高级搜索树

B-树：结构

10-B2

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妻子好合，如鼓瑟琴；兄弟既翕，和乐且湛

等价变换

- R. Bayer & E. McCreight，1970

  平衡的多路搜索树

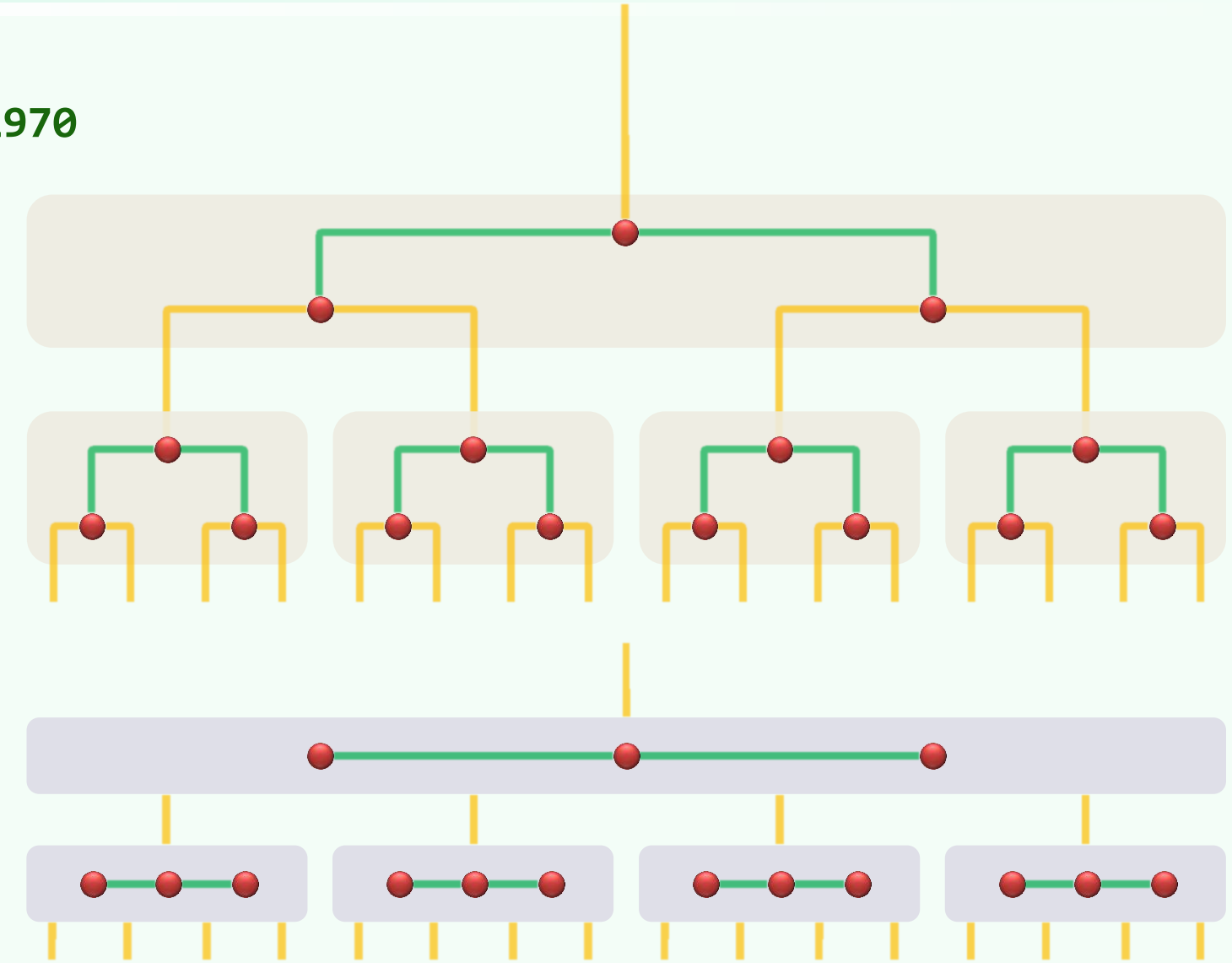
- 每d代合并为超级节点

  - m = 2^d 路

  - m-1 个关键码

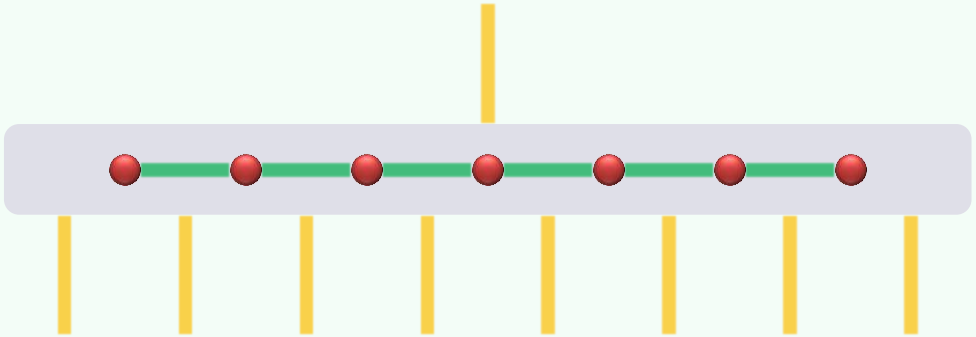
- 逻辑上与BBST完全等价

  既如此，B-树之意义何在？



I/O优化

- 多级存储系统中使用B-树，可针对外部查找，大大减少I/O次数
- 难道，AVL还不够？比如，若有n = 1G个记录...
  - 每次查找需要 $\log_2 10^9 \approx 30$ 次I/O操作
  - 每次只读出单个关键码，得不偿失

