

ELIGIBILITY PROPAGATION IN MULTI-LAYER
RECURRENT SPIKING NEURAL NETWORKS

WERNER VAN DER VEEN

Faculty of Science and Engineering
University of Groningen

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ACRONYMS

INTRODUCTION

A primary goal of artificial intelligence is to develop systems that display intelligent behavior. During the 1980s, with the popularization of backpropagation and trainable Hopfield networks, the focus of the field shifted from expert systems and symbolic reasoning to *connectionist* approaches, such as artificial neural networks (ANNs). ANNs are networks of small computational units that can be trained to perform specific pattern recognition tasks. These networks are based loosely on the human brain.

As computing power and data storage capabilities increased exponentially, and the rise of the internet provided abundant training data, ANNs have become a dominant field in artificial intelligence in the context of deep learning (DL). This has particularly been the case during the the 2010s, when convolutional neural networks (CNNs) and recurrent neural networks (RNNs) approached or exceeded human level performance in some areas (Schmidhuber, 2015). CNNs were also inspired by neuroscience; the connectivity pattern between units in a CNN resembles the organization of the primate visual cortex (Hubel and Wiesel, 1968).

However, DL-based methods are starting to show diminishing returns; training some state-of-the-art models can require so much data and computing power that only a small number of organizations has the resources to train and deploy them. For example, training the 11-billion parameter version of Google’s T5 model (Raffel et al., 2019) is estimated to cost more than \$1.3 million per run Sharir, Peleg, and Shoham, 2020. This contrasts strongly with the energy consumption of the human brain, which is made up of around 86 billion neurons (Azevedo et al., 2009) and 100–500 trillion synapses (Drachman, 2005), consumes approximately 20 W (Sokoloff, 1960), and does not require as much data to learn patterns.

One reason why the human brain is more energy-efficient is that its computational function is realized in an analog and massively parallel physical substrate A Pastur-Romay et al., 2017, in which neurons communicate through sparsely occurring spikes (Bear, Connors, and Paradiso, 2020). Connections in DL models, on the other hand, are represented by multiplications between the matrices of the weight of these connections and the activation values of the efferent units (LeCun, Bengio, and Hinton, 2015). Backpropagation, which has become the de-facto standard for training DL models, is a biologically implausible learning algorithm that trains models by propagating the error back into the network, further raising computational costs.

Spiking neural networks (SNNs) are another step towards biological plausibility of connectionist models. SNNs use neurons that do not relay continuous activation values at every propagation cycle, but spike

once when they reach a threshold value. The concept of SNNs dates back to the 1980s (Hopfield, 1982), but since spike-based activation is differentiable, gradient descent is not as effective as in ANNs to minimize the loss, and the lack of suitable training methods has limited the popularity of SNNs.

Neuromorphic computing is an emerging technology that, like the human brain, performs computation in a physical substrate. This technology has the potential to offer more energy-efficient computation than the von Neumann architecture that is standard in training DL models. Due to the centralized nature of von Neumann computers, SNNs do not enjoy the energy efficiency as networks of biological neurons. In theory, however, the massively parallel and decentralized neuromorphic computers can efficiently run SNNs. This requires a learning algorithm that is both spatially and temporally local (i.e., neurons and synapses can only change their state based on information that is available at the same timestep and immediately adjacent to that neuron or synapse).

This report examines functional modifications to *eligibility propagation* (e-prop), which is a spatially and temporally local learning algorithm for SNNs that suggests a promising approach to train SNNs in a neuromorphic architecture (Bellec et al., 2020).

(cont.)

- Brief historical overview of 3F-Hebbian (use my own literature trace)
- Explain 3F-Hebbian (mention bioplausibility: online & local)

- E-prop approximates BPTT using RSNNs by using eligibility traces and learning signals. Also mathematical link (only intuition!)

