主定理无字证明

Master Theorem's proof without words

$$O(n) = O(2n) = O(kn)$$

$$O(n^2) = O(n^2+n),\ O(n) = O(n+\log n)$$

$$O(\log n) = O(\log_2 n) = O(\log_2 3 \times \log_3 n) = O(\log_3 n)$$

Manim-Kindergarten
有一种悲伤叫颓废

$T(n) = n^2 + n\log n^2 + 1024$

$O(T(n)) = O(n^2 + 2n\log n) = O(n^2)$

$T(n) = 2^n + n!$

$O(T(n)) = O(n!)$

$$n! \rightarrow \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$

$$T(n) = 2^n + 2^{n-1} + 2^{n-2} + \cdots + 1$$

$$O(T(n)) = O(2^n)$$

$$2^n + 2^{n-1} + 2^{n-2} + \cdots + 1 + 1 - 1$$

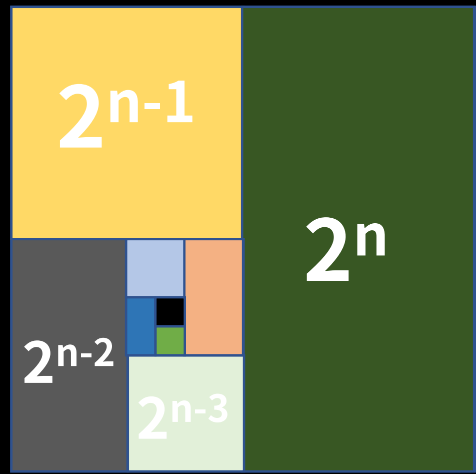
$$= (2^n + 2^{n-1} + 2^{n-2} + \cdots + 1 + 1) - 1$$

$$= 2^{n+1} - 1$$

几何级数的时间复杂度等于最大那一项

$$T(n) = 3^n + 3^{n-1} + 3^{n-2} + \cdots + 1$$

$$O(T(n)) = O(3^n)$$



$$T(n)=aT(n/b)+f(n)$$

$$T(n) = 2T(\frac{n}{2}) + n, T(1) = 1$$

$$= 2(2T(\frac{n}{2^2}) + \frac{n}{2}) + n$$

$$= 2^2T(\frac{n}{2^2}) + 2n$$

$$= \ldots$$

$$= 2^kT(\frac{n}{2^k}) + kn$$

设 $n = 2^k$

$$= nT(1) + kn = n + n\log n, O(T(n)) = O(n\log n)$$

T(n)=aT(n/b)+f(n)

$$
\begin{aligned}
T(n) &= 2T(\frac{n}{2}) + 1, T(1) = 1 \\
&= 2(2T(\frac{n}{2^2}) + 1) + 1 \\
&= 2^2T(\frac{n}{2^2}) + 2 + 1 \\
&= \cdots \\
&= 2^kT(\frac{n}{2^k}) + 2^{k-1} + \cdots + 2 + 1 \\
\text{令 } n = 2^k, &= n + 2^{k-1} + \cdots + 2 + 1 \\
O(T(n)) &= O(n + 2^{k-1} + \cdots + 2 + 1) \\
&= O(n + 2^{k-1}) = O(2n + 2^k) = O(3n) = O(n)
\end{aligned}
$$

T(n)=aT(n/b)+f(n)