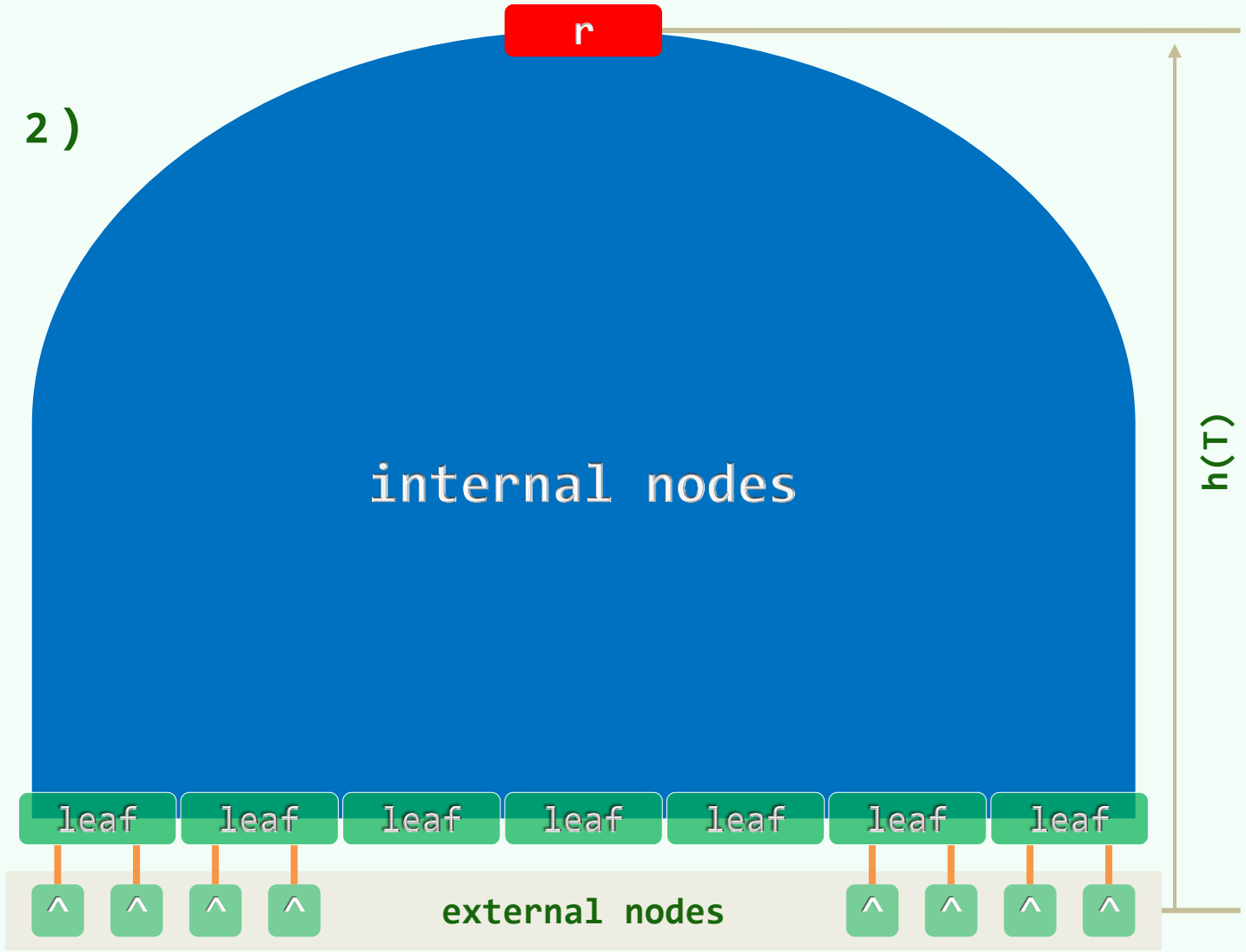


- B-树又能如何？
  - 充分利用外存对批量访问的高效支持，将此特点转化为优点
  - 每下降一层，都以超级节点为单位，读入一组关键码
- 具体多大一组？视磁盘的数据块大小而定，m = #keys / pg
  - 比如，目前多数数据库系统采用 m = 200~300
- 回到上例，若取 m = 256，则每次查找只需 $\log_{256} 10^9 \leq 4$ 次I/O

