate Swap Contracts
   - Currency Swap Contracts
   - Equity Swap Contracts

Forward Contracts

- A **forward contract** is an agreement between two parties in which one party, the buyer, agrees to buy from the other party, the seller, an underlying asset or other derivative, at a future date at a price established at the start of the contract.
- The party to the forward contract that agrees to buy the financial or physical asset has a long forward position and is called the long. The party to the forward contract that agrees to sell/deliver the asset has a short forward position and is called the short.

Price and Value

- The price is the predetermined price in the contract that the long should pay to the short to buy the underlying asset at the settlement date
- **The contract value is zero** to both parties at initiation
- **The no-arbitrage principle:** there should not be a riskless profit to be gained by a combination of a forward contract position with position in other asset.
  - Two assets or portfolios with identical future cash flows, regardless of future events, should have same price
  - The portfolio should yield the risk-free rate of return, if it generates certain payoffs
  - General formula: $FP = S_0 \times (1+R_f)^T$

Generic Pricing: No-Arbitrage Principle

- **Pricing a forward contract** is the process of determining the no-arbitrage price that will make the value of the contract be zero to both sides at the initiation of the contract
  - Forward price=price that would not permit profitable riskless arbitrage in frictionless markets
- **FP=S_0 + Carrying Costs - Carrying Benefits**
- **Valuation of a forward contract** means determining the value of the contract to the long (or the short) at some time during the life of the contract.

Forwards Arbitrage

➢ **Cash-and-Carry Arbitrage When the Forward Contract is Overpriced**

- If $FP > S_0 \times (1+R_f)^T$

At initiation	At settlement date
• Short a forward contract • Borrow S_0 at the risk-free rate • Use the money to buy the underlying bond	• Deliver the underlying to the long • Get FP from the long • Repay the loan amount of $S_0 \times (1+R_f)^T$
	Profit= $FP - S_0 \times (1+R_f)^T$

Forwards Arbitrage

- **Reverse Cash-and-Carry Arbitrage when the Forward Contract is Under-priced**
  - If $FP < S_0 \times (1+R_f)^T$

At initiation	At settlement date
• Long a forward contract • Short sell the underlying bond to get S_0 • Invest S_0 at the risk-free rate	• Pay the short FP to get the underlying bond • Close out the short position by delivering the bond • Receive investment proceeds $S_0 \times (1+R_f)^T$
	$Profit = S_0 \times (1+R_f)^T - FP$

Forward contract value

- **T-bill (zero-coupon bond) forwards**
  - buy a T-bill today at the spot price (S_0) and short a T-month T-bill forward contract at the forward price (FP)

$$\boxed{FP = S_0 \times (1 + R_f)^T}$$

  - Forward value of long position at initiation(t=0), during the contract life(t=t), and at expiration(t=T)

Time	Forward Contract Valuation
t=0	Zero, because the contract is priced to prevent arbitrage
t=t	$S_t - \dfrac{FP}{(1+R_f)^{T-t}}$
t=T	S_T-FP

Equity Forward Contracts

- **Forward contracts on a dividend-paying stock**
  - **Price:** $FP = (S_0 - PVD_0) \times (1 + R_f)^T$
  - **Value:** $V_{long} = S_t - PVD_t - \dfrac{FP}{(1+R_f)^{T-t}}$

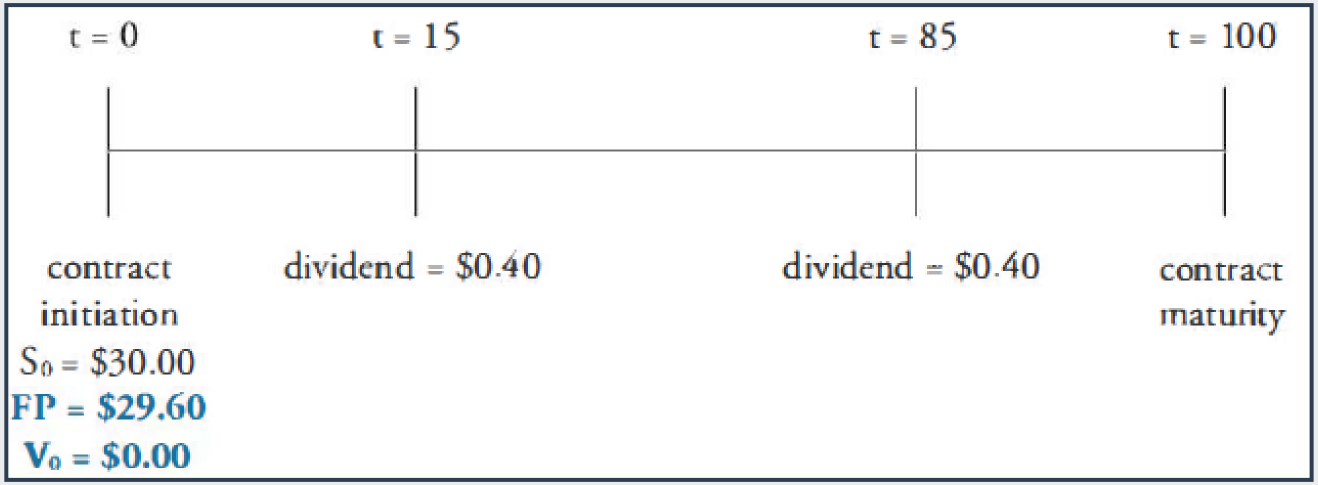
$$Or \quad V_{long} = PV_{t,T}[F_t(T) - F_0(T)]$$

Example

- Calculate the no-arbitrage forward price for a 100-day forward on a stock that is currently priced at $30.00 and is expected to pay a dividend of $0.40 in 15 days, $0.40 in 85 days, and $0.50 in 175 days. The annual risk-free rate is 5%, and the yield curve is flat.
- Ignore the dividend in 175 days because it occurs after the maturity of the forward contract.

$$PVD = \frac{\$0.4}{1.05^{15/365}} + \frac{\$0.4}{1.05^{85/365}} = \$0.7946$$

$$FP = (\$30 - \$0.7946) \times 1.05^{100/365} = \$29.6$$

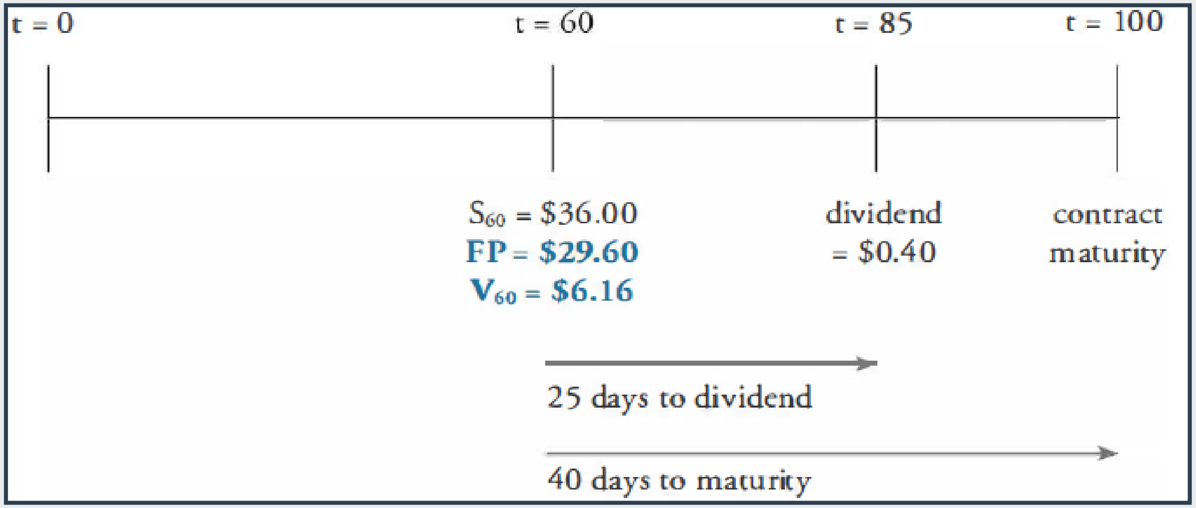


Example

- After 60 days, the value of the stock in the previous example is $36.00. Calculate the value of the equity forward contract to the long position, assuming the risk-free rate is still 5% and the yield curve is flat.
- There's only one dividend remaining (in 25 days) before the contract matures (in 40 days) as shown below, so:

$$PVD_{60} = \frac{\$0.4}{1.05^{25/365}} = \$0.3987$$

$$V_{60}\ (long\ position) = (\$36 - \$0.3987) - \frac{\$29.6}{1.05^{40/365}} = \$6.16$$



Example

- One month ago, Troubadour purchased euro/yen forward contracts with three months to expiration at a quoted price of 100.20 (quoted as a percentage of par). The contract notional amount is ¥100,000,000. The current forward price is 100.05.
- The value of the position is closest to:
  - A. -¥149,925.
  - B. -¥150,000.
  - C. -¥150,075.
- Solutions: A
  - The value of Troubadour's euro/JGB forward position is calculated as

$$V_t(T) = PV_{t,T}\left[F_t(T) - F_0(T)\right]$$

$$V_t(T) = (100.05 - 100.20)\,\text{A/}\,(1 + 0.0030)^{2/12}$$

$$= -0.149925(\textit{per}\ ¥100\ \textit{par value})$$

  - Therefore, the value of the Troubadour's forward position is

$$Vt(T) = \frac{0.149925}{100}(¥100{,}000{,}000) = -¥149{,}925$$

Equity Index Forward Contracts

- **Forward contracts on an equity index**
  - Continuously compounded risk-free rate: $R_f^c = \ln(1 + R_f)$
  - Continuously compounded dividend yield: δ^c
  - Price: $$FP = S_0 \times e^{(R_f^c - \delta^c)\times T}$$
  - Value: $$V_{long} = \left(\frac{S_t}{e^{\delta^c \times (T-t)}}\right) - \left(\frac{FP}{e^{R_f^c \times (T-t)}}\right)$$

Example

- The value of the S&P 500 index is 1,140. The continuously compounded risk-free rate is 4.6% and the continuous dividend yield is 2.1 %. Calculate the no-arbitrage price of a 140-day forward contract on the index.

$$FP = 1{,}140 \times e^{(0.046 - 0.021) \times (140/365)} = 1{,}151$$

- After 95 days, the value of the index in the previous example is 1,025. Calculate the value to the long position of the forward contract on the index, assuming the continuously compounded risk-free rate is 4.6% and the continuous dividend yield is 2.1%.
- After 95 days there are 45 days remaining on the original forward contract:

$$V_{95}\text{(of the long position)} = \frac{1{,}025}{e^{0.021\times(45/365)}} - \frac{1{,}151}{e^{0.046 \times (45/365)}} = -122.14$$

Forward Contracts on Coupon Bonds

- **Coupon bonds**
  - Similar to dividend-paying stocks, but the cash flows are coupons
  - Price: $$FP = (S_0 - PVC_0) \times (1 + R_f)^T$$
  - Value: $$V_{long} = (S_t - PVC_t) - \frac{FP}{(1 + R_f)^{T-t}}$$

$$Or \quad V_{long} = PV_{t,T}[F_t(T) - F_0(T)]$$

Example

- Calculate the price of a 250-day forward contract on a 7% U.S. Treasury bond with a spot price of $ 1,050 (including accrued interest) that has just paid a coupon and will make another coupon payment in 182 days. The annual risk-free rate is 6%.
- Remember that U.S. Treasury bonds make semiannual coupon payments:

$$C = \frac{\$1000 \times 0.07}{2} = \$35$$

$$PVC = \frac{\$35.00}{1.06^{182/365}} = \$34.00$$

- The forward price of the contract is therefore:

$$FP(\text{on a income security}) = (\$1,050 - \$34.00) \times 1.06^{250/365} = 1057.37$$

Currency Forward Contracts

- Price: covered Interest Rate Parity (IRP)

$$FP = S_0 \times \frac{(1+R_D)^T}{(1+R_F)^T}$$

FP and S_0 are quoted in currency D per unit of currency F (i.e., D/F)

- Value:

$$V_{long} = \frac{S_t}{(1+R_F)^{T-t}} - \frac{FP}{(1+R_D)^{T-t}}$$

- If you are given the continuous interest rates

$$FP = S_0 \times e^{(R_D^c - R_F^c)\times T}$$

$$V_{long} = \left(\frac{S_t}{e^{R_F^c \times (T-t)}}\right) - \left(\frac{FP}{e^{R_D^c \times (T-t)}}\right)$$

Example

- Consider the following: The U.S. risk-free rare is 6 percent, the Swiss risk-free rate is 4 percent, and the spot exchange rate between the United States and Switzerland is $0.6667.
  - Calculate the continuously compounded U.S. and Swiss risk-free rates.
  - Calculate the price at which you could enter into a forward contract that expires in 90 days.
  - Calculate the value of the forward position 25 days into the contract. Assume that the spot rate is $0.65.

Currency Forward Contracts

- Answer:
  - r^{fc}= ln(1.04)=0.0392 r^{c}=ln(1.06)=0.0583
  - S_0 = \$0.6667
  - T = 90/365

$$F(0,T) = (\$0.666 \times e^{-0.0392(90/365)})(e^{0.0583(90/365)}) = \$0.6698$$

  - S_t = \$0.65
  - T = 90/365
  - t = 25/365
  - T - t = 65/365

$$V_t(0,T) = (\$0.65 \times e^{-0.0392(65/365)}) - (\$0.6698 \times e^{-0.0583(65/365)}) = -\$0.0174$$

  - The value of the contract is -\$0.0174 per Swiss franc

Forward Rate Agreements (FRAs)

- **A Forward Rate Agreement (FRA) is a forward contract on an interest rate (LIBOR).**
  - The long position can be viewed as the right and the obligation to borrow at the forward rate in the future;
  - The short position can be viewed as the right and the obligation to lend at the forward rate in the future.
  - No loan is actually made, and FRAs are always settled in cash at contract expiration.
- **Let's take a 1×4 FRA for example. A 1×4 FRA is**
  - a forward contract expires in 1 month,
  - and the underlying loan is settled in 4 months,
  - with a 3-month notional loan period.
  - The underlying interest rate is 90-day LIBOR in 30 days from now.

Forward Rate Agreements (FRAs)

- **LIBOR: London Interbank Offered Rate .**
  - an annualized rate based on a 360-day year
  - an add-on rate
  - often used as a reference rate for floating rate U.S dollar-denominated loans worldwide.
  - published daily by the British Banker’s Association
- **Euribor: Europe Interbank Offered Rate, established in Frankfurt, and published by European Central Bank.**

Forward Pricing and Valuation – FRA

- LIBOR, Euribor, and FRAs（续）

交割：settle in cash, but no actual loan is made at the settlement date

- **Payoff定性分析：**
  - ✓ If the reference rate at the expiration date is above the specified contract rate, the long will receive cash payment from the short;
  - ✓ If the reference rate at the expiration date is below the contract rate, the short will receive cash payment from the long
- **Payoff定量分析**

$$\left(\text{Notional principal}\right)\left[\frac{\left(\text{Floating rate at settlement-forward rate}\right)\left[\frac{\text{days}}{360}\right]}{1+\text{Floating rate at settlement}\left[\frac{\text{days}}{360}\right]}\right]$$

Example

- In 30 days, a UK company expects to make a bank deposit of £10,000,000 for a period of 90 days at 90-day Libor set 30 days from today. The company is concerned about a possible decrease in interest rates. Its financial adviser suggests that it negotiate today, at Time 0, a 1 × 4 FRA, an instrument that expires in 30 days and is based on 90-day Libor. The company enters into a £10,000,000 notional amount 1 × 4 receive-fixed FRA that is advanced set, advanced settled. The appropriate discount rate for the FRA settlement cash flows is 0.40%. After 30 days, 90-day Libor in British pounds is 0.55%.

1. If the FRA was initially priced at 0.60%, the payment received to settle it will be closest to:

   A. –£2,448.75.

   B. £1,248.75.

   C. £1,250.00.

Example

➢ **Solution :**

B is correct. In this example, m = 90 (number of days in the deposit), tm = 90/360 (fraction of year until deposit matures observed at the FRA expiration date), and h = 30 (number of days initially in the FRA). The settlement amount of the 1 × 4 FRA at h for receive-fixed is

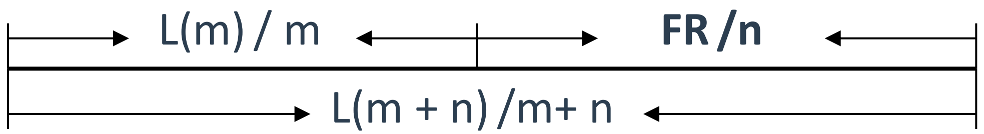
$$NA\{[FRA(0,h,m) - L_h(m)]t_m\}/[1 + D_h(m)t_m]$$

$$= [10{,}000{,}000(0.0060 - 0.0055)(0.25)]/[1 + 0.0040(0.25)] = £1{,}248.75.$$

Because the FRA involves paying floating, its value benefited from a decline in rates.

FRA Pricing

- **The forward price in an FRA is the no-arbitrage forward rate (FR)**
  - If spot rates are known, The FR is just the unbiased estimate of the forward interest rate:



$$(1+L_m \times m/360)\,(1+FR\times n/360) = (1+L_{m+n}\times(m+n)/360)$$

Example

- Calculate the price of a 1×4 FRA. The current 30-day LIBOR is 3% and 120-day LIBOR is 3.9%.
- **Answer:**
  - The actual 30-day rate (Period): $R(30)=0.03\times30/360 = 0.0025$
  - The actual 120-day rate (Period): $R(120)=0.039\times120/360 = 0.013$
  - The actual 90-day forward rate in 30 days from now (period):

    $$(1+R(120))/(1+R(30)) - 1 = 1.013 / 1.0025 - 1= 0.015.$$

  - The annualized forward rate, which is the price of the FRA, is :

    $$R_{FRA}=0.015\times360/90 = 0.042 = 4.2\%.$$

Example

- Suppose we entered a receive-floating 6 × 9 FRA at a rate of 0.86%, with notional amount of C$10,000,000 at Time 0. The six-month spot Canadian dollar (C$) Libor was 0.628%, and the nine-month C$ Libor was 0.712%. Also, assume the 6 × 9 FRA rate is quoted in the market at 0.86%. After 90 days have passed, the three-month C$ Libor is 1.25% and the six-month C$ Libor is 1.35%, which we will use as the discount rate to determine the value at g. We have h = 180 and m = 90.
- Assuming the appropriate discount rate is C$ Libor, the value of the original receive-floating 6 × 9 FRA will be closest to:

  A. C$14,500.

  B. C$14,625.

  C. C$14,651.

Example 8 - Solution

➤ **Solution:**

C is correct. Initially, we have $L_0(h) = L_0(180) = 0.628\%$, $L_0(h + m) = L_0(270) = 0.712\%$, and FRA(0,180,90) = 0.86%. After 90 days (g = 90), we have $L_g(h – g) = L_{90(}90) = 1.25\%$ and $L_g(h + m – g) = L_{90}(180) = 1.35\%$. Interest rates rose during this period; hence, the FRA likely has gained value because the position is receive-floating. First, we compute the new FRA rate at Time g and then estimate the fair FRA value as the discounted difference in the new and old FRA rates. The new FRA rate at Time g, denoted FRA(g,h – g,m) = FRA(90,90,90), is the rate on day 90 of an FRA to expire in 90 days in which the underlying is 90-day Libor. That rate is found as

$$FRA(g,h – g,m) = FRA(90,90,90)$$
$$= \{[1 + L_g(h + m – g)_{th+m–g}]/[1 + L_g(h – g)t_{h–g}] – 1\}/t_m,$$

and based on the information in this example, we have

$$FRA(90,90,90) = \{[1 + L90(180 + 90 – 90)(180/360)]/[1 + L90(180 – 90)(90/360)] – 1\}/(90/360).$$

Substituting the values given in this problem, we find

$$FRA(90,90,90) = \{[1 + 0.0135(180/360)]/[1 + 0.0125(90/360)] – 1\}/(90/360)$$
$$= [(1.00675/1.003125)– 1]4 = 0.0145, \text{ or } 1.45\%.$$

Example 8 - Solution

➢ **Solution:**

Therefore,

$V_g(0,h,m) = V90(0,180,90)$

$= 10{,}000{,}000[(0.0145 – 0.0086)(90/360)]/[1 + 0.0135(180/360)]$

$= 14{,}651.$

Again, floating rates rose during this period; hence, the FRA enjoyed a gain. Notice that the FRA rate rose by roughly 59 bps (= 145 – 86), and 1 bp for 90-day money and a 1,000,000 notional amount is 25. Thus, we can also estimate the terminal value as 10 × 25 × 59 = 14,750. As with all fixed-income strategies, understanding the value of a basis point is often helpful when estimating profits and losses and managing the risks of FRAs.

