09-A1

BST Application

Range Query: 1D

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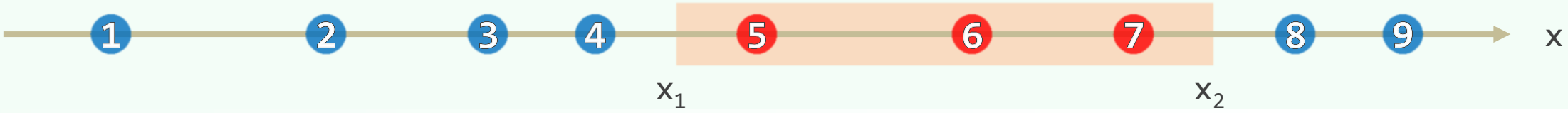
你这个人太敏感了。这个社会什么都需要，唯独不需要敏感。

1D Range Query

- Let $P = \{ p_1, p_2, p_3, \ldots, p_n \}$ be a set of n points on the x-axis
- For any given interval $I = (x_1, x_2]$
  - COUNTING: how many points of P lies in the interval?
  - REPORTING: enumerate all points in $I \cap P$ (if not empty)
- [Online] P is fixed while I is randomly and repeatedly given
- How to PREPROCESS P into a certain data structure s.t.

  the queries can be answered efficiently?

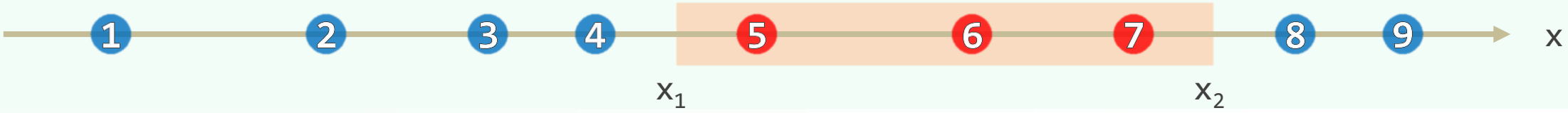


Brute-Force

- For each point p of P, test if $p \in (x_1, x_2]$
- Thus each query can be answered in LINEAR time
- Can we do it faster? It seems we can't, for ...
- In the worst case,

  the interval contains up to $O(n)$ points, which need $O(n)$ time to enumerate
- However, how if we

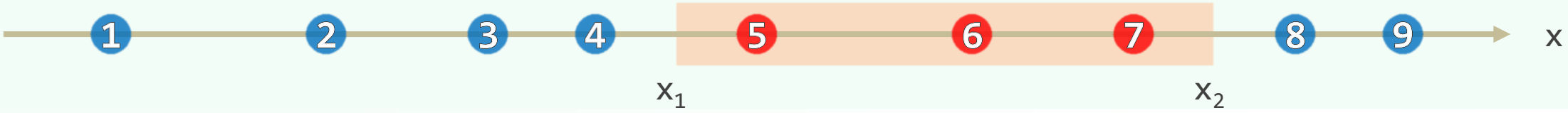
  ignore the time for enumerating and count only the searching time?



Binary Search

- Sort all points into a sorted vector and add an extra sentinel p[0] = -∞
- For any interval $I = (x_1, x_2]$
  - Find t = search(x_2) = max{ i | p[i] ≤ x_2 } //O(logn)
  - Traverse the vector BACKWARD from p[t] and report each point //O(r)

    until escaping from I at point p[s]
  - return r = t - s //output size



Output-Sensitivity

- An enumerating query can be answered in $O(r + \log n)$ time
- p[s] can also be found by binary search in $O(\log n)$ time
- Hence for COUNTING query, $O(\log n)$ time is enough //independent to r
- Can this simple strategy be extended to PLANAR range query?

  TTBOMK, unfortunately, no!