外部节点 + 叶子

- 所谓m阶B-树，即m路平衡搜索树（m ≥ 2）
- 外部节点的深度统一相等

  约定以此深度作为树高h
- 叶节点的深度统一相等（h-1）



内部节点

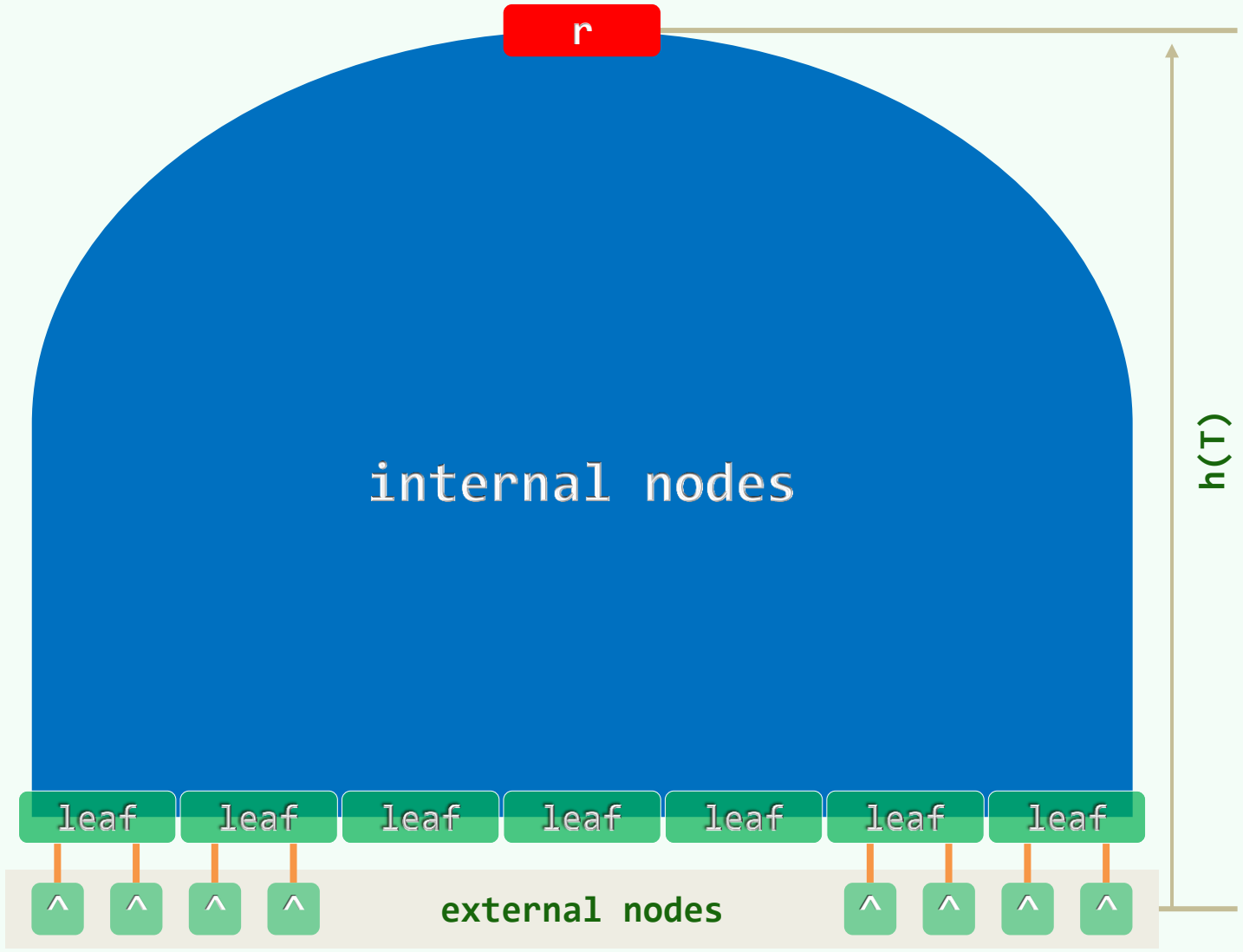
- 各含 n ≤ m-1 个关键码：