- This paper examines the effects of enhancements that may improve the performance of e-prop. Some of these are used successfully in DL. (Argue scientifically why these might improve performance)
- Multilayer. Mention how MLPs were breakthrough on perceptrons.
- Other neuron types
- Regularization that's also observed in brain (e.g. synaptic scaling)

RELATED WORK

2.1 THREE-FACTOR HEBBIAN LEARNING

-3F Hebbian learning

2.1.1 *Spike-timing dependent plasticity*

- Clopath rule - R-STDP - (and other variants if they're relevant)

2.1.2 *Learning signal*

- Biological plausibility, (how does it happen in brain?) - Error-related negativity (see Bellec1)

2.1.2.1 *Broadcasting*

- Broadcasting methods - Broadcast alignment (see Bellec1) - In brain

2.1.3 *Eligibility traces*

- Why necessary? - Bioplausibility

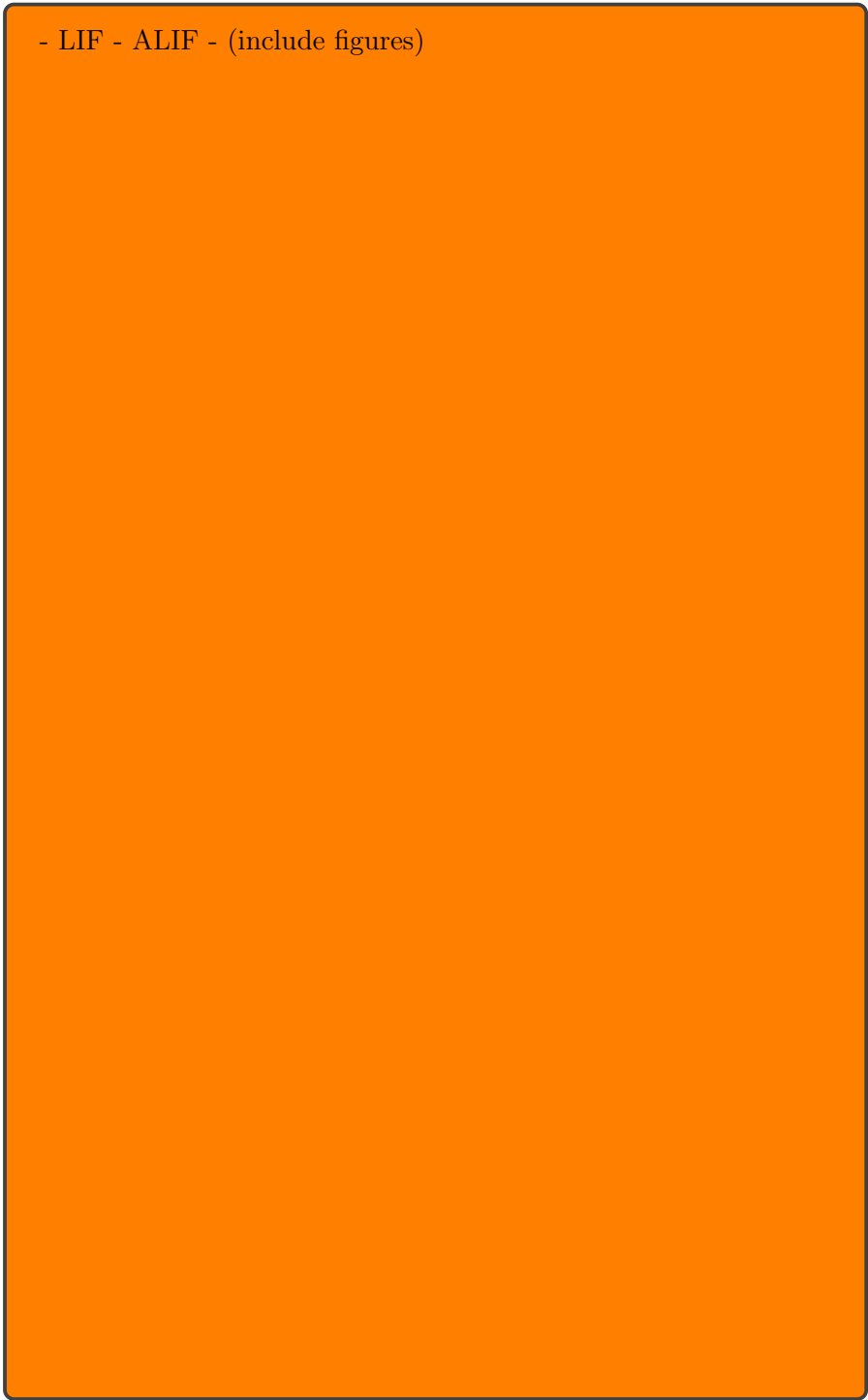
2.2 ELIGIBILITY PROPAGATION

2.2.1 *Network model*

- Network model (formal, see Bellec)

2.2.2 *Neuron models*

- LIF - ALIF - (include figures)



2.2.3 *Learning procedure*

- Neuron variables - Synapse variables

2.2.4 *Deriving e-prop from RNNs*

- Explanation of BPTT. - Derivation from RNN and backprop.

2.3 SYNAPTIC SCALING

- Synaptic Scaling (in brain, if applicable)

2.4 METAPLASTICITY

- Metaplasticity (in brain, if applicable)

2.5 NETWORK TOPOLOGY

- Network topology (e.g. multilayer) (but keep relevant) - Mainly focus on how brain does it. Cite often! Don't hypothesize on effects, just describe with sources. - Also denote a subsection on effects of topologies in related ANNs (preferably (R)SNNs).

METHOD

3.1 DATA PREPROCESSING

3.1.1 The TIMIT speech corpus