$$T(n) = 2T(\frac{n}{2}) + n = 2^k T(\frac{n}{2^k}) + kn$$

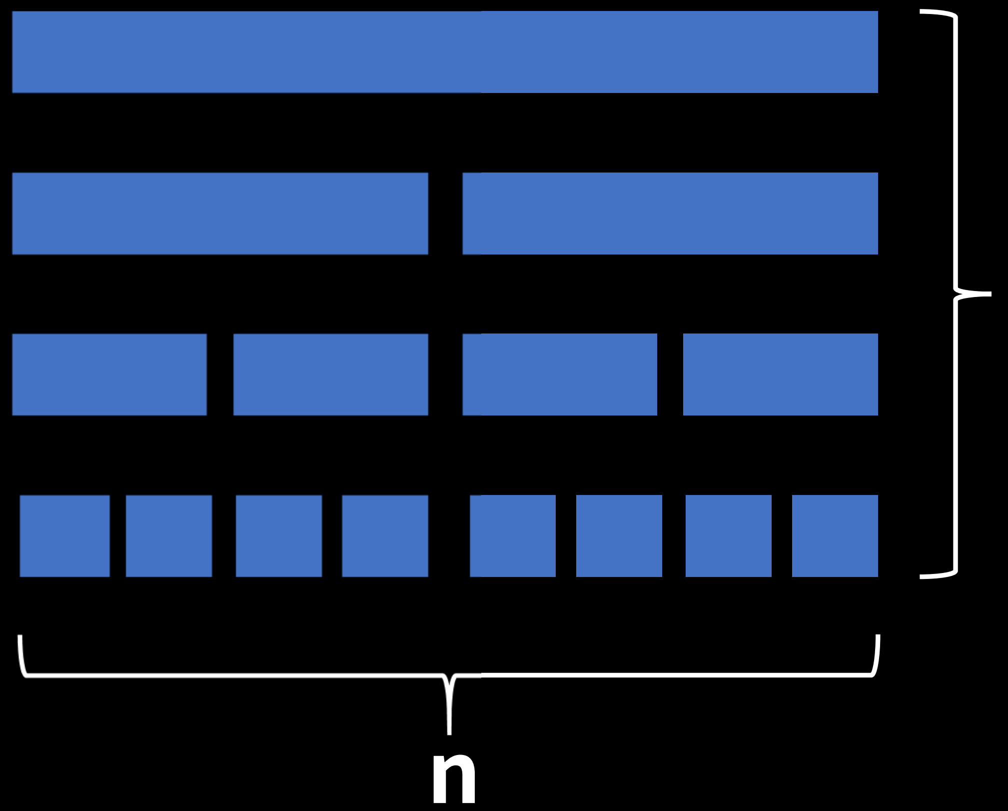
$$T(n) = 2^1 T(\frac{n}{2^1}) + 1n$$

$$T(n) = 2^2 T(\frac{n}{2^2}) + 2n$$

$$T(n) = 2^3 T(\frac{n}{2^3}) + 3n$$

$$T(n) = 2^4 T(\frac{n}{2^3}) + 4n$$

$$O(T(n)) = O(nlogn)$$



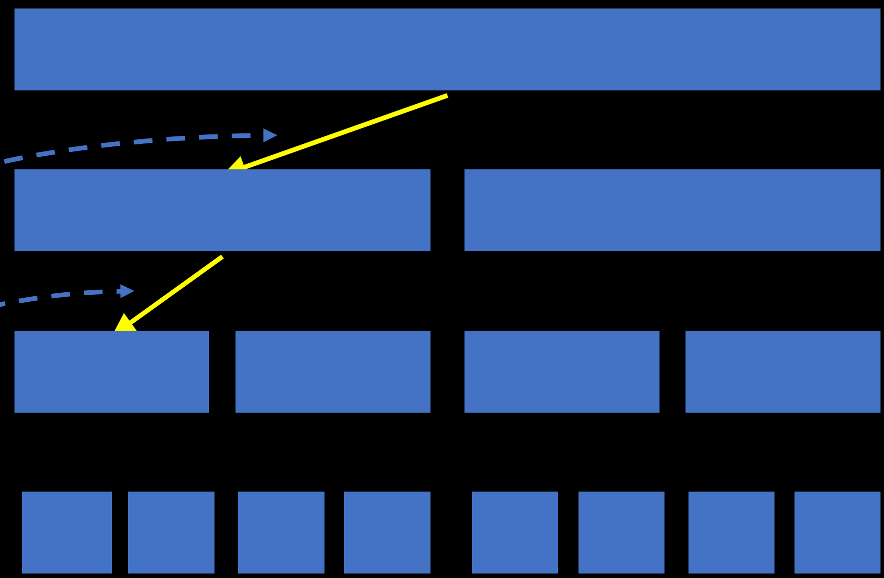
$$T(n) = 2T(\frac{n}{2}) + n = 2^k T(\frac{n}{2^k}) + kn$$

$$T(n) = 2T(\frac{n}{2}) + n$$

$$T(n) = 2(2T(\frac{n}{2^2}) + \frac{n}{2}) + n$$

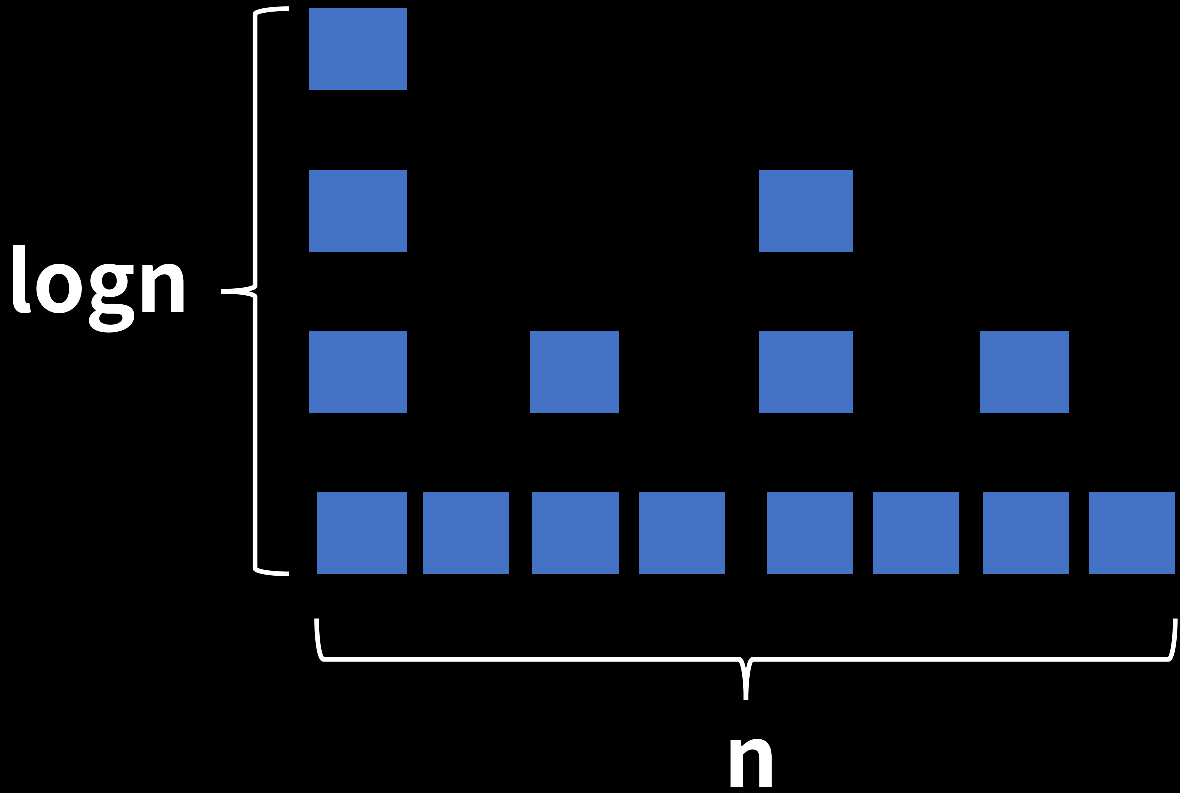
$$T(n) = 2(2(2T(\frac{n}{2^3}) + \frac{n}{2^2})) + \frac{n}{2}) + n$$