$$K_1 < K_2 < K_3 < \cdots < K_n$$

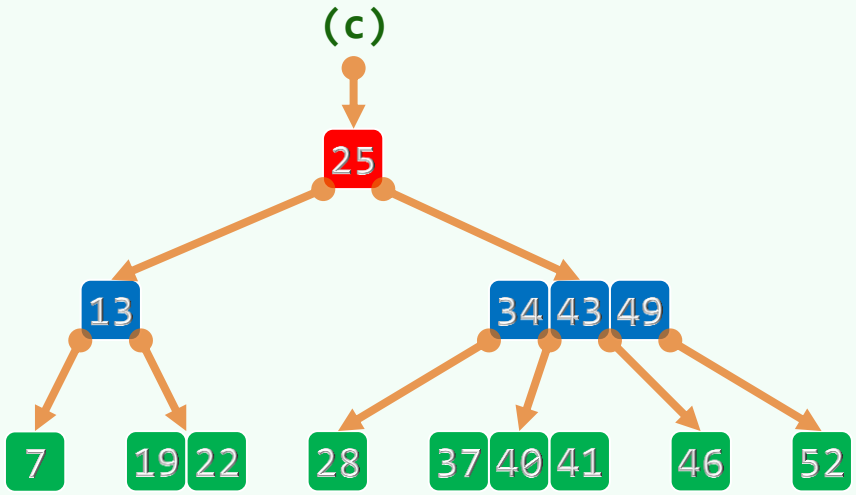
- 各有 n+1 ≤ m 个分支：

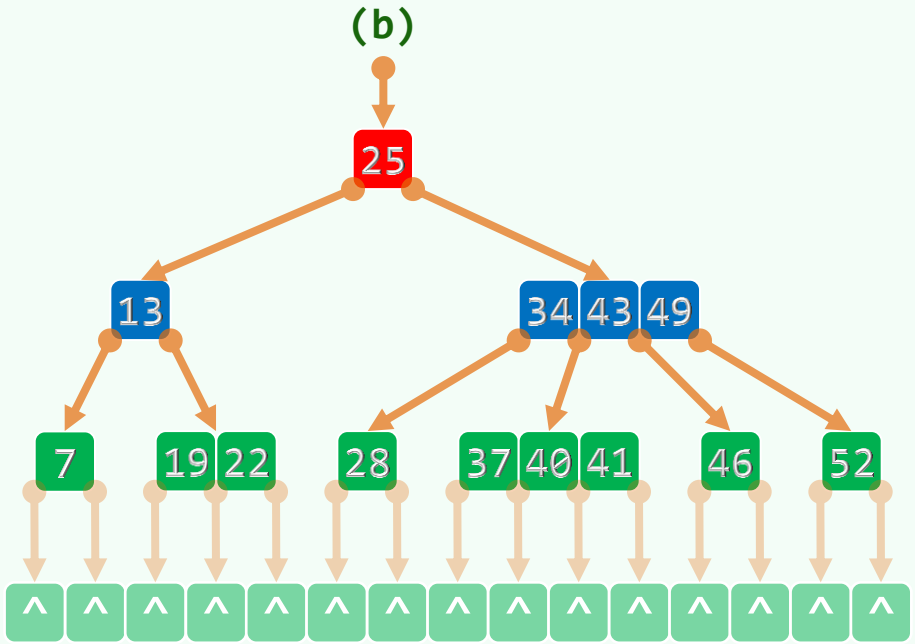
$$A_0,\ A_1,\ A_2,\ A_3,\ \ldots,\ A_n$$