TIMIT is a speech corpus that contains phonemically transcribed speech (Garofolo et al., 1993), comprising 6300 sentences, 10 spoken by each of the 630 speakers. To include a broad range of dialects all speakers lived in 8 different geographical regions in the United States (as categorized in Labov, Ash, and Boberg, 2008) during their childhood years. Table 3.1 breaks down the precise composition of the dialect distribution.

DIALECT REGION	# MALE	# FEMALE	TOTAL
1 (New England)	31 (63%)	18 (27%)	49 (8%)
2 (Northern)	71 (70%)	31 (30%)	102 (16%)
3 (North Midland)	79 (67%)	23 (23%)	102 (16%)
4 (South Midland)	69 (69%)	31 (31%)	100 (16%)
5 (Southern)	62 (63%)	36 (37%)	98 (16%)
6 (New York City)	30 (65%)	16 (35%)	46 (7%)
7 (Western)	74 (74%)	26 (26%)	100 (16%)
8	22 (67%)	11 (33%)	33 (5%)
All	438 (70%)	192 (30%)	630 (100%)

Table 3.1: Distribution of speakers’ dialect regions and sexes. Speakers of the innominate dialect region 8 relocated often during their childhood.

The sentence text can be categorized into 2 *dialect* sentences, 450 *phonetically compact* sentences, and 1890 *phonetically diverse* sentences.

The dialect sentences, which are spoken by all speakers, are designed to expose the dialectical variants of the speakers. The phonetically compact sentences are designed to include many pairs of phones. The phonetically diverse sentences are taken from the Brown Corpus (Kucera, Kučera, and Francis, 1967) and the Playwrights Dialog (Hultzsich et al., 1964) in order to maximize the number of allophones (i.e., different phones used to pronounce the same phoneme). Table 3.2 lists an overview of the distribution of the number of speakers per sentence type.

Each of the sentences is encoded in as a waveform signal in .wav format, and is accompanied by a corresponding text file indicating which phones are pronounced in the waveform, and between which pairs of sample points.

SENTENCE TYPE	#SENTENCES	#SPEAKERS	TOTAL
Dialect	2	630	1260
Compact	450	7	3150
Diverse	1890	1	1890
Total	2342		6300

Table 3.2: Distribution of sentence types.

