09-XA

BST Application

Range Tree

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BBST<BBST<T>> --> BBST<List<T>>

❖ Note that each y-search is just

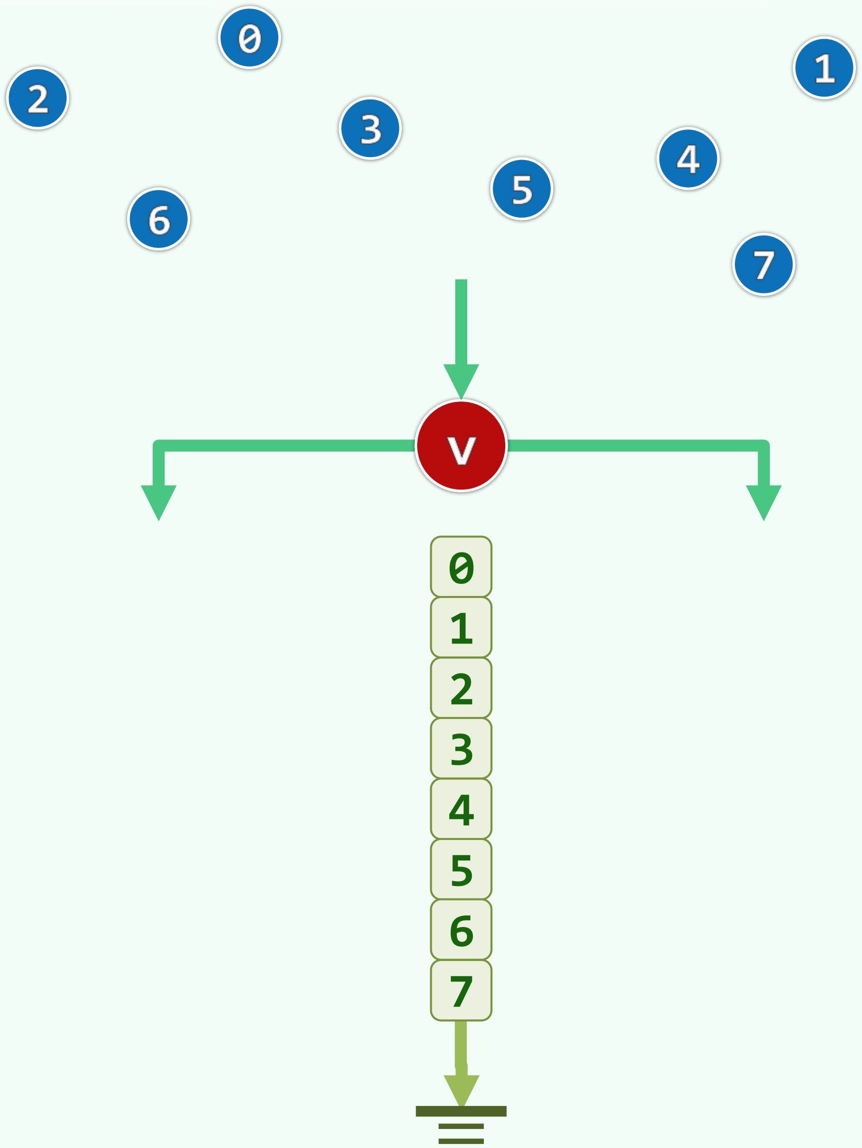
a 1D query without further recursions

❖ So it not necessary