Framework

1. Forward
   - Principle of Arbitrage-free Pricing
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Futures Contract Value

- The value of a futures contract is zero at contract inception.
- Futures contracts are marked to market daily, **the value just after marking to market is reset to zero.**
- Between the times at which the contract is marked to market, the value can **be different from zero**.
  - V (long) = current futures price − futures price at the last mark-to-market time.
- Another view of futures: **settle previous futures, and then open another new futures with same date of maturity.**

T-bond Futures Contracts

- Underlying: Hypothetical 30 year treasury bond with 6% coupon rate
- Bond can be deliverable: $100,000 par value T-bonds with any coupon but with a maturity of at least 15 years.
- The quotes are in points and 32nds: A price quote of 95-18 is equal to 95.5625 and a dollar quote of $95,562.50
- The short has a delivery option to choose which bond to deliver. Each bond is given a **conversion factor (CF),** which means a specific bond is equivalent to CF standard bond underlying in futures contract.
- The short designates which bond he will deliver (**cheapest-to-deliver** bond).
- For a specific Bond A:

$$FP_{标准} = FP_A \times \frac{1}{CF_A}$$

Quoted futures price

- **Bond price is usually quoted as clean price**
  - Clean price=full price-accrued interest
- **First, the futures price can be written as**

$$FP=(S_0-PVC_0)\times(1+R_f)^T=S_0\times(1+R_f)^T-FVC$$

- **If S_0 is given by clean price(quoted price)**

$$FP=(S_0+AI_0)\times(1+R_f)^T-FVC$$

- **If the futures price is quoted as clean price**

$$FP_{clean}=(S_0+AI_0)\times(1+R_f)^T-FVC-AI_T$$

  - Noted that $AI_T \neq AI_0*(1+R_f)^T$
- **The quoted futures price is adjusted with conversion factor**

$$QFP=\left[\left(S_0+AI_0\right)\times(1+R_f)^T-AI_T-FVC\right]\times\frac{1}{CF}$$

Example

- Euro-bund futures have a contract value of €100,000, and the underlying consists of long-term German debt instruments with 8.5 to 10.5 years to maturity. They are traded on the Eurex. Suppose the underlying 2% German bund is quoted at €108 and has accrued interest of €0.083 (one-half of a month since last coupon). The euro-bund futures contract matures in one month. At contract expiration, the underlying bund will have accrued interest of €0.25, there are no coupon payments due until after the futures contract expires, and the current one-month risk-free rate is 0.1%. The conversion factor is 0.729535. In this case, we have T = 1/12, CF(T) = 0.729535, $B_0(T + Y) = 108$, $FVCI_{0,T} = 0$, $AI_0 = 0.5(2/12) = €0.083$, $AI_T = 1.5(2/12) = 0.25$, and r = 0.1%. The equilibrium euro-bund futures price based on the carry arbitrage model will be closest to:

  A. €147.57.

  B. €147.82.

  C. €148.15.

Example - Solution

- **Solution:**

B is correct. The carry arbitrage model for forwards and futures is simply the future value of the underlying with adjustments for unique carry features. With bond futures, the unique features include the conversion factor, accrued interest, and any coupon payments. Thus, the equilibrium euro-bund futures price can be found using the carry arbitrage model in which

$$F_0(T) = FV_{0,T}(S0) – AIT – FVCI_{0,T}$$

or

$$QF_0(T) = [1/CF(T)]\{FV_{0,T}[B_0(T + Y) + AI_0] – AI_T – FVCI_{0,T}\}$$

Thus, we have

$$QF_0(T) = [1/0.729535][(1 + 0.001)^{1/12}(108 + 0.083) – 0.25 – 0]$$

$$= 147.82$$

In equilibrium, the euro-bund futures price should be approximately €147.82 based on the carry arbitrage model.

Example

- Troubadour identifies an arbitrage opportunity relating to a fixed-income futures contract and its underlying bond. Current data on the futures contract and underlying bond are presented in Exhibit . The current annual compounded risk-free rate is 0.30%.

Exhibit 1 Current Data for Futures and Underlying Bond

Futures Contract		Underlying Bond	
Quoted futures price	125.00	Quoted bond price	112.00
Conversion factor	0.90	Accrued interest since last coupon payment	0.08
Time remaining to contract expiration	Three months	Accrued interest at futures contract expiration	0.20
Accrued interest over life of futures contract	0.00		

Example

- Based on Exhibit and assuming annual compounding, the arbitrage profit on the bond futures contract is closest to:
  - A. 0.4158.
  - B. 0.5356.
  - C. 0.6195.
- Solutions: B The no-arbitrage futures price is equal to the following:

$$F_0(T) = FV_{0,T}(T)\left[B_0(T+Y) + AI_0 - PVCI_{0,T}\right]$$

$$F_0(T) = (1+0.003)^{0.25}(112.00 + 0.08 - 0)$$

$$F_0(T) = (1+0.003)^{0.25}(112.08) = 112.1640$$

The adjusted price of the futures contract is equal to the conversion factor multiplied by the quoted futures price:

$$\mathrm{F}_0(\mathrm{T}) = \mathrm{CF}(\mathrm{T})\mathrm{QF}_0(\mathrm{T})$$

$$\mathrm{F}_0(\mathrm{T}) = (0.90)(125) = 112.50$$

- Adding the accrued interest of 0.20 in three months (futures contract expiration) to the adjusted price of the futures contract gives a total price of 112.70. This difference means that the futures contract is overpriced by 112.70 – 112.1640 = 0.5360. The available arbitrage profit is the present value of this difference: 0.5360/(1.003)0.25 = 0.5356.

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   - Equity Swap Contracts

Pricing a plain vanilla swap

- **A plain vanilla swap** is an interest rate swap in which one party pays a fixed rate and the other pays a floating rate.
  - Pricing a plain vanilla swap means calculating the fixed rate (**swap rate**) that makes the contract value zero at initiation.
- Since a floating-rate bond has a value equal to its par value at initiation, what we will do is to find a fixed-rate bond with a value equal to the same par value at initiation. Denote C as the coupon rate of the n-period fixed-rate bond, we have:

Pricing a plain vanilla swap

- $1 = C \times B_1 + C \times B_2 + C \times B_3 + \ldots\ldots + C \times B_n + 1 \times B_n$
- And then we can get the C as:

$$C = \frac{1 - B_n}{B_1 + B_2 + \cdots + B_n}$$

- Recall that B_n is the discount factor, which is the present value of $1 in n periods. It's important to note that the answer C is a periodic rate, and you must annualize it to get the annual swap rate.

Example

- Calculate the swap rate of a 1-year quarterly-pay plain vanilla swap. The LIBOR spot rates are: R(90-day)=2.5%; R(180-day)=3%; R(270-day)=3.5%; R(360-day)=4%.
- Answer:
  - **Step 1: Calculate the discount factors:**

    B1=1/(1+2.5%×90/360)= 0.9938; B2=1/(1+3%×180/360) = 0.9852

    B3=1/(1+3.5%×270/360)= 0.9744; B4=1/(1+4%×360/360)= 0.9615
  - **Step 2: Calculate the periodic swap rate, C:**

    C = (1−B4)/(B1+B2+B3+B4)

    = (1−0.9615)/(0.9938+0.9852+0.9744+0.9615)

    = 0.98%

Example

- **Step 3: Calculate the annualized swap rate:**

  swap rate = 0.98%×360/90 = 3.92%

  The swap rate can be viewed as the average of the spot rates. So every time you get a swap rate result, always check if it is within the range of the spot rates. For example, we get the result of 3.92% and this value is within the range of 2.5% and 4%. Another trick is that the swap rate is usually very close to the last spot rate (4% here).

Valuing a plain vanilla swap

- The market value of a swap to the pay-fixed side is the difference between the value of a floating-rate bond and the value of a fixed-rate bond at any time during the life of the swap.

Paying fixed rate

Company X $\longrightarrow$ Company Y

Company X $\longleftarrow$ Company Y

Paying floating rate

$$V_{swap}(X) = B_{flt} - B_{fix} \quad V_{swap}(Y) = B_{fix} - B_{flt}$$

- Notes: the value of a floating rate bond will be equal to the notional amount at any of its periodic settlement dates when the next payment is set to the market rate (floating).

Example

- Calculate the value of the plain vanilla swap to the pay-fixed side in the previous example after 30 days. The swap rate is 3.92% and the notional principal is \$ 1 million. Assume after 30 days the LIBOR spot rates are: R(60-day)=3%; R(150-day)=3.5%; R(240-day)=4%; R(330-day)=4.5%.
- Answer
  - Step 1: Calculate the new discount factors 30 days later:

    $B1 = 1/(1+3\%\times 60/360) = 0.9950$; $B2 = 1/(1+3.5\%\times 150/360) = 0.9856$

    $B3 = 1/(1+4\%\times 240/360) = 0.9740$; $B4 = 1/(1+4.5\%\times 330/360) = 0.9604$
  - Step 2: Calculate the value of the fixed-rate bond:

    $P(\text{fixed}) = 0.98\%\times(0.9950+0.9856+0.9740+0.9604) + 1\times 0.9604 = 0.998767$

Example

- **Step 3: Calculate the value of the floating-rate bond:**

  60 days later when the first payment date comes, the floating-rate bond price will be \$1. The first payment is known at initiation: 2.5%×90/360=0.00625. So we get the floating-rate bond price as:

  P(floating) = (1+0.00625)×0.9950 = 1.001219

- **Step 4: Calculate the swap value to the pay-fixed side:**

  V = [P(floating) −P(fixed)]×notional principal

  = (1.001219-0.998767)×\$1 million = \$2452

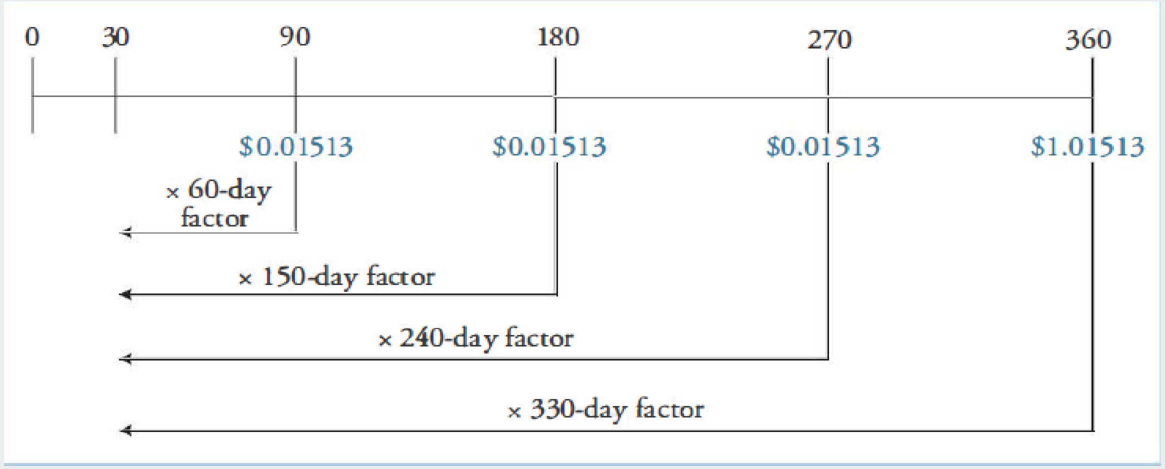
Example

- Consider a 1-year LIBOR swap with quarterly payments priced at 6.052% at initiation when 90-day LIBOR was 5.5%. The annualized LIBOR rates and discount factors 30 days later are shown in the following figure. The notional principal amount is $30,000,000. Calculate the value of the swap to the fixed-rate payer after 30 days . LIBOR and Discount Factors for $30 Million, 1-Year Interest Rate Swap

	Rate	Present Value Factor
60-day LIBOR	6.0%	0.99010
150-day LIBOR	6.5%	0.97363
240-day LIBOR	7.0%	0.95541
330-day LIBOR	7.5%	0.93567

Example

- Answer
  - The quarterly payments per \$ of notional principal are:
    0.06052 ×90 /360= \$0.01513
  - The hypothetical fixed-rate bond will pay four coupon payments of \$0.01513 in 60,150, 240, and 330 days. It will also make a final principal payment of \$ 1 .0000 in 330 days. The time line of the cash Rows is shown in the following figure. Remember that we're valuing the swap on day 30.



Valuing a plain vanilla swap between payment dates

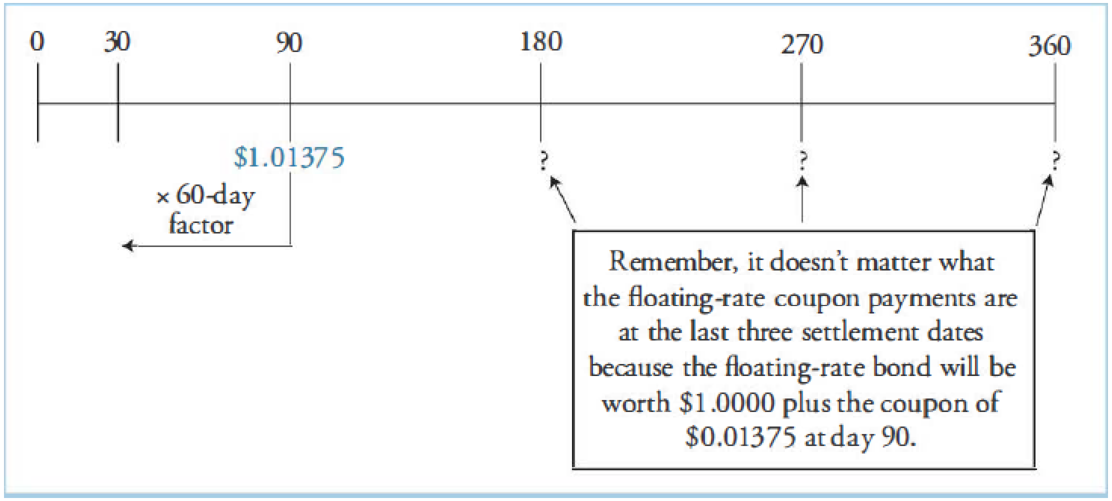
- ***Answer***: We can calculate the value of the fixed-rate side of the swap as $0.993993, the present value of the expected coupon payments and the face value payment, as shown in the following figure.

Day	Cash Flow	Discount Factor	Present Value
90	$0.01513	0.99010	$0.014980
180	$0.01513	0.97363	$0.014731
270	$0.01513	0.95541	$0.014455
360	$0.01513+1	0.93567	$0.949827
		Total	$0.993993

- To value the floating-rate bond at day 30, we need only note two things. First, at the payment date on day 90, the bond value will be $ 1.00 because the coupon rate will be reset to the current market rate. Therefore, we don't need to know what the floating rate bond's cash flows are after day 90. Second, the first payment (per $ of notional principal) is known at inception:
- $0.055 \times 90/360 = \$0.01375$

Valuing a plain vanilla swap between payment dates

- Because the next payment date is 60 days out, we need to discount the first floating-rate payment and the $ 1 .00 bond value immediately after the payment is made by the current 60-day rate of 6%. The time line is as shown. The 60-day discount factor is 0.99010. The value of the floating-rate bond at day 30 (per $ of notional principal) is therefore $1.01375 × 0.99010 = $1.003714.
  - Calculation of Value for the Floating Rate Side of an Interest Rate Swap



  - The value of the pay-fixed side of the swap is the difference between the values of the fixed and floating sides (per $ of notional principal):

    swap value to fixed-rate payer = $1.003714 - $0.993993 = $0.009721
  - The total value of the $30,000,000 notional principal swap to the fixed-rate payer is:

    swap value to fixed-rate payer = 30,000,000 × $0.009721 = $291 ,630

Pricing a currency swap

- Consider a swap involving two currencies, the US dollar ($) and the Euro (€). The exchange rate is now 0.80€/$. As we do in a plain vanilla swap, we can get the fixed rate that will make the fixed $ payments equal to $1, and the fixed rate that will make the fixed € payments equal to €1. For example, if the US term structure is: R(90-day)=5.2%, R(180-day)=5.4%, R(270-day)=5.55%, R(360-day)=5.7%, we can get the fixed rate of 5.56%. If the Euro term structure is: R(90-day)=3.45%, R(180-day)=3.58%, R(270-day)=3.7%, R(360-day)=3.75%, we can get the fixed rate of 3.68%. Our currency swap involving dollars for Euros would have a fixed rate of 5.56% in dollars and 3.68% in Euros. The notional principal would be $1 and €0.8. There are four ways to construct the swap:

Pricing a currency swap

- **There are four ways to construct the swap:**
  - Swap 1: pay dollar fixed at 5.56% and receive Euro fixed at 3.68%.
  - Swap 2: pay dollar fixed at 5.56% and receive Euro floating.
  - Swap 3: pay dollar floating and receive Euro fixed at 3.68%.
  - Swap 4: pay dollar floating and receive Euro floating.
    - ✓ In the swap 4 (floating for floating), there is no pricing problem because there is no fixed rate. We should only set the notional principal to €0.8 for every 1$.

Example

- Some days later, the term structures in both countries will change, and the exchange rate will also be different. You can calculate the fixed- and floating-rate bond prices in both currencies, and then you will get the swap value just as you do in a plain vanilla swap.

$$\text{Company A} \xrightarrow{\text{GBP } 6\%} \text{Company B}$$

$$\text{Company B} \xrightarrow{\text{USD } 5\%} \text{Company A}$$

$$\textit{the value of the swap in USD to Company A is:}$$

$$V_{swap}(USD) = B_{USD} - (S_0 \times B_{GBP})$$

$$\textit{where:}$$

$$S_0 = \textit{spot rate in USD per GBP}$$

Example

- Consider a two-year currency swap with semiannual payments. The domestic currency is the U.S. dollar, and the foreign currency is the U.K. pound. The current exchange rate is $1.41 per pound.

