

04-XB

栈与队列

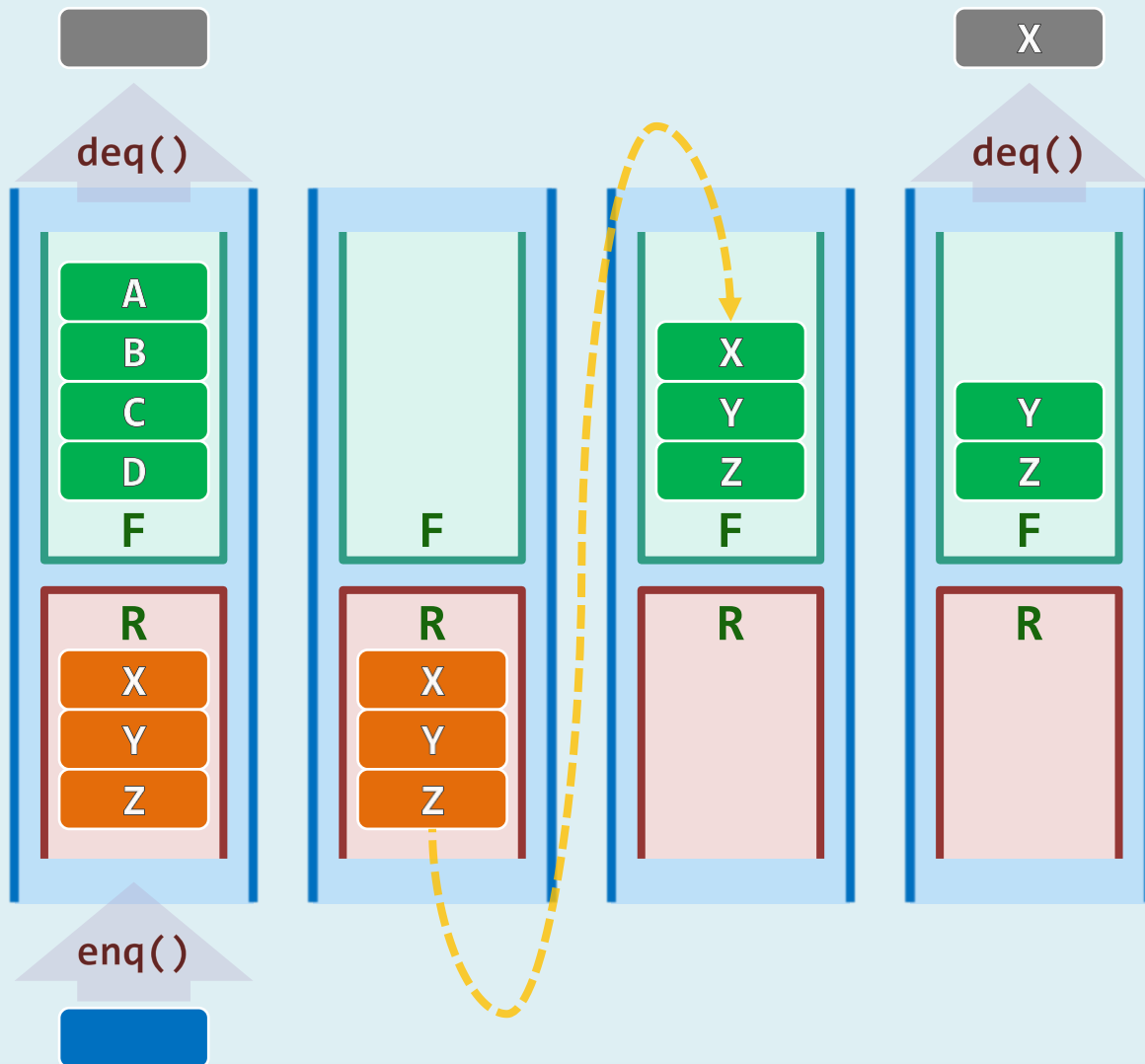
双栈当队

我们也不用当场付款，要了什么东西都由店家记在一个小账本上，每两星期结一次账。

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Queue = Stack x 2



❖ `def Q.enqueue(e)`

`R.push(e);`

❖ `def Q.dequeue() # 0 < Q.size()`

`if (F.empty())`

`while (!R.empty())`

