# 08-D5

# 二叉搜索树

## AVL树：(3+4)-重构

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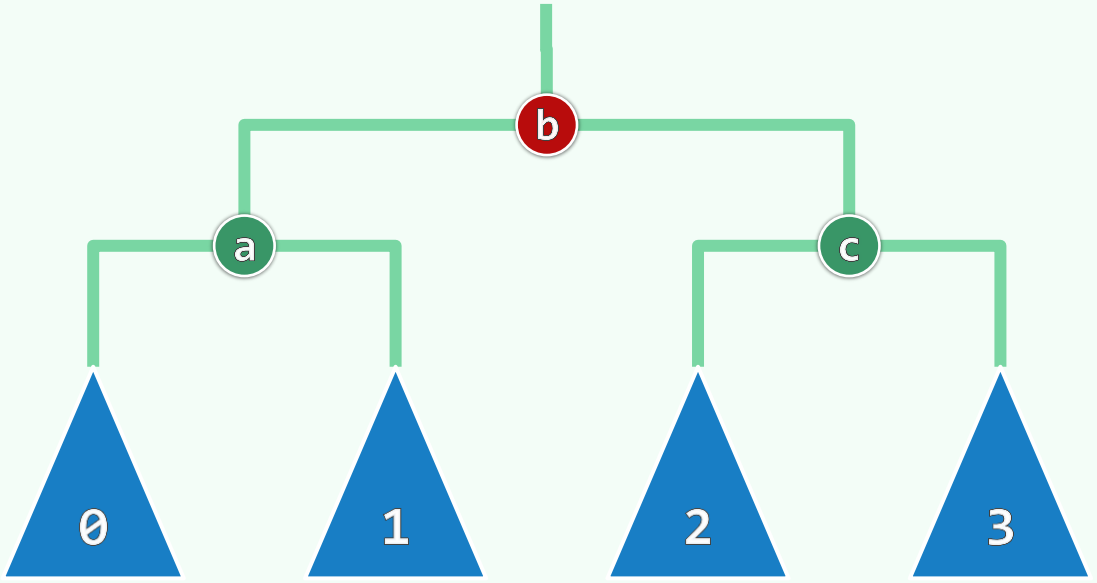
# 返璞归真

❖ 设g为最低的失衡节点，沿最长分支考察祖孙三代：g ~ p ~ v

按中序遍历次序，重命名为：a < b < c

❖ 它们总共拥有四棵子树（或为空）

按中序遍历次序，重命名为：$T_0$ < $T_1$ < $T_2$ < $T_3$




❖ 将原先以g为根的子树S，替换为一棵新子树S'

$$\mathrm{root}(S') = b$$

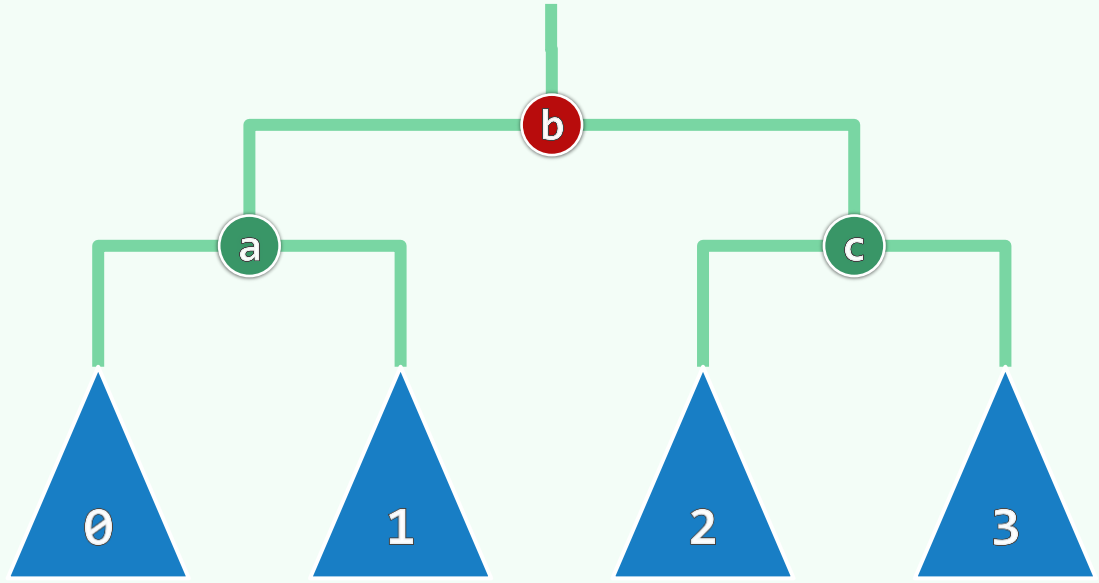
$$\mathrm{lc}(b) = a \qquad \mathrm{rc}(b) = c$$