to store each canonical subset

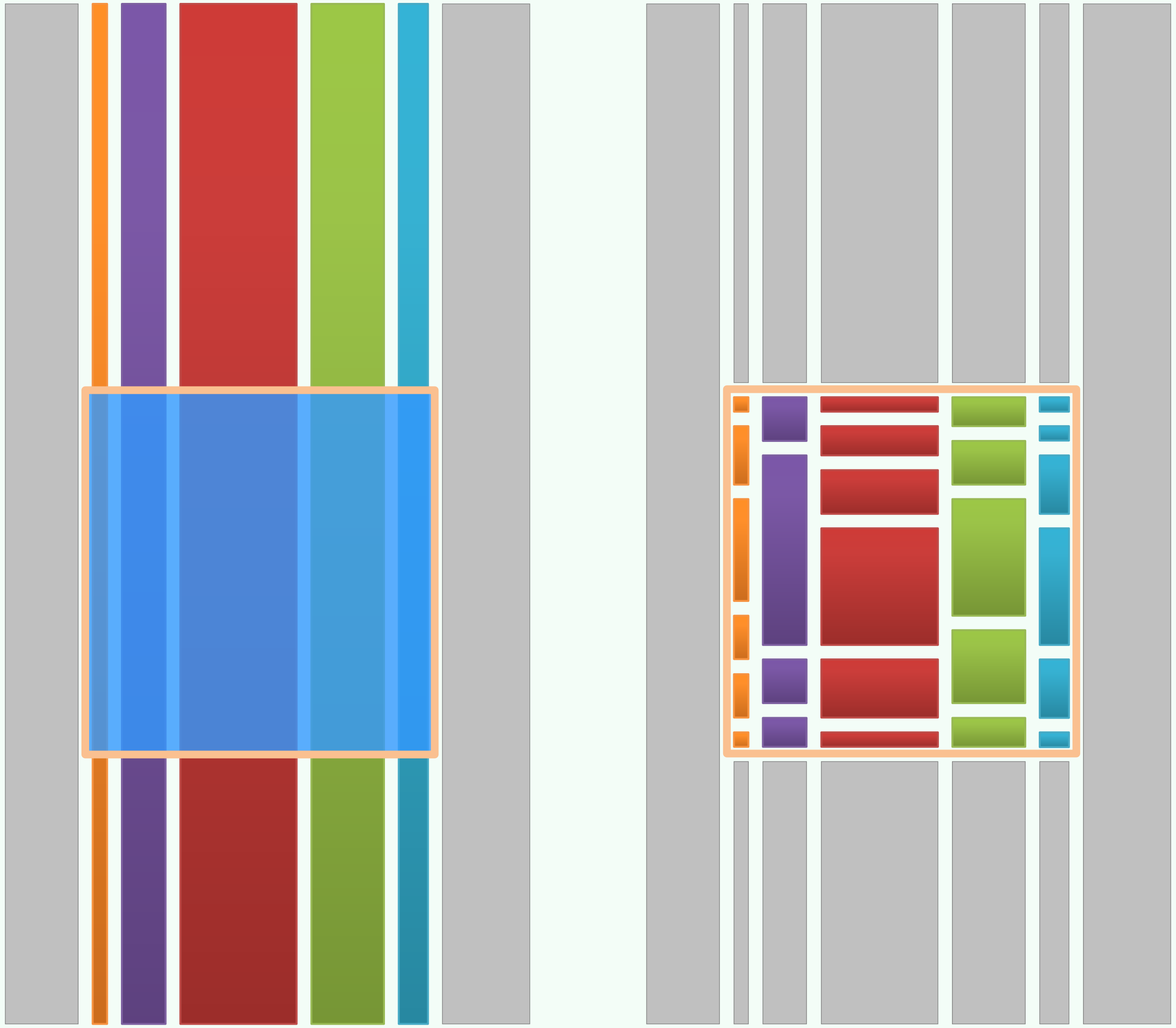
as a BBST

❖ Instead, a sorted y-list simply works



Coherence

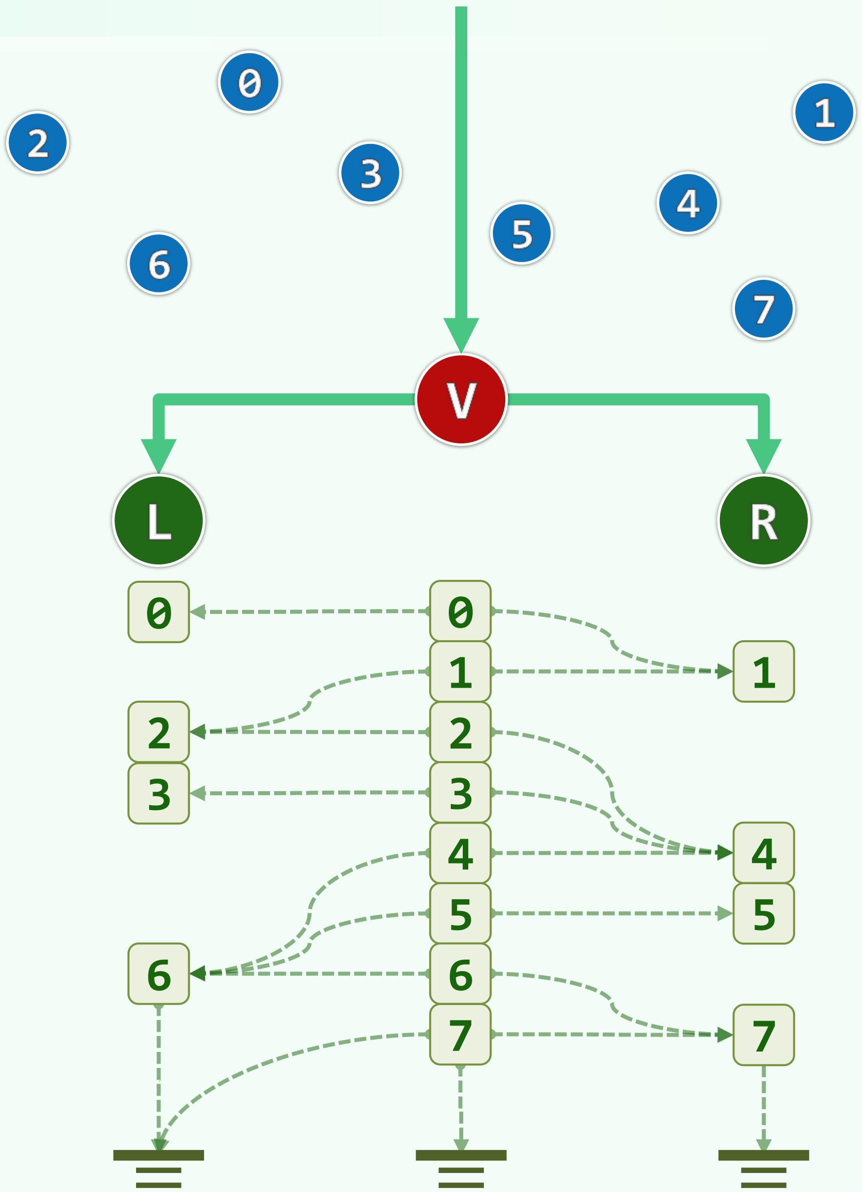
- ❖ Observe further that, for each query
  - we need to repeatedly search DIFFERENT y-lists,