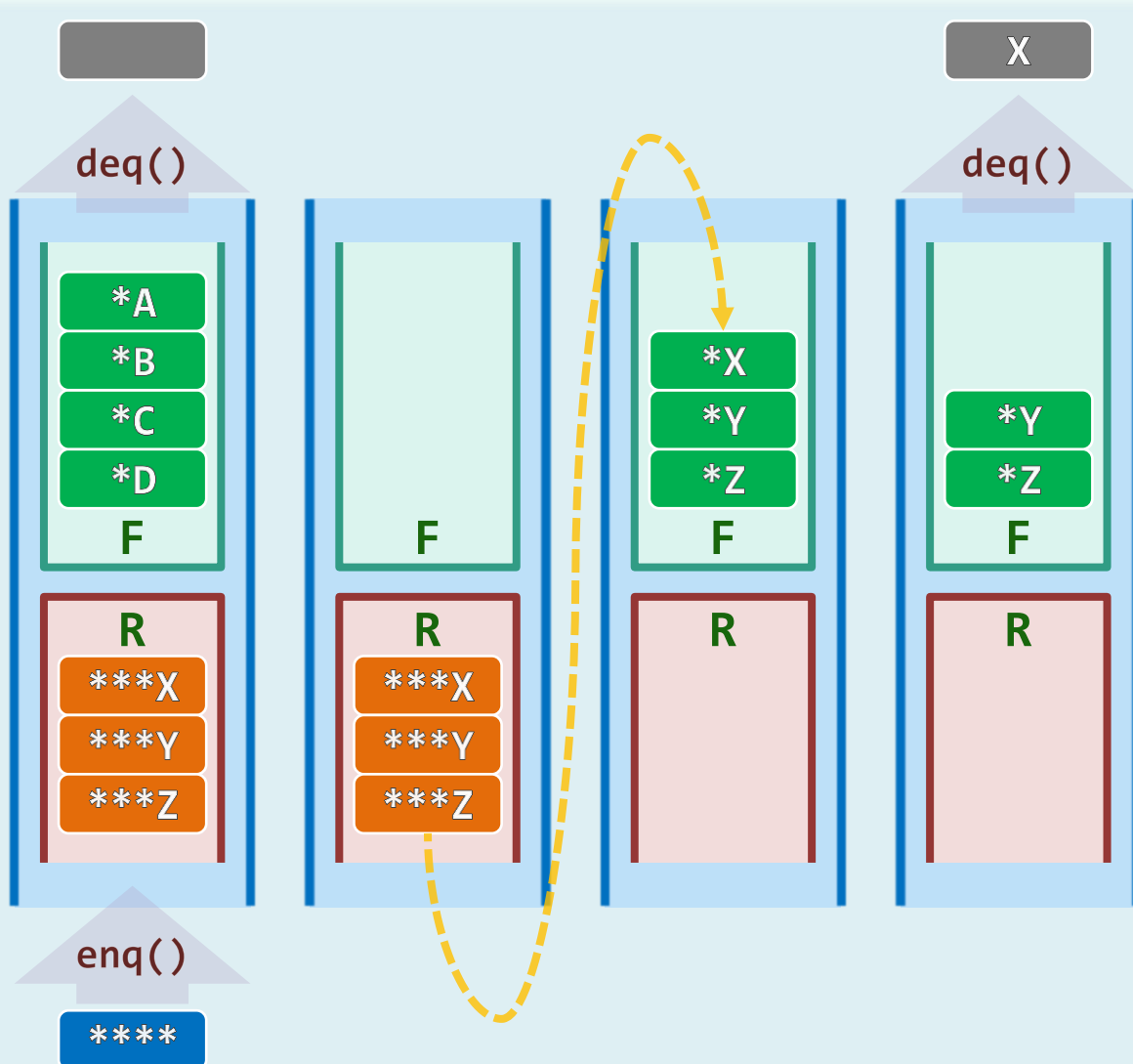
`F.push(R.pop());`

`return F.pop();`

❖ Best/worst case: $O(1)/O(n)$

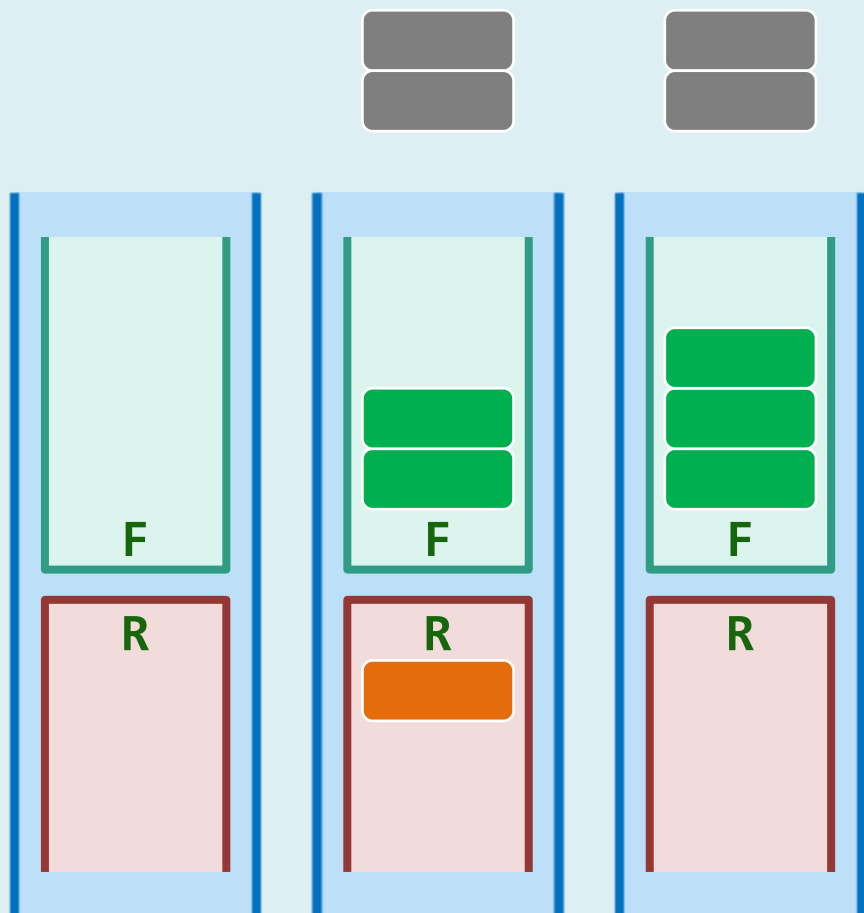
Average? Amortization!

Amortization By Accounting



- ❖ Assign each new element with **4 coins** **//deposit**
 - **1** for its **enqueue()**
 - **2** for transfer, and
 - the last **1** for **dequeue()**
- ❖ Hence every operation is **pre-paid** and ...
- ❖ The structure will never run out of credit
- ❖ **Amortized cost** of any operation sequence involving **n** ITEMS is $4n = O(n)$