$$O(T(n)) = O(nlogn)$$

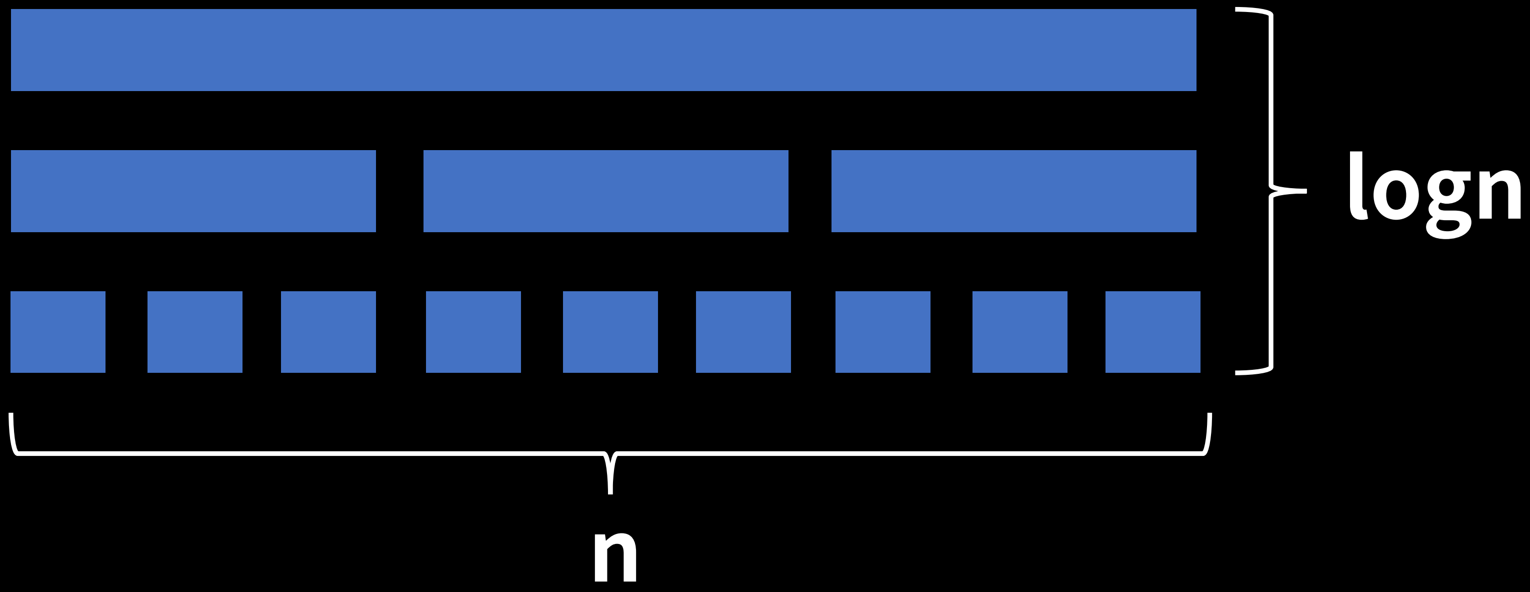


$$2^k = n, k = \log n$$

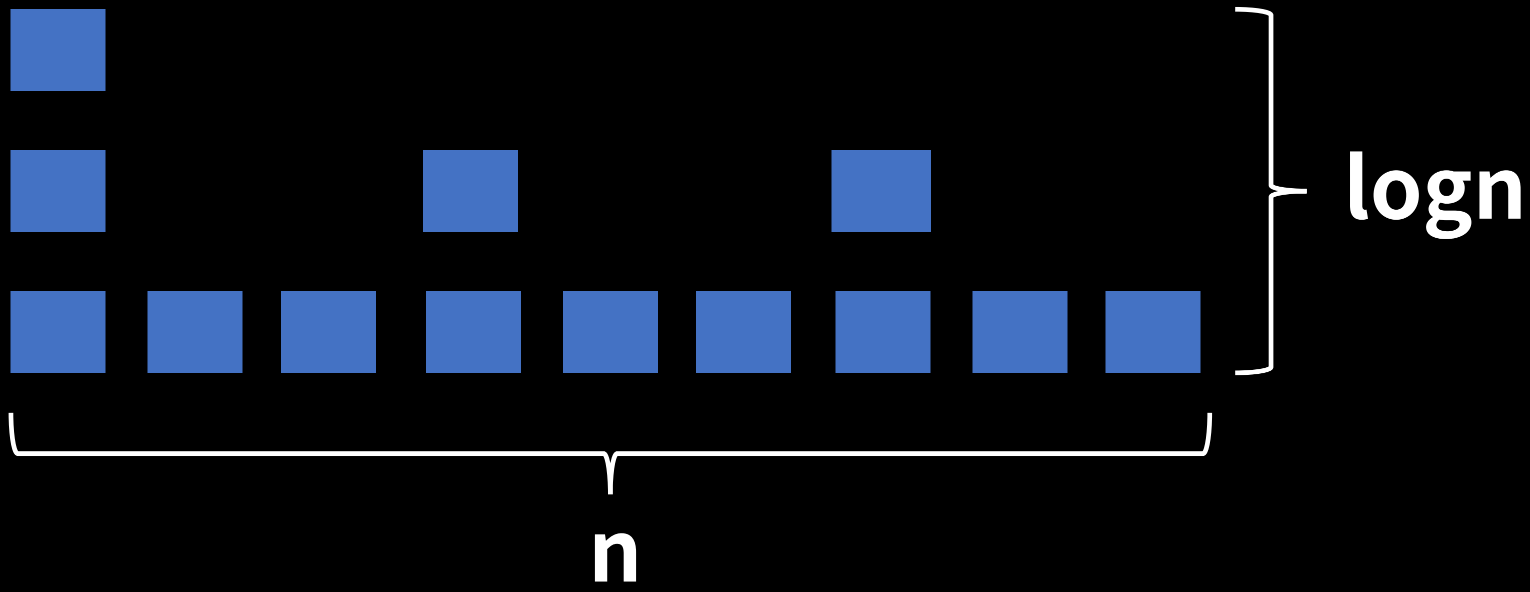
$$T(n)=2T(n/2)+1$$



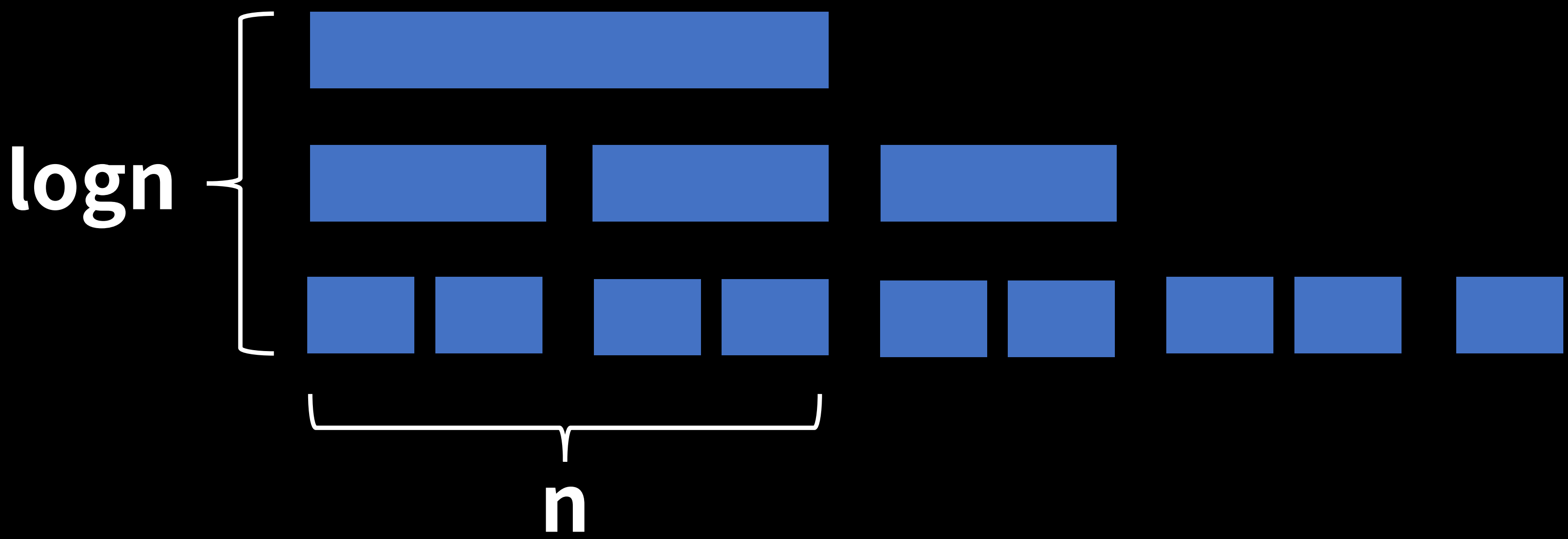
$$T(n)=3T(n/3)+n$$



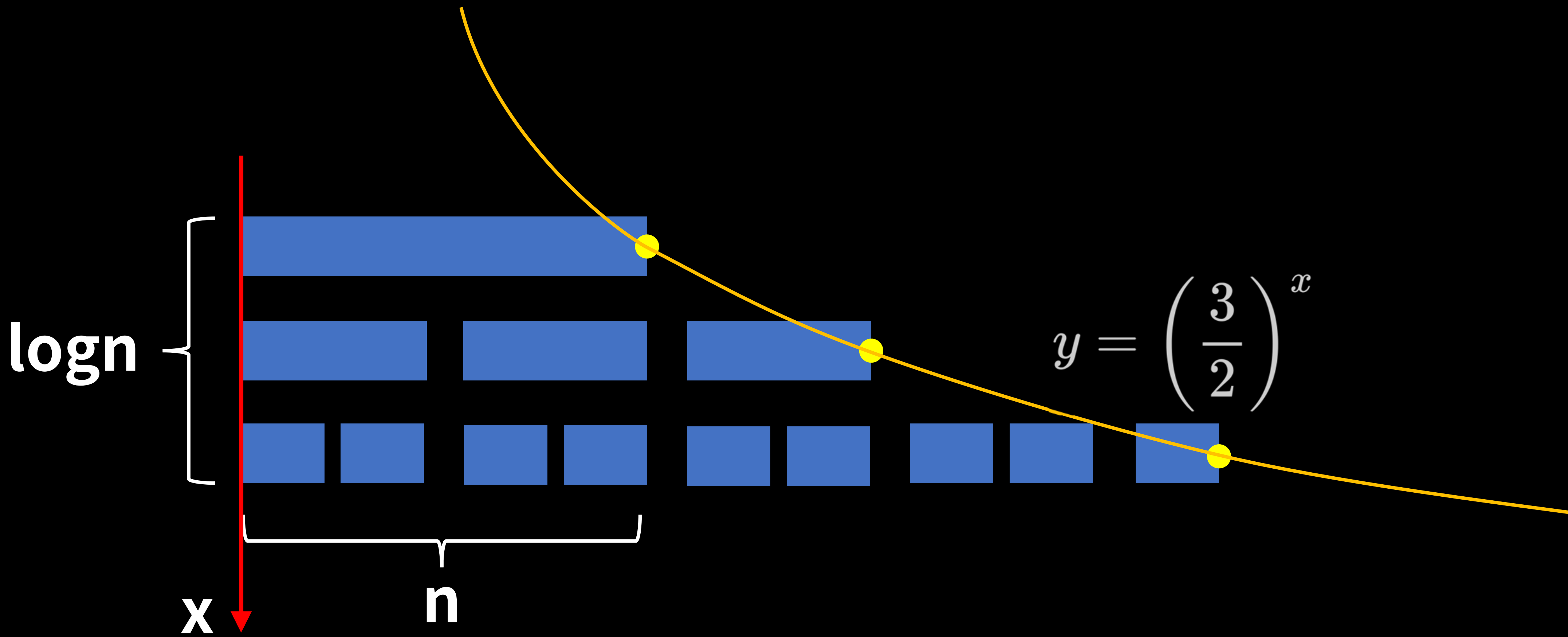
