

09-A2

BST Application

Range Query: 2D

昔者明王必尽知天下良士之名；既知其名，又知其数；
既知其数，又知其所在。

邓俊辉

deng@tsinghua.edu.cn

Planar Range Query

❖ Let $P = \{ p_1, p_2, p_3, \dots, p_n \}$ be a planar set

❖ Given $R = (x_1, x_2] \times (y_1, y_2]$

- COUNTING: $|R \cap P| = ?$

- REPORTING: $R \cap P = ?$

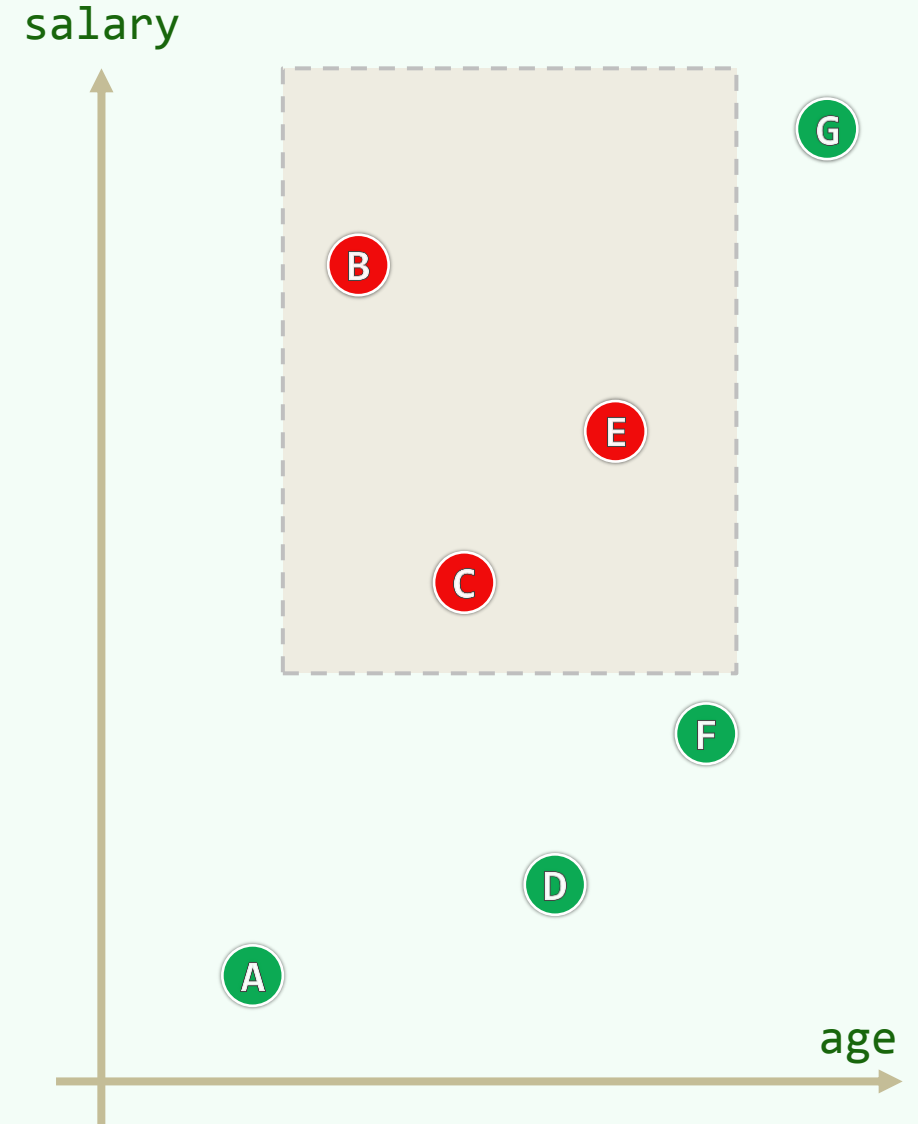
❖ Binary search

doesn't help this kind of query