  - but always with the SAME key
- ❖ However, such an essential fact is not used yet



Links Between Lists

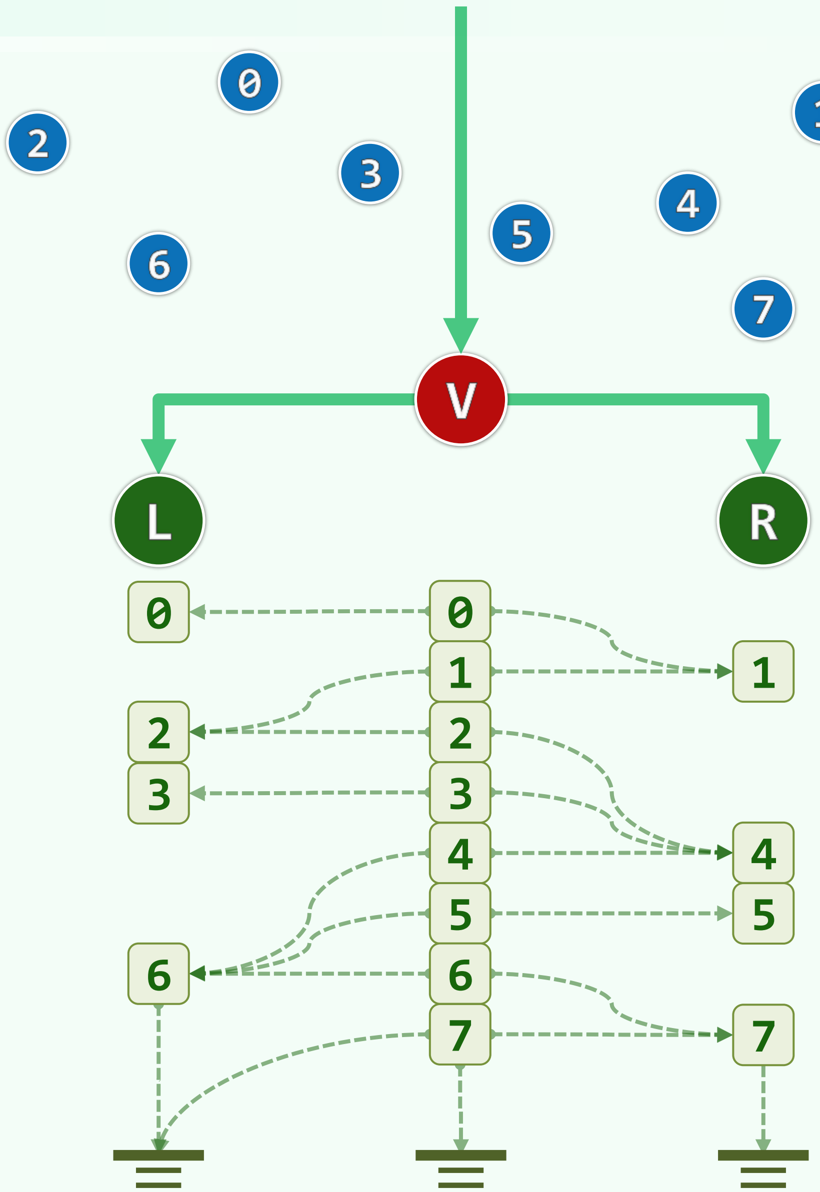
- The idea for an improvement is that
  we **CONNECT** all the different lists
  into a **SINGLE** massive list

