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Including STDP to eligibility propagation in multi-layer recurrent spiking neural networks.

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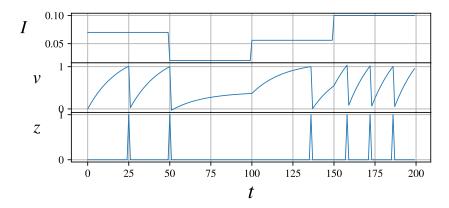
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Introduction - context

- Biological inspiration to Al
- ANNs and backpropagation
- Deep learning
 - → high energy consumption
- Fundamental differences between DL and biological learning
 - Continuous vs. binary communication
 - Backpropagation/BPTT vs. learning signals

Introduction - SNNs

- Binary spikes \rightarrow no backpropagation
- Good performance, but not competitive with DL
- Leaky integrate-and-fire (LIF) neuron



Introduction - neuromorphic computing

- Physical embedding of neural network in an analog medium
- Colocalized memory and computation
 - ightarrow no backpropagation
- SNNs as ideal biologically inspired NC paradigm for efficient computation
- Analog SNNs are extremely fast and energy efficient
- Problem: learning rule must be local and online
 - \rightarrow revalue biological learning

Introduction – biological learning

- Hebbian learning
 - ightarrow runaway excitation
- STDP
 - \rightarrow how to teach?
- Learning signals: three-factor Hebbian learning, R-STDP
- Biological learning signals: neurotransmitters
 - \rightarrow credit assignment
- Eligibility traces

Introduction – eligibility propagation

- Mathematical approximation to BPTT
- Local learning signals and eligibility traces
- Applicable to any SNN topology and multiple neuron models
- Also applicable to neuromorphic VLSIs
- Competitive with LSTMs on phone classification
- E-prop: only ALIF so far
- New research: STDP-LIF and Izhikevich neuron display STDP

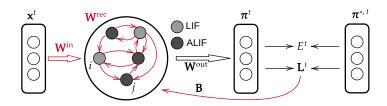
Introduction – this research

Research question:

Does including STDP-like behavior in e-prop lead to faster and more accurate phone classification?

- Reproduce results of original e-prop paper
 - ightarrow explicitly instead of automatic differentiation
- Extend STDP-LIF to STDP-ALIF
- Experimentally verify the STDP-ALIF and Izhikevich neuron on phone classification task
- Examine effects of e-prop in multi-layer recurrent SNN

Methods – technical framework (e-prop)

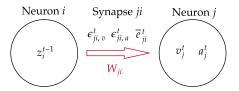


E-prop model $\mathcal{M} = \langle M, f \rangle$

$$\mathbf{h}_{j}^{t} = M\left(\mathbf{h}_{j}^{t-1}, \mathbf{z}^{t-1}, \mathbf{x}^{t}, \mathbf{W}_{j}\right)$$
(1)

$$z_j^t = f\left(\mathbf{h}_j^t\right) \tag{2}$$

Methods – the ALIF neuron model

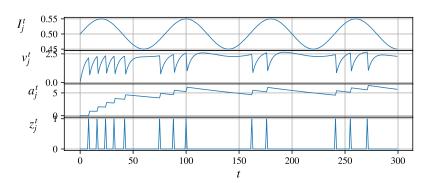


Methods - the ALIF neuron model

$$z_j^t = H\left(v_j^t - v_{\mathsf{th}} - \beta a_j^t\right) \tag{3}$$

$$v_j^{t+1} = \alpha v_j^t + \sum_{i \neq j} W_{ji}^{\text{rec}} z_i^t + \sum_i W_{ji}^{\text{in}} x_i^{t+1} - z_j^t v_{\text{th}}$$
(4)

$$a_j^{t+1} = \rho a_j^t + z_j^t \tag{5}$$



Methods – e-prop

$$\frac{dE}{dW_{ji}} = \sum_{t} L_{j}^{t} e_{ji}^{t}$$

$$\frac{dE}{dW_{ji}} = \sum_{t} \frac{dE}{dz_{j}^{t}} \underbrace{\frac{\partial z_{j}^{t}}{\partial \mathbf{h}_{j}^{t}} \underbrace{\sum_{t \geq t'} \frac{\partial \mathbf{h}_{j}^{t}}{\partial \mathbf{h}_{j}^{t-1}} \cdots \frac{\partial \mathbf{h}_{j}^{t+1}}{\partial \mathbf{h}_{j}^{t'}} \cdot \frac{\partial \mathbf{h}_{j}^{t'}}{\partial W_{ji}}}_{e_{ij}^{t}}$$