$$T(n)=3T(n/3)+1$$



$$T(n)=3T(n/2)+n$$



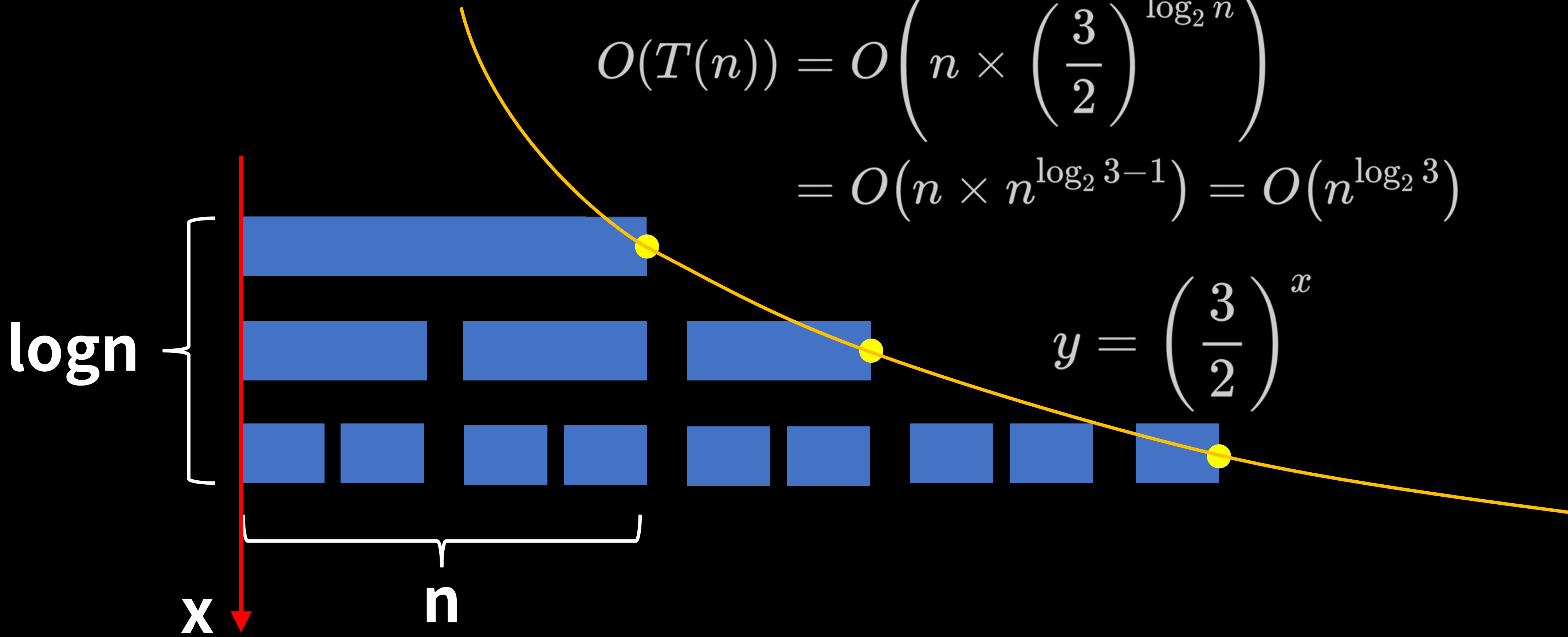
$T(n)=3T(n/2)+n$



T(n)=3T(n/2)+n

$$O(T(n)) = O\left(n \times \left(\frac{3}{2}\right)^{\log_2 n}\right)$$

$$= O\left(n \times n^{\log_2 3 - 1}\right) = O\left(n^{\log_2 3}\right)$$