$L_0^{\$}(180) = 0.0585$

$L_0^{\$}(360) = 0.0605$

$L_0^{\$}(540) = 0.0624$

$L_0^{\$}(720) = 0.0665$

The comparable set of £ rates are

$L_0^{£}(180) = 0.0493$

$L_0^{£}(360) = 0.0506$

$L_0^{£}(540) = 0.0519$

$L_0^{£}(720) = 0.0551$

  - A. Calculate the annualized fixed rates for dollars and pounds:

Example

- Answer for A:
  - First calculate the fixed payment in dollars and pounds. The dollar present value factors for 180, 360, 540, and 720 days are as follows

$$B_0(180) = \frac{1}{1+(0.0585 \times \frac{180}{360})} = 0.9716 \qquad B_0^{£}(180) = \frac{1}{1+(0.0493 \times \frac{180}{360})} = 0.9759$$

$$B_0(360) = \frac{1}{1+(0.0605 \times \frac{360}{360})} = 0.9430 \qquad B_0^{£}(360) = \frac{1}{1+(0.0505 \times \frac{360}{360})} = 0.9519$$

$$B_0(540) = \frac{1}{1+(0.0624 \times \frac{540}{360})} = 0.9144 \qquad B_0^{£}(540) = \frac{1}{1+(0.0519 \times \frac{540}{360})} = 0.9278$$

$$B_0(720) = \frac{1}{1+(0.0665 \times \frac{720}{360})} = 0.8826 \qquad B_0^{£}(720) = \frac{1}{1+(0.0551 \times \frac{720}{360})} = 0.9007$$

  - The semiannual and annualized fixed payment of per $1 of notional principal is $\frac{1-0.8826}{0.9716+0.9430+0.9144+0.8826} = 0.0316$ or 0.0632 for annualized
  - Similarly The semiannual and annualized fixed payment of per £1 of notional principal is: 0.0264 or 0.0528 for annualized

Example

- Now move forward 120 days. The new exchange rate is \$1.35 per pound, and the new U.S. term structure is:

$L_{120}^{\$}(60) = 0.0613$		$L_{120}^{£}(60) = 0.0517$
$L_{120}^{\$}(240) = 0.0629$	The comparable set of	$L_{120}^{£}(240) = 0.0532$
$L_{120}^{\$}(420) = 0.0653$	£ rates are	$L_{120}^{£}(420) = 0.0568$
$L_{120}^{\$}(600) = 0.0697$		$L_{120}^{£}(600) = 0.0583$

- B. Assume that the notional principal is \$1 or the corresponding amount in British pounds. Calculate the market values of the following swaps:
  - ✓ Pay £ fixed and receive \$ fixed;
  - ✓ Pay £ floating and receive \$ fixed
  - ✓ Pay £ floating and receive \$ floating

Example

- Answer for B:
  - The new dollar and pound factors for 60, 240, 420, and 600 days are as follows:

$$B_{120}(180)=\frac{1}{1+(0.0613\times\frac{60}{360})}=0.9899 \qquad B_{120}^{£}(180)=\frac{1}{1+(0.0517\times\frac{60}{360})}=0.9915$$

$$B_{120}(360)=\frac{1}{1+(0.0629\times\frac{240}{360})}=0.9430 \qquad B_{120}^{£}(360)=\frac{1}{1+(0.0532\times\frac{240}{360})}=0.9657$$

$$B_{120}(540)=\frac{1}{1+(0.0653\times\frac{420}{360})}=0.9292 \qquad B_{120}^{£}(540)=\frac{1}{1+(0.0568\times\frac{420}{360})}=0.9379$$

$$B_{120}(720)=\frac{1}{1+(0.0697\times\frac{600}{360})}=0.8959 \qquad B_{120}^{£}(720)=\frac{1}{1+(0.0583\times\frac{600}{360})}=0.9114$$

Example

- In terms of $ payments:
  - ✓ The present value of the remaining fixed payments plus the $1 notional principal is 0.0316(0.9899 + 0.9598 + 0.9292 + 0.8959) + 1(0.8959) = 1.0152.
  - ✓ The present value of the floating payments plus hypothetical $1 notional principal discounted back 120 days is [0.0585(180/360) + 1](0.9899) = 1.0189.

Example

- In terms of £ payments:
  - ✓ The present value of the remaining fixed payments plus the £1 notional principal is 0.0264(0.9915 + 0.9657 + 0.9379 + 0.9114) + 1(0.9114) = 1.0119.
  - ✓ Convert this amount to the equivalent of $1 notional principal and convert to dollars at the current exchange rate: 1/1.41 * 1.35 * 1.0119 = $0.9688.

Example

- ✓ The present value of the floating payments plus hypothetical £1 notional principal is [0.0493(180/360) + 1](0.9915) = 1.016. Convert to dollars at the current exchange rate 1/1.41 * 1.35 * 1.016 = $0.9728.

- The market values based on notional principal of $1 are as follows:
  - ✓ Pay £ fixed and receive $ fixed = $0.0464 = 1.0152 - 0.9688
  - ✓ Pay £ floating and receive $ fixed = $0.0424 = 1.0152 - 0.9728
  - ✓ Pay £ floating and receive $ floating = $0.0461 = 1.0189 - 0.9728
  - ✓ Pay £ fixed and receive $ floating = $0.0501 = 1.0189 - 0.9688

Pricing an equity swap

- There are three types of equity swaps: (1) pay fixed rate and receive equity return; (2) pay floating rate and receive equity return; (3) pay one equity return and receive another equity return. We only need to price the first type of swaps because there are no fixed rates in the other two.
- We have the same formula as for the plain vanilla swap to get the periodic swap rate of an equity swap:

$$C = \frac{1 - B_n}{B_1 + B_2 + \Lambda + B_n}$$

- Why is this the case? We can exchange a fixed-rate bond with periodic coupon rate of C for a stock or an index with the notional amount equal to the par value of the bond, because the bond value at inception is equal to par.

Valuing an equity swap

- Example: An equity swap has the annual swap rate of 3.92% and the notional principal of $ 1 million. The underlying is an index, currently trading at 1000. Assume after 30 days the index becomes 1100 and the LIBOR spot rates are: R(60-day)=3%; R(150-day)=3.5%; R(240-day)=4%; R(330-day)=4.5%. Calculate the value of the equity swap to the fixed-rate payer.

Example

Answer:

- **Step 1: Calculate the new discount factors 30 days later:**

  $B_1 = 1/(1+3\%\times60/360) = 0.9950$; $B_2 = 1/(1+3.5\%\times150/360) = 0.9856$

  $B_3 = 1/(1+4\%\times240/360) = 0.9740$; $B_4 = 1/(1+4.5\%\times330/360) = 0.9604$

- **Step 2: Calculate the value of the fixed-rate bond:**

  $P(\text{fixed}) = 0.98\%\times(0.9950+0.9856+0.9740+0.9604)+1\times0.9604 = 0.998767$

- **Step 3: Calculate the value of the index investment:**

  $P(\text{index}) = 1\times1100/1000 = 1.1$

- **Step 4: Calculate the swap value to the fixed-rate payer:**

  $V = [P(\text{index}) - P(\text{fixed})]\times\text{notional principal} = (1.1-0.998767)\times\$1\text{ million}$

  $=\$101{,}233$

Swaption contracts

- A *swaption* is an option to enter into a swap. We will focus on the plain vanilla interest rate swaption. The notation for swaptions is similar to FRAs. For example, a swaption that matures in 2 years and gives the holder the right to enter into a 3-year swap at the end of the second year is a 2×5 swaption.
- A *payer swaption* is an option to enter into a swap as the fixed-rate payer.
  - If interest rate increases, the payer swaption value will go up. So a payer swaption is equivalent to a put option on a coupon bond. *Another view?*
  - *It's also equivalent to a call option on floating rate.*
- A *receiver swaption* is an option to enter into a swap as the fixed-rate receiver (the floating-rate payer). If interest rate increases, the receiver swaption value will go down. So a receiver swaption is equivalent to a call option on a coupon bond.

Swaption contracts: Valuation at expiration

- Valuing a plain vanilla swaption at expiration is equivalent to valuing the underlying (off-market) swap. However, if the swap value is lower than zero at expiration, the value of a plain vanilla swaption will be zero.

- Example: A 1-year quarterly-pay payer swaption with the swap rate of 3.84% and the notional principal of $1 million comes to its expiration date today. The LIBOR spot rates are: R(90-day)=2.5%; R(180-day)=3%; R(270-day)=3.5%; R(360-day)=4%. The current rate on an interest rate swap is 3.92%. Calculate the value of the payer swaption.

Swaption contracts: Valuation at expiration

Answer:

- ***Step 1: Calculate the discount factors:***

  $B_1 = 1/(1+2.5\%\times 90/360) = 0.9938$; $B_2 = 1/(1+3\%\times 180/360) = 0.9852$

  $B_3 = 1/(1+3.5\%\times 270/360) = 0.9744$; $B_4 = 1/(1+4\%\times 360/360) = 0.9615$

- ***Step 2: Calculate the net cash savings at each payment date:***

  $(3.92\%-3.84\%)\times 90/360\times \$1 \text{ million} = \$200$

- ***Step 3: Calculate the present value of the savings:***

  $\$200\times(0.9938+0.9852+0.9744+0.9615) = \$783$

- The owner of the swaption can exercise the swaption and enter into a swap. Or he can agree to receive quarterly payments of $200 from the counterpart. He can also agree to receive $783 today to terminate the contract.
- If the swap rate is 4% in the swaption, the value of the underlying swap is −$783 and the swaption holder will not exercise the swaption

Reading 41

Valuation of Contingent Claims

Framework

1. Binomial Model——The Expectations Approach
   - The one –period binomial stock model
   - The two –period binomial stock model
   - Interest rate binomial model
2. Black-Scholes-Merton Model
3. Option Greeks and Implied Volatility
4. Binomial Model——The No-arbitrage Approach
5. Black Option Valuation Model
   - Option On Futures
   - Interest Rate Option
   - Swaptions

Put-call parity for European options

- **A <u>fiduciary call</u> is a portfolio consisting of:**
  - A long position in a European call option with an exercise price of X that maturities in T years on a stock.
  - A long position in a pure-discount riskless bond that pays X in T years.
- **The cost a fiduciary call** is the cost of the call (C_0) plus the cost of the bond (the present value of X). **The payoff to a fiduciary call** will be X if the call is out-of-the-money and S_T if the call is in-the-money, as shown in the following:

	$S_T \leq X$ (*Call is out-of or at-the-money*)	$S_T > X$ (*Call is in-the-money*)
Long call payoff	0	$S_T - X$
Long bond payoff	X	X
Total payoff	X	S_T

Put-call parity for European options

- **A <u>protective put</u> is a portfolio consisting of :**
  - A long position in a European put option with an exercise price of X that maturities in T years on a stock.
  - A long position in the underlying stock.
- **The cost of a protective put** is the cost of the put (P_0) plus the cost of the stock(S_0). **The payoff to a protective put** is X if the put is in-the-money and ST if the put is out-of-the-money. as shown in the following:

	$S_T < X$ (put is in-the-money)	$S_T \geq X$ (put is out-of or at-the-money)
Long put payoff	$X - S_T$	0
Long stock payoff	S_T	S_T
Total payoff	X	S_T

Create synthetic instruments

- **There are four synthetic instruments:**
  - A synthetic European call option:

    synthetic call = put + stock – bond

$$C_0 = P_0 + S_0 - \frac{X}{(1+R_f)^T}$$

  - A synthetic European put option:

    synthetic put = call + bond – stock

$$P_0 = C_0 + \frac{X}{(1+R_f)^T} - S_0$$

  - A synthetic pure-discount risk-less bond:

    synthetic bond = put + stock – call

$$\frac{X}{(1+R_f)^T} = P_0 + S_0 - C_0$$

  - A synthetic stock position:

    synthetic stock = call + bond – put

$$S_0 = C_0 + \frac{X}{(1+R_f)^T} - P_0$$

Create synthetic instruments

- **There are two reasons why investors might wants to create synthetic positions in the securities.**
  - To price options by using combinations of the other instruments with known prices.
  - To earn arbitrage profits by exploiting relative mispricing among the four securities. If put-call parity doesn't hold, an arbitrage profit is available.

Exploit violations of put-call parity

- As with all arbitrage trades, you want to “buy low and sell high.” if put-call parity doesn’t hold (if the cost of a fiduciary call does not equal the cost of a protective put), then you buy (go long in) the underpriced position and sell (go short) in the overpriced position.
- Example: Exploit violations of put-call parity
  - 90-day European call and put options with a strike price of $45 is priced at $7.50 and $3.70. The underlying is priced at $48 and makes no cash payments during the life of the options. The risk-free rate is 5%. Calculate the no-arbitrage price of the call option, and illustrate how to earn an arbitrage profit.

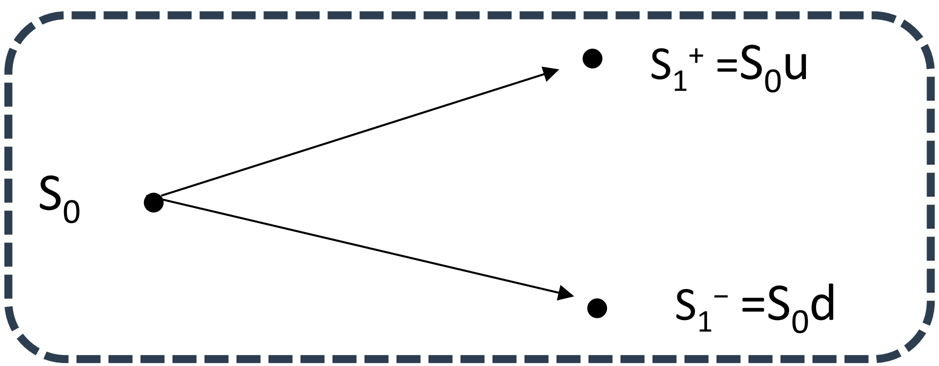
Exploit violations of put-call parity

Answer:

- $C_0 = P_0 + S_0 - X/(1+R_f)^T = \$3.70 + \$48 - \$45/1.05^{90/365} = \$7.24 < \$7.5$
- Since the call is overpriced
  - we should sell the call for \$7.50 and buy the synthetic call for \$7.24.
  - To buy the synthetic call, buy the put for \$3.70, buy the underlying for \$48, and issue (sell short) a 90-day zero-coupon bond with a face value of \$45.
  - The transaction will generate an arbitrage profit of \$0.26 today.

One-period binomial model

- **A binomial model** is based on the idea that, over the next period, some value will change to one of two possible values (binomial). To construct a binomial model, we need to know the beginning asset value, the size of the two possible changes, and the probabilities of each of these changes occurring.
- We start off by having only one binomial period, which means that the underlying price moves to two new prices at option expiration. We let S_0 be the price of the underlying stock now. One period later, the stock price can move up to S_1^+ or down to S_1^-. We then identify a factor, u, as the up move on the stock and d as the down move. Thus, $S_1^+ = S_0u$ and $S_1^- = S_0d$. <u>We further assume that u = 1/d.</u>



Expectation approach

- Risk-neutral probability of an up move is π_u ; Risk-neutral probability of an down move is π_d=1- π_u:

$$\pi_u = \frac{1+R_f-d}{u-d}$$

- We start with a call option. If the stock goes up to S_1^+ , the call option will be worth C_1^+. If the stock goes down to S_1^-, the call option will be worth C_1^-. We know that the value of a call option will be its intrinsic value on expiration date. Thus we get: C_1^+ = Max (0, S_1^+ −X) ; C_1^- = Max (0, S_1^- −X)

$$\text{value of an option: c} = \left[\pi_u C_1^+ + \pi_d C_1^-\right] \times \frac{1}{(1+R_f)^T}$$

- $$\text{h}\left(\text{hedge ratio}\right) = \frac{C_1^+ - C_1^-}{S_1^+ - S_1^-}\text{(shares per option)}$$

Example

- Calculate the value today of a 1-year call option on the stock with the strike price of $20. The price of the stock is $20 now, and the size of an up-move is 1.25. The risk-free rate is 7%.
- Answer:
  - Step 1: Calculate the parameters:
    - ✓ $u=1.25$; $d=1/u=0.8$; $S_u=20\times1.25=25$; $S_d=20\times0.8=16$
    - ✓ $C^+ = \text{Max}(0, 25-20) = 5$; $C^- = \text{Max}(0, 16-20) = 0$
  - Step 2: Calculate risk-neutral probabilities, π_u and $\pi_d = 1-\pi_u$:
    - ✓ $\pi_u = (1+0.07-0.8)/(1.25-0.8) = 0.6$
    - ✓ $\pi_d = 1-\pi_u = 0.4$
  - Step 3: Calculate call option price:
    - ✓ $C_0 = (5*0.6+0*0.4)/(1+7\%) = 2.80$

Example

- Step 3: Draw the one-period binomial tree:

$S^{+}=S_u=25$; $C^{+}=5$

(C=?) 0.6

S=20

0.4

$S^{-}=S_d=16$; $C^{-}=0$

today year 1

Example

- Pricing a put option is similar to that of a call. The only difference is that $P^+ = \text{Max}(0, X-S^+)$ and $P^- = \text{Max}(0, X-S^-)$.
- Example: Use the information in the previous example to calculate the value today of a put on the same stock with the strike price of \$20.
- Answer:

  $P^+ = \text{Max}(0, 20-25) = 0$; $P^- = \text{Max}(0, 20-16) = 4$

  $P = (0.6\times0 + 0.4\times4)/1.07 = 1.6/1.07 = 1.50$

The two –period binomial model

$S^{++} = S_{uu}$

$C^{++} = \text{Max}\,(0, S^{++} - X)$

$S^{+} = S_{u}$

C^{+}

S

$(C=?)$

$S^{+-} = S^{-+} = S_{ud} = S_{du} = S$

$C^{+-} = C^{-+} = \text{Max}\,(0, S^{+-} - X)$

$S^{-} = S_{d}$

C^{-}

$S^{--} = S_{dd}$

$C^{--} = \text{Max}\,(0, S^{--} - X)$

today **year 1** **year 2**

Example

- Calculate the value today of a 2-year call option on the stock with the strike price of $18. The price of the stock is $20 now, and the size of an up-move is 1.25. The risk-free rate is 7%.