Amortization By Aggregate



❖ Consider the moment when

d dequeue()'s and **e** enqueue()'s

have been done // $d \leq e$

❖ The time cost

for **ALL** the operations is

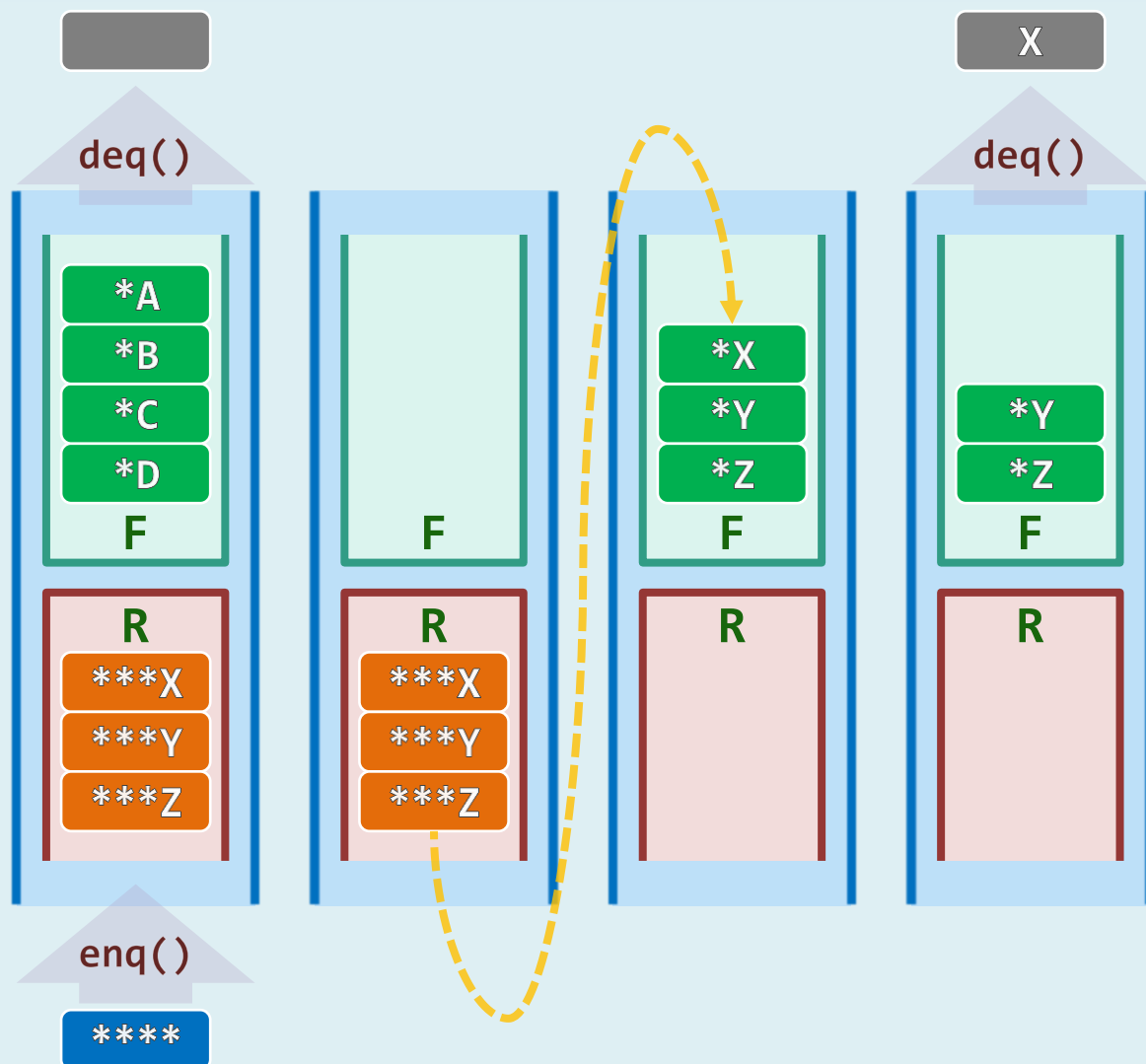
$$\leq 4 \cdot d + 3 \cdot (e - d) = 3e + d$$

❖ The amortized cost

for each OPERATION is

$$\frac{3e + d}{e + d} \leq 3$$

Amortization By Potential



❖ Consider the k^{th} operation

❖ Define

$$\Phi_k = |R_k| - |F_k|$$

❖ Then

$$A_k = T_k + \Phi_k - \Phi_{k-1} \equiv 2$$

❖ Hence

$$2n \equiv \sum_{k=1}^n A_k = \sum_{k=1}^n T_k + \Phi_n - \Phi_0$$

$$2n = T(n) + \Phi_n - \Phi_0 > T(n) - n$$

$$T(n) < 3n = \mathcal{O}(n)$$