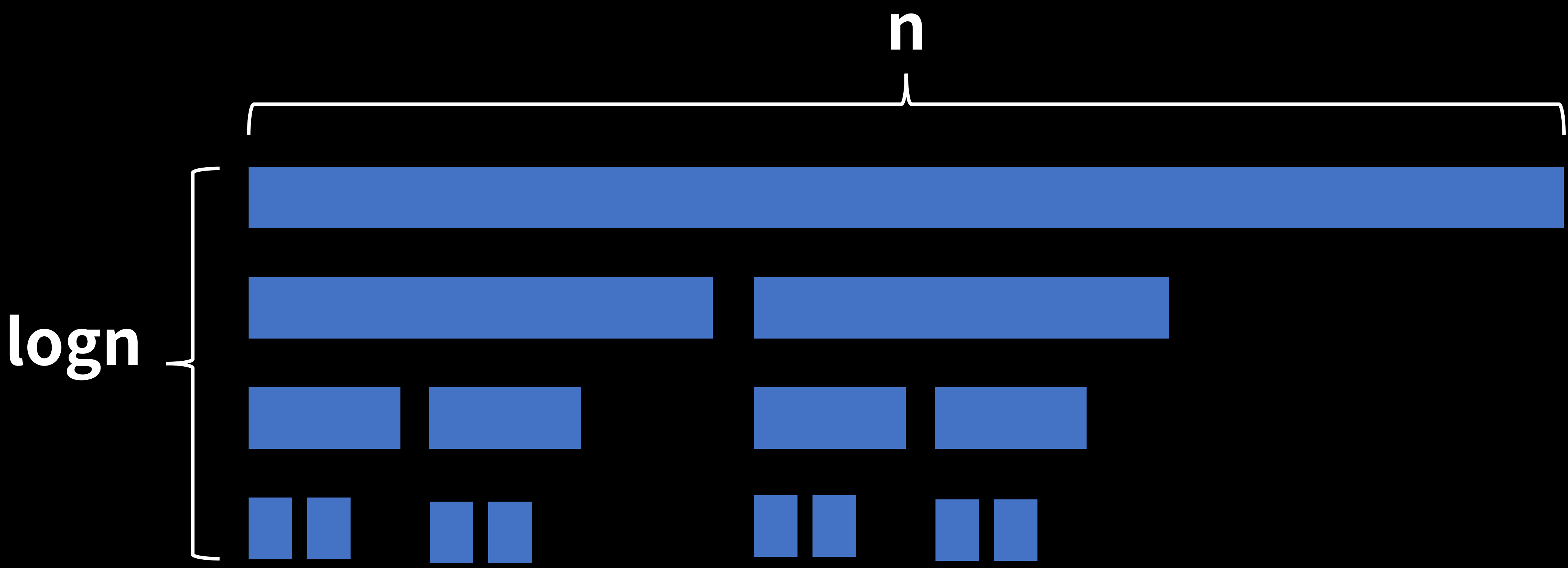


$$y = \left(\frac{3}{2}\right)^x$$

$$
\begin{aligned}
T(n) &= 3T(\frac{n}{2}) + n \\
&= 3(3T(\frac{n}{2^2}) + \frac{n}{2}) + n \\
&= 3^2T(\frac{n}{2^2}) + \frac{3}{2}n + n \\