3.1.2 Data splitting

The TIMIT dataset is split into a training, validation and testing set as in Graves and Schmidhuber, 2005 and Bellec et al., 2020. The training set is used to train the network synaptic weights according to the e-prop algorithm. The validation set is used to obtain a well-performing set of hyperparameters, and to anneal the learning rate (see Section ??). The testing set is used to evaluate the performance of the network after the hyperparameters are obtained.

The TIMIT corpus documentation offers a suggested partitioning of the training and testing data, which is based on the following criteria:

1. 70%–80% of the data is used for training, and the remaining 20%–30% for testing.
2. No speaker appears in both the training and testing portions.
3. Both subsets include at least 1 male and 1 female speaker from every dialect region.
4. There is a minimal overlap of text material in the two subsets.
5. The test set should contain all phonemes in as many allophonic contexts as possible.

In accordance with these criteria, the TIMIT corpus includes a “core” test set that contains 2 male speakers and 1 female speaker from each dialect, summing up to 24 speakers. Each of these speakers read a different set of 5 phonetically compact sentences, and 3 phonetically diverse sentences that were unique for each speaker. Consequently, the test set comprises 192 sentences ($24 \times (5 + 3)$) and was selected such that it contains at least one occurrence of each phoneme. In this report, the TIMIT core test set is used, thereby meeting the criteria listed above.

The remaining 4096 sentences are randomly partitioned into 3696 training sentences and 400 validation sentences.

3.1.3 Engineering features

In this subsection, we describe the preprocessing pipeline as in Fayek, 2016, which can be summarized by applying a pre-emphasis filter on

the waveforms, then slicing the waveform in short frames, taking their short-term power spectra, computing 26 filterbanks, and finally obtain 12 Mel-Frequency Cepstrum Coefficients (MFCCs). We align these MFCCs with the phones found in the TIMIT dataset. An example of a waveform signal is given in Figure ??.

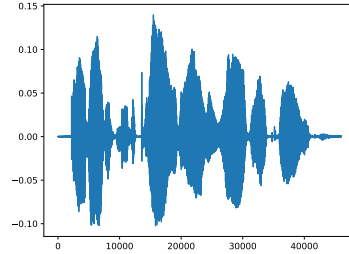


Figure 3.1: A raw waveform signal from the TIMIT dataset.

PRE-EMPHASIS In speech signals, high frequencies generally have smaller magnitudes than lower frequencies. To balance the magnitudes over the range of frequencies in the signal, we apply a pre-emphasis filter $y(t)$ on the waveform signal $x(t)$ defined in Equation 3.1.

$$y(t) = x(t) - 0.97x(t - 1) \quad (3.1)$$

This procedure yields the additional benefit of improving the signal-to-noise ratio. An example of a pre-emphasized signal is given in Figure ??.

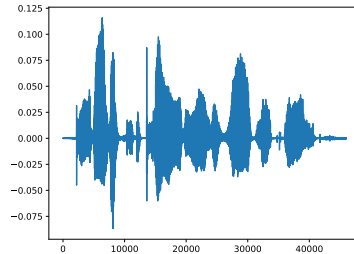


Figure 3.2: A signal after the pre-emphasis filter of Equation 3.1 was applied to it.

FRAMING The waveforms, which are sampled at a rate f_s of 16 kHz, cannot be directly used as input to the model, because they are too long—a typical sentence waveform contains in the order of tens of thousands of samples. Furthermore, the samples are not very informative, because they represent the sound wave of the uttered sound. These sounds are filtered by the shape of the vocal tract, which manifests itself in the envelope of the short time power spectrum of the sound. This power spectrum representation describes the power of the frequency components of the

signal over a brief interval. We assume the frequency components to be stationary over short intervals—in contrast to the full sentence, which carries its meaning because it is non-stationary. Therefore, we transform the waveform signals into series of frequency coefficients of short-term power spectra. To obtain multiple short-term power spectra over the duration of the waveform, we slice it up into brief overlapping frames.

