

Let X_1, X_2, \dots, X_n are n independent variables, $p_{x_1}(x_1), p_{x_2}(x_2), \dots, p_{x_n}(x_n)$ are their *p. d. fs* respectively. Find the *p. d. fs* of

(1) $Y = \min(X_1, X_2)$

(2) $Y = \max(X_1, X_2)$

(3) $Y = h(X)$

(4) $Y_1 = h_1(X_1, X_2), Y_2 = h_2(X_1, X_2)$

Solution:

(1)

$$\begin{aligned} F(y) &= P(\min(X_1, X_2) \leq y) \\ &= 1 - P(X_1, X_2 \geq y) \\ &= 1 - P(X_1 \geq y)P(X_2 \geq y) \\ &= 1 - (1 - F_{x_1}(y))(1 - F_{x_2}(y)) \\ &= -F_{x_1}(y)F_{x_2}(y) + F_{x_1}(y) + F_{x_2}(y) \\ \Rightarrow f(y) = F'(y) &= -p_{x_1}(y)F_{x_2}(y) - p_{x_2}(y)F_{x_1}(y) + p_{x_1}(y) + p_{x_2}(y) \end{aligned}$$

(2)

$$\begin{aligned} F(y) &= P(\max(X_1, X_2) \leq y) \\ &= P(X_1 \leq y)P(X_2 \leq y) \\ &= F_{x_1}(y)F_{x_2}(y) \\ \Rightarrow f(y) = F'(y) &= p_{x_1}(y)F_{x_2}(y) + p_{x_2}(y)F_{x_1}(y) \end{aligned}$$

(3)

$$\begin{aligned} F(y) &= P(h(X) \leq y) \\ &= P(X_1 \leq h^{-1}(y)) \\ &= F_{x_1}(h^{-1}(y)) \\ \Rightarrow f(y) = F'(y) &= p_{x_1}(h^{-1}(y))(h^{-1}(y))' \end{aligned}$$

(4)真没见过，不会了