Investment Returns and Risks

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1. Aim

Linear Programming is an effective way to solve problems which contains several constrains and requires to get an optimal solution. Depending on the specific constraints, the conditions of optimal solution can be one, more than one, many equivalent optimal solutions or no optimal solution.

This project is going to implement the optimal solution of a case of Investment Returns and Risks with some known constraints. Assuming there are going to have needs from different types of customers, constraints will be changed in some way to get different types of optimal results.

2. Introduction

There are **N** types of assets **S**i(i=1,2...N) in the market which can be chosen. Let the total amount of Capital be **M** which is invested for a period, and the amount of Capital invested for each of the assets **S**i is **X**i respectively. These **N** assets which purchasing **S**i have an average Return Rate of **R**i, with the Risk Rate of **Q**i. The more the Investments are dispersed, the less the Overall Risk will be. Overall Risk can be represented by the maximum value of **Q**i in **S**i. Purchasing asset **S**i will cost Transaction Fee and the Transaction Rate is **P**i.

1. The **Objective Function** is <u>**Z**=Max∑(**R**i-**P**i)**X**i</u> (i=1~N);
2. The **Constraints** are:

(1) <u>**Q**i**X**i/M<=a</u>(a is the constant which represents the up limit for all the **Q**i***X**i)

(2) ∑(1+**P**i)**X**i=M

Where **X**i>=0 and i=1~N.

3. The **Expected Output** is the Maximum Solution of **Z**.

The following table shows the default values of **S**i, **R**i, **Q**i and **P**i.

Si	**Ri(%)**	**Qi**	**Pi(%)**	**Ri-Pi**
S1	28	2.5	1	27
S2	21	1.5	2	19
S3	23	5.5	4.5	18.5
S4	25	2.1	6.5	18.5
S5	22	2.0	5.0	17

Select S1 and S2 as the constraints for the problem, then standard form and slack form will be like the following. Let M=1 and a=2.

Standard Form:
Maximum Z=0.27*X1+0.19*X2
Subject to:
    2.5*X1<=2
    1.5*X2<=2
    1.01*X1+1.02*X2<=1
    -1.01*X1-1.02*X2<=-1
    Xi>=0(i=1,2)

Slack Form:
Z=0.27*X1+0.19*X2
    X3=2-2.5*X1
    X4=2-1.5*X2
    X5=1-1.01*X1-1.02*X2
    X6=-1+1.01*X1+1.02*X2
    Xi>=0(i=1~6)

3. Methods

All the solutions generated from 2.5*X1<=2, 1.5*X2<=2 and 1.01*X1+1.02*X2=1 are **feasible solutions**. (e.g. X1=0, X2=1/1.02, Z=0+0.19*1/1.02=0.1863). As R2-P2=19<R1-P1=27, so that in order to increase the returns, we prefer S1 as much as possible. However, Q1=2.5>2>Q2=1.5, which means that we have to purchase some S2 instead to reduce risk to make each QiXi/M<=2. Then the largest possible value of X1 is when 2.5*X1=2, which leads to X1=0.8. At this time, X2=0.1882 and **Optimal Solution** Z=0.2518

