第五章作业

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5.1:

$$f(x) = e^{3x}\cos(2x)$$

$$f(0) = 1 \quad f(\tfrac{1}{2}) = e^{\frac{3}{2}}\cos(1) \quad f(\tfrac{1}{4}) = e^{\frac{3}{4}}\cos(\tfrac{1}{2})$$

计算精确值：0.9082

中矩公式：$I = \frac{1}{2}f(\frac{1}{4}) = 0.9289$ (*Loss:2.2%*)

梯形公式：$I = \frac{1}{4}[f(0) + f(\frac{1}{4})] = 0.8553$ (*Loss:5.8%*)

Simpson 公式：

$I = \frac{1}{6}\frac{1}{2}[f(0) + 4f(\frac{1}{4}) + f(\frac{1}{2})] = 0.9044$ (*Loss:0.4%*)

5.2:

Simpson 公式：$I = \frac{b-a}{2}[f(a) + 4f(\frac{a+b}{2}) + f(b)]$

变形为：$I = \frac{b-a}{2}[f(a) + f(\frac{\sqrt[3]{4}(a+b)}{2}) + f(b)]$

代入题目数据，得到：$I \approx 0.5002$

5.4:

运行以下 python 程序：

运行结果：

1.6351257241231425

1.635227835816294

```python
from math import *
import numpy as np

def function(x):  5 用法
    return x / log(x + 1)
class Trapezium:  1 个用法
    def __init__(self, a, b, h):
        self.a = a
        self.b = b
        self.h = h
    def ans(self):  1 个用法
        result = 0
        for x in np.arange(self.a, self.b, self.h):
            result += (function(x) + function(x + self.h)) * self.h / 2
        return result
class Simpson:  1 个用法
    def __init__(self, a, b, h):
        self.a = a
        self.b = b
        self.h = h
    def ans(self):  1 个用法
        result = 0
        for x in np.arange(self.a, self.b, self.h):
            result += (function(x) + 4 * function(x + self.h / 2) + function(x + self.h)) * self.h / 6
        return result

trapezium = Trapezium( a: 1,  b: 2, 1 / 6)
simpson = Simpson( a: 1,  b: 2, 1 / 6)
print(trapezium.ans(), simpson.ans())
```

图 1: Code