&= 3^kT(\frac{n}{2^k}) + \left(\frac{3^k}{2^k} + \cdots + \frac{3}{2}\right)n + n
\end{aligned}
$$

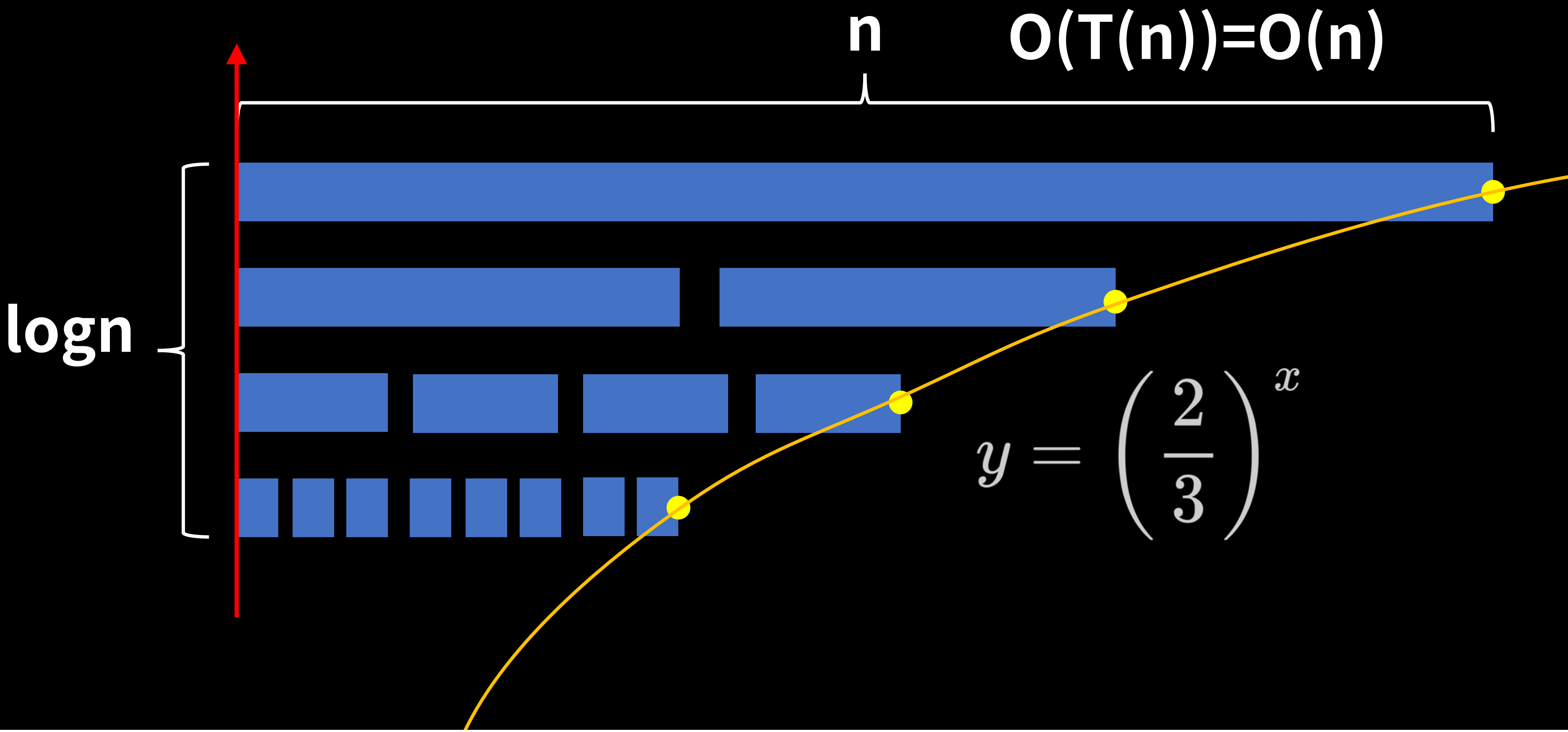
$$
\begin{aligned}
O(T(n)) &= O(3^kT(\frac{n}{2^k}) + \frac{3^k}{2^k}n + n) \\
&= O(n^{\log_2 3} + n^{\log_2 3} + n) = O(n^{\log_2 3})
\end{aligned}
$$