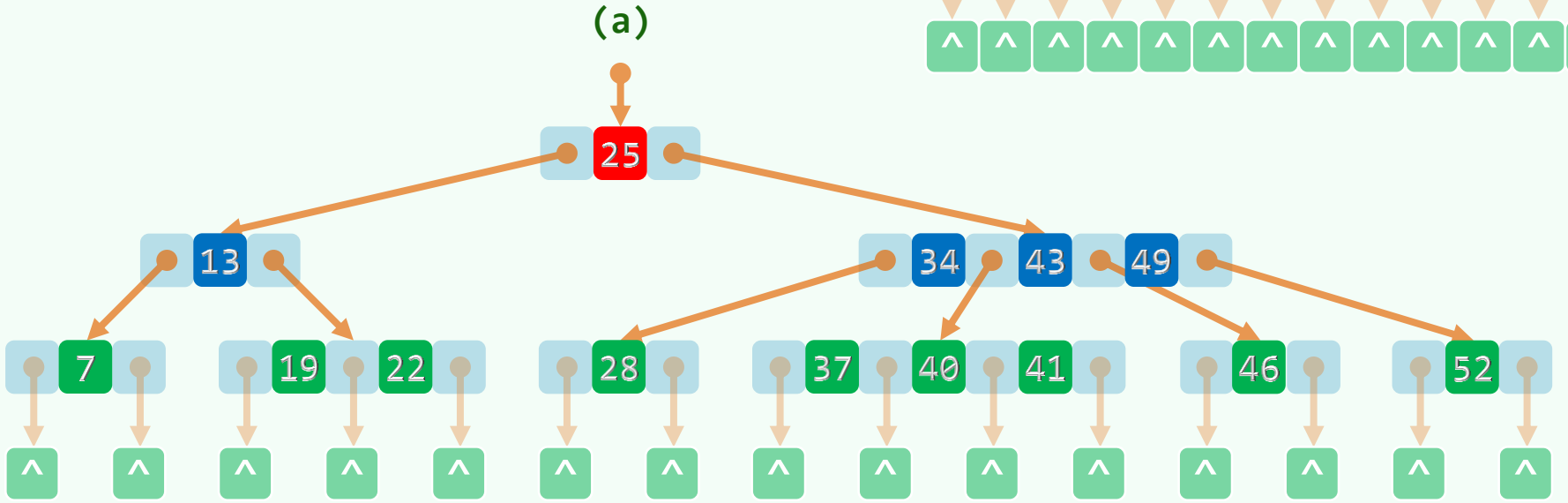
- 反过来，分支数也不能太少
  - 树根：$2 \le n+1$
  - 其余：$\lceil m/2 \rceil \le n+1$
- 故亦称作(⌈m/2⌉,m)-树
  - (3,5)-树
  - (9,18)-树
  - ...



