森普生（Thomas Simpson，1710--1761）英国数学家

生于英格兰，并卒于同地。父亲是一位纺织工人，所以他主要靠自学成材，而他的第一份工作也是纺织。

他对数学的兴趣最初是由一次日蚀所引发起的，他在一位占卜师的指导之下，他学会了算术和基本的代数。其后他放弃了纺织的工作,凭藉他刻苦和持久的努力，他证明了他在数学方面的能力. 1735年他解决了数个有关微积分的问题，1737年他开始撰写有关数学的文章。1754年到伦敦的乌尔威治并出任数学教授一职，直至逝世。

森普生最为人熟悉的贡献是他在插值法及数值积分方面，在概率方面也有一定的工作. 森普生所发表的文章充份证明他刻苦力学以及超卓的才华。当中最有名的著作便是1750年出版的两册《流数的意义及其应用》，当中包括了Cotes的一些结果及数个包括物理和天文学的应用例子；在1740年推出的《机会的特性和法则》.

他共出版了8篇文章，涉及天文学、概率论、微积分、代数学等，其中得出了一条有关月球轨迹的微分方程。

Roger Cotes (1682--1716)， English mathematician

Cotes was born in Burbage. At first Roger attended Leicester School where his mathematical talent was recognised. His aunt Hannah had married Rev. John Smith, and Smith took on the role of tutor to encourage Roger's talent. Cotes later studied at St Paul's School in London and entered Trinity College, Cambridge in 1699. He graduated BA in 1702 and MA in 1706.

From 1709 to 1713, Cotes became heavily involved with the second edition of Newton's Principia. The first edition of Principia had only a few copies printed and was in need of revision to include Newton's works and principles of lunar and planetary theory.

Newton at first had a casual approach to the revision, since he had all but given up scientific work. However, through the vigorous passion displayed by Cotes, Newton's scientific hunger was once again reignited. The two spent nearly three and half years collaborating on the work, in which they fully deduce, from Newton's laws of motion, the theory of the moon, the equinoxes, and the orbits of comets. Only 750 copies of the second edition were printed. However, a pirate copy from Amsterdam met all other demand. As reward to Cotes, he was given a share of the profits and 12 copies of his own. Cotes's original contribution to the work was a preface which supported the scientific superiority of Newton's principles over the then popular vortex theory of gravity advocated by Descartes (笛卡尔). Cotes concluded that the Newton's law of gravitation was confirmed by observation of celestial phenomenon that were inconsistent with the vortex phenomena that Cartesian critics alleged

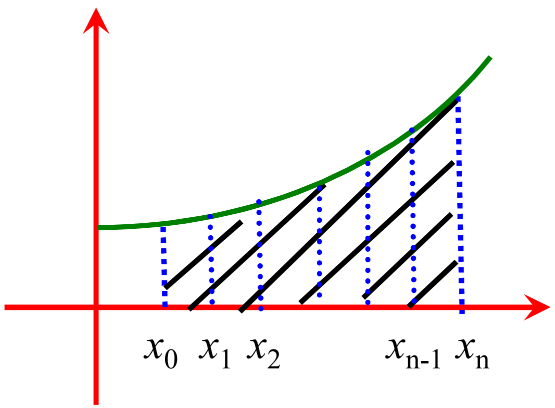
Cotes's major original work was in mathematics, especially in the fields of integral calculus, logarithms, and numerical analysis. He published only one scientific paper in his lifetime, in which he successfully constructs the logarithmic spiral. After his death, many of Cotes's mathematical papers were hastily edited by Robert Smith and published in a book. Although Cotes's style was somewhat obscure, his systematic approach to integration and mathematical theory was highly regarded by his peers. Cotes discovered an important theorem on the nth roots of unity, foresaw the method of least squares, and he discovered a method for integrating rational fractions with binomial denominators. He was also praised for his efforts in numerical methods, especially in interpolation methods and his table construction techniques. He was regarded as one of the few British mathematicians capable of following the powerful work of Sir Isaac Newton.

Cotes died from a violent fever in Cambridge in 1716 at the early age of 33. Isaac Newton remarked, "If he had lived we would have known something."

