14-C2

排序

希尔排序：Shell序列 + 输入敏感性

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Shell's Sequence

- Shell 1959：$\mathcal{H}_{shell} = \{ 1, 2, 4, 8, \ldots, 2^k, \ldots \}$
- 实际上，采用 $\mathcal{H}_{\text{shell}}$，在最坏情况下需要运行 $\Omega(n^2)$ 时间...
- 考查由子序列 A = unsort[0, 2^{N-1}) 和 B = unsort[2^{N-1}, 2^N) **交错**而成的序列

11 4 14 3 10 0 15 1 9 6 8 7 13 2 12 5

- 在做**2**-sorting时，A、B各成一列；故此后必然各自有序

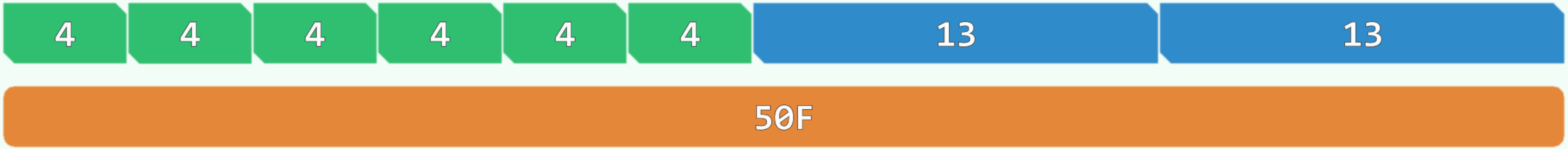
8 0 9 1 10 2 11 3 12 4 13 5 14 6 15 7

- 然而其中的逆序对依然很多，最后的**1**-sorting仍需$1 + 2 + 3 + \cdots + 2^{N-1} = \Omega(n^2/4)$ 时间
- 根源在于，$\mathcal{H}_{\text{shell}}$中各项并不**互素**，甚至**相邻项**也非互素

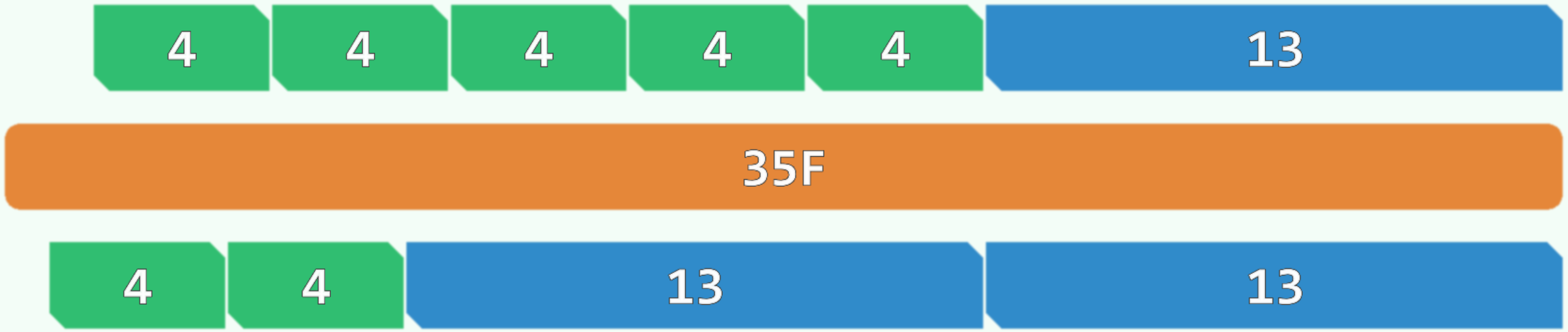
Postage Problem

❖ The postage for a letter is **50F**, and a postcard **35F**

But there are only stamps of **4F** and **13F** available



❖ Possible to stamp the letter and the postcard EXACTLY?