紧凑表示


(c)
25
13
34 43 49
7
19 22
28
37 40 41
46
52


(b)
25
13
34 43 49
7
19 22
28
37 40 41
46
52


(a)
25
13
34
43
49
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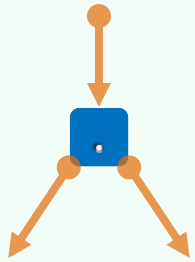
37
40
41
46
52

实例

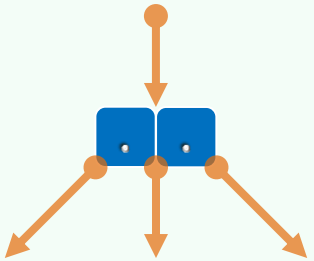
❖ m = 3：2-3-树，(2,3)-树，最简单的B-树 //J. Hopcroft, 1970

- 各（内部）节点的分支数，可能是2或3
- 各节点所含key的数目，可能是1或2

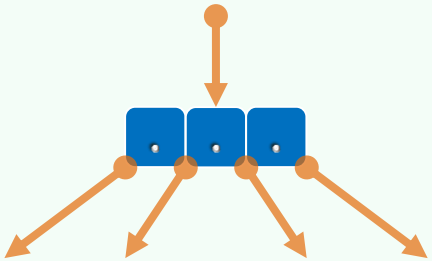


❖ m = 4：2-3-4-树，(2,4)-树

- 各节点的分支数，可能是2、3或4
- 各节点所含key的数目，可能是1、2或3

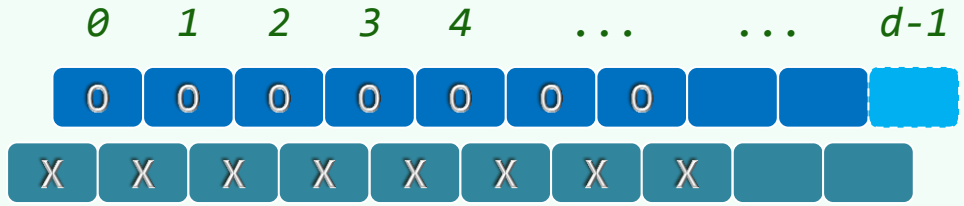


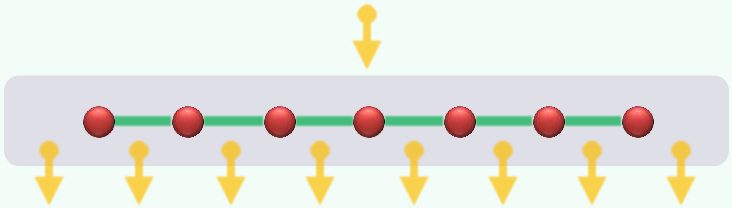
❖ 留意把玩4阶B-树，稍后对于理解红黑树大有裨益



BTNode

```
❖ template <typename T> struct BTNode { //B-树节点
     BTNodePosi(T) parent; //父
     Vector<T> key; //关键码（总比孩子少一个）
     Vector< BTNodePosi(T) > child; //孩子
     BTNode() { parent = NULL; child.insert( 0, NULL ); }
     BTNode( T e, BTNodePosi(T) lc = NULL, BTNodePosi(T) rc = NULL ) {
        parent = NULL; //作为根节点，而且初始时
        key.insert( 0, e ); //仅一个关键码，以及
        child.insert( 0, lc ); child.insert( 1, rc ); //两个孩子
        if ( lc ) lc->parent = this; if ( rc ) rc->parent = this;
     }
  };
```





BTree

- `#define BTNodePosi(T) BTNode<T>* //B-树节点位置`
- `template <typename T> class BTree { //B-树`

```
protected:
    int _size; int _order; BTNodePosi(T) _root; //关键码总数、阶次、根
    BTNodePosi(T) _hot; //search()最后访问的非空节点位置
    void solveOverflow( BTNodePosi(T) ); //因插入而上溢后的分裂处理
    void solveUnderflow( BTNodePosi(T) ); //因删除而下溢后的合并处理
public:
    BTNodePosi(T) search(const T & e); //查找
    bool insert(const T & e); //插入
    bool remove(const T & e); //删除
};
```