复化求积公式

- 将区间[a,b]适当分割成若干个字区间，对每个子区间使用求积公式，构成所谓的复化求积公式，这是提高积分精度的一个常用的方法。

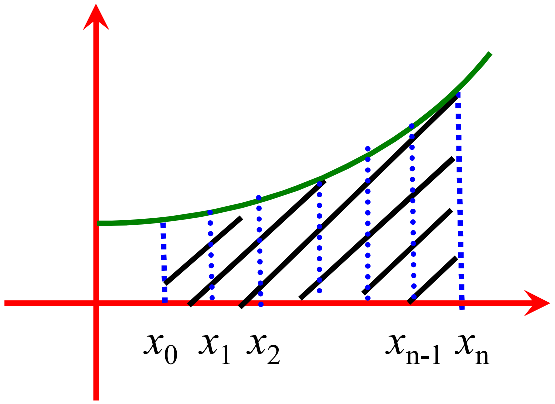
$$I=\sum_{i=0}^{n-1} I_i \approx ?$$



定步长复化求积公式

梯形公式

$$T_k^{(n)}(h)=\frac{h}{2}\left(f(x_k)+f(x_{k+1})\right)$$



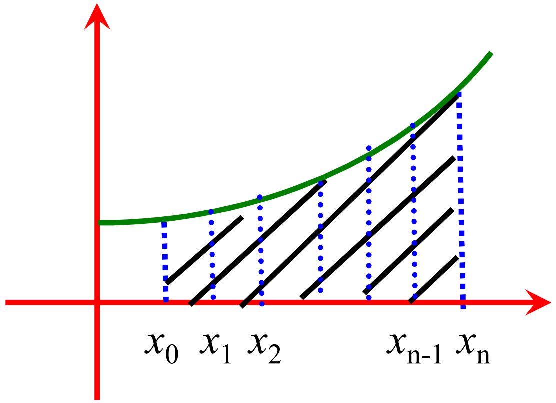
$$T^{(n)}=\frac{h}{2}\sum_{k=0}^{n-1}\left[f(x_k)+f(x_{k+1})\right]$$

$$=\frac{h}{2}\left[f(a)+f(x_1)+f(x_1)+f(x_2)+\ldots+f(x_{n-1})+f(b)\right]$$

$$=\frac{h}{2}\left[f(a)+f(b)+2\sum_{k=1}^{n-1}f(x_k)\right]$$

Simpson公式

$$S_k^{(n)} = \frac{h}{6}\left[f(x_k) + 4f(x_{k+1/2}) + f(x_{k+1})\right]$$



$$S^{(n)} = \frac{h}{6}\left[(f_0 + 4f_{1/2} + f_1) + (f_1 + 4f_{3/2} + f_2) + \ldots + (f_{n-1} + 4f_{n-1/2} + f_n)\right]$$

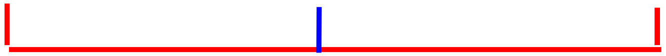
$$= \frac{h}{6}\left\{f_0 + 4\left[f_{1/2} + f_{3/2} + \ldots + f_{n-1/2}\right] + 2\left[f_1 + f_2 + \ldots + f_{n-1}\right] + f_n\right\}$$

$$= \frac{h}{3}\left[\frac{f(a) + f(b)}{2} + 2\sum_{k=0}^{n-1} f\left(x_{k+1/2}\right) + \sum_{k=1}^{n-1} f(x_k)\right]$$

变步长求积公式

$x_0^{(1)} = a$ $\qquad$ $x_1^{(1)} = b$

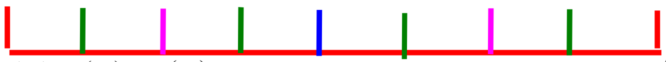
$$n = 1: \quad T_0^{(1)} = \frac{h^{(1)}}{2}\left[f(x_0^{(1)}) + f(x_1^{(1)}) \right]$$



$x_0^{(2)} = a$ $\qquad$ $x_1^{(2)}$ $\qquad$ $x_2^{(2)} = b$

$$x_1^{(2)} = x_{1/2}^{(1)}$$

$$T_0^{(2)} = \frac{h^{(2)}}{2}\left[f(x_0^{(2)}) + 2f(x_1^{(2)}) + f(x_2^{(2)}) \right]$$



$x_0^{(n)}$ $x_1^{(n)}$ $x_2^{(n)}$ $\qquad$ $x_n^{(n)}$

$$x_{2k+1}^{(n)} = x_{k+1/2}^{(n-1)} \qquad x_{2k}^{(n)} = x_k^{(n-1)}$$

$$T_0^{(n)} = \frac{h^{(n)}}{2}\sum_{k=0}^{n-1}\left[f(x_k) + f(x_{k+1}) \right]$$

$$T_0^{(2n)} = \frac{1}{2}\left[T_0^{(n)} + H_0^{(n)} \right] \qquad H_0^{(n)} = h^{(n)}\sum_{k=0}^{n-1} f\left(x_{k+1/2}^{(n)}\right)$$

