科学计算引论作业(一)

谢悦晋　U202210333

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1.5 若以$\frac{355}{113}$作为圆周率π的逼近值，问此逼近值具有多少位有效数字

解：$x=\frac{325}{113}=0.314159204\ldots\times10^{1}$

$|x-\pi|=2.6676\ldots\times10^{-7}<0.5\times10^{-6}=0.5\times10^{1-7}$

∴有7位有效数字

2.1 使用二分法求方程$x=2^{-x}$在$[0,1]$内的根，精确到10^{-8}

解：迭代次数$k>\frac{\ln(b-a)-\ln2\varepsilon}{\ln2}=25.57\Rightarrow k=26$,用计算机模拟计算：

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[1]:0.0000000000:f(a)=-1.0000000000, 1.0000000000:f(b)=0.5000000000, 0.5000000000:f(c)=-0.2071067812
[2]:0.5000000000:f(a)=-0.2071067812, 1.0000000000:f(b)=0.5000000000, 0.7500000000:f(c)=0.1553964425
[3]:0.5000000000:f(a)=-0.2071067812, 0.7500000000:f(b)=0.1553964425, 0.6250000000:f(c)=-0.0234197773
[4]:0.6250000000:f(a)=-0.0234197773, 0.7500000000:f(b)=0.1553964425, 0.6875000000:f(c)=0.0665710940
[5]:0.6250000000:f(a)=-0.0234197773, 0.6875000000:f(b)=0.0665710940, 0.6562500000:f(c)=0.0217245214
[6]:0.6250000000:f(a)=-0.0234197773, 0.6562500000:f(b)=0.0217245214, 0.6406250000:f(c)=-0.0008100080
[7]:0.6406250000:f(a)=-0.0008100080, 0.6562500000:f(b)=0.0217245214, 0.6484375000:f(c)=0.0104666108
[8]:0.6406250000:f(a)=-0.0008100080, 0.6484375000:f(b)=0.0104666108, 0.6445312500:f(c)=0.0048306462
[9]:0.6406250000:f(a)=-0.0008100080, 0.6445312500:f(b)=0.0048306462, 0.6425781250:f(c)=0.0020109061
[10]:0.6406250000:f(a)=-0.0008100080, 0.6425781250:f(b)=0.0020109061, 0.6416015625:f(c)=0.0006005959
[11]:0.6406250000:f(a)=-0.0008100080, 0.6416015625:f(b)=0.0006005959, 0.6411132812:f(c)=-0.0001046693
[12]:0.6411132812:f(a)=-0.0001046693, 0.6416015625:f(b)=0.0006005959, 0.6413574219:f(c)=0.0002479724
[13]:0.6411132812:f(a)=-0.0001046693, 0.6413574219:f(b)=0.0002479724, 0.6412353516:f(c)=0.0000716538
[14]:0.6411132812:f(a)=-0.0001046693, 0.6412353516:f(b)=0.0000716538, 0.6411743164:f(c)=-0.0000165072
[15]:0.6411743164:f(a)=-0.0000165072, 0.6412353516:f(b)=0.0000716538, 0.6412048340:f(c)=0.0000275735
[16]:0.6411743164:f(a)=-0.0000165072, 0.6412048340:f(b)=0.0000275735, 0.6411895752:f(c)=0.0000055332
[17]:0.6411743164:f(a)=-0.0000165072, 0.6411895752:f(b)=0.0000055332, 0.6411819458:f(c)=-0.0000054870
[18]:0.6411819458:f(a)=-0.0000054870, 0.6411895752:f(b)=0.0000055332, 0.6411857605:f(c)=0.0000000231
[19]:0.6411819458:f(a)=-0.0000054870, 0.6411857605:f(b)=0.0000000231, 0.6411838531:f(c)=-0.0000027319
[20]:0.6411838531:f(a)=-0.0000027319, 0.6411857605:f(b)=0.0000000231, 0.6411848068:f(c)=-0.0000013544
[21]:0.6411848068:f(a)=-0.0000013544, 0.6411857605:f(b)=0.0000000231, 0.6411852837:f(c)=-0.0000006657
[22]:0.6411852837:f(a)=-0.0000006657, 0.6411857605:f(b)=0.0000000231, 0.6411855221:f(c)=-0.0000003213
[23]:0.6411855221:f(a)=-0.0000003213, 0.6411857605:f(b)=0.0000000231, 0.6411856413:f(c)=-0.0000001491
[24]:0.6411856413:f(a)=-0.0000001491, 0.6411857605:f(b)=0.0000000231, 0.6411857009:f(c)=-0.0000000630
[25]:0.6411857009:f(a)=-0.0000000630, 0.6411857605:f(b)=0.0000000231, 0.6411857307:f(c)=-0.0000000199
[26]:0.6411857307:f(a)=-0.0000000199, 0.6411857605:f(b)=0.0000000231, 0.6411857456:f(c)=0.0000000016
[27]:0.6411857307:f(a)=-0.0000000199, 0.6411857456:f(b)=0.0000000016, 0.6411857381:f(c)=-0.0000000092
Approximate root: 0.6411857418715954
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Figure 1: 代码运行图

迭代27，更新26次，最终结果为:0.6411857418715954