❖ You might consider to

expand the counting method using

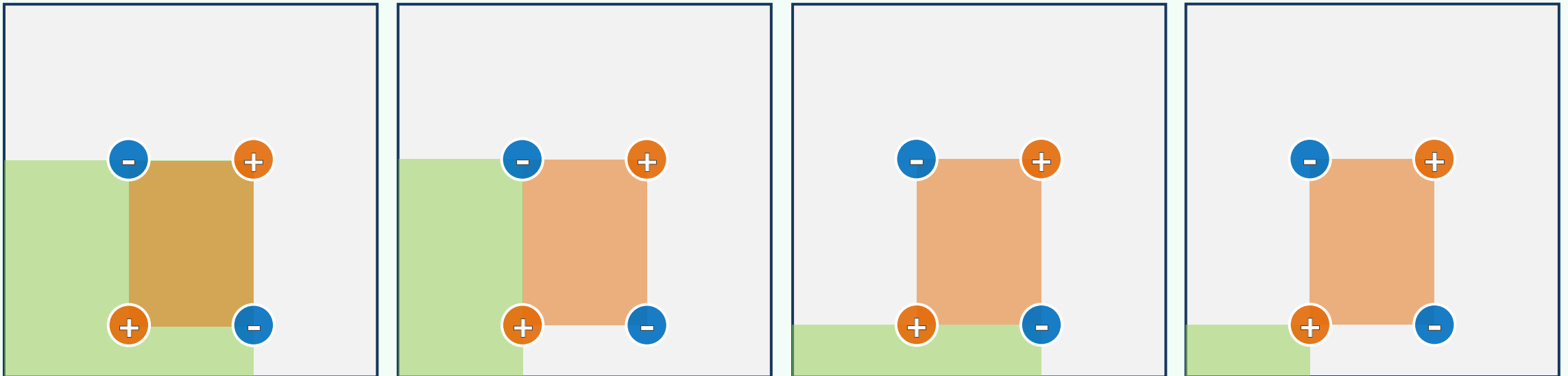
the **Inclusion-Exclusion Principle**



Preprocessing

❖ \forall point (x, y) , let $n(x, y) = |((0, x] \times (0, y]) \cap P|$

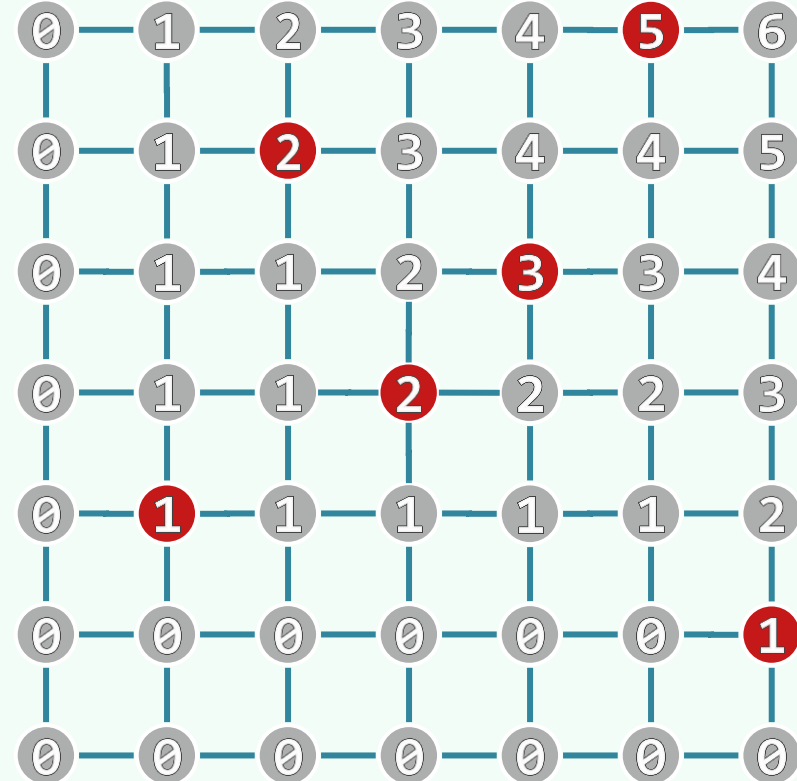
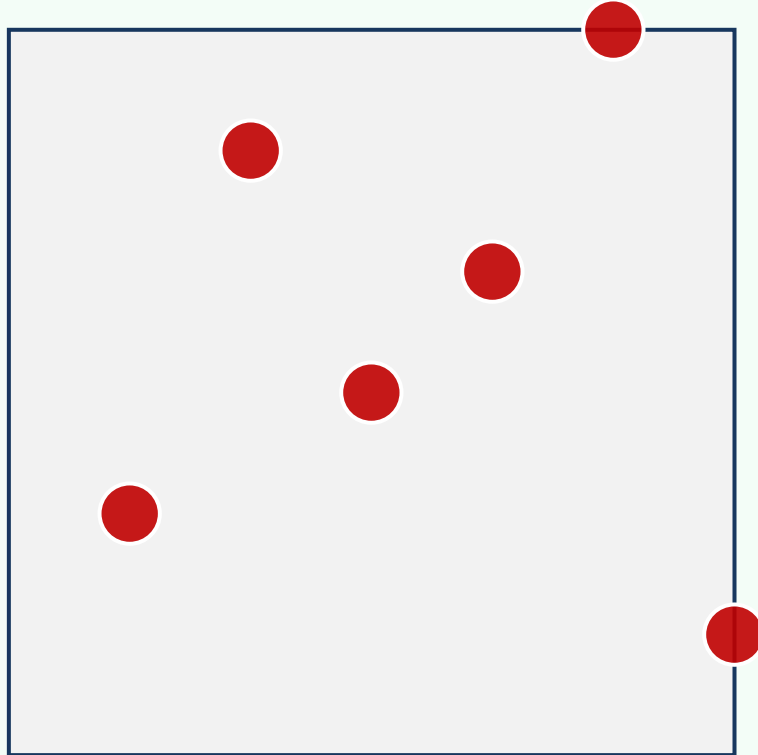
❖ This requires $\mathcal{O}(n^2)$ time/space