- Thus, once a parent y-list is searched,
  we can get, in $\mathcal{O}(1)$ time,
  the entry for child y-list by
  following the link between them



Using Coherence

- To answer a 2D range query, we will
  do an expensive ($\mathcal{O}(\log n)$) search
  on the y-list for the root
- Thereafter, while descending the x-tree, we can
  keep track of the position of y-range
  in each successive list in $\mathcal{O}(1)$ time
- This technique is called Fractional Cascading



Fractional Cascading

❖ For each item in A_v,

we store two pointers to

the item of NLT value

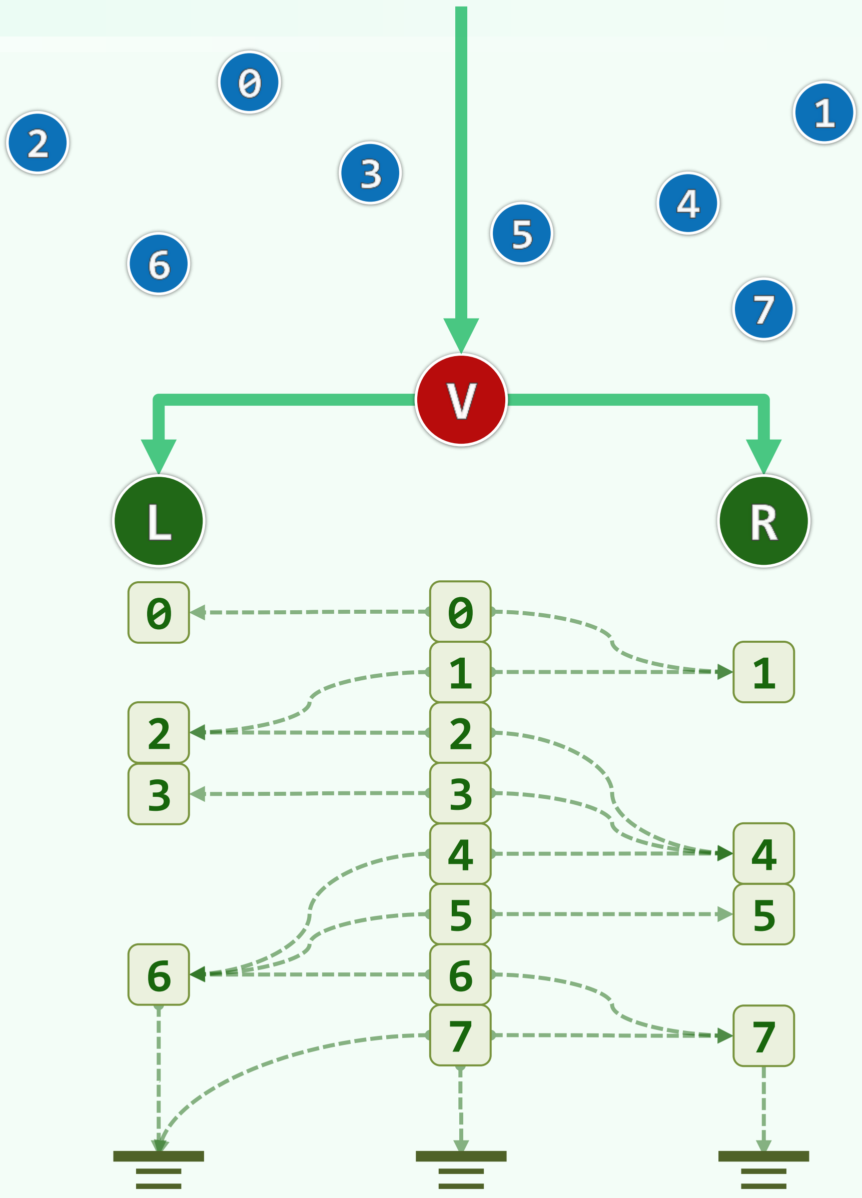
in A_L and A_R resp.

