09-B6

BST Application

kd-Tree: Complexity

肉眼看不清细节，但他们都知道那是木星所在的位置，这颗太阳系最大的行星已经坠落到二维平面上了。

有人嘲笑这种体系说：为了能发现这个比例中项并组成政府共同体，按照我的办法，只消求出人口数字的平方根就行了。

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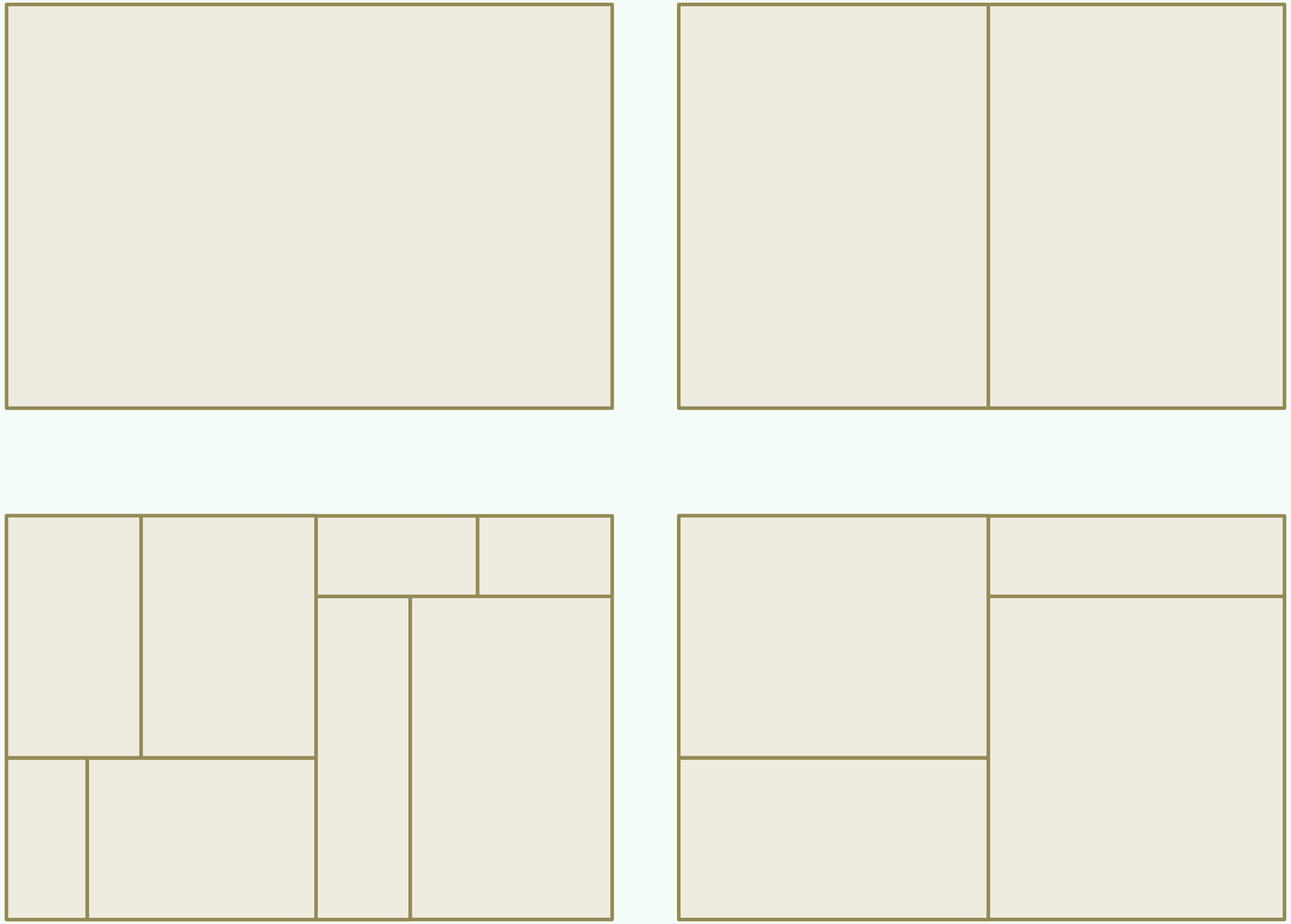
Preprocessing