$$\mathrm{lT}(a) = T_0 \quad \mathrm{rT}(a) = T_1 \quad \mathrm{lT}(c) = T_2 \quad \mathrm{rT}(c) = T_3$$

❖ 等价变换，保持中序遍历次序：$T_0 < a < T_1 < b < T_2 < c < T_3$

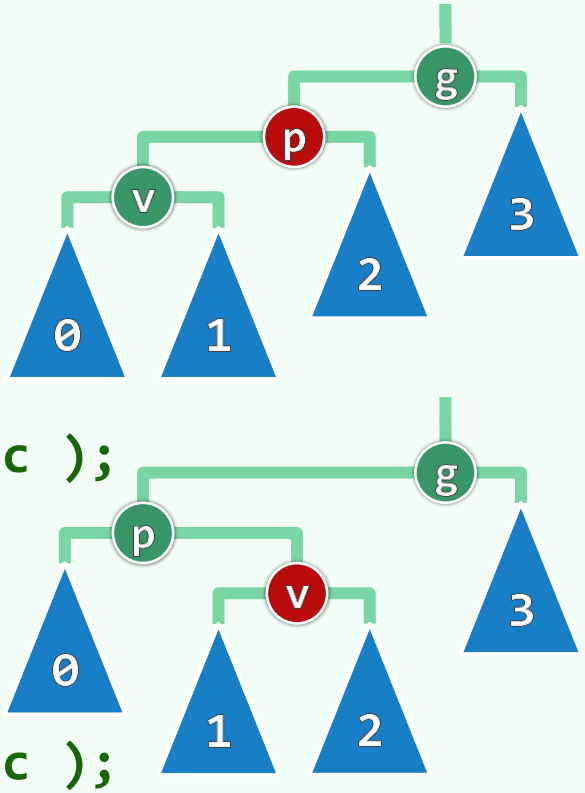
# 实现

```
template <typename T> BinNodePosi(T) BST<T>::connect34(
    BinNodePosi(T) a, BinNodePosi(T) b, BinNodePosi(T) c,
    BinNodePosi(T) T0, BinNodePosi(T) T1,
    BinNodePosi(T) T2, BinNodePosi(T) T3)
{
    a->lc = T0; if (T0) T0->parent = a;
    a->rc = T1; if (T1) T1->parent = a;
    c->lc = T2; if (T2) T2->parent = c;
    c->rc = T3; if (T3) T3->parent = c;
    b->lc = a; a->parent = b; b->rc = c; c->parent = b;
    updateHeight(a); updateHeight(c); updateHeight(b); return b;
}
```



# 统一调整

```
template<typename T> BinNodePosi(T) BST<T>::rotateAt( BinNodePosi(T) v ) {
   BinNodePosi(T) p = v->parent, g = p->parent;
   if ( IsLChild( * p ) ) //zig
      if ( IsLChild( * v ) ) { //zig-zig
         p->parent = g->parent; //向上联接
         return connect34( v, p, g, v->lc, v->rc, p->rc, g->rc );
      } else { //zig-zag
         v->parent = g->parent; //向上联接
         return connect34( p, v, g, p->lc, v->lc, v->rc, g->rc );
      }
   else { /*.. zag-zig & zag-zag ..*/ }
}
```



# AVL：综合评价

❖ 优点　无论查找、插入或删除，最坏情况下的复杂度均为$O(\log n)$

$O(n)$的存储空间

❖ 缺点　借助高度或平衡因子，为此需改造元素结构，或额外封装

实测复杂度与理论值尚有差距

- 插入/删除后的旋转，成本不菲
- 删除操作后，最多需旋转$\Omega(\log n)$次（Knuth：平均仅0.21次）
- 若需频繁进行插入/删除操作，未免得不偿失

单次动态调整后，全树拓扑结构的变化量可能高达$\Omega(\log n)$

❖ 有没有更好的结构呢？ //保持兴趣