Example

Answer:

- Step 1: Calculate the parameters:

  $U = 1.25$; $d = 1/u = 0.8$; $S_u = 20\times1.25 = 25$; $S_d = 20\times0.8 = 16$

  $S_{uu} = 20\times1.25\times1.25 = 31.25$;

  $S_{ud} = 20$;

  $S_{dd} = 20\times0.8\times0.8 = 12.8$

  $C^{++} = \text{Max}(0, 31.25-18) = 13.25$;

  $C^{+-} = \text{Max}(0, 20-18) = 2$

  $C^{--} = \text{Max}(0, 12.8-18) = 0$

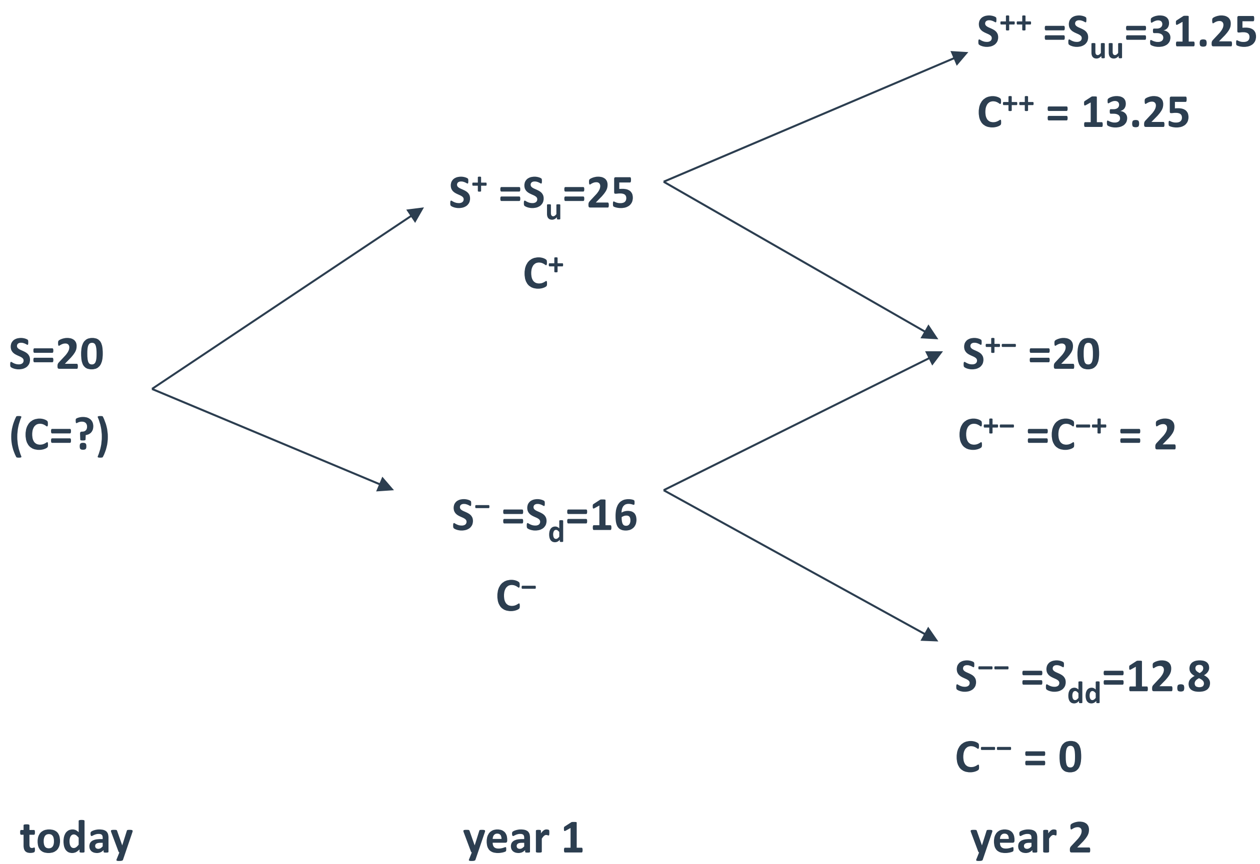
- Step 2: Calculate risk-neutral probabilities, π and $1-\pi$:

  $\pi_u = (1+0.07-0.8)/(1.25-0.8) = 0.6$

  $\pi_d = 1-\pi = 0.4$

A call with a two-period binomial tree

- Step 3: Draw the two-period binomial tree:



A call with a two-period binomial tree

- **Step 4: Calculate the call option value at year 1:**

  $C^{+} = (13.25\times0.6 + 2\times0.4)/1.07 = 8.1776$

  $C^{-} = (2\times0.6 + 0\times0.4)/1.07 = 1.1215$

- **Step 5: Calculate the call option value today:**

  $C = (8.1776\times0.6 + 1.1215\times0.4)/1.07 = 5.0048$

Example

- Consider a two-period binomial model in which a stock currently trades at a price of $65. The stock price can go up 20 percent or down 17 percent each period. The risk-free rate is 5 percent. Calculate the price of a put option expiring in two periods with exercise price of $60

Example

Answer:

- Step 1: Calculate the parameters:

  $U = 1.2$; $d = 0.83$; $Su = 65\times1.2 = 78$; $Sd = 65\times0.83 = 53.95$

  $S_{uu} = 65\times1.2\times1.2 = 93.6$;

  $S_{ud} = 65\times1.2\times0.83 = 64.74$

  $S_{dd} = 65\times0.83\times0.83 = 44.78$

  $P^{++} = \text{Max}(0, 60-93.6) = 0$

  $P^{+-} = \text{Max}(0, 60 - 64.74) = 0$

  $P^{--} = \text{Max}(0, 60-44.78) = 15.22$

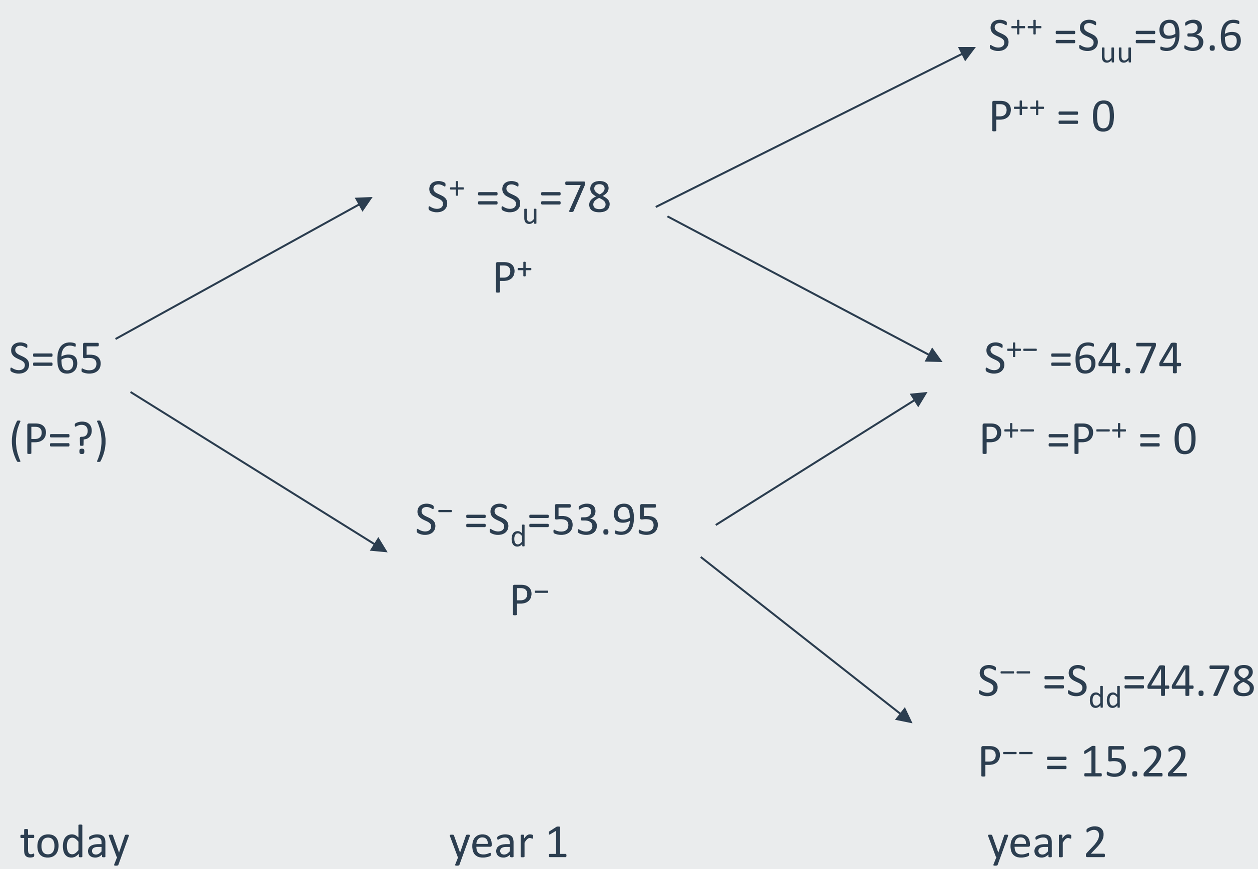
- Step 2: Calculate risk-neutral probabilities, π and $1-\pi$:

  $\pi_u = (1+0.05-0.83)/(1.2-0.83) = 0.5946$

  $\pi_d = 1-\pi = 0.4054$

Example

➢ Step 3: Draw the two-period binomial tree:



Example

- Step 4: Calculate the put option value at year 1:

  $P^+ = (0\times0.5946 + 0\times0.4054)/1.05 = 0$

  $P^- = (0\times0.5946 + 15.22\times0.4054)/1.05 = 5.88$

- Step 5: Calculate the put option value today:

  $P = (0\times0.5946 + 5.88\times0.4054)/1.05 = 2.27$

two-period binomial tree (American Call)

$S^{++} = S_{uu}$
$C^{++} = \text{Max}(0, S^{++} - X)$

$S^{+} = S_{u}$
C^{+}

S
$(C=?)$

$S^{+-} = S^{-+} = S_{ud} = S_{du} = S$
$C^{+-} = C^{-+} = \text{Max}(0, S^{+-} - X)$

$S^{-} = S_{d}$
C^{-}

$S^{--} = S_{dd}$
$C^{--} = \text{Max}(0, S^{--} - X)$

today year 1 year 2

Example

- You observe a €50 price for a non-dividend-paying stock. The call option has two years to mature, the periodically compounded risk-free interest rate is 5%, the exercise price is €50, u = 1.356, and d = 0.744. Assume the call option is European-style.

1. The probability of an up move based on the risk-neutral probability is closest to:
   A. 30%.
   B. 40%.
   C. 50%.
2. The current call option value is closest to:
   A. €9.53.
   B. €9.71.
   C. €9.87.
3. The current put option value is closest to:
   A. €5.06.
   B. €5.33.
   C. €5.94.

Example - Solution

- **Solution to 1:**
  - C is correct. Based on the RN probability equation, we have:

    $\pi = [FV(1) - d]/(u - d) = [(1 + 0.05) - 0.744]/(1.356 - 0.744) = 0.5$ or 50%
- **Solution to 2:**
  - B is correct. The current call option value calculations are as follows:

    $c^{++} = \text{Max}(0,u^2S - X) = \text{Max}[0,1.356^2(50) - 50] = 41.9368$

    $c^{-+} = c^{+-} = \text{Max}(0,udS - X) = \text{Max}[0,1.356(0.744)(50) - 50] = 0.44320$

    $c^{--} = \text{Max}(0,d^2S - X) = \text{Max}[0,0.744^2(50) - 50] = 0.0$

    With this information, we can compute the call option value:

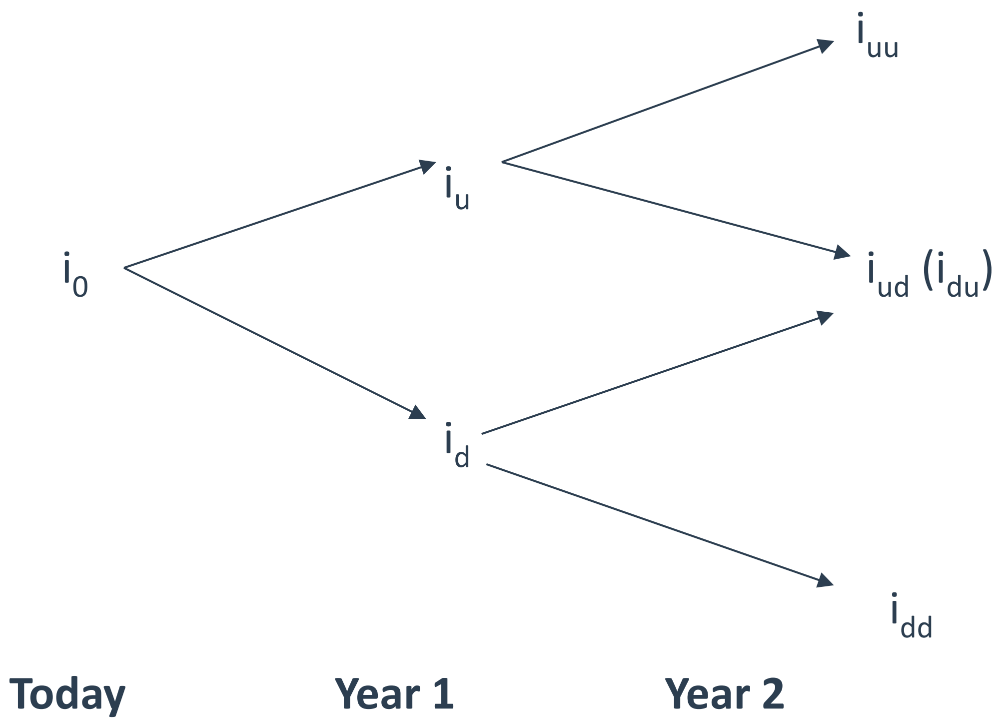
    $c = PV[E(c2)] = PV[\pi^2c^{++} + 2\pi(1 - \pi)c^{+-} + (1 - \pi)^2c^{--}]$

    $= [1/(1 + 0.05)]^2[0.5^2 41.9368 + 2(0.5)(1 - 0.5)0.44320 + (1 - 0.5)^2 0.0]$

    $= 9.71$
- **Solution to 3:**
  - A is correct. The put option value can be computed simply by applying put–call parity or $p = c + PV(X) - S = 9.71 + [1/(1 + 0.05)]^2 50 - 50 = 5.06$. Thus, the current put price is €5.06.

The binomial interest rate tree

- The **binomial interest rate tree** is a set of possible interest rate paths that we use to value options on bonds or interest rates.
- **Two-Period Binomial Interest Rate Tree**



The binomial interest rate tree

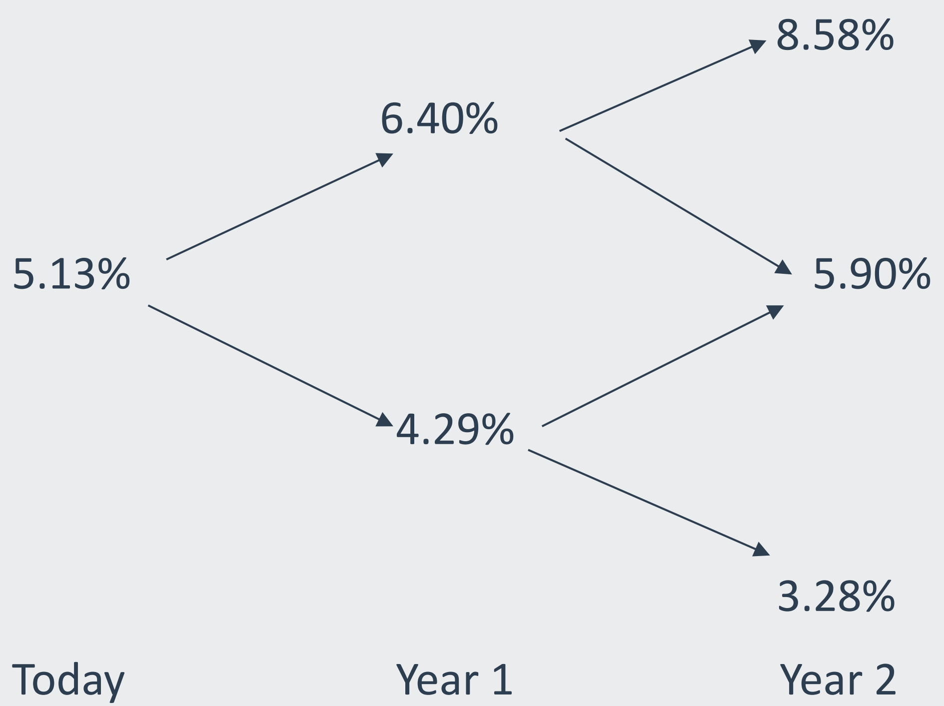
- You don’t need to know how to construct an interest rate tree, since it will be given to you on the exam.
- You should know that the interest rates at each node **are <u>one-year forward rates</u>.**
- You also should know that **<u>risk-neutral probabilities</u>** <u>for</u> The binomial interest rate tree , π and 1−π, are always 0.5.

Interest rate option

- **Interest rate call option**
  - An option which has a positive payoff if the reference interest rate, usually the market interest rate, is greater than the exercise rate.
  - Payoff=max{0, reference rate-exercise rate}*notional principal
- **Interest rate put option**
  - An option which has a positive payoff if the reference interest rate, usually the market interest rate, is smaller than the exercise rate.
  - Payoff=max{0, exercise rate-reference rate}*notional principal

Example

Example: Calculate the value today of a 2-year interest rate call option with the exercise rate of 5%. The notional principal is 1 million. The interest rate tree for the first two years are illustrated below.



Example

Answer:

- Step 1: Calculate the bond price and call option intrinsic value at year 2:

  C^{++} = Max (0, 8.58%−5%) * $1,000,000= $35,800

  C^{+-} = Max (0, 5.90%−5%) * $1,000,000 = $9,000

  C^{--} = Max (0, 3.28%−5%) * $1,000,000 = 0

- Step 2: Calculate the call option value at year 1:

  C^{+} = (35,800×0.5 + 9,000×0.5)/1.0640 = $21,053

  C^{-} = (9,000×0.5 + 0×0.5)/1.0429 = $4,315

- Step 3: Calculate the call option value today:

  C = (21,053×0.5 + 4,315×0.5)/1.0513 = $12,065

Example

- Value a call option on Alpha Company with a one-period binomial option pricing model. It is a non-dividend-paying stock, and the inputs are as follows.
  - The current stock price is 50, and the call option exercise price is 50.
  - In one period, the stock price will either rise to 56 or decline to 46.
  - The risk-free rate of return is 5% per period.
  - Based on the model, Rocha asks Sousa to estimate the hedge ratio, the risk-neutral probability of an up move, and the price of the call option.

Example

- The optimal hedge ratio for the Alpha Company call option using the one-period binomial model is *closest* to:

  A. 0.60.

  B. 0.67.

  C. 1.67.

- Solutions: A

  - The hedge ratio requires the underlying stock and call option values for the up move and down move. $S^+ = 56$, and $S^- = 46$. $c^+ = \text{Max}(0, S^+ - X) = \text{Max}(0, 56 - 50) = 6$, and $c^- = \text{Max}(0, S^- - X) = \text{Max}(0, 46 - 50) = 0$. The hedge ratio is

$$h = \frac{c^+ - c^-}{s^+ - s^-} = \frac{6-0}{56-46} = \frac{6}{10} = 0.60$$

Example

- The risk-neuvtral probability of the up move for the Alpha Company stock is *closest* to:

  A. 0.06.

  B. 0.40.

  C. 0.65.

- Solutions：C

  - For this approach, the risk-free rate is *r* = 0.05, the up factor is $u = S^+/S$ = 56/50 = 1.12, and the down factor is $d = S^-/S$ = 46/50 = 0.92. The risk-neutral probability of an up move is

$$\pi = [FV(1) - d]/(u - d) = (1 + r - d]/(u - d)$$

$$\pi = (1 + 0.05 - 0.92)/(1.12 - 0.92) = 0.13/0.20 = 0.65$$

Framework

1. Binomial Model——The Expectations
   - The one –period binomial model
   - The two –period binomial model
   - Interest rate binomial model
2. Black-Scholes-Merton Model
3. Option Greeks and Implied Volatility
4. Binomial Model——The No-arbitrage Approach
5. Black Option Valuation Model
   - Option On Futures
   - Interest Rate Option
   - Swaptions

Continuous-time option pricing model

- As the period covered by a **binominal model** is divided into arbitrarily small, discrete time periods, the model results converge to those of continuous-time model.
- The famous **Black-Scholes-Merton model** (BSM model) values options in continuous-time and is derived from the same no-arbitrage assumption used to value the binominal model.

Continuous-time option pricing model

- The **underlying assumptions** of the BSM model are:
  - The price of the underlying asset follows a lognormal distribution.
  - The (continuous) risk-free rate is known and constant.
  - The volatility of the underlying asset is known and constant.
  - The markets are frictionless.
  - There are no cash flows on the underlying asset.
  - The options valued are European options.