❖ Given a postage P , determine whether $P \in \{ n \cdot 4 + m \cdot 13 \mid n, m \in \mathcal{N} \}$

Linear Combination

❖ Let $g, h \in \mathcal{N}$

❖ For any $n, m \in \mathcal{N}$, $n \cdot g + m \cdot h$ is called a linear combination of g and h

❖ Denote $\mathbf{C}(g,h) \; = \; \{ \, ng + mh \mid n, m \in \mathcal{N} \, \}$

$\mathbf{N}(g,h) \; = \; \mathcal{N} \backslash \mathbf{C}(g,h)$ //numbers that are NOT combinations of g and h

$\mathbf{x}(g,h) \; = \; max\{ \, \mathbf{N}(g,h) \, \}$ //always exists?

❖ Theorem: when g and h are relatively prime, we have

$$\mathbf{x}(g,h) \; = \; (g-1) \cdot (h-1) - 1 \; = \; gh - g - h$$

e.g. $\mathbf{x}(3,7) \; = \; 11, \quad \mathbf{x}(4,9) \; = \; 23, \quad \mathbf{x}(\boxed{4}, \boxed{13}) \; = \; \boxed{35}, \quad \mathbf{x}(5,14) \; = \; 51$

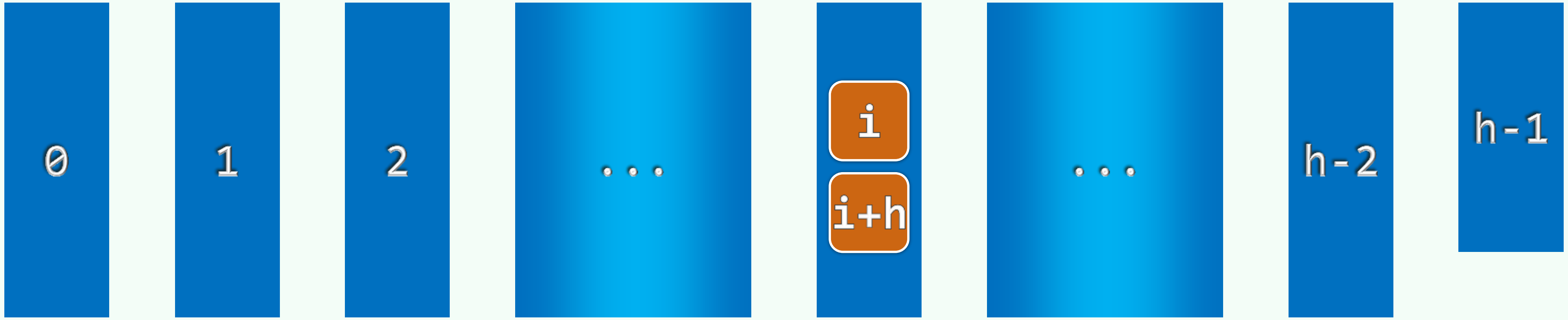
h-sorting & h-ordered

❖ A sequence S[0,n) is called **h**-ordered if $S[i] \le S[i+h],\ \forall\ 0 \le i < n-h$

❖ A 1-ordered sequence is sorted

❖ **h**-sorting: an h-ordered sequence is obtained by

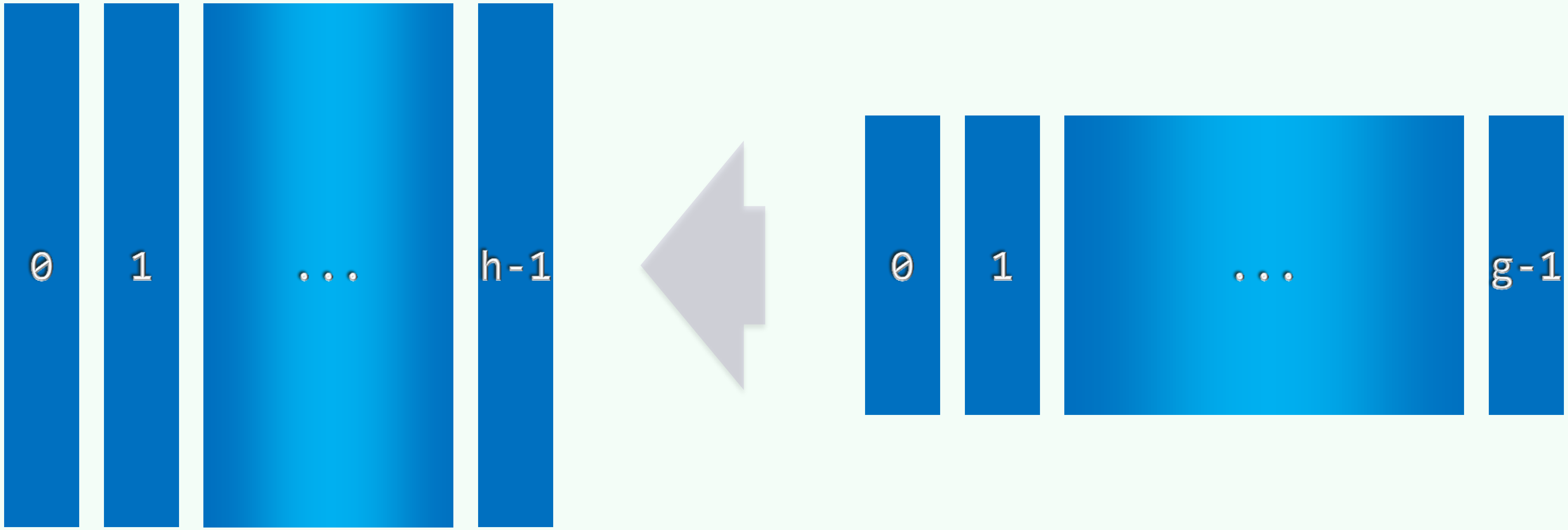
- arranging S into a 2D matrix with **h** columns and
- sorting each column respectively



Theorem K

❖ [Knuth, ACP Vol.3 p.90] //习题解析[12-12, 12-13]

A g-ordered sequence REMAINS g-ordered after being h-sorted.