Every 160 samples (equivalent to 10 ms) of a pre-emphasized signal we take an interval frame of 400 samples (equivalent to 25 ms). This means that the frames overlap by 25 ms. The waveform is zero-padded such that the last frame also has 400 samples. By this process, we obtain signal frames $x_i(n)$, where n ranges over 1–400, and i ranges over the number of frames in the waveform.

Then, we apply a Hamming window with the form

$$w[n] = a_0 - a_1 \cos\left(\frac{2\pi n}{N-1}\right), \quad (3.2)$$

where N is the window length of 400 samples, $0 \leq n < N$, $a_0 = 0.53836$, and $a_1 = 0.46164$. A plot of this window is given in Figure ??.

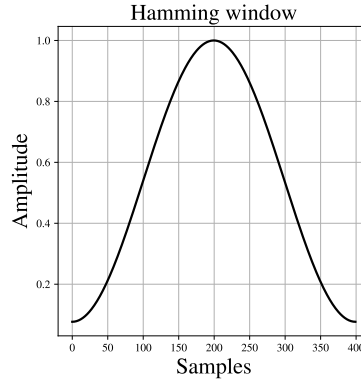


Figure 3.3: The magnitudes of the DFT of a frame.

window is applied to reduce the spectral leakage, which manifests itself though sidelobes in the power spectra. Applying the Hamming window reduces the sidelobes to near-equiripple conditions (Smith, [accessed <date>](#)).

plot for illustration

SHORT-TERM POWER SPECTRA We obtain the power spectra P_i for each frame by first taking the absolute K -point discrete Fourier transform (DFT) of the frame samples $x_i(n)$

don't bother with eqn, just call `mathbb{F}`

$$X_k = \left| \sum_{n=0}^{N-1} x_i(n) \cdot e^{-i\frac{2\pi}{N}kn} \right|, \quad (3.3)$$

where $K = 512$. This yields the magnitudes of the DCT of the frames (an example is illustrated in Figure ??).

We obtain the power spectrum using the equation

$$P = \frac{X_k^2}{K}, \quad (3.4)$$

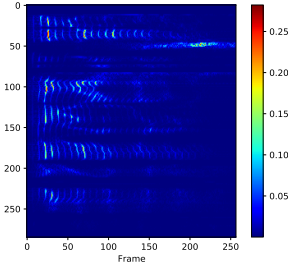


Figure 3.4: The magnitudes of the DFT of a frame.

an example of which is shown in Figure ??.

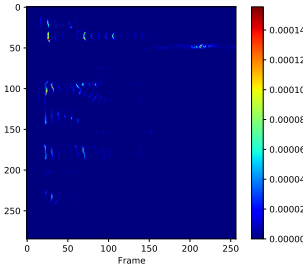


Figure 3.5: A power spectrum of a frame.

MEL FILTERBANK We then transform the short-term power spectra to Mel-spaced filterbanks. The Mel scale is a scale of pitches that are perceptually equal in distance (Stevens, Volkman, and Newman, 1937). This is in contrast to the frequency measurement, in which the human cochlea can better distinguish lower frequencies better than higher ones. The aim of converting to the Mel scale is to make every filterbank coefficient feature equally informative, thereby improving the learning performance of the model.

The Mel-spaced filterbank is a set of 40 triangular filters that we apply to each frame in P .

To compute the Mel-spaced filterbank we choose lower and upper band edges of 0 Hz and $f_s/2 = 8$ kHz, respectively, and convert these to Mels using

$$m(f) = 2595 \log_{10} \left(1 + \frac{f}{700} \right), \quad (3.5)$$

where f is the frequency in Hz. We obtain a lower band edge of 0 Mels and an upper band edge of approximately 2835 Mels.