Romberg求积公式

基本思想:以较小的计算代价,提高精度

$f(x+\frac{h}{2})=f(x)+\frac{h}{2}f'(x)+\frac{1}{2}(\frac{h}{2})^2f''+\cdots$

$f(x-\frac{h}{2})=f(x)-\frac{h}{2}f'(x)+\frac{1}{2}(\frac{h}{2})^2f''$

$$G(h)=\frac{f(x+h/2)-f(x-h/2)}{h}\approx f'(x).$$

$$E(h)=f'(x)-G(h)=\beta_2h^2+\beta_4h^4+\ldots+\beta_{2k}h^{2k}+\ldots \qquad \beta_{2j}=\frac{f^{(2j+1)}(x)}{2^j(2j+1)!}$$

$$E(\frac{h}{2})=f'(x)-G(h/2)=\beta_2\left(\frac{h}{2}\right)^2+\beta_4\left(\frac{h}{2}\right)^4+\ldots+\beta_{2k}\left(\frac{h}{2}\right)^{2k}+\ldots$$

$G(h)=h\cdot f'(x)+\frac{1}{3}(\frac{h}{2})^3f''(x)$

$G(\frac{h}{2})=\frac{f(x+\frac{h}{4})-f(x-\frac{h}{4})}{h/2}$

$$^{*}f'(x)$$

$$G\left(\frac{h}{2}\right)$$

$$G(h)$$

$$G(h)+\lambda\left[G\left(\frac{h}{2}\right)-G(h)\right]$$

$$4E\left(\frac{h}{2}\right)-E(h)=4\left[f'(x)-G\left(\frac{h}{2}\right)\right]-\left[f'(x)-G(h)\right]$$

$$3f'(x)-\left[4G\left(\frac{h}{2}\right)-G(h)\right]=\beta_4'h^4+\cdots$$

$$f'(x)-\frac{1}{3}\left[4G\left(\frac{h}{2}\right)-G(h)\right]=\beta_4^{(1)}h^4+\beta_6^{(1)}h^6$$

$$-\left[\underbrace{G(h)+\frac{4}{3}\left[G\left(\frac{h}{2}\right)-G(h)\right]}\right]$$

$$
\begin{tikzcd}[column sep=small, row sep=tiny]
G(h) \arrow[dr] & & \\
G(\frac{h}{2}) \arrow[r, dash] \arrow[dr, dash] & G^{(1)}(h) \arrow[dr, dash] & \\
 & & G^{(2)}(h). \\
G(\frac{h}{4}) \arrow[r, dash] & G^{(1)}(\frac{h}{2}), \arrow[ur, dash] &
\end{tikzcd}
$$

$$G^{(1)}(h) = \beta_4^{(1)} h^4 + \beta_6^{(1)} h^6 + \dots$$

$$G^{(1)}\left(\frac{h}{2}\right) = \beta_4^{(1)} \left(\frac{h}{2}\right)^4 + \beta_6^{(1)} \left(\frac{h}{2}\right)^6 \dots$$

$$\frac{16\, G^{(1)}(\frac{h}{2}) - G^{(1)}(h)}{15-1} = G^{(2)}(h).$$

$$4(f'(x) - G(h)) - (f'(x) - G(\frac{h}{2}))$$

$$= r_4 h^4 + r_6 h^6 + \ldots + r_{2k} h^{2k} + \ldots = f'(x) - G_1(h)$$

$$G_1(h) = \frac{4G(\frac{h}{2}) - G(h)}{4-1}$$

同理可以有 $G_1(h)$和$G_1(\frac{h}{2})$构造 $G_2(h)$

$$G_2(h)=\frac{4^2G_1(h)-G_1(\frac{h}{2})}{4^2-1}$$

这时 $f'(x)-G_2(h)=O(h^6)$, 依此类推有：

$$\begin{cases} G_0(h)=G(h) \\ G_{m+1}(h)=\dfrac{4^{m+1}G_m(h)-G_m(\frac{h}{2})}{4^{m+1}-1} \qquad m=0,1,2,.. \end{cases}$$