❖ Hence for any y-query with q_y,

once we know its entry in A_v, we can

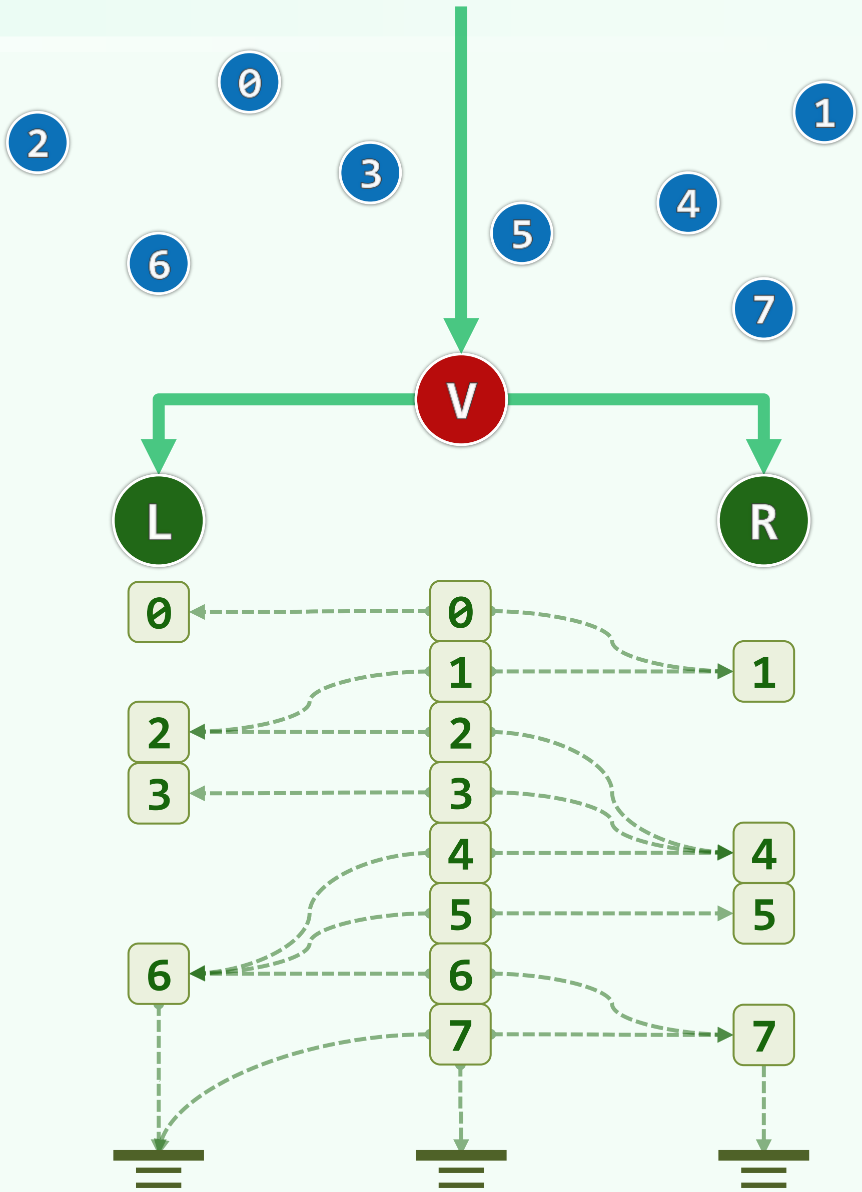
determine its entry in either A_L or A_R

in $\mathcal{O}(1)$ additional time



Construction By 2-Way Merging

- Let V be an internal node in the **x**-tree
  with L/R its left/right child resp.
- Let A_v be the **y**-list for v and
  A_L/A_R be the **y**-lists for its children
- Assuming no duplicate **y**-coordinates, we have
  - A_v is the disjoint union
    of A_L and A_R, and hence
  - A_v can be obtained by
    merging A_L and A_R (in linear time)