Domination

❖ A point (u, v) is called to be **DOMINATED** by point (x, y) if

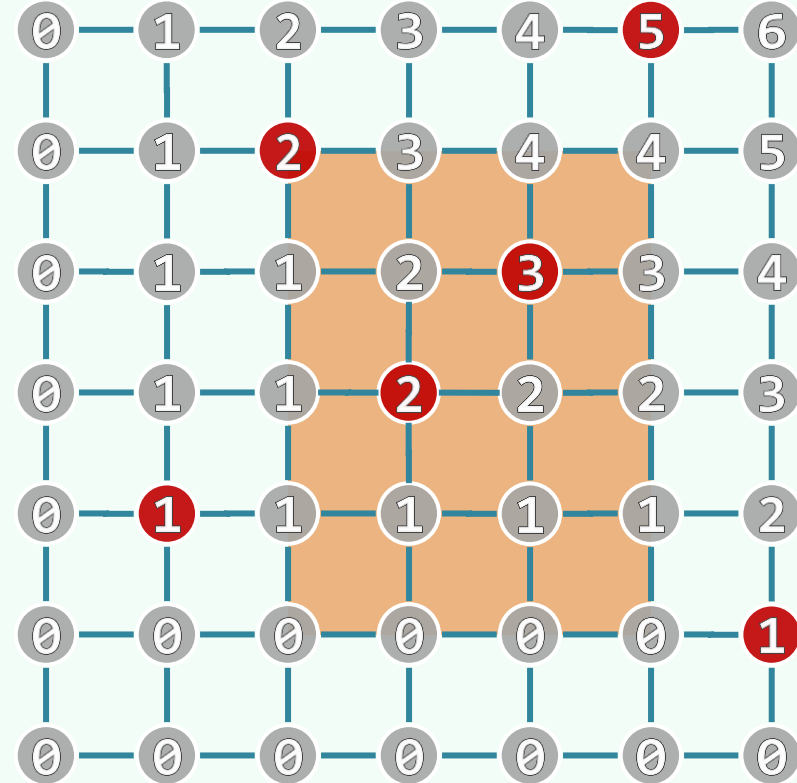
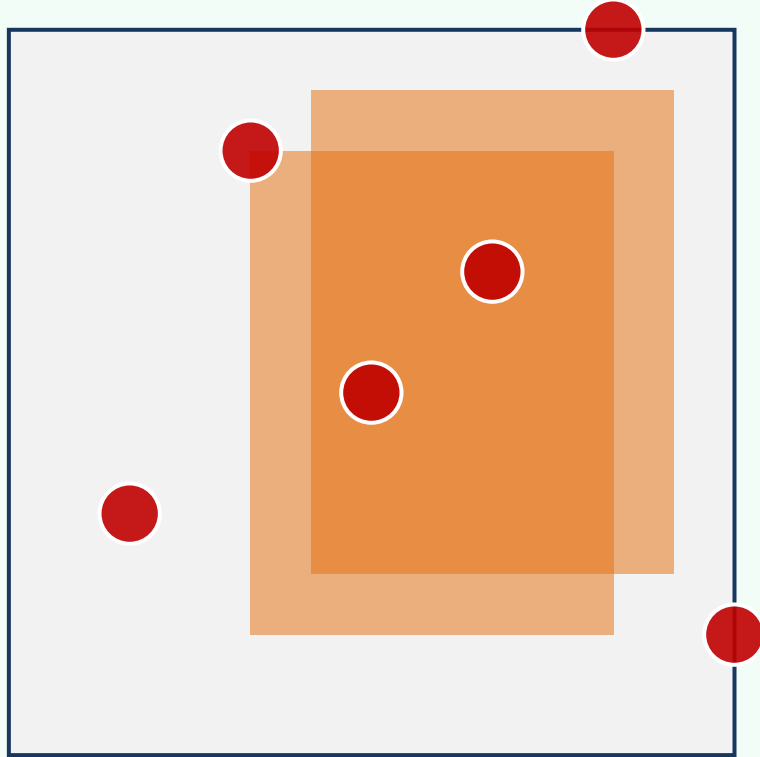
$$u \leq x \text{ and } v \leq y$$



Inclusion-Exclusion Principle

❖ Then for any rectangular range $\mathcal{R} = (x_1, x_2] \times (y_1, y_2]$, we have

$$|\mathcal{R} \cap \mathcal{P}| = n(x_1, y_1) + n(x_2, y_2) - n(x_1, y_2) - n(x_2, y_1)$$



Performance

- ❖ Each query needs only $\mathcal{O}(\log n)$ time
- ❖ Uses $\Theta(n^2)$ storage and even more for higher dimensions
- ❖ To figure out a better solution, let's go back to the **1D** case ...