Methods - e-prop for ALIF

Hidden state:

$$\mathbf{h}_{j}^{t} = \begin{pmatrix} v_{j}^{t} \\ a_{j}^{t} \end{pmatrix} \tag{6}$$

Pseudo-derivative:

$$\psi_j^t = \gamma \max\left(0, 1 - \left| \frac{v_j^t - v_{\mathsf{th}} - \beta a_j^t}{v_{\mathsf{th}}} \right| \right) \tag{7}$$

Eligibility vector:

$$\begin{pmatrix} \epsilon_{ji,v}^{t+1} \\ \epsilon_{ji,a}^{t+1} \end{pmatrix} = \begin{pmatrix} \alpha \cdot \epsilon_{ji,v}^t + z_i^{t-1} \\ \psi_j^t \epsilon_{ji,v}^t + \left(\rho - \psi_j^t \beta\right) \epsilon_{ji,a}^t \end{pmatrix} \tag{8}$$

Eligibility trace:

$$e_{ji}^{t} = \psi_{j}^{t} \left(\epsilon_{ji,v}^{t} - \beta \epsilon_{ji,a}^{t} \right) \tag{9}$$

Methods - e-prop weight update

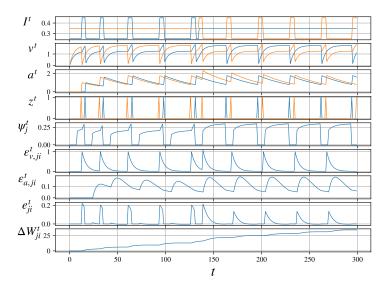
$$\Delta W_{ji} = -\eta \sum_{t} \underbrace{\sum_{k} B_{jk} \left(\pi_k^t - \pi_k^{*,t} \right)}_{=L_j^t} \underbrace{\sum_{t' \le t} \kappa^{t'-t} e_{ji}^{t'}}_{\stackrel{\text{def}}{=} \bar{e}_{i,j}^t}$$
(10)

where $y_k^t = \kappa y_k^{t-1} + \sum_j W_{kj}^{\text{out}} z_j^t + b_k$ and $\pi_k^t = \sigma_k (y_1^t, \dots, y_K^t)$.

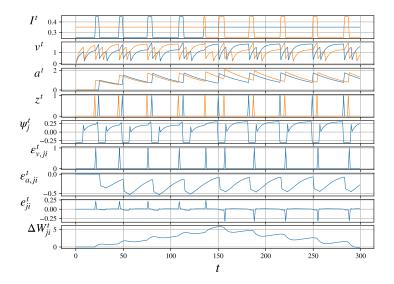
$$\Delta W_{kj}^{\text{out}} = -\eta \sum_{t} \left(\pi_k^t - \pi_k^{*,t} \right) \sum_{t' \le t} \kappa^{t'-t} z_j^t \tag{11}$$

