

算法结构

对自变量取值范围进行离散化

构造双线性形式a(u,v)以及J(u)

选择合适基函数

Ritz方法:根据极小位能原理,构造方程组使得J(u)达到最小;

Galerkin方法:根据虚功原理构造方程组使得J(u)达到最小;

具体形式

代码

**1.计算a**

%基函数是sin(i\*pi\*x)

%计算a(φi,φj)

function result\_integral\_a = Ritz\_Galerkin\_integral\_a(i,j)

syms x

fail = sin(i\*pi\*x)\*sin(j\*pi\*x) + i\*pi\*cos(i\*pi\*x)\*j\*pi\*cos(j\*pi\*x);

result\_integral\_a = int(fail,x,0,1);

end

**2.计算a**

%基函数是sin(i\*pi\*x),f = x^2

%注题初边值条件非齐次,构造u0(x) = x

function result\_integral\_f = Ritz\_Galerkin\_integral\_f(i)

syms x

u = x^2\*sin(i\*pi\*x);%(f,φi)

w = x\*sin(i\*pi\*x) + i\*pi\*cos(i\*pi\*x);%a(u(0),φi)

result\_integral\_f = int(u,x,0,1) - ( int(w,x,0,1) );%

end

**3.** **主程序**

function result = Ritz\_Galerkin(n)

%主程序

%推导可得a(u,v) = uv+u'v'

%f(x) = x^2

for i = 1:n

for j = 1:i

a(i,j) = Ritz\_Galerkin\_integral\_a(i,j);

a(j,i) = a(i,j);

end

end

for i = 1:n

f(i) = Ritz\_Galerkin\_integral\_f(i);

end

c = a\f';%解出c

syms x

result = 0;

for i = 1:n

result = result + c(i)\*sin(i\*pi\*x);%输出表达式

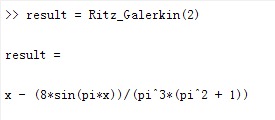
end

result = result + x;

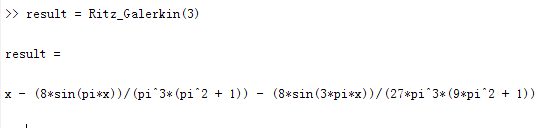
end

结果

取n=2时,得到的n次近似:

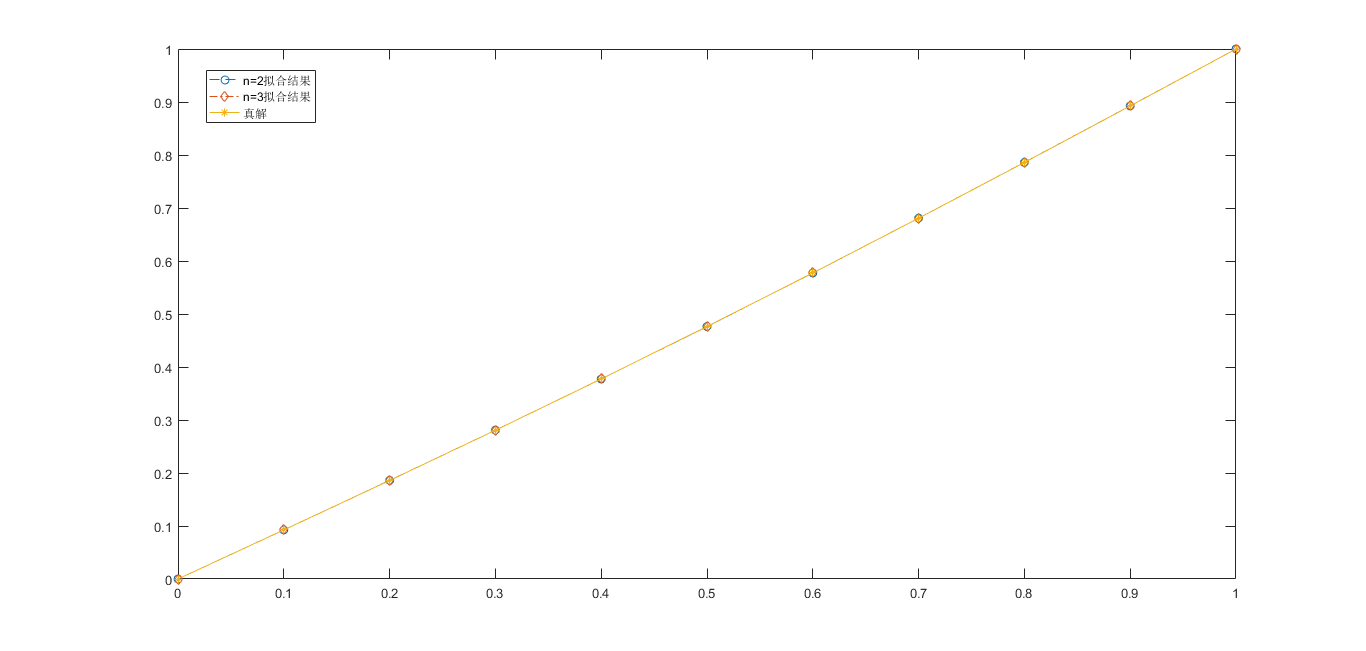


取n=3时,得到的n次近似:



选取n=2,3时,得到的n次近似来计算,如下表(x取0:0.1:1):

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
| n=2 | 0 | 0.092665 | 0.186048 | 0.280796 | 0.377425 | 0.476263 | 0.577425 | 0.680796 | 0.786048 | 0.892665 | 1 |
| n=3 | 0 | 0.092579 | 0.185947 | 0.280763 | 0.377487 | 0.476369 | 0.577487 | 0.680763 | 0.785947 | 0.892579 | 1 |
| 真解 | 0 | 0.092569 | 0.185948 | 0.280771 | 0.377487 | 0.476362 | 0.577487 | 0.680771 | 0.785948 | 0.892569 | 1 |

作出n=2,3以及真解的图像: