练习一

1.Solution:

边界条件：$u|_{x=0}=u|_{x=L}=0$

初始条件：$u|_{t=0}=f(x),\left.\dfrac{\partial u}{\partial x}\right|_{t=0}=g(x)$

其他：$|u(x,t)|<\infty$

2.Solution:

(1) $u|_{x=0}=g(t)$

(2)$\left.\dfrac{\partial u}{\partial x}\right|_{x=L}=0$

3.Solution:

$xu_x-yu_y=xf'(xy)y-yf'(xy)x=0$，$u=f(xy)$是原方程的解

练习二

1.Solution

$u(x,t)=e^{-8t}\sin 2x$，$\dfrac{\partial u}{\partial t}=-8e^{-8t}\sin 2x$，$\dfrac{\partial^2 u}{\partial x^2}=e^{-8t}(-4\sin 2x)\Rightarrow\dfrac{\partial u}{\partial t}=2\dfrac{\partial^2 u}{\partial x^2}$

容易验证：$u(0,t)=u(\pi,t)=0,u(x,0)=\sin 2x$

2.Solution:

(1)
$4y_{tt}-25y_{xx}=4[25F''(2x+5t)+25G''(2x-5t)]-25[4F''(2x+5t)+4G''(2x-5t)]=0$

(2)由题目条件可得:

$$\begin{cases}F(5t)+G(-5t)=0\\F(2\pi+5t)+G(2\pi-5t)=0\\F(2x)+G(2x)=\sin 2x\\5F'(2x)-5G'(2x)=0\end{cases}\Rightarrow\begin{cases}F(x)+G(-x)=0\\F(x)+G(x)=\sin x\\F'(x)-G'(x)=0\end{cases}\Rightarrow\begin{cases}F(x)=\dfrac{1}{2}\sin x+C\\G(x)=\dfrac{1}{2}\sin x-C\end{cases}$$

故: $y(x,t)=\dfrac{1}{2}\sin(2x+5t)+\dfrac{1}{2}\sin(2x-5t)$

3.Solution:

(1)先对 x 积分: $z_y=\dfrac{1}{3}x^3y+\varphi(y)$, 再对 y 积分: $z=\dfrac{1}{6}x^3y^2+\displaystyle\int\varphi(y)\mathrm{d}y+\rho(x)$=
$z=\dfrac{1}{6}x^3y^2+\phi(y)+\rho(x)$, $\phi(x):=\displaystyle\int\varphi(y)\mathrm{d}y$

(2)由题目条件可得:

$$\begin{cases}\phi(0)+\rho(x)=x^2\\\dfrac{1}{6}y+\phi(y)+\rho(1)=\cos y\end{cases}\Rightarrow\begin{cases}\rho(x)=x^2-\phi(0)\Rightarrow\rho(1)+\phi(0)=1\\\phi(y)=\cos y-\dfrac{1}{6}y-\rho(1)\end{cases}$$

故: $z(x,y)=\dfrac{1}{6}x^3y^2+\cos y-\dfrac{1}{6}y-\rho(1)+x^2-\phi(0)=\dfrac{1}{6}x^3y^2+\cos y-\dfrac{1}{6}y+x^2-1$