$$\Delta b_k = -\eta \sum_{t} \left(\pi_k^t - \pi_k^{*,t} \right). \tag{12}$$

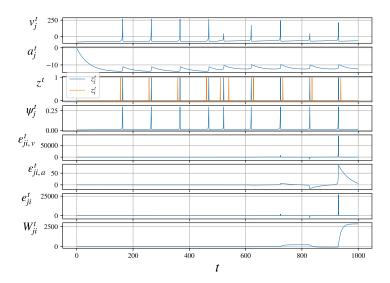
Methods – Visualizing the ALIF neuron



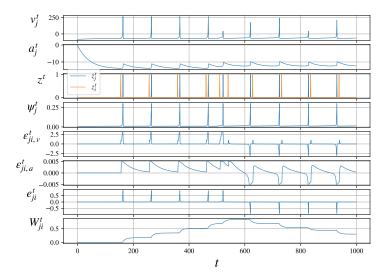
Methods - STDP-ALIF



Methods - Izhikevich



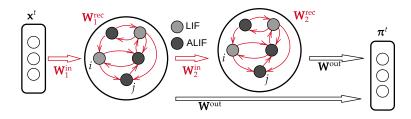
Methods - Izhikevich fixed



Methods - Multilayer

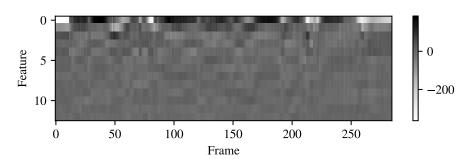
$$\mathbf{h}_{rj}^{t} = \begin{cases} M\left(\mathbf{h}_{rj}^{t-1}, \mathbf{z}_{r}^{t-1}, \mathbf{x}^{t}, \mathbf{W}_{rj}\right) & \text{if } r = 1\\ M\left(\mathbf{h}_{rj}^{t-1}, \mathbf{z}_{r}^{t-1}, \mathbf{z}_{r-1}^{t}, \mathbf{W}_{rj}\right) & \text{otherwise,} \end{cases}$$
(13)

where $r \in [1 \dots R]$.



Methods - Task

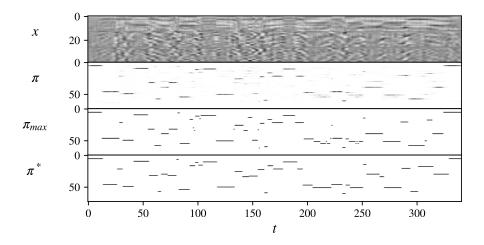
- Classify frame-wise phones from speech signals.
- There are 61 different class labels, and approximately 4000 speech sentences containing around 200-300 frames each.
- Speech is preprocessed into MFCCs (2–13) with their 1st and 2nd deltas, standardized per channel



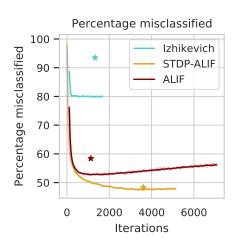
Methods – Other settings

- Adam optimizer for e-prop
- Firing rate regularization
- L2 regularization

Results – Example

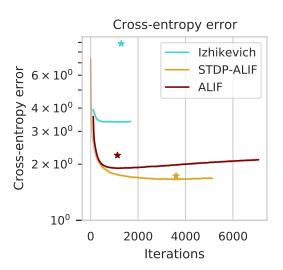


Results – Accuracy per neuron type



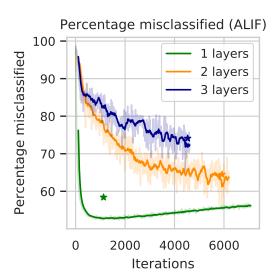
Test scores: ALIF = 58.4%, STDP-ALIF = 48.3%, Izhikevich = 93.5%

Results – Cross-entropy per neuron type

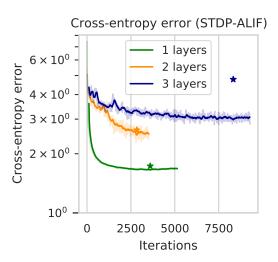


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Results – Multi-layer effects



Results – Multi-layer effects



Discussion

- Izhikevich neuron unsuitable
- More layers = less efficient
- Including STDP-like behavior:
 - improves the classification performance of the e-prop model
 - more closely resembles biological learning.

Discussion - future research

- Smoothen output
- Distributed parameters
- Custom connectivity graphs
- Dynamic pruning and growing
- Synaptic delay

Conclusion

- Including STDP in e-prop improves classification performance and efficiency. \rightarrow one of the best biologically plausible learning algorithms for SNNs.
- Good trade-off between running cost and performance
- Interesting neurocomputational model
- More bioplausibility \rightarrow easier in NC but not always better performance
- More research is needed on applying e-prop in other types of learning tasks, and many ideas exist that may improve performance

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