We begin obtaining the 40 filterbanks by spacing 42 points \mathbf{m} linearly between these bounds (inclusive). Hence, we obtain 42 points spaced exclusively between the bounds.

Then, we convert each point m back to Hz using

$$f = 700 \left(10^{m/2595} - 1 \right). \quad (3.6)$$

We round each resulting Mel-spaced frequency f to their nearest Fourier transform bin b using

$$b = \lfloor (K + 1)f/fs \rfloor \quad (3.7)$$

The resulting 40 filterbanks with their corresponding Mels and frequencies are listed in Table A.1.

not sure

The i^{th} filter in filterbank H_i is a triangular filter that has its lower boundary at b_i Hz, its peak at b_{i+1} Hz, and its upper boundary at b_{i+2} Hz. For other frequencies, they are 0. Therefore, the filterbank can be described by

$$H_i(k) = \begin{cases} 0 & k < b_i \\ \frac{k-b_i}{b_{i+1}-b_i} & b_i \leq k < b_{i+1} \\ 1 & k = b_{i+1} \\ \frac{b_{i+2}-k}{b_{i+2}-b_{i+1}} & b_{i+1} < k \leq b_{i+2} \\ 0 & b_{i+2} < k \end{cases}, \quad (3.8)$$

where $0 \leq k \leq \frac{K}{2}$. These Mel-spaced filters are shown in Figure ??.

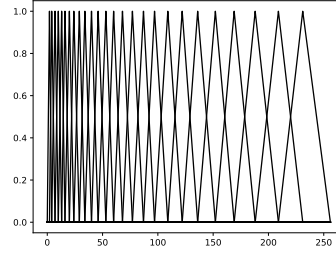


Figure 3.6: The Mel-spaced filterbanks.

We obtain a spectrogram S of the frame (see e.g. Figure ??) after applying the filterbank to the short-term power spectrum.

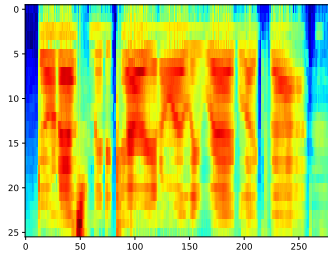


Figure 3.7: An example of a spectrogram.

MEL-FREQUENCY CEPSTRAL COEFFICIENTS We observe that the coefficients in the spectrograms are strongly correlated, which would negatively impact the learning performance of the model.

why?

Therefore, we apply the DCT again to decorrelate the coefficients and obtain the power cepstrum C of the speech frame:

$$C(n) = \left| \sum_{k=0}^{N-1} S(k) \cdot e^{-\frac{i2\pi}{N}kn} \right|. \quad (3.9)$$

We discard the first coefficient in C , because it is the average power of the input signal and therefore carries little meaning. We also discard coefficients higher than 13, because they represent only fast changes in the spectrogram and increase the complexity of the input signal while adding increasingly less meaning to it. An example of the remaining MFCC components is shown in Figure ??.

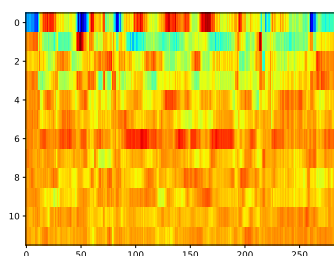


Figure 3.8: An example of Mel-frequency cepstral coefficients that are given as input to the system.

Then, we balance the final MFCCs by centering each frame around the value 0. Next, the trailing frames that are labeled as ‘silent’ are trimmed from the end of the input and target sequences. Finally, to reflect the speed of the original waveform signal, the input sequences are stretched by a factor of 5, interpolating linearly between frames. The target sequences are also stretched by this factor, but proximally interpolated to retain its one-hot encoding. An example of the final MFCCs is given in ??.

TARGET OUTPUT The target output of the model is a frame-wise representation of the phones that are uttered in a sentence. The TIMIT corpus contains text files indicating in what order phones occur in a sentence, and their starting and ending sample points.