*Richardson*外推法

例　试用 Richardson 外推算法，计算 $f(x)=\sqrt{x}$ 在 $x=1$ 的导数值。

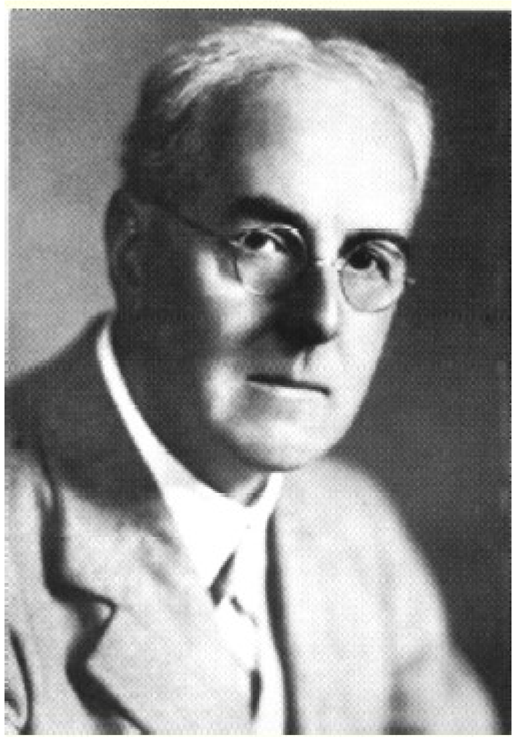
解

$$G(h)=\frac{\sqrt{x+\frac{h}{2}}-\sqrt{x-\frac{h}{2}}}{h}$$

取 $h=0.4$,可算得结果如下表所示：

K	$G(\frac{h}{2^k})$	$G_1(\frac{h}{2^k})$	$G_2(\frac{h}{2^k})$
0	0.50254481		
1	0.50062775	0.49998873	
2	0.50015642	0.49999931	0.50000015

与真值 $f'(1)=0.5$ 相比，仅两次外推，效果极为显著。



Lewis Fry Richardson (1881-1953)
英国数学家

Richardson was an English mathematician, physicist, meteorologist(气象学家), psychologist(心理学) and pacifist who pioneered modern mathematical techniques of weather forecasting, and the application of similar techniques to studying the causes of wars and how to prevent them. He is also noted for his pioneering work on fractals and a method for solving a system of linear equations known as modified Richardson iteration.

Richardson's working life reflected his eclectic interests:

National Physical Laboratory (1903–1904)
University College Aberystwyth (1905–1906)
chemist, National Peat Industries (1906–1907)
National Physical Laboratory (1907–1909)
manager of the physical and chemical laboratory, Sunbeam Lamp Company (1909–1912)
Manchester College of Technology (1912–1913)
Meteorological Office - as superintendent of Eskdalemuir Observatory (1913–1916)
Friends Ambulance Unit in France (1916–1919)
Meteorological Office at Benson, Oxfordshire (1919–1920)
Head of the Physics Department at Westminster Training College (1920–1929)
Principal, Paisley Technical College, now part of the University of the West of Scotland (1929–1940)
In 1926, he was elected to the Fellowship of the Royal Society

Richardson's attempt at numerical forecast: One of Richardson's most celebrated achievements is his attempt in hind sight to forecast the weather during a single day — 20 May 1910 — by direct computation

Mathematical analysis of war: Richardson also applied his mathematical skills in the service of his pacifist principles, in particular in understanding the roots of international conflict. For this reason, today he is considered the founder, or co-founder (with Quincy Wright and Pitirim Sorokin as well as others such as Kenneth Boulding, Anatol Rapaport and Adam Curle), of the scientific analysis of conflict

Research on the length of coastlines and borders: Richardson demonstrated that measured length of coastlines and other natural features appears to increase without lim the unit of measurement is made smaller.[13] Today this is known as the Richardson effe

Patents for detection of icebergs: In April 1912, shortly after the loss of the Tita Richardson filed a patent for iceberg detection using acoustic echolocation in air.

Since 1997, the Lewis Fry Richardson Medal is been awarded by the European Geoscie Union for "exceptional contributions to nonlinear geophysics in general

Werner Romberg (1909–2003), 德国数学家

Born in Berlin, Werner Romberg was a numerical analysis pioneer. He started his teaching career in the autumn of 1938 when he joined the University of Oslo.

In 1949, Romberg joined the Norwegian Institute of Technology in Trondheim as an associate professor of physics

In 1955, he originated a procedure for improving the accuracy of the trapezoidal rule by eliminating the successive terms in the asymptotic expansion. It was based on the extrapolation process devised by Lewis Fry Richardson (1881-1953).

This procedure can now be found in every textbook on numerical analysis without any reference to Romberg's original paper. Besides its usefulness, Romberg's method also showed to non-specialists that convergence acceleration methods can be quite powerful.