$$T(n)=2T(n/3)+n$$



T(n)=2T(n/3)+n

O(T(n))=O(n)



$$
\begin{aligned}
T(n) &= 2T(\frac{n}{3}) + n \\
&= 2(2T(\frac{n}{3^2}) + \frac{n}{3}) + n \\
&= 2^2T(\frac{n}{3^2}) + \frac{2}{3}n + n \\
&= 2^kT(\frac{n}{3^k}) + \left(\frac{2^k}{3^k} + \cdots + \frac{2}{3}\right)n + n \\
O(T(n)) &= O(2^kT(\frac{n}{3^k}) + \frac{2^k}{3^k}n + n) \\
&= O(n^{\log_3 2} + n^{\log_3 2} + n) = O(n)
\end{aligned}
$$

$T(n)=aT(n/b)+f(n)$, $f(n)=O(n^d)$

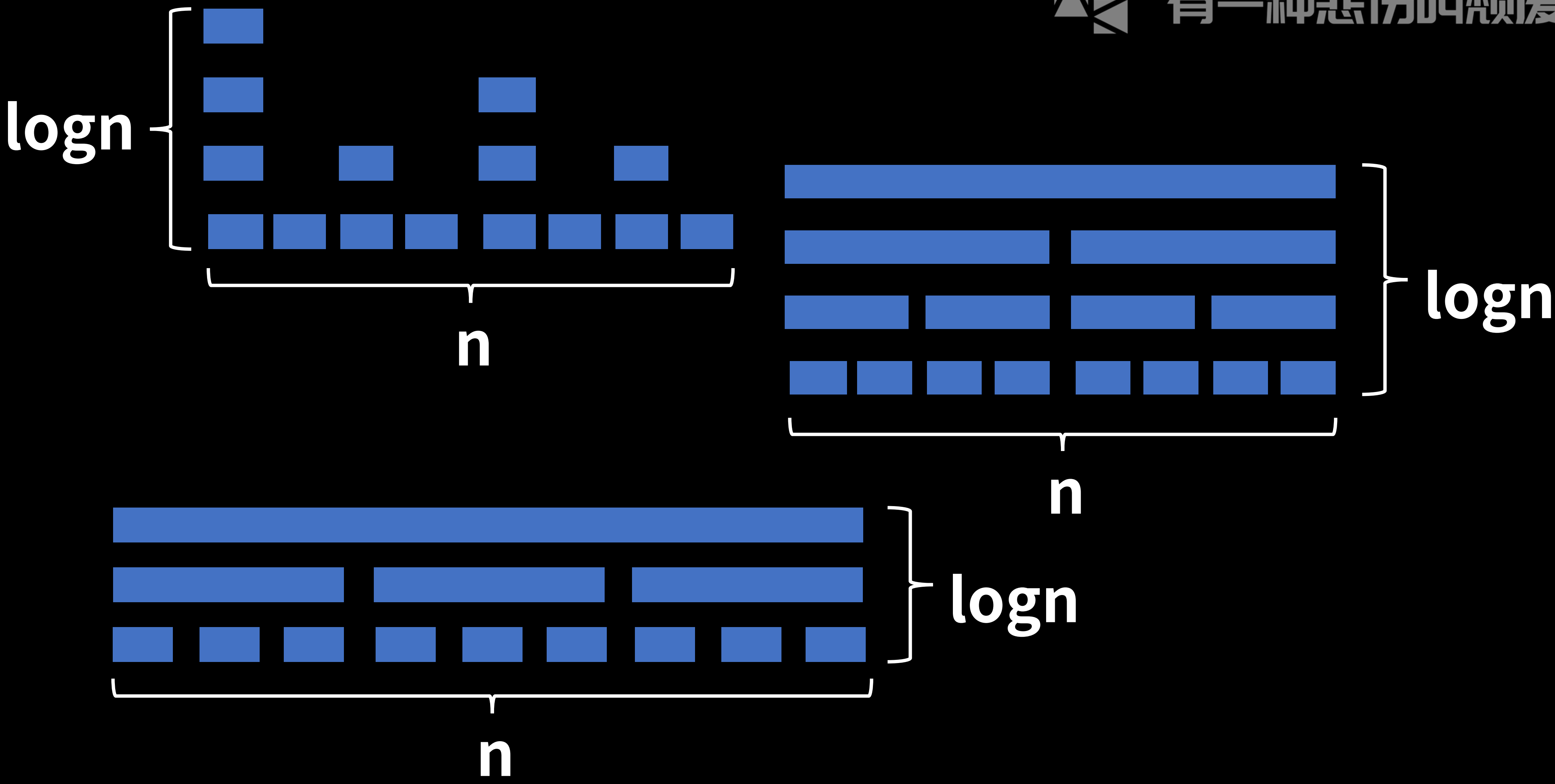
$$T(n) = \begin{cases} O(n^d), & d > \log_b a \\ O(n^d \log n), & d = \log_b a \\ O(n^{\log_b a}), & d < \log_b a \end{cases}$$