Visualization of S1&S2

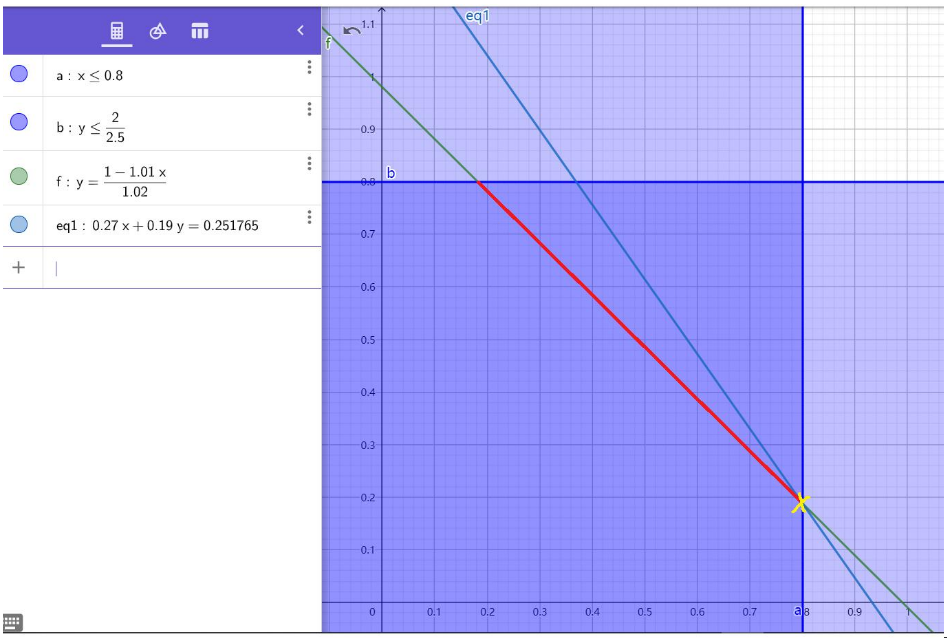

Fig1

In Fig1, x-axis represents X1 and y-axis represents X2. the red line shows the combination of all the feasible solutions of Z, and the yellow “X” shows the position of the optimal solution. The value of solutions increases in ascending with the x-axis. As X1 has a larger value of Ri-Pi, it earns more Returns rather than X2 in unit. So that as the Capital M is constant, Z will increase with X1. When X1 reaches the largest possible value 0.8, Z is then also at the optimal value, which is 0.2518.

4. Development

This step is going to **vary the constraints** and hence analyze the new solutions and graphs. This time we choose S4 and S5 as the constraints. Let M=2 and a=2.05. Then the formula is:

Maximum Z=0.185*X1+0.17*X2

Subject to

2.1*X1/2<=2.05

2*X2/2<=2.05

1.065*X1+1.05*X2=2

Xi>=0(i=1,2)

Which S4 is correspond to X1 and S5 is correspond to X2.

Visualization of S4&S5

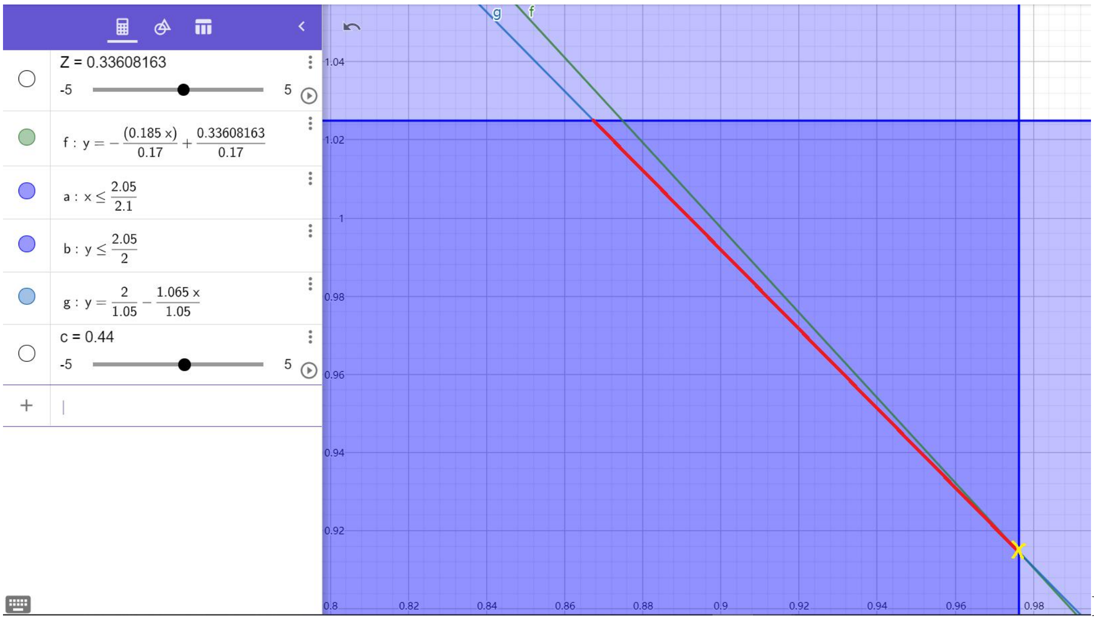

Fig2

In Fig2, x-axis represents X1 and y-axis represents X2. the red line shows the combination of all the feasible solutions of Z, and the yellow “X” shows the position of the optimal solution. The value of solutions increases in ascending with the x-axis. It is obvious to find that when X1 is greater, Z is greater also. This can be because S4’s Returns after Transaction Fee over per unit Risk is higher than that of S5(18.5/2.1>17/2.0), so that although S4 has a higher Risk Rate, it is still more valuable than S5. Then when following the constraints, Z prefer as much S4 as possible, and hence the optimal Z appears at the point when X1 is at its peak.

Optimal Solution: X1=0.9762, X2=0.9146 and Z=0.3361

5. Conclusion

The solutions show that assets of larger Returns(after deducting Transaction Fee) or smaller Risk Rate are preferred. If an asset satisfies only 1 requirement, the algorithm will hence value it by its

Returns per unit Risk to judge whether it is better or worse than another one. Optimal Solution will expect assets which have a high value of Returns per unit Risk. Assets which have the same value of it can be seen as enjoy equivalent preference, and at that time the objective function can have numerous optimal solutions within a specific bounded area.