The Value of European option using the BSM

- The BSM formulas for the prices of European call and put options are:

$$C_0 = [S_0 \times N(d_1)] - [X \times e^{-R_f^c \times T} \times N(d_2)]$$

$$P_0 = C_0 - S_0 + \left(X \times e^{-R_f^c \times T}\right)$$

Remember that you can always use put-call parity to calculate the put value

- where:
  - S_0 = underlying asset price; X = strike price; T = time to maturity
  - R_f^c = continuously compounded risk-free rate
  - σ = volatility of the underlying asset
  - N( ) = cumulative normal probability

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left[R_f^c + (0.5 \times \sigma^2)\right] \times T}{\sigma \times \sqrt{T}}$$

$$d_2 = d_1 - (\sigma \times \sqrt{T})$$

Example

- The underlying asset price now is 68.5 with a volatility of 0.38. The continuously compounded risk-free rate is 4%. Consider the value of a 110-day European call and put with the exercise price of 65.
- Answer:
  - We should first calculate d_1 and d_2: （110 days equal to 0.3014 year）

    $d_1 = [\ln(68.5/65)+(0.04+0.38^2\times0.5)\times0.3014]/(0.38\times\sqrt{0.3014}) = 0.41$

    $d_2 = 0.41-0.38\times\sqrt{0.3014} = 0.20$
  - Then look up $N(d_1)$ and $N(d_2)$ in the cumulative normal probability table:

    $N(0.41)=0.6591$ $N(0.20)=0.5793$
  - Then we can get the value of the call:

    $C_0 = 68.5\times0.6591-65e^{-0.04\times0.3014}\times0.5793 = 7.95$
  - We can use put-call parity or the BSM formula to get the put price:

    $P_0 = C_0+PV(X)-S_0 = 7.95+65\,e^{-0.04\times0.3014}-68.5 = 3.67$

    $P_0 = 65e^{-0.04\times0.3014}\times(1-0.5793)-68.5\times(1-0.6591) = 3.67$

Features of BSM

- **Leveraged stock investment**
  - Borrow money to invest in stock

$$C_0 = S_0 \times N(d_1) - X \times e^{-R_f^c \times T} \times N(d_2)$$

  - Buying the bond(or lend) with the proceeds from short selling the underlying

$$P_0 = X \times e^{-R_f^c \times T} \times N(-d_2) - S_0 \times N(-d_1)$$

- **N(d_2)**
  - The prob. of an in-the-money call
- **The present value of option payoff**

$$C_0 = e^{-R_f^c \times T} \left\{ [S_0 e^{R_f^c \times T} \times N(d_1)] - [X \times N(d_2)] \right\}$$

Options on dividend paying stock or currency

- Options on dividend paying stock

$$C_0 = [S_0 \times e^{-\delta \times T} \times N(d_1)] - [X \times e^{-R_f^c \times T} \times N(d_2)]$$

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left[R_f^c - \delta + (0.5 \times \sigma^2)\right] \times T}{\sigma \times \sqrt{T}} \qquad d_2 = d_1 - (\sigma \times \sqrt{T})$$

- Options on currencies

$$C_0 = [S_0 \times e^{-r(Foreign) \times T} \times N(d_1)] - [X \times e^{-r(Domestic) \times T} \times N(d_2)]$$

Example

- Suppose we are given the following information on call and put options on a stock: S = 100, X = 100, r = 5%, T = 1.0, and σ = 30%. Thus, based on the BSM model, it can be demonstrated that PV(X) = 95.123, d1 = 0.317, d2 = 0.017, N(d1) = 0.624, N(d2) = 0.507, N(–d1) = 0.376, N(–d2) = 0.493, c = 14.23, and p = 9.35.

1. The initial trading strategy required by the no-arbitrage approach to replicate the call option payoffs for a buyer of the option is:
   A. buy 0.317 shares of stock and short sell –0.017 shares of zero-coupon bonds.
   B. buy 0.624 shares of stock and short sell 0.507 shares of zero-coupon bonds.
   C. short sell 0.317 shares of stock and buy 0.017 shares of zero-coupon bonds.

2. Identify the initial trading strategy required by the no-arbitrage approach to replicate the put option payoffs for a buyer of the put.
   A. Buy 0.317 shares of stock and short sell –0.017 shares of zero-coupon bonds.
   B. Buy 0.624 shares of stock and short sell 0.507 shares of zero-coupon bonds.
   C. Short sell 0.376 shares of stock and buy 0.493 shares of zero-coupon bonds.

Example Solution

- **Solution to 1:**
  - B is correct. The no-arbitrage approach to replicating the call option involves purchasing nS = N(d1) = 0.624 shares of stock partially financed with nB = –N(d2) = –0.507 shares of zero-coupon bonds priced at $B = X^{e-rT}$ = 95.123 per bond. Note that by definition the cost of this replicating strategy is the BSM call model value or $n_S S + n_B B = 0.624(100) + (-0.507)95.123 = 14.17$. Without rounding errors, the option value is 14.23.
- **Solution to 2:**
  - C is correct. The no-arbitrage approach to replicating the put option is similar. In this case, we trade $n_S = -N(-d1) = -0.376$ shares of stock—specifically, short sell 0.376 shares—and buy $n_B = N(-d2) = 0.493$ shares of zero-coupon bonds. Again, the cost of the replicating strategy is $n_S S + n_B B = -0.376(100) + (0.493)95.123 = 9.30$. Without rounding errors, the option value is 9.35. Thus, to replicate a call option based on the BSM model, we buy stock on margin. To replicate a put option, we short sell stock and lend part of the proceeds.

Example

- The underlying stock price is 225 with a volatility of 0.15. The continuously compounded risk-free rate is 5.25% and the continuously compounded dividend yield is 2.7%. Consider the value of a 3-year European call with the exercise price of 200.

Example

- Answer:
  - We should first calculate the adjusted $S_0 = S_0 \times e^{-\delta T} = 225 \times e^{-0.027 \times 3} = 207.49$
  - Then calculate d_1 and d_2:

    $d_1 = [\ln(225/200)+(0.0525-0.027+0.15^2 \times 0.5) \times 3]/(0.15 \times \sqrt{3}) = 0.88$

    $d_2 = 0.88 - 0.15 \times \sqrt{3} = 0.62$
  - Then look up $N(d_1)$ and $N(d_2)$ in the cumulative normal probability table:

    N(0.488)=0.8106   N(0.62)=0.7324
  - Then we can get the value of the call:

    $C_0 = 225 \times e^{-0.027 \times 3} \times 0.8106 - 200e^{-0.0525 \times 3} \times 0.7324 = 43.06$

Example

- Suppose we are given the following information on an underlying stock and options: S = 60, X = 60, r = 2%, T = 0.5, δ = 2%, and σ = 45%. Assume we are examining European-style options.

1. Which answer best describes how the BSM model is used to value a call option with the parameters given?

   A. The BSM model call value is the exercise price times N(d1) less the present value of the stock price times N(d2).

   B. The BSM model call value is the stock price times $e^{-\delta T}N(d1)$ less the exercise price times $e^{-rT}N(d2)$.

   C. The BSM model call value is the stock price times $e^{-\delta T}N(-d1)$ less the present value of the exercise price times $e^{-rT}N(-d2)$.

Example

2. Which answer best describes how the BSM model is used to value a put option with the parameters given?
   - A. The BSM model put value is the exercise price times N(d1) less the present value of the stock price times N(d2).
   - B. The BSM model put value is the exercise price times $e^{-\delta T}N(-d2)$ less the stock price times $e^{-rT}N(-d2)$.
   - C. The BSM model put value is the exercise price times $e^{-rT}N(-d2)$ less the stock price times $e^{-\delta T}N(-d1)$.
3. Suppose now that the stock does not pay a dividend—that is, $\delta = 0\%$. Identify the correct statement.
   - A. The BSM model option value is the same as the previous problems because options are not dividend adjusted.
   - B. The BSM model option values will be different because there is an adjustment term applied to the exercise price, that is $e^{-\delta T}$, which will influence the option values.
   - C. The BSM model option value will be different because d1, d2, and the stock component are all adjusted for dividends.

Example

- **Solution to 1:**
  - B is correct. The BSM call model for a dividend-paying stock can be expressed as $Se^{-\delta T}N(d1) - X^{e-rT}N(d2)$.

- **Solution to 2:**
  - C is correct. The BSM put model for a dividend-paying stock can be expressed as $Xe^{-rT}N(-d2) - Se^{-\delta T}N(-d1)$.

- **Solution to 3:**
  - C is correct. The BSM model option value will be different because d1, d2, and the stock component are all adjusted for dividends.

Example

- Suppose that we have some Bank of China shares that are currently trading on the Hong Kong Stock Exchange at HKD4.41. Our view is that the Bank of China's stock price will be steady for the next three months, so we decide to sell some three-month out-of-the-money calls with exercise price at 4.60 in order to enhance our returns by receiving the option premium. Risk-free government securities are paying 1.60% and the stock is yielding HKD 0.24%. The stock volatility is 28%. We use the BSM model to value the calls.
- Which statement is correct? The BSM model inputs (underlying, exercise, expiration, risk-free rate, dividend yield, and volatility) are:

  A. 4.60, 4.41, 3, 0.0160, 0.0024, and 0.28.

  B. 4.41, 4.60, 0.25, 0.0160, 0.0024, and 0.28.

  C. 4.41, 4.41, 0.3, 0.0160, 0.0024, and 0.28.

- **Solution:**
  - B is correct. The spot price of the underlying is HKD4.41. The exercise price is HKD4.60. The expiration is 0.25 years (three months). The risk-free rate is 0.016. The dividend yield is 0.0024. The volatility is 0.28.

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Option Greeks

- **The Black-Scholes-Merton model has five inputs:** underlying asset price, volatility, risk-free rate, time to expiration, and strike price. The relationship between each input and the European option price is captured by a sensitivity factor called the option Greeks.
- Direction of BSM European option prices for a change in the five model inputs.

Option Greeks

Sensitivity Factor (Greek)	Input	Calls	Puts
Delta	Underlying price (S)	Positively related (Delta>0)	Negatively related (Delta<0)
Vega	Volatility (σ)	Positively related (Vega>0)	Positively related (Vega>0)
Rho	Risk-free rate (r)	Positively related (Rho>0)	Negatively related (Rho<0)
Theta	Time to expiration (T)	Value $0 as call → maturity, Theta<0	Value usually $0 as put→maturity, Theta < 0*
	Strike price (X)	Negatively related	Positively related

* There is an exception to the general rule that European put option thetas are negative. The put value may increases as the option approaches maturity if the option is deep in-the-money and close to maturity.

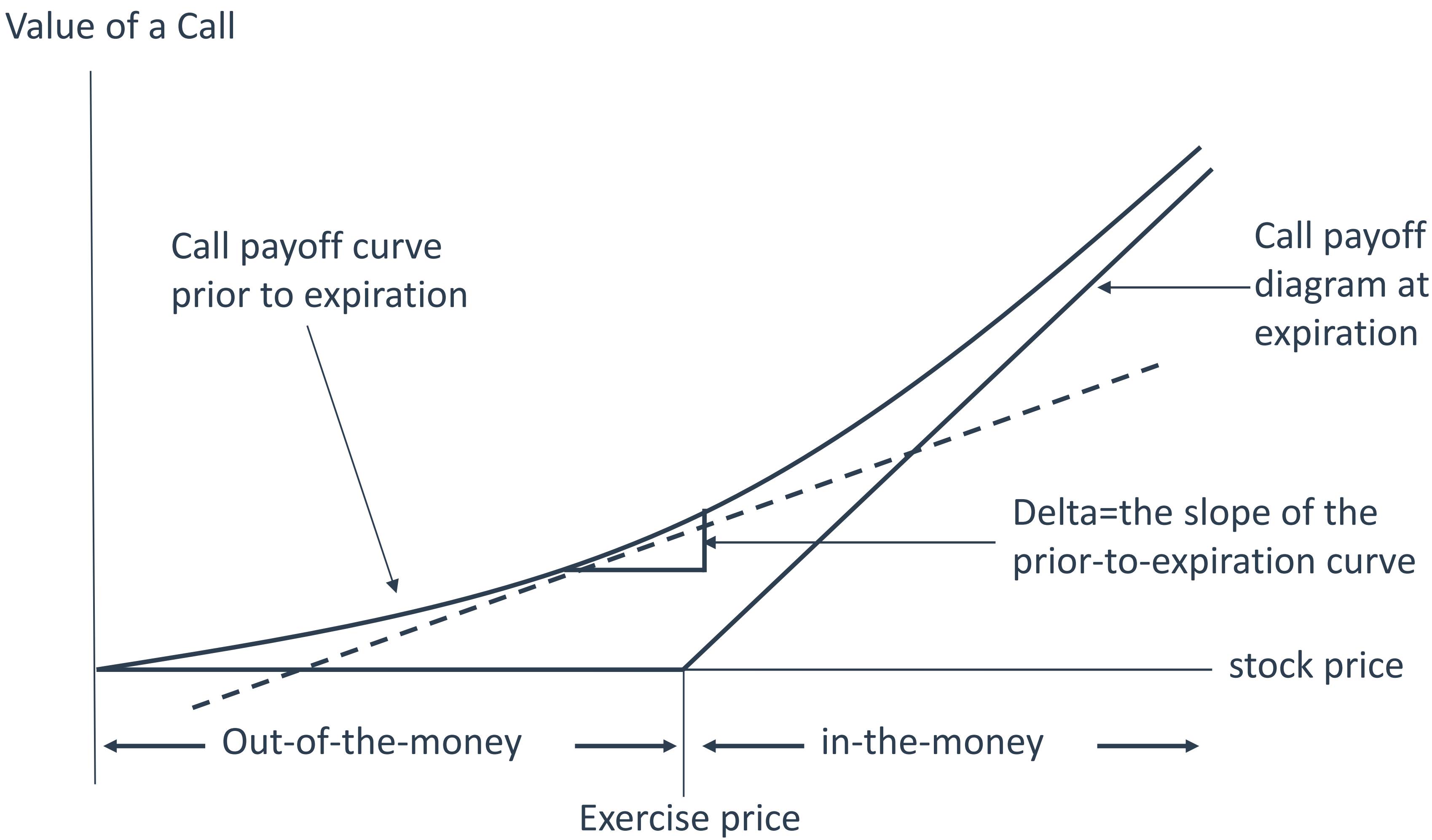
The Option Delta

- **The relationship between the option price and the price of the underlying asset has a special name: the delta. The option delta is defined as:**
  - delta = Change in option price / Change in underlying price
- **The delta defines the sensitivity of the option price to a change in the price of the underlying asset. Another view, one option is equivalent to delta stock**
- **The call option's delta is defined as:**
  - $\text{delta}_{call} = (C_1 - C_0)/(S_1 - S_0) = \Delta C/\Delta S$
- **The delta of a put option is the call option's delta minus one:**
  - $\text{delta}_{put} = (P_1 - P_0)/(S_1 - S_0) = \Delta P/\Delta S = \text{delta}_{call} - 1$
- **The call option's delta is also equal to $N(d_1)$ from the BSM model, and the put option's delta equals $N(d_1) - 1$.**

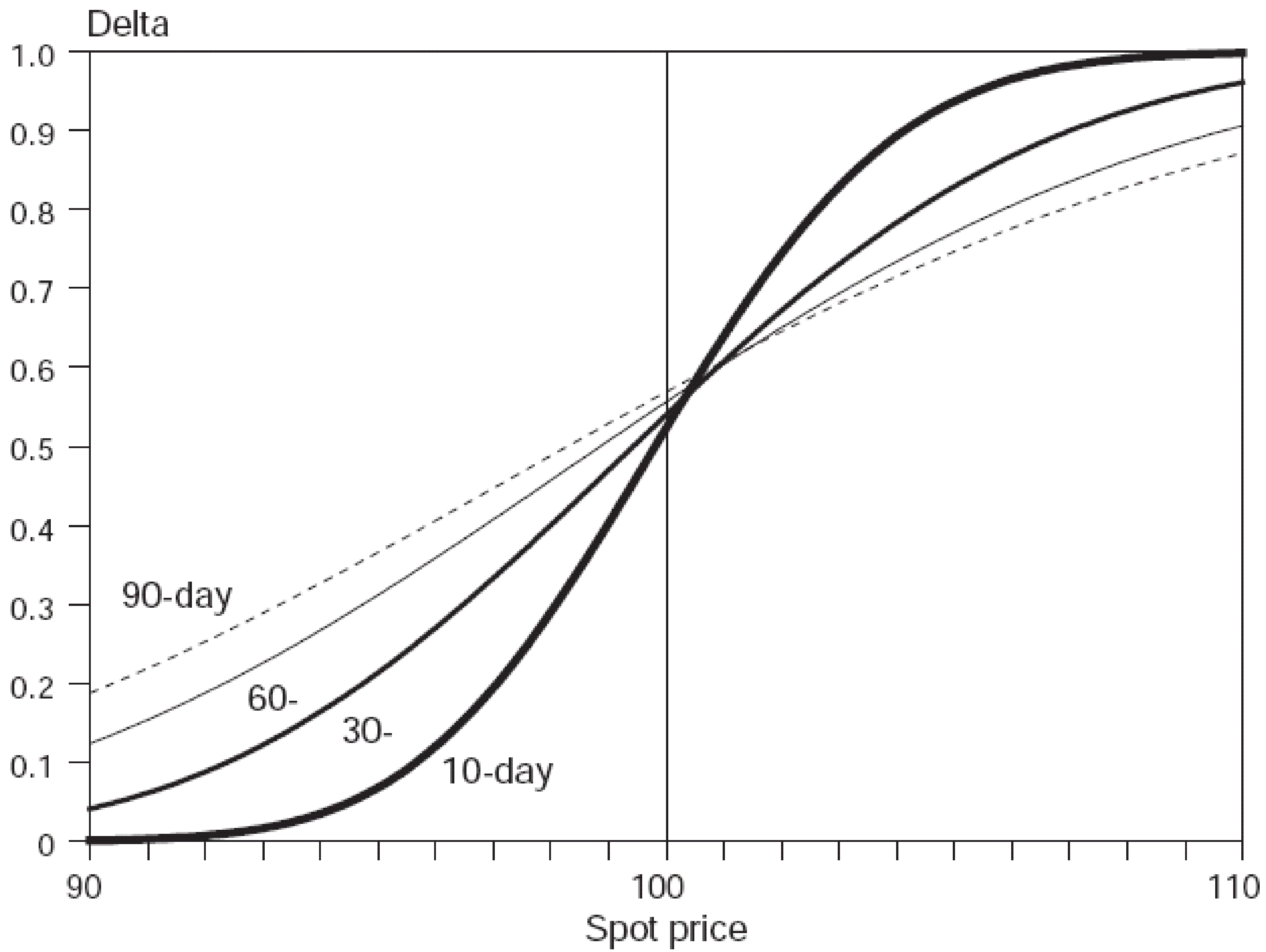
The Option Delta

- **The call delta increases from 0 to 1 as stock price increases.**
  - When the call option is deep out-of-the-money, the call delta is close to zero. The option price changes a very small amount for a given change in the stock price.
  - When the call option is deep in-the-money, the call delta is close to one. The option price changes almost one dollar for a one-dollar change in the stock price.
- **The put delta increases from −1 to 0 as stock price increases.**
  - When the put option is deep in-the-money, the put delta is close to −1.
  - When the put option is deep out-of-the-money, the put delta is close to zero.
- **When, t approaches maturity, a in-the-money call's delta is close to 1,while a out-of-the-money call's delta is close to 0.**

The Option Delta



The Option Delta



Dynamic hedging

- If we long a stock and short 1/delta calls, the value of the portfolio does not change, when the value of the stock changes. For example, if the stock price increases by n dollar, the call price will **decrease by delta×n dollar and then 1/delta calls decrease n dollar.**
- We make money by buying the stock and lose money by selling the call options; the value of the portfolio remains unchanged. This portfolio is referred to as a delta-neutral portfolio.