$T(n)=aT(n/b)+f(n),\ f(n)=O(n^d)$

几何级数第一项大

$$T(n)=\begin{cases} O(n^d), & d>\log_b a \\ O(n^d\log n), & d=\log_b a \\ O(n^{\log_b a}), & d<\log_b a \end{cases}$$

几何级数最后一项大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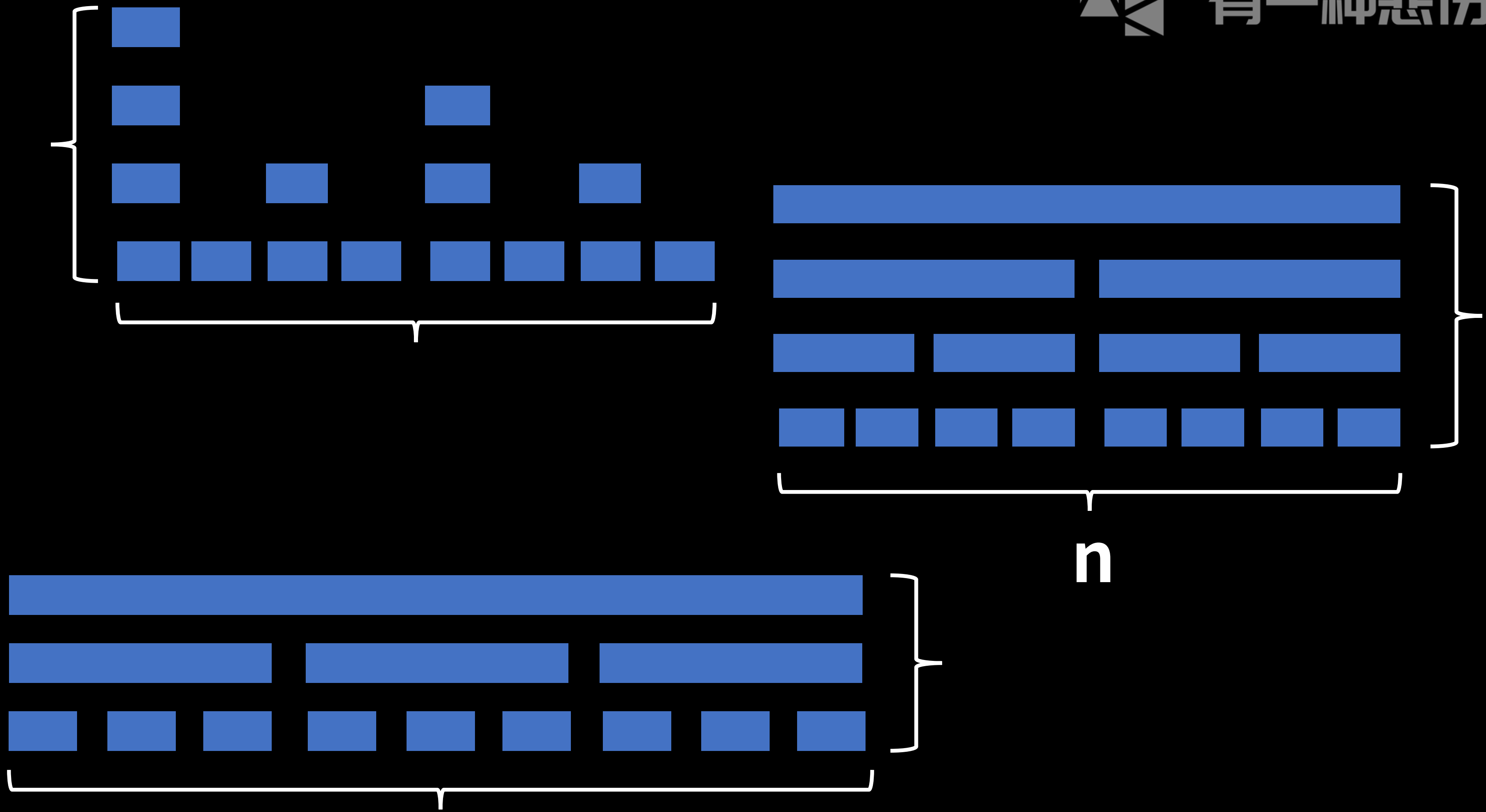
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