Complexity

❖ An MLST with fractional cascading is called a **range tree**

❖ A **y-search** for root is done in $\mathcal{O}(\log n)$ time

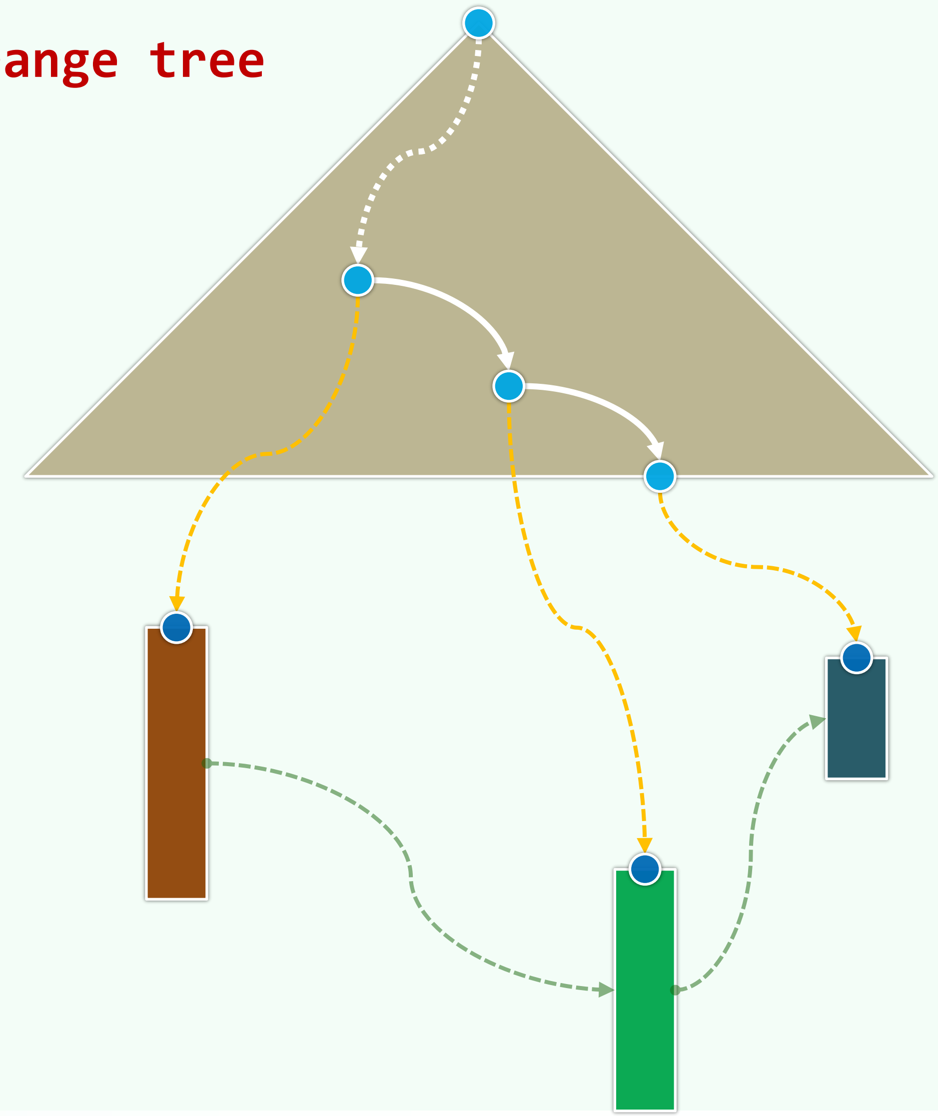
❖ To drop down to each next level, we can

get, in $\mathcal{O}(1)$ time, the current **y-interval**

from that of the **prior** level

❖ Hence, each 2D orthogonal range query

- can be answered in $\mathcal{O}(r + \log n)$ time
- from a data structure of size $\mathcal{O}(n \cdot \log n)$,
- which can be constructed in $\mathcal{O}(n \cdot \log n)$ time



Beyond 2D

- Unfortunately, the trick of fractional cascading

  can **ONLY** be applied to

  the **LAST** level the search structure

- Given a set of n points in $\mathcal{E}^d$,

  an orthogonal range query

  - can be answered in $\mathcal{O}(r + \log^{d-1} n)$ time
  - from a data structure of size $\mathcal{O}(n \cdot \log^{d-1} n)$,
  - which can be constructed in $\mathcal{O}(n \cdot \log^{d-1} n)$ time