Dynamic hedging

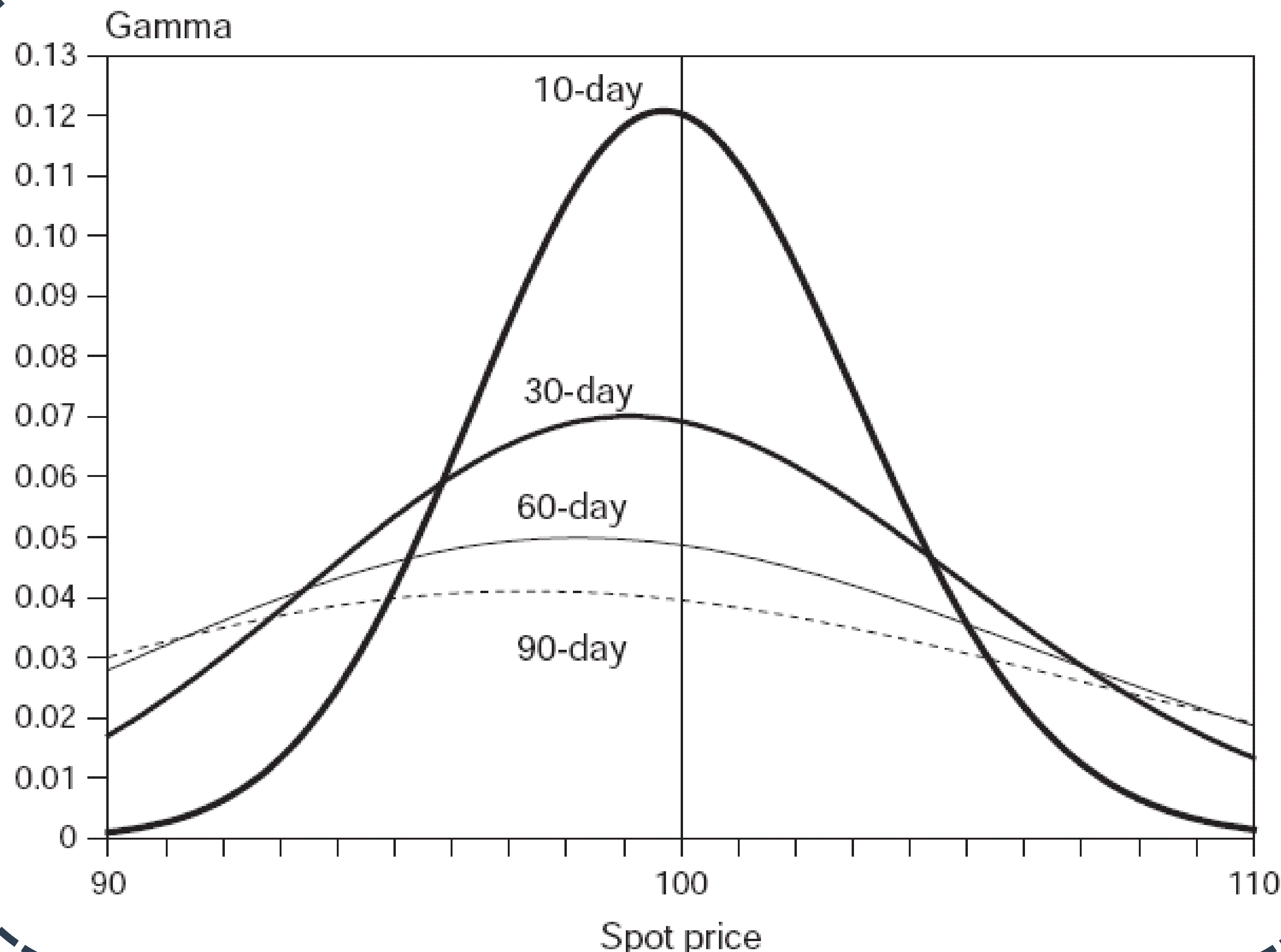
- However, the delta-neutral hedging is a **dynamic process**, since **the delta is constantly changing.** The delta will change if the underlying stock price changes. Even if the underlying stock price does not change, the delta would still change as the option moves toward the expiration day. As the delta changes, the number of calls that should be sold to construct the delta-neutral portfolio changes. Delta-neutral hedging is often referred to as dynamic hedging.

$$
\begin{aligned}
\textit{number of options needed to delta hedge} &= \frac{\textit{number of shares hedged}}{\textit{delta of call option}} \\
&= \textit{number of shares hedged} \times \textit{hedge ratio}
\end{aligned}
$$

The Option Gamma

- **The gamma defines <u>the sensitivity of the option delta to a change in the price of the underlying asset.</u>** The option gamma is defined as:
  - gamma = $(\text{delta}_1 - \text{delta}_0)/(S_1 - S_0) = \Delta\ \text{delta}/\Delta S$
- **Call and put options on the same stock with the same T and X have equal gammas.**
  - A long position in calls or puts will have a positive gamma.
  - Gamma is largest when the option is at-the-money.
  - If the option is deep in- or out-of-the-money, gamma approaches zero.
- **Gamma** is largest when the option is at the money, which means delta is very sensitive to a change in the stock price. Then we must rebalance the delta-neutral portfolio more frequently. This leads to higher transaction cost .

The Option Gamma



Vega, Theta and Rho

- **Vega**
  - Vega defines the sensitivity of the option value to a change in the volatility of the underlying asset.
  - Positive to the long, both call option and put option
- **Theta**
  - Theta defines the sensitivity of the option value to a change in the calendar time.
  - Negative to the long, both call option and put option, usually refers to time decay
- **Rho**
  - Rho defines the sensitivity of the option value to a change in the risk free rate.
  - The rho of the call is positive while the call of a put is negative(leverage effect)
  - Small impact compared to vega

Example

- Klein, CFA, has owned 100,000 shares of a biotechnology company A, A is waiting for its approving from FDA who will release its decision in six months, now the current is share price is $38, and if the latest pharmaceutical is rejected by the FDA, it will have a great impact on the share price of company in six month and will eventually become $26. Expected annual volatility of return is 20%, current annual risk-free rate is 1.12%.
- Klein wants to use BSM model to implement a hedging strategy in using 6-month European option if company A's new product rejected by the FDA and gathers the data below:

Option	W	X	Y	Z
Type of Option	Call	Call	Put	Put
Exercise Price	$38	$46	$38	$36
$N(d_1)$	0.56	0.30	0.56	0.64
$N(d_2)$	0.45	0.21	0.45	0.53

Example

- Using the data above, the number of option X contracts that Klein would have to sell to implement the hedge strategy would be closet to:
  - The required number of call options to sell = Number of shares of underlying to be hedged / $N(d_1)$, where $N(d_1)$ is the estimated delta used for hedging a position with call options. There are 100,000 shares to be hedged and the $N(d_1)$ for Option X from Exhibit 2 is 0.30. Thus, the required number of call options to sell is 100,000 / 0.30 =333,333
- Based on the data above, which of the following options is most likely to exhibit the largest gamma measure?
  - The gamma will tend to be large when the option is at-the-money. The exercise price of Option Y is equal to the underlying price, hence at-the-money, whereas both Option X and Option Z are out-of-the-money.

Example

- If Klein using Option Z from the data above, and company A's share price subsequently dropped to \$36, Klein would most likely need to take the following action to maintain the same hedged position:
  - The required number of put options = Number of shares of underlying to be hedged / [$N(d_1)$ - 1], where $N(d_1)$ - 1 is the estimated delta used for hedging a position with put options (otherwise known as the put delta). As the share price drops to \$36, the delta of a put position will decrease toward -1.0, requiring less put options than the original position.
- If company A pays a dividend, holding all other factors constant, what would be the most likely effect on the price of option W?
  - Including the effect of cash flows lowers the underlying price. Lowering the underlying price causes the price of the call option to decrease.

The critical role of volatility

- Historical volatility and implied volatility
  - Historical volatility is using historical data to calculate the variance and standard deviation of the continuously compounded returns.

$$S_{R_i^c}^2 = \frac{\sum_{i=1}^{N}\left(R_i^c - \overline{R}_i^c\right)^2}{N-1}$$

$$\sigma = \sqrt{S_{R_i^c}^2}$$

  - If we have S_0, X, R_f , and T, we can set the BSM price equal to the market price and then work backwards to get the volatility. This volatility is called the **implied volatility**. The most basic method to get the implied volatility is trial and error.

Framework

No-arbitrage approach with binomial tree

- **Hedge Ratio:**

$$h\left(\text{hedge ratio}\right)=\frac{C_1^+ - C_1^-}{S_1^+ - S_1^-}(\text{shares per option})$$

- If the portfolio is constructed as long 1 call and short h stock, the portfolio should exist no delta risk.
  - $c_0=hS_0 + PV(-hS^- +c^-)= hS_0 + PV(-hS^+ +c^+)$
  - $p_0=hS_0 + PV(-hS^- +p^-)= hS_0 + PV(-hS^+ +p^+)$
- **Memory Tips——Similar to BSM: Borrow money to invest in stock**

$$C_0 = S_0 \times N(d_1) - X \times e^{-R_f^c \times T} \times N(d_2)$$

- **Buying the bond(or lend) with the proceeds from short selling the underlying**

$$P_0 = X \times e^{-R_f^c \times T} \times N(-d_2) - S_0 \times N(-d_1)$$

Example

- For the Alpha Company option, if the call option is overpriced relative to the model. The positions to take advantage of the arbitrage opportunity are to write the call and:

  A. short shares of Alpha stock and lend.

  B. buy shares of Alpha stock and borrow.

  C. short shares of Alpha stock and borrow.

- Solutions：B
  - You should sell (write) the overpriced call option and then go long (buy) the replicating portfolio for a call option. The replicating portfolio for a call option is to buy h shares of the stock and borrow the present value of ($hS^- - c^-$).
  - $c = hS + PV(-hS^- + c^-)$.

Example

- Calculate the value today of a 1-year call option on the stock with the strike price of $20. The price of the stock is $20 now, and the size of an up-move is 1.25. The risk-free rate is 7%.
- Answer:
  - Step 1: Calculate the parameters:
    - ✓ $u=1.25$; $d=1/u=0.8$; $S_u=20\times1.25=25$; $S_d=20\times0.8=16$
    - ✓ $C^+ = \text{Max}(0, 25-20) = 5$; $C^- = \text{Max}(0, 16-20) = 0$
  - Step 2: Calculate hedge ratio,:
    - ✓ $h=(5-0)/(25-16)=5/9$
  - Step 3: Calculate call option: $c_0=hS_0 + PV(-hS_1^- +c_1^-)= hS_0 + PV(-hS_1^+ +c_1^+)$
    - ✓ $C_0 =5/9*20+(-5/9*25+5)/(1+7\%)=2.80$
    - ✓ Or: $C_0 =5/9*20+(-5/9*16+0)/(1+7\%)=2.80$

Framework

1. Binomial Model——The Expectations
   - The one –period binomial model
   - The two –period binomial model
   - Interest rate binomial model
2. Black-Scholes-Merton Model
3. Option Greeks and Implied Volatility
4. Binomial Model——The No-arbitrage Approach
5. Black Option Valuation Model
   - Option On Futures
   - Interest Rate Option
   - Swaptions

Black Model: Options on Futures

- We assume that the futures price also follows **geometric Brownian motion**. We ignore issues like margin requirements and marking to market. Black proposed the following model for European-style futures options:

$$C_0 = e^{-R_f^c \times T}\left[FP \times N(d_1) - X \times N(d_2)\right]$$

$$P_0 = e^{-R_f^c \times T}\left[X \times N(\text{-}d_2)\ \text{-}\ FP \times N(\text{-}d_1)\right]$$

$$d_1 = \frac{\ln\left(\frac{FP}{X}\right) + 0.5 \times \sigma^2 \times T}{\sigma \times \sqrt{T}} \qquad d_2 = d_1 - (\sigma \times \sqrt{T})$$

- Interpretation
  - Call option is the present value of the difference between part of futures price and part of exercise price
  - Leverage
    - ✓ Call option can be thought as long futures financed by bond

Example

- The S&P 500 Index (a spot index) is presently at 1,860 and the 0.25 expiration futures contract is trading at 1,851.65. Suppose further that the exercise price is 1,860, the continuously compounded risk-free rate is 0.2%, time to expiration is 0.25, volatility is 15%, and the dividend yield is 2.0%. Based on this information, the following results are obtained for options on the futures contract.
- We ignore the effect of the multiplier. As of this writing, the S&P 500 futures option contract has a multiplier of 250. The prices reported here have not been scaled up by this amount. In practice, the option cost would by 250 times the option value.

Options on Futures	
Calls	Puts
N(d1) =0.491	N(–d1) = 0.509
N(d2) = 0.461	N(–d2) = 0.539
c = US$51.41	p = US$59.76

Example

1. Identify the statement that best describes how the Black model is used to value a European call option on the futures contract just described.
   A. The call value is the present value of the difference between the exercise price times 0.461 and the current futures price times 0.539.
   B. The call value is the present value of the difference between the current futures price times 0.491 and the exercise price times 0.461.
   C. The call value is the present value of the difference between the current spot price times 0.491 and the exercise price times 0.461.

Example

2. Which statement best describes how the Black model is used to value a European put options on the futures contract just described?
   A. The put value is the present value of the difference between the exercise price times 0.539 and the current futures price times 0.509.
   B. The put value is the present value of the difference between the current futures price times 0.491 and the exercise price times 0.461.
   C. The put value is the present value of the difference between the current spot price times 0.491 and the exercise price times 0.461.

3. What are the underlying and exercise prices to use in the Black futures option model?
   A. 1,851.65; 1,860
   B. 1,860; 1,860
   C. 1,860; 1,851.65

Example Solution

- **Solution to 1:**
  - B is correct. Recall Black's model for call options can be expressed as $c = e^{-rT}[F_0(T)N(d1) - XN(d2)]$.

- **Solution to 2:**
  - A is correct. Recall Black's model for put options can be expressed as $p = e^{-rT}[XN(-d2) - F_0(T)N(-d1)]$.

- **Solution to 3:**
  - A is correct. The underlying is the futures price of 1,851.65 and the exercise price was given as 1,860.

Black Model: Interest Rate Options

- **Interest rate Option**
  - Option Underlying: Forward rate or FRA rate;
  - Strike interest rate are fixed in advance;
  - Interest rate call option gains when rates rise and put option gains when rate fall

$$C_0 = NP \times (accrual\ period) e^{-r \times N \times \frac{30}{360}} \left[FR_{M \times N} \times N(d_1) - X \times N(d_2) \right]$$

$$accrual\ period = \frac{(N - M) \times 30}{360}$$

Example

- Suppose you are a speculative investor in Singapore. On 15 May, you anticipate that some regulatory changes will be enacted, and you want to profit from this forecast. On 15 June, you intend to borrow 10,000,000 Singapore dollars to fund the purchase of an asset, which you expect to resell at a profit three months after purchase, say on 15 September. The current three-month Sibor (that is, Singapore Libor) is 0.55%. The appropriate FRA rate over the period of 15 June to 15 September is currently 0.68%. You are concerned that rates will rise, so you want to hedge your borrowing risk by purchasing an interest rate call option with an exercise rate of 0.60%.

1. In using the Black model to value this interest rate call option, what would the underlying rate be?
   A. 0.55%
   B. 0.68%
   C. 0.60%
2. The discount factor used in pricing this option would be over what period of time?
   A. 15 May–15 June
   B. 15 June–15 September
   C. 15 May–15 September

Example Solution

- **Solution to 1:**
  - B is correct. In using the Black model, a forward or futures price is used as the underlying. This approach is unlike the BSM model in which a spot price is used as the underlying.

- **Solution to 2:**
  - C is correct. You are pricing the option on 15 May. An option expiring 15 June when the underlying is three-month Sibor will have its payoff determined on 15 June, but the payment will be made on 15 September. Thus, the expected payment must be discounted back from 15 September to 15 May.

Example

- Solomon forecasts the three-month Libor will exceed 0.85% in six months and is considering using options to reduce the risk of rising rates. He asks Lee to value an interest rate call with a strike price of 0.85%. The current three-month Libor is 0.60%, and an FRA for a three-month Libor loan beginning in six months is currently 0.75%.

- The valuation inputs used by Lee to price a call reflecting Solomon's interest rate views should include an underlying FRA rate of:

  A. 0.60% with six months to expiration.

  B. 0.75% with nine months to expiration.

  C. 0.75% with six months to expiration.

Solutions: C

- Solomon's forecast is for the three-month Libor to exceed 0.85% in six months. The correct option valuation inputs use the six-month FRA rate as the underlying, which currently has a rate of 0.75%.

Black Model: Interest Rate Options

- **Option on Swaptions**
  - Gives investor the right to enter into an interest rate swap with a pre-specified exercise swap rate;
  - Option Underlying: Forward swap rate
    - ✓ Payer swaption

$$V_{payer} = NP \times (AP) \times PVA\left[R_{FIX} \times N(d_1) - R_X \times N(d_2)\right]$$

    - ✓ Receiver swaption

$$V_{payer} = NP \times (AP) \times PVA\left[R_X \times N(-d_2) \text{ - } R_{FIX} \times N(-d_1)\right]$$

- R_{FIX} = Fixed swap rate starting when the swaption expires;
- R_X = Exercise swap rate;
- AP = 1/# of settlement periods per year in the underlying swap;

Example

- Suppose you are an Australian company and have ongoing floating-rate debt. You have profited for some time by paying at a floating rate because rates have been falling steadily for the last few years. Now, however, you are concerned that within three months the Australian central bank may tighten its monetary policy and your debt costs will thus increase. Rather than lock in your borrowing via a swap, you prefer to hedge by buying a swaption expiring in three months, whereby you will have the choice, but not the obligation, to enter a five-year swap locking in your borrowing costs. The current three-month forward, five-year swap rate is 2.65%. The current five-year swap rate is 2.55%. The current three-month risk-free rate is 2.25%.
- With reference to the Black model to value the swaption, which statement is correct?
  - A. The underlying is the three-month forward, five-year swap rate.
  - B. The discount rate to use is 2.55%.
  - C. The swaption time to expiration, T, is five years.
- **Solution:**
  - A is correct. The current five-year swap rate is not used as a discount rate with swaptions. The swaption time to expiration is 0.25, not the life of the swap.

Reading 42

Derivatives Strategies

149-186

Framework

1. Derivatives Application
   - Risk Management by Using Derivatives
   - Synthetic Asset
2. Option Strategies
   - Covered Call Strategy
   - Protective Put Strategy
   - Bull Spread
   - Bear Spread
   - Calendar Spread
   - Straddle
   - Collar

Risk Management by Using Derivatives

- **Interest rate derivatives**
  - Futures
  - Swap
    - ✓ Duration of payer swap = duration of floating rate note - duration of a fixed bond
    - ✓ Duration of receiver swap = duration of fixed rate bond - duration of floating bond
- **Equity derivatives**
  - Futures
  - Swap
- **Currency derivatives**
  - Futures
  - Swap
- **Index derivatives**
  - Futures

Example 1

1. A US bond portfolio manager who wants to hedge a bond portfolio against a potential rise in domestic interest rates could best hedge by:
   A. buying Treasury bond futures.
   B. paying a fixed rate in an interest rate swap.
   C. selling foreign currency futures on the home currency.

➢ **Solution to 1:**

- B is correct. In an interest rate swap, if someone pays the fixed rate he or she would receive the floating rate. A floating-rate asset would most likely have a lower duration than a fixed-rate asset, and duration is a direct measure of interest rate risk. The swap would lower the portfolio duration.