These phones are discretized into frames such that they align correctly with the MFCCs. They are represented in one-hot vector encoding. Since the dataset contains 61 different phones, this is also the length of these vectors.

Figure ?? illustrates the waveform data and its framewise aligned MFCCs and target output.

do we take
absolute?

source?

better word-
ing: re-
approximate?

side-by-side
with original
text and
phonemes,
label as
fig:source_mfcc_target

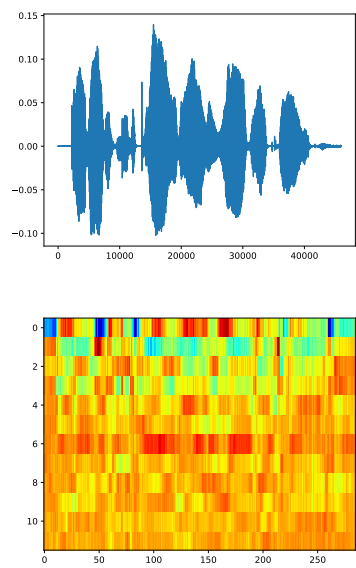


Figure 3.9: An alignment of a sample signal with its MFCCs and target phones.

3.2 ENHANCING E-PROP



3.2.1 *Multilayer architecture*

- English, visual and formal descriptions.

3.2.2 *Other neuron types*

- Shoutout to Traub - Argue in favor of different models (refer to brain, simplicity vs. plausibility tradeoff). E.g.: built-in refractory

3.2.2.1 *STDP-LIF*

- STDP-LIF (intuition, maths, graphs) - Mention Bellec's reset too.

3.2.2.2 *Izhikevich neuron*

- STDP-LIF (intuition, maths, graphs)

3.3 REGULARIZATION

- What, why, how?

3.3.1 *Firing rate regularization*

- What, why, how? - Bioplausible?

3.3.2 *Ridge regression*

- What, why, how? - Bioplausible?

3.3.3 *Weight decay*

- What, why, how? - Bioplausible?

3.3.4 *Synaptic scaling*

- What, why, how? - Bioplausible?

3.3.5 *Metaplasticity*

- What, why, how? - Bioplausible?

3.4 SYNAPTIC DELAY

- I/A - What, why, how?

3.5 BIDIRECTIONAL NETWORK

- I/A - What, why, how?

3.6 OPTIMIZER

- Show how Adam works, and how it replaces SGD

3.7 LEARNING RATE ANNEALING

- I/A - How, what, why?

3.8 HYPERPARAMETER OPTIMIZATION

- What, why, how?

4

RESULTS

- RESULTS PLACEHOLDERS

- RESULTS PLACEHOLDERS

- RESULTS PLACEHOLDERS

- RESULTS PLACEHOLDERS

5

DISCUSSION

- DISCUSSION PLACEHOLDERS

- DISCUSSION PLACEHOLDERS

- DISCUSSION PLACEHOLDERS

- DISCUSSION PLACEHOLDERS

6

CONCLUSION

- CONCLUSION PLACEHOLDERS

- CONCLUSION PLACEHOLDERS

MELS	HZ	FILTERBANK
0	0	0
105	68.5	2
210	143.7	4
315	226.2	7
420	316.8	10
525	416.3	13
630	525.5	16
735	645.4	20
840	777	24
945	921.5	29
1050	1080.1	34
1155	1254.4	40
1260	1445.4	46
1365	1655.3	53
1470	1885.7	60
1575	2138.6	68
1680	2416.3	77
1785	2721.2	87
1890	3055.9	97
1995	3423.3	109
2100	3826.7	122
2205	4269.5	136
2310	4755.7	152
2415	5289.4	169
2520	5875.3	188
2625	6518.6	209
2730	7224.8	231
2835	8000	256

Table A.1: Conversion table between linearly spaced Mels and their corresponding frequencies and filterbank boundaries.

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