

绪论

动态规划：记忆法

圣人不记事，所以常记得；今人忘事，以其记事

有人建议不妨置备一本签名簿，供来访者留下自己的名字，就像怀特山那样；可是，天哪！我的记性非常好，用不着那个玩意儿。

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fib() : 递归

$$fib(n) = fib(n-1) + fib(n-2) \quad \boxed{0} \boxed{1} \boxed{1} \boxed{2} \boxed{3} \boxed{5} \boxed{8} \boxed{13} \boxed{21} \boxed{34} \boxed{55} \boxed{89} \dots$$

❖ `int fib(n) { return (2 > n) ? n : fib(n - 1) + fib(n - 2); }` //为何这么慢?

❖ 复杂度: $T(0) = T(1) = 1; \quad T(n) = T(\boxed{n-1}) + T(\boxed{n-2}) + 1, \quad \forall n > 1$

- 令 $S(n) = [T(n) + 1]/2$
- 则 $S(0) = 1 = fib(1), \quad S(1) = 1 = fib(2)$
- 故 $S(n) = S(n-1) + S(n-2) = fib(n+1)$

$$T(n) = 2 \cdot S(n) - 1 = 2 \cdot fib(n+1) - 1 = \mathcal{O}(fib(n+1)) = \mathcal{O}(\phi^n)$$

- 其中 $\phi = (1 + \sqrt{5})/2 \approx 1.618$

封底估算

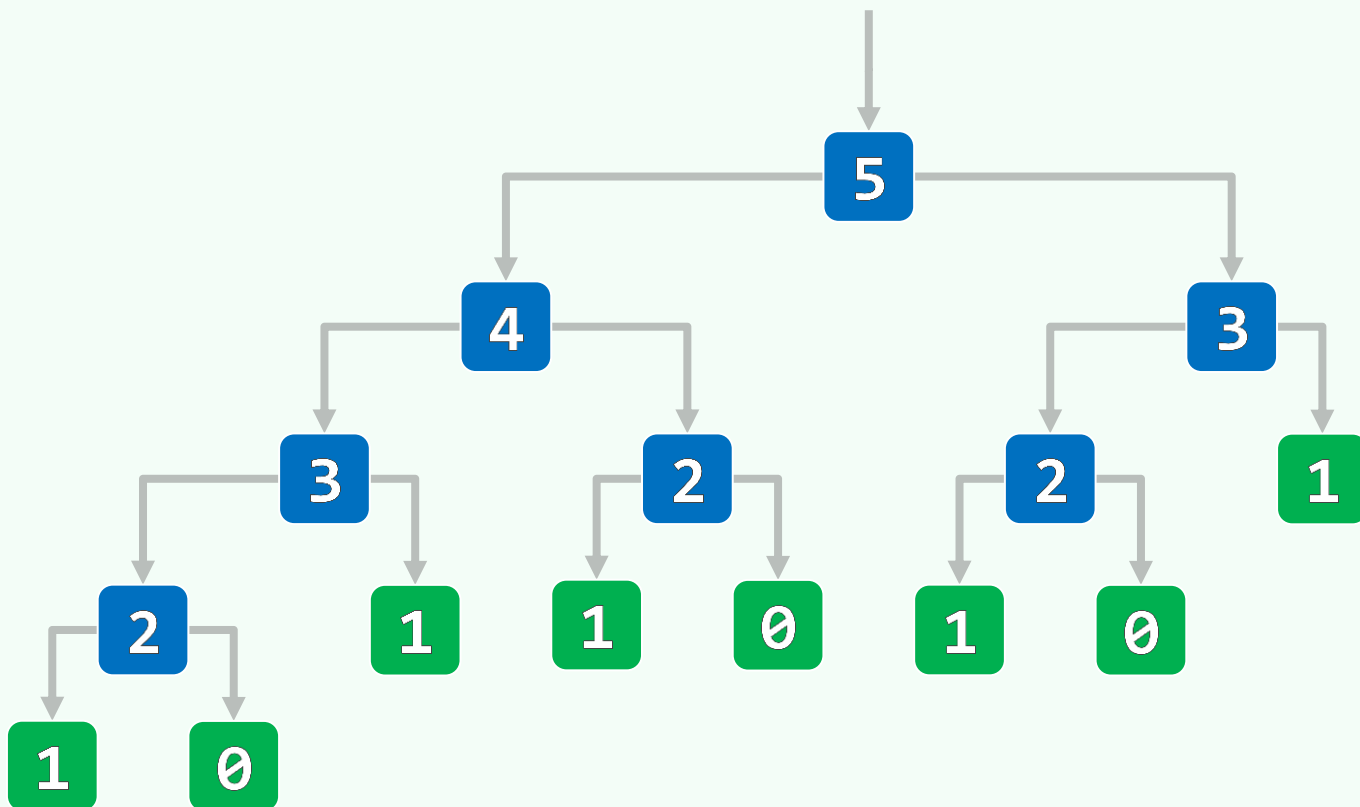
$$\phi^{36} \approx 2^{25} \quad \phi^{43} \approx 2^{30} \approx 10^9 \text{ flo} = 1 \text{ sec}$$

$$\phi^5 \approx 10 \quad \phi^{67} \approx 10^{14} \text{ flo} = 10^5 \text{ sec} \approx 1 \text{ day}$$

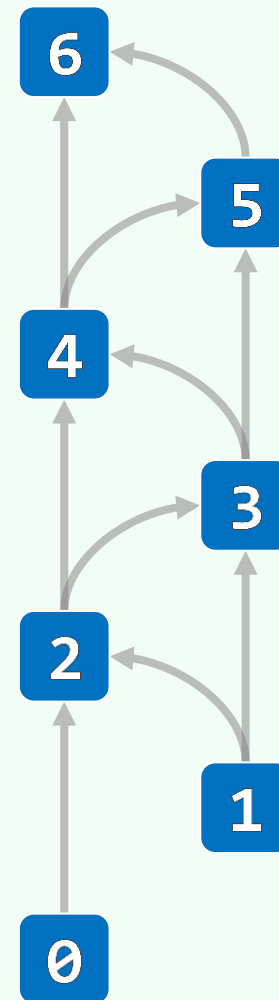
$$\phi^{92} \approx 10^{19} \text{ flo} = 10^{10} \text{ sec} \approx 10^5 \text{ day} \approx 3 \text{ century}$$

递归

❖ 递归版fib()低效的根源在于，各递归实例均被大量地重复调用



❖ 先后出现的递归实例，共计 $\mathcal{O}(\phi^n)$ 个；而去除重复之后，总共不过 $\mathcal{O}(n)$ 种



Memoization



```
def f(n)
    if ( n < 1 ) return trivial( n );

    return f(n-X) + f(n-Y)*f(n-Z);
```

❖ T **M**[N]; #init. with UNDEFINED

```
def f(n)
    if ( n < 1 ) return trivial( n );

    # recur only when necessary &
    # always write down the result
    if ( M[n] == UNDEFINED )
        M[n] = f(n-X) + f(n-Y)*f(n-Z);

    return M[n];
```

Memoization: fib()