Synthetic stock index fund and Synthetic Cash

- Synthetic long/short asset
  - Long call + short put= long asset
  - Long put + short call= short asset
- Synthetic call/put
  - Long call = long asset + short put
  - Long put = short asset + short call
- Synthetic stock
  - Synthetic Equity = Long risk-free asset + Long stock futures
- Synthetic Cash
  - Synthetic risk-free asset = Long stock + Short stock futures

Example

1. Which of the following is most similar to a long put position?
   A. Buy stock, write call
   B. Short stock, buy call
   C. Short stock, write call
2. Which of the following is most similar to a long call position?
   A. Buy stock, buy put
   B. Buy stock, write put
   C. Short stock, write put
3. Which option portfolio with the same exercise price for both options is most similar to a long stock position?
   A. Short call, long put
   B. Long call, short put
   C. Short call, short put

Example - Solution

- **Solution to 1:**
  - B is correct. The long call "cuts off" the unlimited losses from the short stock position.
- **Solution to 2:**
  - A is correct. The long put provides a floor value to the position, making the maximum loss flat below the exercise price. The profit and loss diagram is the same shape as a long call.
- **Solution to 3:**
  - B is correct. When both options have the same exercise price, a short put and long call produce a profit and loss diagram that is the same as a long stock position.

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Framework

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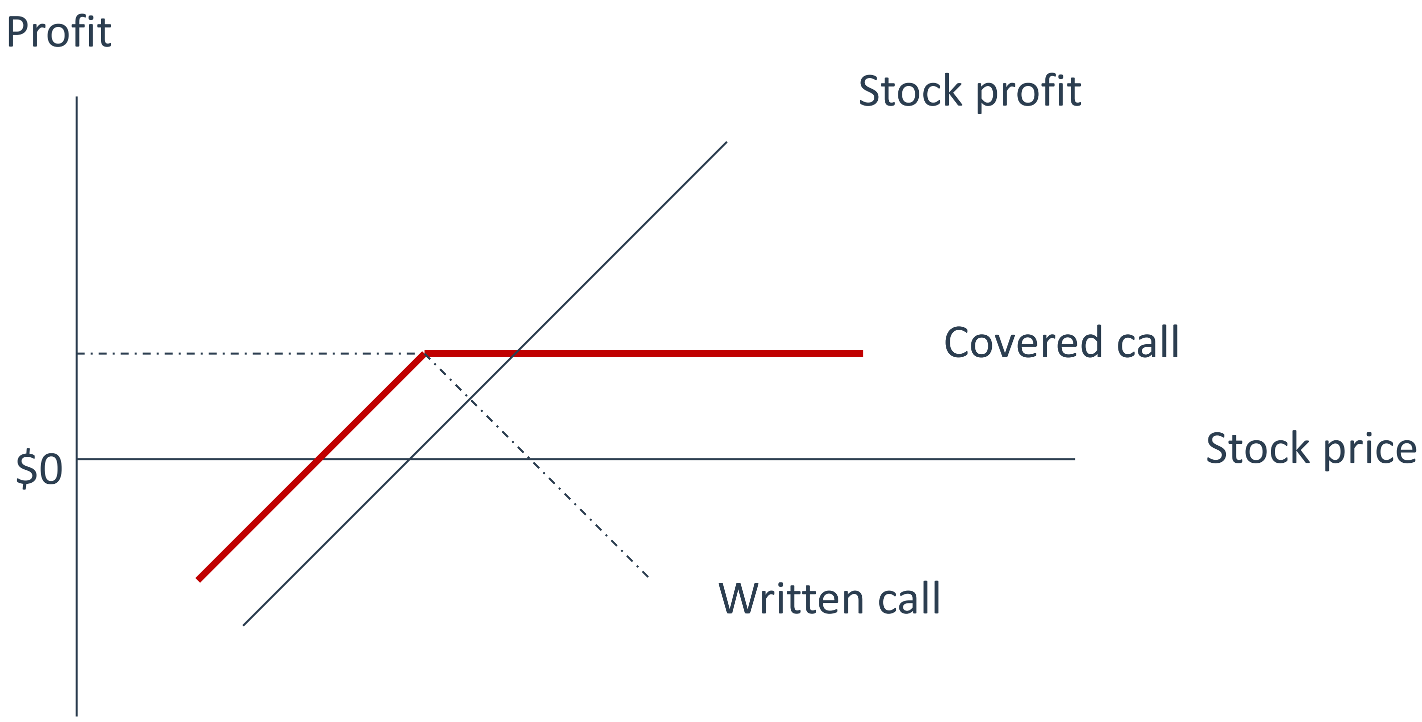
   - Calendar Spread
   - Straddle
   - Collar

Covered call strategy

- An investor creates covered call position by selling a call option on a stock that is owned by the option writer.
  - Income generation
    - ✓ By writing an out-of-the-money call option, the combined position caps the upside potential at the strike price.
  - Improving on the market
    - ✓ Sell the underlying with **a better price** if the investor writes a in-the-money call option at the same time.
  - Target price realization
    - ✓ An investor can **continually write** call option with the exercise price equals to his target price until it is realized.

Covered call strategy



Profit profile for a covered call

Covered call strategy

- 有担保的看涨期权（Covered call）：持有股票的多头并卖出看涨期权，损益情况可以表示为：

$$(S_T - S_0) - \max\{0, (S_T - X)\} + C$$

- 分析结论：
  - 当股票价格S_T涨到期权协议价X以上时，可得最大利润：

$$(S_T - S_0) - \max\{0, (S_T - X)\} + C = (S_T - S_0) - (S_T - X) + C = X - S_0 + C$$

  - 当股票价格跌到0时，损失最大：

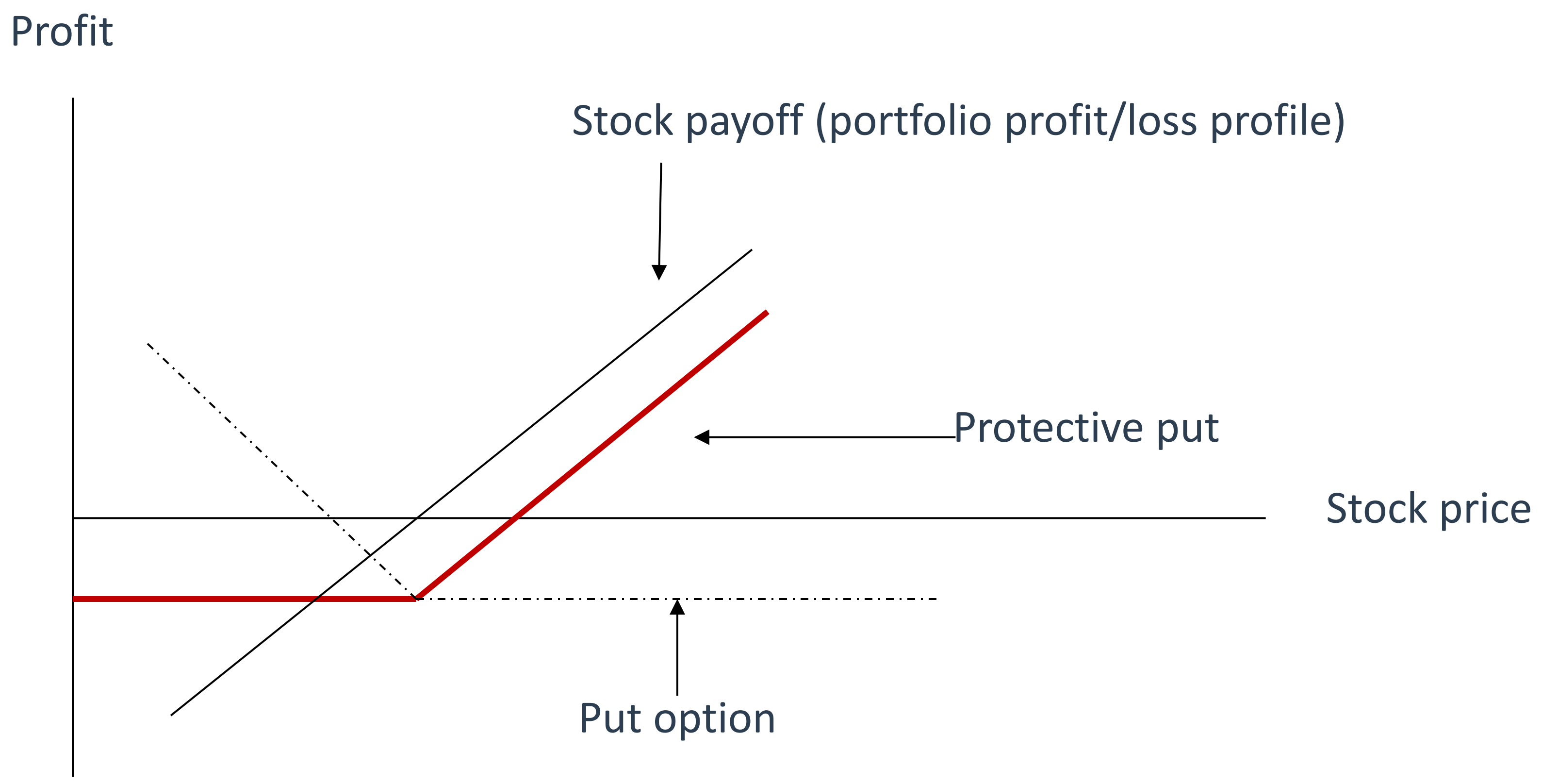
$$(S_T - S_0) - \max\{0, (S_T - X)\} + C = (0 - S_0) - 0 + C = C - S_0$$

  - 盈亏平衡点为：$S_T = S_0 - C$

Protective put strategy

- **A protective put** (also called portfolio insurance or a hedged portfolio) is constructed by holding a long position in the underlying security and buying a put option.
  - **You can use a protective put limit the downside risk at the cost of the put premium, P_0.**
  - You will see by the diagram that the investor will still be able to benefit from increases in the stock's price, but it will be lower by the amount paid for the put, P_0.
  - Notice that the combined strategy looks very much like a call option.

Protective put strategy



Protective put strategy

Protective put strategy

- 有保护的看跌期权（Protective put）:持有股票的多头并买入一份看跌期权，损益情况为：

$$(S_T\text{-}S_0)+\max\{0,(X\text{-}S_T)\}\text{-}P$$

- 分析结论：
  - 当股票价格S_T涨到期权协议价X以上时，可得最大利润 (无上限)
  - 当股票价格跌到0时，损失最大：

$$(S_T\text{-}S_0)+\max\{0,(X\text{-}S_T)\}\text{-}P=0\text{-}S_0+X\text{-}P=X\text{-}S_0\text{-}P$$

  - 盈亏平衡点为：$S_T=S_0+P$

Applications

- **Delta of the strategy**
  - Delta of covered call = delta of stock - delta of call stock
  - Delta of protective put = delta of stock + delta of put
- **Cash secured put**
  - Write an put option and secured with depositing the amount equals to exercise price in an account

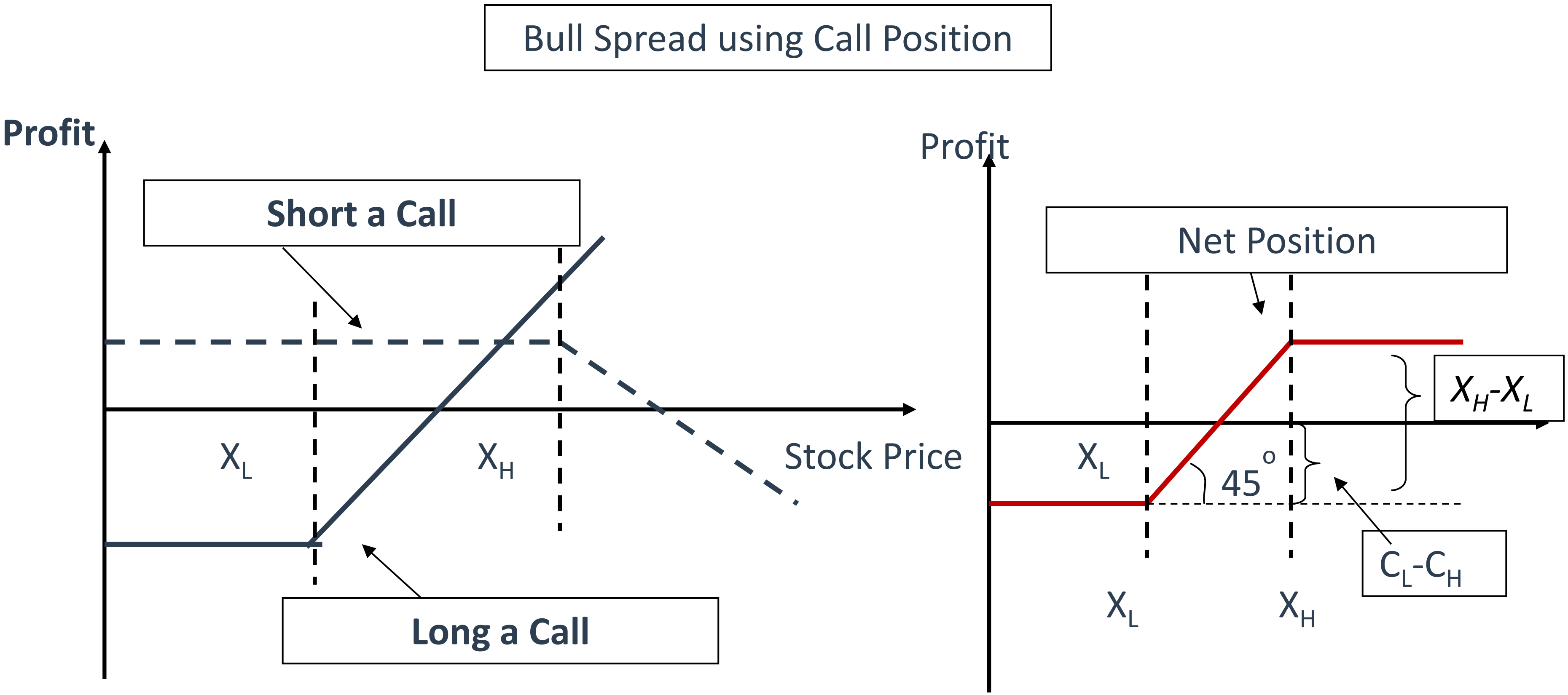
Example

- Marve is contemplating using a covered call strategy, protective put strategy or forwards in conjunction with a long position in 100 shares of Xunoty, Inc. The call option that Marve would use to construct the covered call position has a delta of 0.7. The put option that Marve would use to construct the protective put position has a delta of -0.8.
- What forward position would ensure that Marve has the same position delta as the covered call? Protective put?
- Answer
  - Covered call delta = delta of stock - delta of call option = 1.0 - 0.7 = 0.3.
  - Protective put delta = delta of stock + delta of put option = 1.0 - 0.8 = 0.2.
  - To replicate the delta of a covered call, Marve would go short in a forward contract for 70 shares.
  - Long stock delta + short forward delta = 1 - 0.7 = 0.3.
  - To replicate the delta of protective put, Marve would take the short position in a forward contract for 80 shares.
  - Long stock delta + short forward delta =1 - 0.8 = 0.2.

Bull spread

- In a bull spreads using call , the buyer of the spread purchases a call option with a low exercise price, X_L, and subsidizes the purchase price of the call by selling a call with a higher exercise price, X_H.
- The buyer of a bull spread using call expects the stock price to rise and the purchased call to finish in-the-money. However, the buyer does not believe that the price of the stock will rise above the exercise price for the out-of-the-money written call.
- **A bull spread limit the downside risk at a cost of giving up the upside return.**

Bull spread


Bull Spread using Call Position
Profit
Short a Call
X_L
X_H
Stock Price
Long a Call
Profit
Net Position
X_H-X_L
X_L
45°
C_L-C_H
X_L
X_H

