Computer Problems

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Problem 1. Let A be the 1000×1000 matrix with entries $A(i,i) = i, A(i,i+1) = A(i+1,i) = \frac{1}{2}, A(i,i+2) = A(i+2,i) = \frac{1}{2}$ for all i that fit within the matrix.

- Solve the system with $Ax = [1, 1, \cdots, 1]^\top$ by the following methods in 15 steps:
  1. The Jacobi Method;
  2. The Gauss-Seidel Method;
  3. SOR with $\omega = 1.1$;
  4. The Conjugate Gradient Method;
  5. The Conjugate Gradient Method with Jacobi preconditioner.
- Report the errors of every step for each method.

Problem 2. Let $f(x) = e^{-2x}$ and the interval to be $[-1, 1]$.

1. Write a program generating the Newton's divided difference formula;
2. Use the program to generate a degree n polynomial with evenly spaced points and Chebyshev points for $n = 10$, 20 and 40;
3. Plot the polynomials for the above types (see Figure 3.8);
4. By sampling at a 0.05 step, create the empirical interpolation errors for each type, and plot a comparison (see Figure 3.11).

Problem 3. Given two inconsistent systems as follows:

(a) $\begin{bmatrix} 3 & -1 & 2 \\ 4 & 1 & 0 \\ -3 & 2 & 1 \\ 1 & 1 & 5 \\ -2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ -5 \\ 15 \\ 0 \end{bmatrix}$ and (b) $\begin{bmatrix} 4 & 2 & 3 & 0 \\ -2 & 3 & -1 & 1 \\ 1 & 3 & -4 & 2 \\ 1 & 0 & 1 & -1 \\ 3 & 1 & 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 2 \\ 0 \\ 5 \end{bmatrix}$.

1. Write a program that implements classical Gram-Schmidt to find the full QR factorization, and report the matrices Q and R.
2. Repeat the first question, but implement Householder reflections and report each Householder reflector H_i of every step, the matrices Q and R.
3. Report the least squares solution and 2-norm error.

Problem 4. 1. Implement a simple web crawler;

2. Acquire the google matrix of the first 500 web pages from the main webpage of any university via the above web crawler, and give their adjacencymatrix;

3. Implement the Power Method;

4. Compute the dominant eigenvector of the google matrix;

5. List the top 20 web pages.