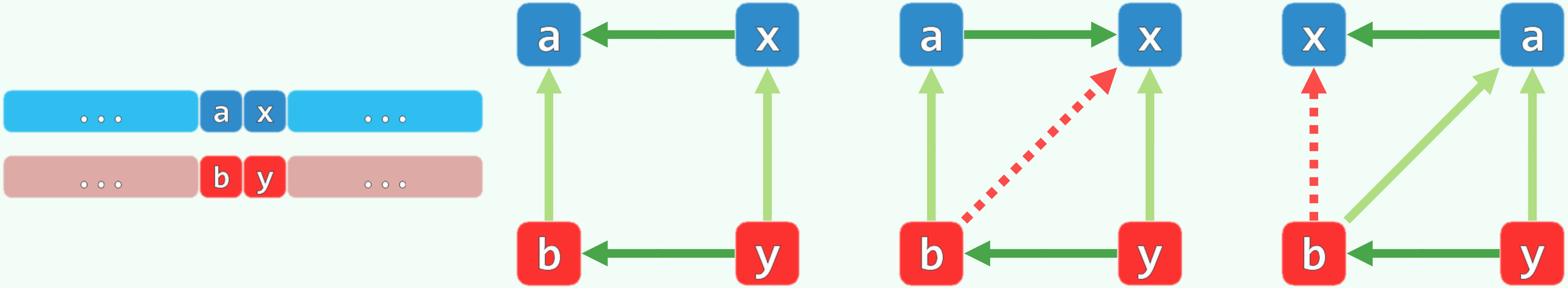


Order Preservation

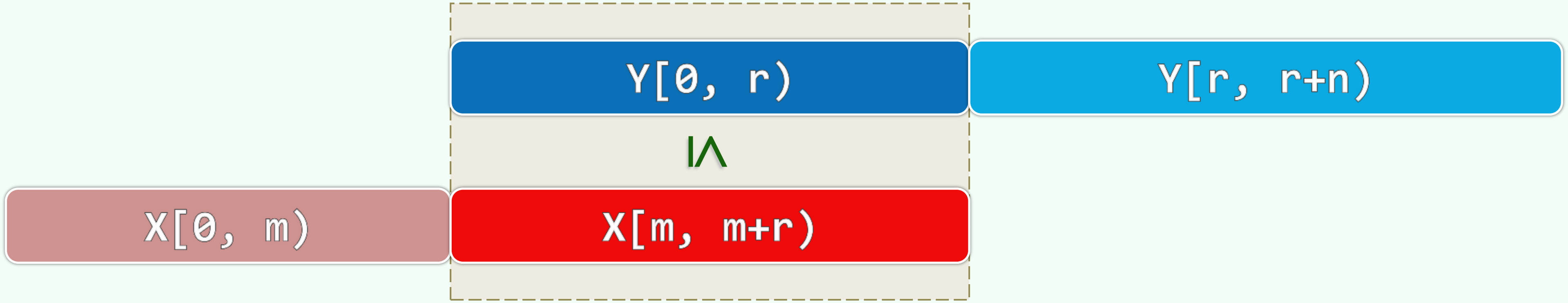
5	8	7	3	5
1	5	2	8	8
0	9	4	6	2
6	3	1	4	7

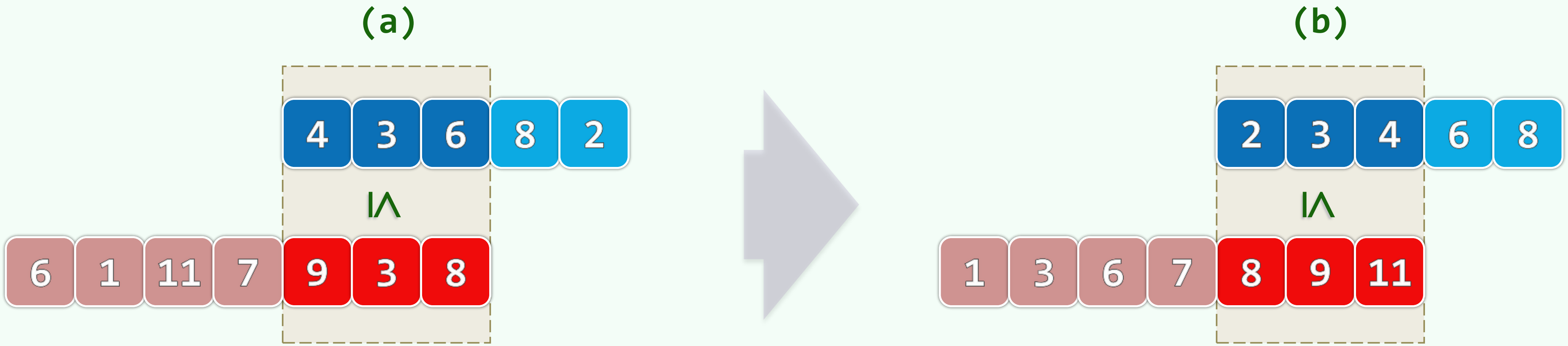
0	3	1	3	2
1	5	2	4	5
5	8	4	6	7
6	9	7	8	8

0	1	2	3	3
1	2	4	5	5
4	5	6	7	8
6	7	8	8	9



Lemma L



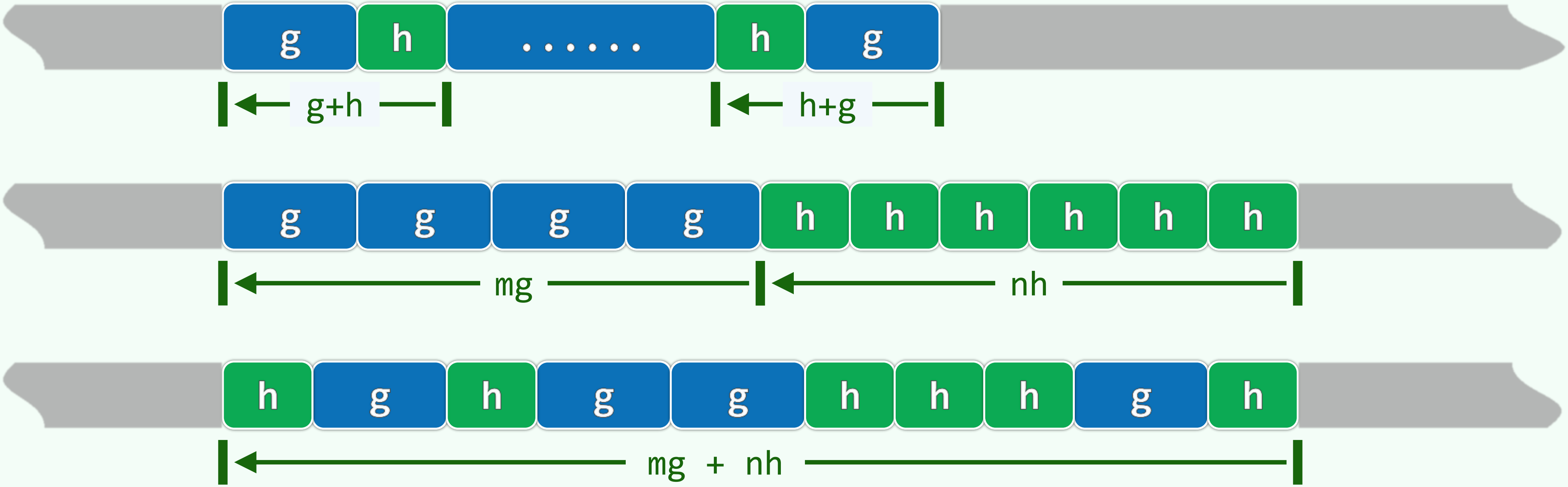


Linear Combination

❖ A sequence that is both g-ordered and h-ordered

is called (g,h)-ordered, which must be both

(g+h)-ordered and (mg+nh)-ordered for any m, n ∈ N