Bull spread

- 牛市看涨差价期权：由两份资产标的相同、到期日相同但执行价不同的看涨期权构成，具体为买入执行价低的看涨期权，卖出执行价高的看涨期权，其损益表示为：

$$\max\{0,(S_T-X_L)\}-\max\{0,(S_T-X_H)\}-C_L+C_H$$

- 结论：
  - 当资产价格S_T上涨到X_H以上时，获得最大利润：

$$\max\{0,(S_T-X_L)\}-\max\{0,(S_T-X_H)\}-C_L+C_H$$
$$=(S_T-X_L)-(S_T-X_H)-C_L+C_H=X_H-X_L+C_H-C_L$$

  - 当资产价格S_T跌到X_L以下时，损失最大：

$$\max\{0,(S_T-X_L)\}-\max\{0,(S_T-X_H)\}-C_L+C_H$$
$$=0-0-C_L+C_H=C_H-C_L$$

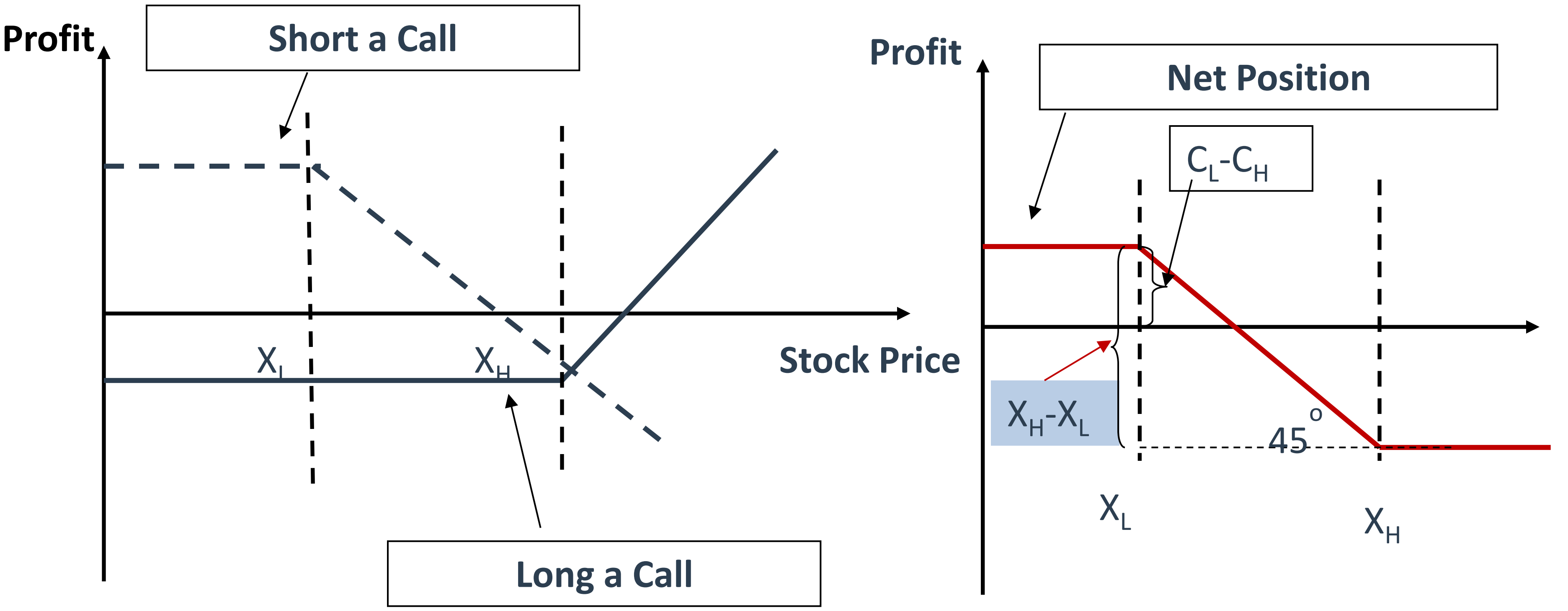
  - 盈亏平衡点为：$S_T=X_L+C_L-C_H$

Bear spread

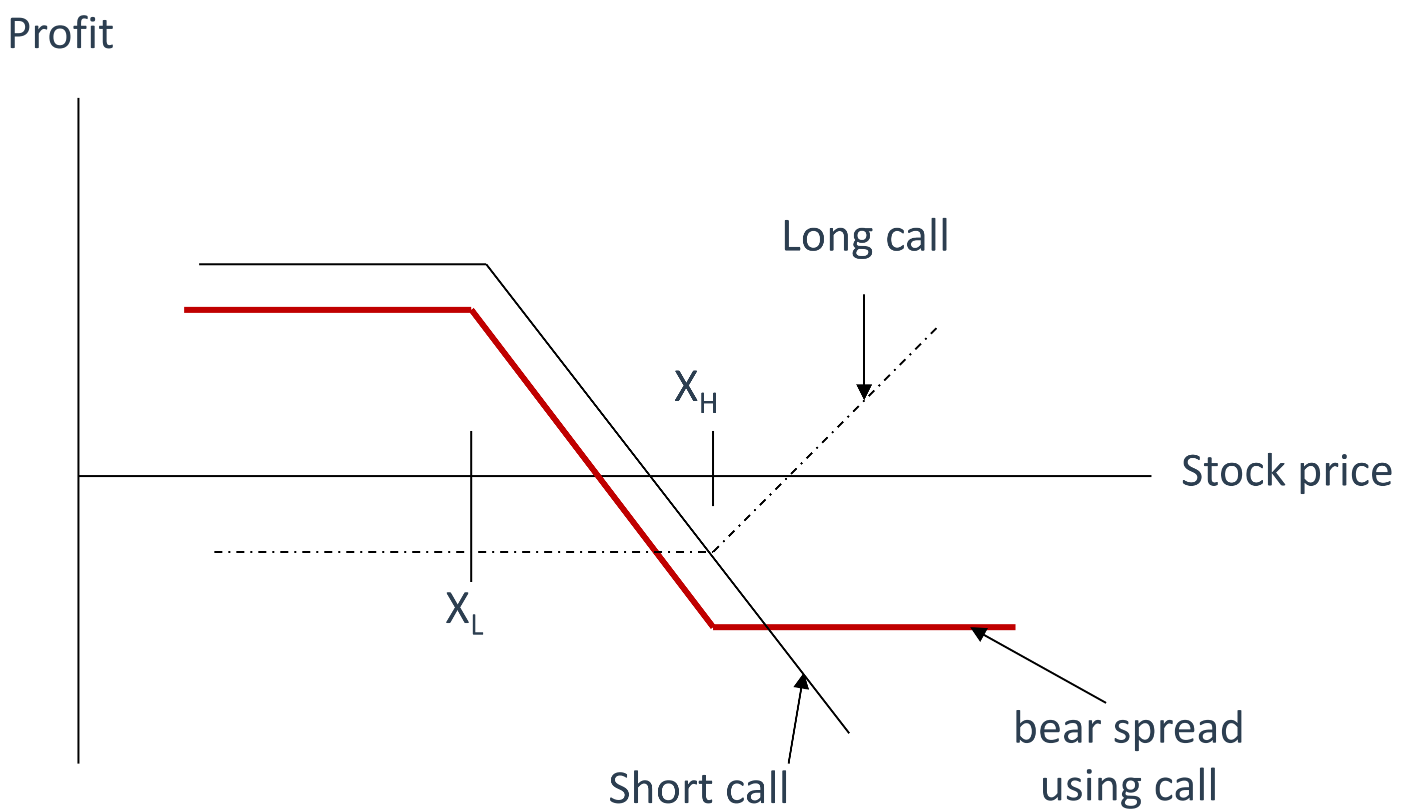
- A **bear spread** using call is the sale of a bull spread. That is the bear spread trader will purchase the call with the higher exercise price and sell the call with the lower exercise price.
  - This strategy is designed to profit from falling stock prices.
  - As stock prices fall, the investor keeps the premium from the written call, net of the long call's cost.
  - The purpose of the long call is to protect from sharp increases in stock prices.
  - The payoff is the opposite (mirror image) of the bull spread using call and is shown in following figure.
- **A bear spread limit the downside risk at a cost of giving up the upside return.**

Bear spread

Bear Spread using Call Position



Bear spread



bear spread using call

Bear spread

- **熊市看涨差价期权：买入执行价格高的看涨期权，卖出执行价低的看涨期权，损益如下：**

$$\max(0,S_T-X_H)-\max(0,S_T-X_L)+C_L-C_H$$

- **分析结论：**
  - 当资产价格S_T跌到执行价格X_L以下时，获得最大利润：

$$\max(0,S_T-X_H)-\max(0,S_T-X_L)+C_L-C_H=0-0+C_L-C_H=C_L-C_H$$

  - 当资产价格S_T涨到执行价格X_H以上时，损失最大：

$$\max(0,S_T-X_H)-\max(0,S_T-X_L)+C_L-C_H=(S_T-X_H)-(S_T-X_L)+C_L-C_H$$
$$=X_L-X_H+C_L-C_H$$

  - 盈亏平衡点为：$S_T=X_L+C_L-C_H$

Example

- Suppose
  - S0 = 44.50
  - OCT 45 call = 2.55 OCT 45 put = 2.92
  - OCT 50 call = 1.45 OCT 50 put = 6.80

1. What is the maximum gain with an OCT 45/50 bull call spread?
   A. 1.10
   B. 3.05
   C. 3.90
2. What is the maximum loss with an OCT 45/50 bear put spread?
   A. 1.12
   B. 3.88
   C. 4.38
3. What is the breakeven point with an OCT 45/50 bull call spread?
   A. 46.10
   B. 47.50
   C. 48.88

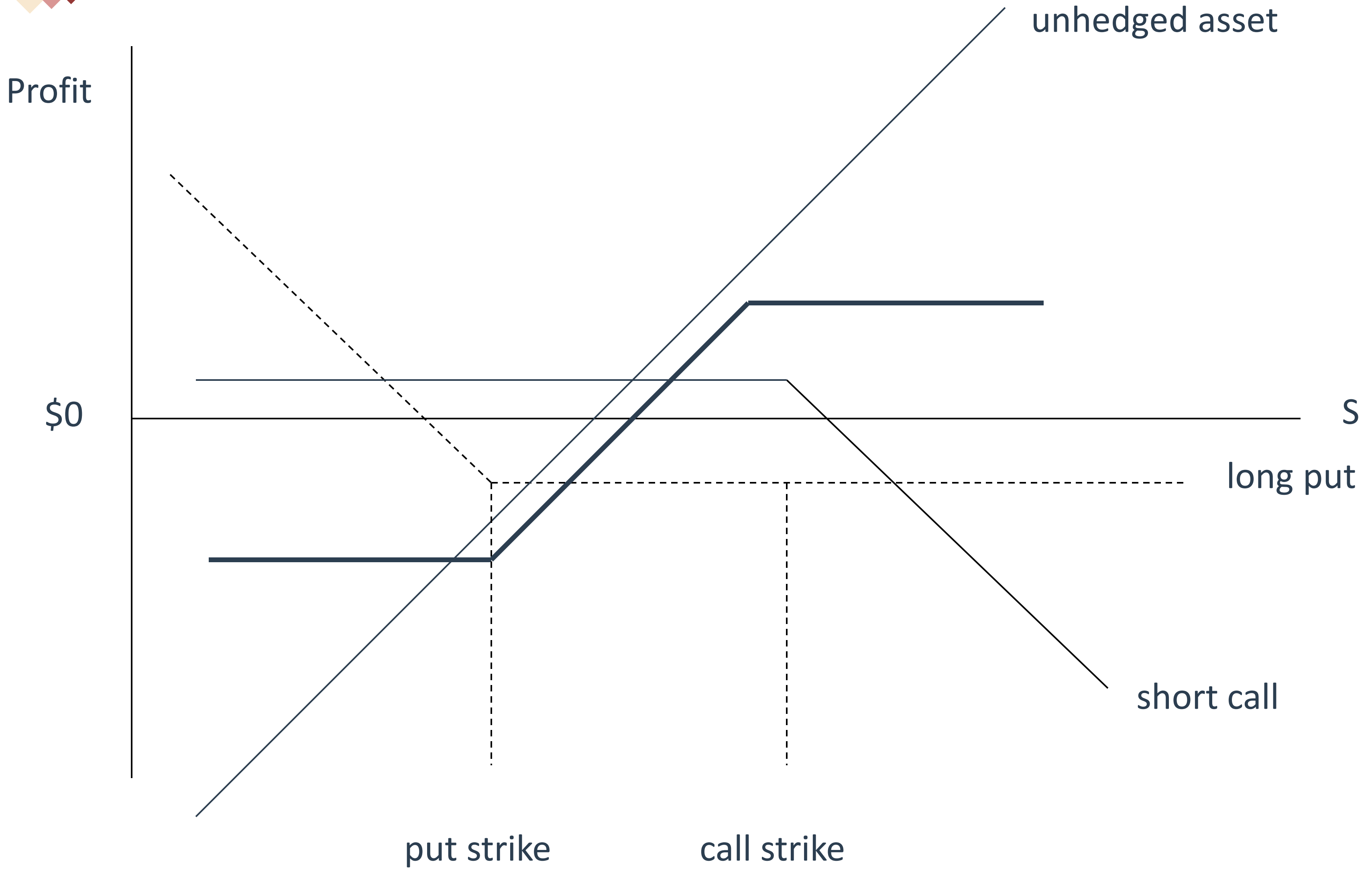
Example - Solution

- **Solution to 1:**
  - C is correct. With a bull spread, the maximum gain occurs at the high exercise price. At an underlying price of 50 or higher, the spread is worth the difference in the strikes, or 50 – 45 = 5. The cost of establishing the spread is the price of the lower-strike option minus the price of the higher-strike option: 2.55 – 1.45 = 1.10. The maximum gain is 5.00 – 1.10 = 3.90.
- **Solution to 2:**
  - B is correct. With a bear spread, you buy the higher exercise price and write the lower exercise price. When this strategy is done with puts, the higher exercise price option costs more than the lower exercise price option. Thus, you have a debit spread with an initial cash outlay, which is the most you can lose. The initial cash outlay is the cost of the OCT 50 put minus the premium received from writing the OCT 45 put: 6.80 – 2.92 = 3.88.
- **Solution to 3:**
  - A is correct. You buy the OCT 45 call for 2.55 and sell the OCT 50 call for 1.45, for a net cost of 1.10. You breakeven when the position is worth the price you paid. The long call is worth 1.10 at a stock price of 46.10, and the OCT 50 call would expire out of the money and thus be worthless. The breakeven point is the lower exercise price of 45 plus the 1.10 cost of the spread, or 46.10.

Collar

- **A collar** is the combination of a protective put and covered call.
- The usual goal is for the owner of the underlying asset to **buy a protective put** and then **sell a call to pay** for the put.
- If the premium of the two are equal, it is called a **zero-cost collar.**
- **Profit and loss for a collar**(where the strike price of the put(X_L) is usually smaller than that of the call(X_H))
  - Profit=S_T-S_0+max{0,X_L-S_T}-P_0-max{0,S_T-X_H}+C_0
  - Maximum profit=X_H-S_0-P_0+C_0
  - Maximum loss=S_0-X_L+P_0-C_0
  - Breakeven price=S_0+P_0-C_0
- **A collar limit the downside risk at a cost of giving up the upside return.**

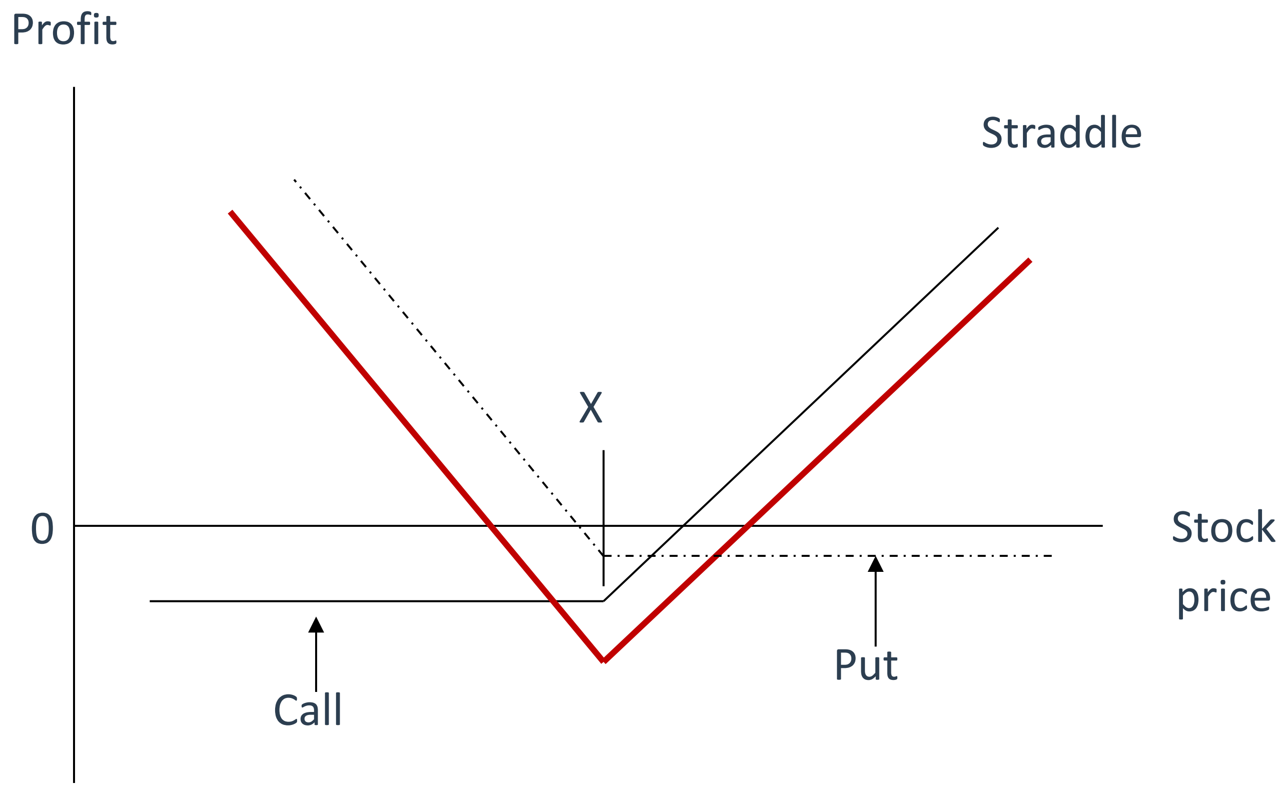
Collar


unhedged asset
Profit
$0
S
long put
short call
put strike
call strike

Straddle

- **A long straddle is created by purchasing a call and a put with the same strike price and expiration.**
  - Both options have the same exercise price and expiration.
  - This strategy is profitable when the stock price moves strongly in either direction.
  - This strategy bets on volatility.
- **A short straddle sells both options and bets on little movement in the stock.**
  - A short straddle bets on the same thing as the butterfly spread, except the losses are not limited.
  - It is a bet that will profit more if correct but also lose more if it is incorrect.
  - Straddles are symmetric around the strike price.

Straddle



Long straddle profit/loss

Straddle

- **Profit and loss for a straddle**
  - Profit=max{0, S_T-X}- C_0 -max{0,X-S_T}+P_0
  - Maximum profit: unlimited
  - Maximum loss=P_0+C_0
  - Breakeven price=X-(P_0+C_0) or X+(P_0+C_0)

Example

➢ Suppose：

- XYZ stock = 100.00
- 100-strike call = 8.00
- 100-strike put = 7.50
- Options are three months until expiration

1. If Smith buys a straddle on XYZ stock, he is best described as expecting a:
   A. high volatility market.
   B. low volatility market.
   C. average volatility market.
2. This strategy will break even at expiration stock prices of:
   A. 92.50 and 108.50.
   B. 92.00 and 108.00.
   C. 84.50 and 115.50.
3. Reaching a breakeven point implies an annualized rate of return closest to:
   A. 16%.
   B. 31%.
   C. 62%.

Example - Solution

- **Solution to 1:**
  - A is correct. A straddle is directionally neutral; it is neither bullish nor bearish. The straddle buyer wants volatility and wants it quickly, but does not care in which direction. The worst outcome is for the underlying asset to remain stable.
- **Solution to 2:**
  - C is correct. To break even, the stock price must move enough to recover the cost of both the put and the call. These premiums total \$15.50, so the stock must move up to \$115.50 or down to \$84.50.
- **Solution to 3:**
  - C is correct. The price change to a breakeven point is 15.50 points, or 15.5% on a 100 stock. This is for three months. This outcome is equivalent to an annualized rate of 62%, found by multiplying by 4 (15.5% × 4 = 62%).

Calendar Spread

- A strategy in which someone **sells a near-dated call and buys a longer-dated one** on the same underlying asset and with the same strike is commonly referred to as a calendar spread.
  - When the investor buys the more distant option, it is a long calendar spread. The investor could also buy a near-term option and sell a longer-dated one, which would be a short calendar spread.
  - As discussed previously, a portion of the option premium is time value. Time value decays over time and approaches zero as the option expiration date approaches. **Taking advantage of this time decay is a primary motivation behind a calendar spread.**
    - ✓ **Time decay is more pronounced for a short-term option** than for one with a long time until expiration.
    - ✓ A calendar spread trade seeks to exploit this characteristic by purchasing a longer-term option and writing a shorter-term option.

Calendar Spread

- Here is an example of how someone might use such a spread.
  - Suppose XYZ stock is trading at 45 a share in August. XYZ has a new product that is to be introduced to the public early the following year. A trader believes this new product introduction is going to have a positive impact on the shares.
  - Until the excitement associated with this announcement starts to affect the stock price, the trader believes that the stock will languish around the current level.
  - Based on the bullish outlook for the stock going into January, the trader purchases the XYZ JAN 45 call at 3.81. Noting that the near-term price forecast is neutral, the trader also decides to sell a XYZ SEP 45 call for 1.55.
- Now move forward to the September expiration and assume that XYZ is trading at 45. The September option will now expire with no value, which is a good outcome for the calendar spread trader.
- If the trader still believes that XYZ will stay around 45 into October before starting to move higher, the trader may continue to execute this strategy. An XYZ OCT 45 call might be sold for 1.55 with the hope that it also expires with no value.

Breakeven price analytics

- Breakeven price for each strategy can be used to determine the volatility that needs to be breakeven following the steps.
  - Calculate the breakeven price for the strategy
  - Calculate breakeven price deviation

$$\%\Delta P = \frac{|breakeven\ price - current\ price|}{current\ price}$$

  - Calculate annual breakeven volatility

$$\sigma_{annual} = \%\Delta P \times \sqrt{\frac{252}{trading\ days\ until\ maturity}}$$

Breakeven price analytics

- Suppose the underlying stock sells for 50, and an investor selects 30-day options with an exercise price of 50. The call sells for 2.29 and the put for 2.28, for a total investment of 4.57.
- the underlying stock typically has an annual volatility of 30%. An investor can obtain some information on the likelihood of reaching the breakeven points before entering into the trade.
- In order for the straddle to be profitable at expiration, the stock must move up or down by 4.57 units from the current price of 50, which is a 9.14% movement. Expiration is in 30 days, but this includes four weekends and possibly a holiday. Suppose there are only 21 trading days until expiration. We convert a 9.14% movement in 21 days to an annual volatility by multiplying by the square root of the number of 21-day periods in a 252-day "year":

$$\sigma_{annual}=0.0914\times\sqrt{\frac{252}{21}}=32.6\%$$

Breakeven price analytics

- **Analysis**
  - The required price movement to the breakeven point represents an annual volatility that is only slightly greater than the historical level, so someone contemplating establishing the straddle might view this scenario favorably.
  - If, instead, the straddle costs 7 to establish, it would require a 14% move to reach a breakeven point. Using the formula just presented, this move is about 48.5% on an annual basis. You might not believe that such a price change could reasonably be expected in a 30-day period and thus elect not to enter into the strategy.

It’s not the end but just beginning.

Thought is already is late, exactly is the earliest time.

感到晚了的时候其实是最早的时候。