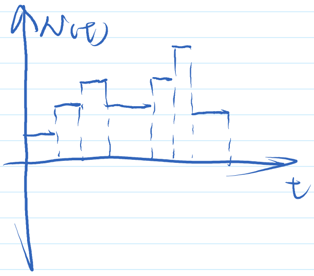
第二次作业

2024年3月9日 20:07

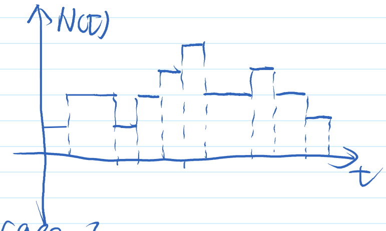
2.2 Consider a checkout line at a local grocery store. Let $N(t)$ denote the number of customers (including the one being served) waiting in line at time t. Sketch three typical sample functions for this random process. What is the primary difference between these sample functions and that of Figure 2–5?

Solution:

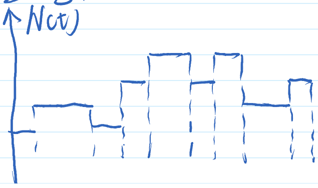
Case 1:



Case 2:



Case 3:



The main difference between them is that $N(t)$ may drop but $X(t)$ not.

2.5 Determine the classification of the random processes in each of the situations that follow. In each case, indicate whether the process is a continuous- or discrete-time process and whether it is a continuous- or discrete-amplitude process.

(a) A manufacturing process begins at time 0, and we are interested in the number of defects that have occurred up to time t for all positive values of t.

(b) A computation is carried out in a sequence of steps in a special-purpose digital computer, and the content of a particular shift register is converted to decimal form and recorded at the end of each clock cycle.

(c) A continuous-time signal is sampled and quantized every T_0 seconds, and we are interested in the quantization error at the sampling times.

(d) A continuous-time signal is sampled and quantized every T_0 seconds, and we are interested in the quantized value of the signal at each sampling time.

(e) Because of interference and thermal noise in an analog FM receiver, the demodulated audio signal differs from the transmitted audio signal. We are interested in the error signal (the difference between the transmitted and demodulated audio signals).

Sol:
(a) continuous-time, discrete-amplitude process
(b) discrete-time, discrete-amplitude process
(c) discrete-time, continuous-amplitude process
(d) discrete-time, discrete-amplitude process
(e) continuous-time, continuous-amplitude process

2.7 Suppose X is a random variable that is uniformly distributed on $[0, 1]$. The random process $Y(t), t > 0$, is defined by $Y(t) = \exp\{-Xt\}$. Find the one-dimensional distribution function $F_{Y,1}(u;t)$ for the random process $Y(t)$. Find the one-dimensional density function $f_{Y,1}(u;t)$.

Solution: $X \sim U(0,1) \Rightarrow F_X(x) = \begin{cases} 0 & x<0 \\ x & x\in[0,1] \\ 1 & 1 \end{cases}, \quad f_X(x) = \begin{cases} 0 & x\in(-\infty,0) \\ 1 & x\in[0,1] \\ 0 & x\in(1,+\infty) \end{cases}$

$$F_{Y,1}(u;t) = P(Y(t)\le u) = P(-Xt \le \ln u)$$
$$= P\left(X \ge -\frac{\ln u}{t}\right) = 1 - P\left(X < -\frac{\ln u}{t}\right) = \begin{cases} 1, & -\frac{\ln u}{t} < 0 \\ 1+\frac{\ln u}{t}, & -\frac{\ln u}{t} \in [0,1] \\ 0, & -\frac{\ln u}{t} > 1 \end{cases}$$

$\therefore f_{Y,1}(u;t) = \begin{cases} \frac{1}{ut}, & -\frac{\ln u}{t} \in [0,1] \\ 0, & \text{else} \end{cases}$

2.9 Suppose that $X(t)$ is a zero-mean, wide-sense stationary, continuous-time Gaussian random process with autocorrelation function $R_X(\tau)$. Let the random process $Y(t)$ be given by $Y(t) = c_1X(t) + c_2X(t - T)$. Find the probability that $Y(t_0)$ is greater than some threshold γ. Express your answer in terms of the standard Gaussian distribution function Φ, the autocorrelation function R_X, and the parameters c_1, c_2, γ, and T.

Solution: $E[Y(t_0)] = c_1E[X(t_0)] + c_2E[X(t_0-T)] = 0$

$$Var[Y(t_0)] = Var[c_1X(t_0) + c_2X(t_0-T)] = E[(c_1X(t_0) + c_2X(t_0-T))^2]$$

$$= c_1^2E[X^2(t_0)] + 2c_1c_2E[X(t_0)X(t_0-T)] + c_2^2E[X^2(t_0-T)]$$

$$= c_1^2R_X(0) + c_2^2R_X(0) + 2c_1c_2R_X(T)$$

$$P(Y(t_0) \geq \gamma) = 1 - P(Y(t_0) \leq \gamma) = 1 - \Phi\left(\frac{\gamma}{\sqrt{Var[Y(t_0)]}}\right)$$

$$= 1 - \Phi\left(\frac{\gamma}{\sqrt{(c_1^2+c_2^2)R_X(0) + 2c_1c_2R_X(T)}}\right)$$

2.14 A random process $X(t)$ has a constant mean and an autocorrelation function

$$R_X(t,s) = \cos(\omega_0t)\cos(\omega_0s) + \sin(\omega_0t)\sin(\omega_0s).$$

Is the random process wide-sense stationary? Prove your answer.

Prove:

$R_X(t,s) = \cos(\omega_0(t-s)) = R_X(\tau)$

$\because E[X(t)] =$ constant

$\therefore X(t)$ is wide-sense stationary