Inversion

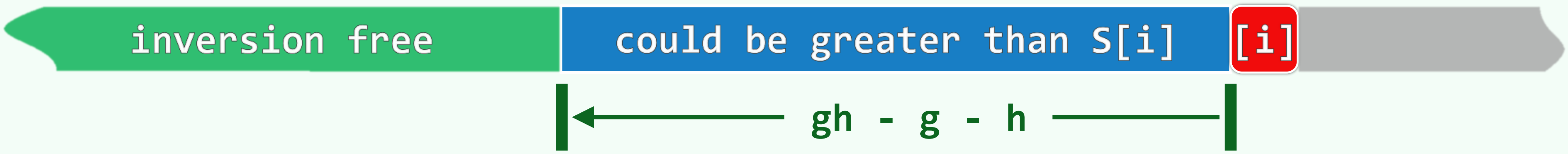
❖ Let S[0,n) be a (g,h)-ordered sequence, where g and h are **relatively prime**

❖ Then for all elements S[j] and S[i], we have

$$i - j \ > \ \mathbf{x}(g,h) \qquad \text{only if} \qquad S[j] \ \leq \ S[i]$$

❖ This implies that to the **LEFT** of each element,

only the previous $\mathbf{x}(g,h)$ elements could be **GREATER**



❖ There would be no more than $n \cdot \mathbf{x}(g,h)$ **INVERSIONs** altogether