Physics CST (2021-22) Homework 1

Please send the completed file to my mailbox yy.lam@qq.com by September 22nd, with using the filename format:

2020xxxxxx_yourname_cst_hw1

Please answer the questions by filling on these sheets. It would be perfect if you are able to write on the sheets directly (by using a stylus or a pen with the computer software, e.g. Microsoft Edge). If you do not have the appropriate hardware (I mean "pen"), you may handle the questions as usual by using pieces of blank papers, then take the photos and paste onto the blank spaces of these question sheets.

1. Find the unit vector of the cross product of the vectors $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ and $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$.

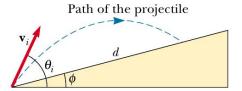
2. A mobile phone is accidently dropped from a hot-air balloon that is 320 m above the ground and rising at 6.50 m/s upward. For the phone, find (a) the maximum height reached, (b) its position and velocity 3.50 s after being released, and (c) the time before it hits the ground.

3. A projectile is fired up an inclined plane with an initial speed v_i at an angle θ_i as shown.

(a) Show that

$$d = \frac{2v_i^2 \cos \theta_i \sin(\theta_i - \phi)}{g \cos^2 \phi}.$$

(b) Find θ_i when d is maximum. What is the maximum value of d?



4. Given a vector ${\bf v}$ in 2-dimensional plane written in terms of the orthonormal basis in polar coordinates

$$\mathbf{v} = 5t\mathbf{e}_r + 2t^2\mathbf{e}_\theta,$$

Find the derivative of \mathbf{v} with respect to t.

5. A bus starts from rest and moves with constant acceleration. First observed that the velocity is 10 ms⁻¹, then it becomes 30 ms⁻¹ after 20 s. (a) Calculate the average velocity and the constant acceleration over the time interval. (b) Calculate the position of the bus at 6 s after the first observation. (c) Find the time required for the bus to reach 300 m from rest. (d) Find the velocity of the bus at 250 m from rest.

6. The velocity of a particle moving along the x axis varies in time according to the expression $v_x = (35 - 2t^2) \ ms^{-1}$, where t is in seconds. (a) Find the average acceleration in the time interval t = 0 to t = 2 s along the direction 15° from the x-axis. (b) Determine the acceleration at t = 2 s. (c) What is the acceleration along the y-axis?

7. Please use dimensional analysis to find the expression of the angular momentum L of a particle with mass m in uniform circular motion at an angular speed ω with radius r.