- $T(n)$

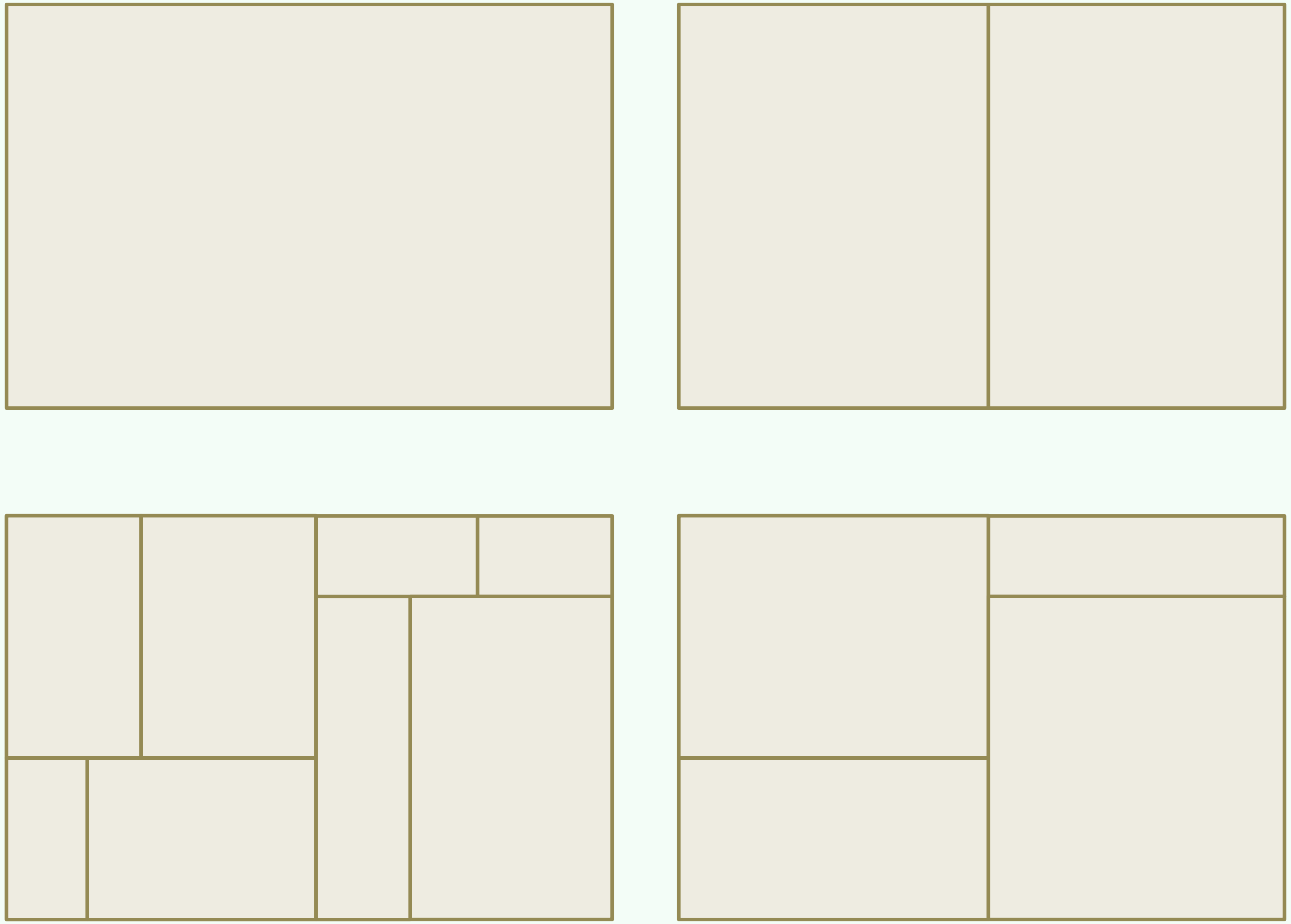
  $= 2*T(n/2) + \mathcal{O}(n)$

  $= \mathcal{O}(n\log n)$



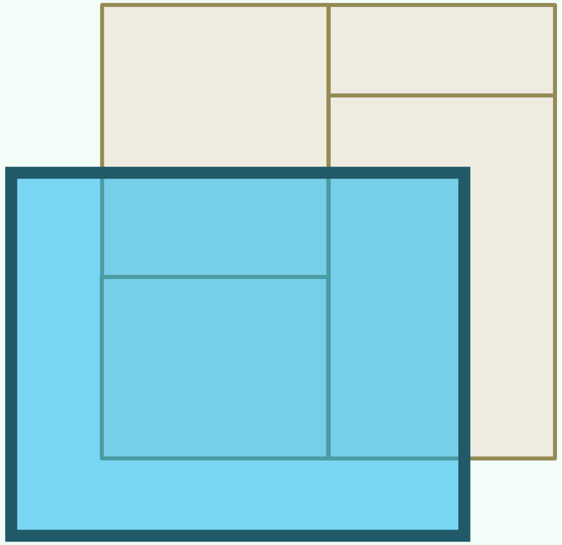
Storage

- The tree has a height of $\mathcal{O}(\log n)$
- $$\begin{aligned} & 1 \\ + & \ 2 \\ + & \ 4 \\ + & \ \ldots \\ + & \ \mathcal{O}(2^{\log n}) \\ = & \ \mathcal{O}(n) \end{aligned}$$



Query Time

❖ Claim: Report + Search = $\mathcal{O}(r+\sqrt{n})$

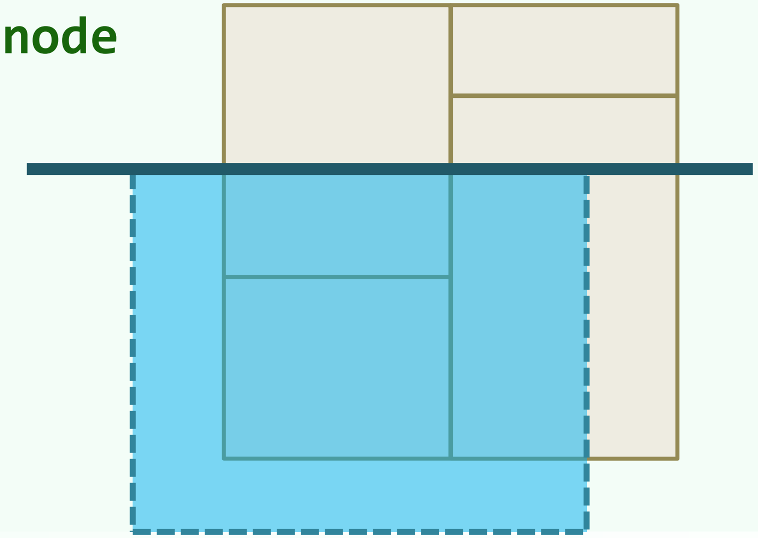


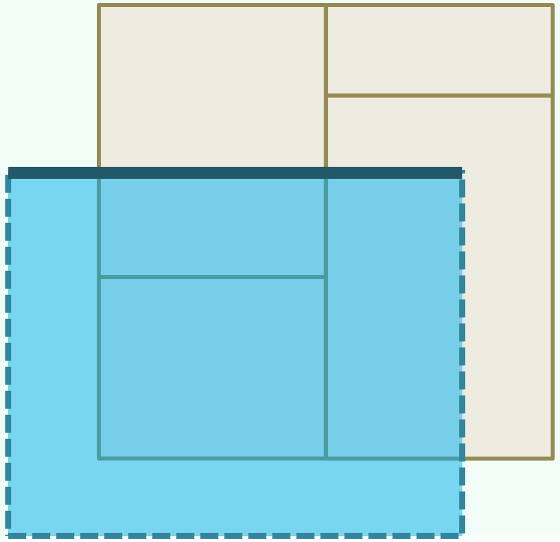
❖ The searching time depends on Q(n), the number of

- recursive calls, or
- sub-regions **intersecting with** R (at all levels)

❖ No more than **2** of the **4** grandchildren of each node will recurse

- $Q(1) = \mathcal{O}(1)$
- $Q(n) = 2 + 2 \cdot Q(n/4)$





❖ Solve to $Q(n) = \mathcal{O}(\sqrt{n})$

Beyond 2D

❖ Can 2d-tree be extended to kd-tree and help HIGHER dimensional GRS?

If yes, how efficiently can it help?

❖ A kd-tree in k-dimensional space is constructed by

recursively divide $\mathcal{E}^d$ along the $\boxed{1^{st},\ 2^{nd},\ \ldots,\ kth}$ dimensions

❖ An orthogonal range query on a set of n points in $\mathcal{E}^d$

- can be answered in $\mathcal{O}(r + n^{1-1/d})$ time,
- using a kd-tree of size $\mathcal{O}(n)$, which
- can be constructed in $\mathcal{O}(n \log n)$ time