In 1960, Romberg was appointed to chair the applied mathematics department at the Norwegian Institute of Technology. He held this position for ten years until he accepted a professorship at the University of Heidelberg. There, he organized a teaching program in applied mathematics, and built a strong research program in numerical analysis.

Romberg求积算法

复化梯形求积公式的截断误差的 Euler-Maclaurin 公式。
设函数 $f(x) \in C^{2k+2}[a,b]$ ，则复化梯形求积公式余式为

$$R_T[f,h] = \int_a^b f(x)dx - T_n(h)$$

$$= \beta_2 h^2 + \beta_4 h^4 + \ldots \beta_{2k} h^{2k} + E$$

其中： $h = \dfrac{(b-a)}{n}$

$$T_n(h) = h\left[\frac{f(a)+f(b)}{2} + \sum_{j=1}^{n} f(a+jh)\right]$$ ；

$\beta_{2j} = -b_{2j}[f^{(2j-1)}(b) - f^{(2j-1)}(a)]$ 为与 h 无关的常数；

b_{2j} $\left(j = 1, 2, \cdots, k, k+1\right)$ 为与 h， $f\left(x\right)$ 无关的常数

$E = -b_{2k+2}(b-a) f^{(2k+2)}(\xi) \quad \xi \in [a, b]$。

证明过于复杂，这里从略。

由于复化梯形求积公式的截断误差有 Euler-Maclaruin 级数形式，且 h^{2j} 项的系数是与 h 无关的常数，因此满足 Richardson 外推算法的条件，有

$$
\begin{cases}
T_0^{(1)}(h)=\dfrac{b-a}{2}[f(a)+f(b)] \\[2ex]
T_1^{(2)}=\dfrac{1}{2}[T_1(\dfrac{h}{2^{k-1}})+h\displaystyle\sum_{i=1}^{n}f(a+h(i-\dfrac{1}{2})] \quad k=1,2,3... \\[2ex]
T_{m+1}(\dfrac{h}{2^{j-1}})=\dfrac{4^m T_m(\dfrac{h}{2^j})-T_m(\dfrac{h}{2^{j-1}})}{4^m-1} \quad (m=1,2,...k;j=k-m+1,...,2,1)
\end{cases}
$$

其中，$h=\dfrac{b-a}{n}$，$n=2^{k-1}$， Romberg 求积公式。

$$\int_0^1 \frac{\sin x}{x} dx, (\varepsilon = \frac{1}{2} \times 10^{-5})$$

k	$T_1(\frac{h}{2^k})$	$T_2(\frac{h}{2^k})$	$T_3(\frac{h}{2^k})$	$T_4(\frac{h}{2^k})$
0	0.920735492			
1	0.939793284	0.946145881		
2	0.944513521	0.946086933	0.946083003	
3	0.945690863	0.94608331	0.946083068	0.946083069

Romberg 算法外推次数也不宜过多，一般以 $m=4$ 或 5 为宜。如果 m 过大，则，$\frac{4^{m}}{4^{m}-1}\approx 1,\frac{1}{4^{m}-1}\approx 0$，也就是说，外推所得新序列与前一次序列值无多大差别，即精度不再有明显改善。

$I=\int_0^1 \frac{1}{1+x^2}\,dx$. $\quad \varepsilon<10^{-4}$.

K	h		$\frac{4}{3}\cdot - \frac{1}{3}\cdot$	$\frac{16}{15}\cdot - \frac{1}{15}\cdot$
0	1	0.75		
1	0.5	0.775	0.7833	
2	0.25	0.7828	0.7854	0.78554
3	0.125	0.7848	0.7855	0.78551

Romberg求积算法

1． 输入：a,b,ξ；

2．计算 $T(h)$

（1） $k=0$； $n=1$； $h=b-a$；

（2） $T(0)=\frac{h}{2}[f(a)+f(b)]$ ， $c=T(0)$；

3．计算 $T_1(\frac{h}{2^k})$

（1） $k=k+1$；

（2） $T(k)=\frac{1}{2}[T(k-1)+h\sum_{i=1}^{n}f(a+h(i-\frac{1}{2})]$ ；

3) $d=1$；

4． 对于 $j=k,k-1,\cdots,2,1$ 做

（1） $d=4d$；

（2） $T(j-1)=T(j)+\left(T(j)-T(j-1)\right)/(d-1)$；

5． **if** $\left|c-T(0)\right|<\varepsilon$,输出 $T(0)$，停止计算；

6． $n=2n$;$h=h/2$;$c=T(0)$;返回 3。