

图

概述

06-A

黄药师道：“你拿了这图，到临安府找一家客店或是寺观住下，三月之后，我派人前来取回。图中一切，只许心记，不得另行抄录印摹。”

邓俊辉

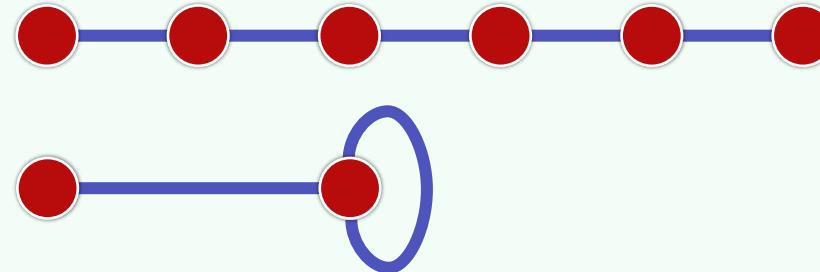
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基本术语

❖ $G = (V; E)$

vertex: $n = |V|$

edge|arc: $e = |E|$



❖ 同一条边的两个顶点，彼此邻接 (adjacency)

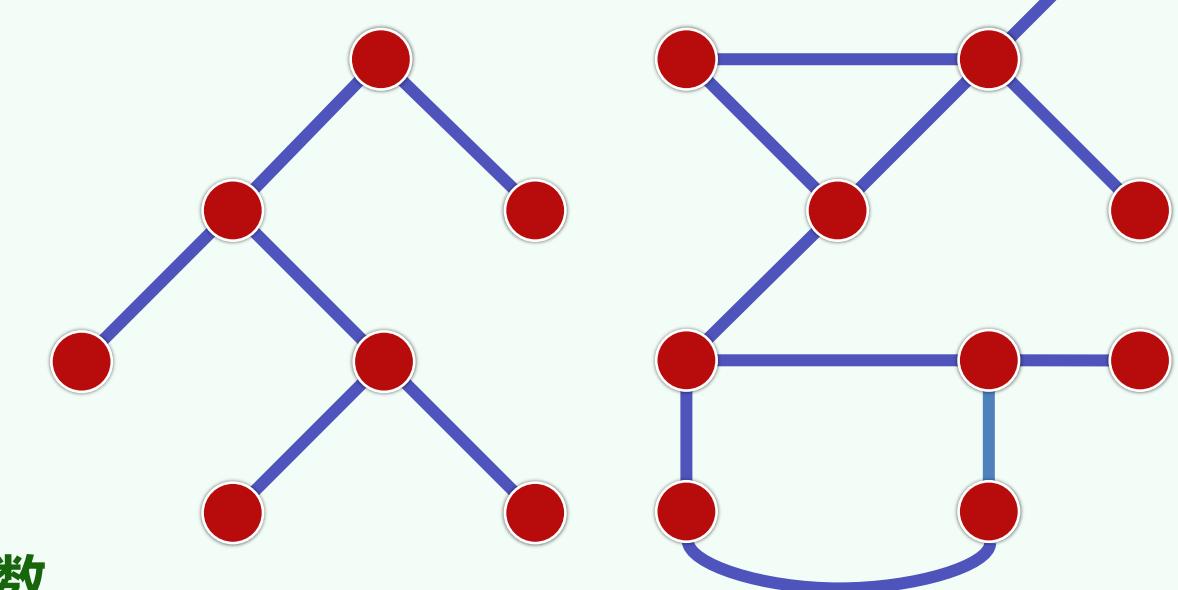
同一顶点自我邻接，构成自环 (self-loop)

不含自环，即为简单图 (simple graph)

非简单 (non-simple) 图，暂不讨论

❖ 顶点与其所属的边，彼此关联 (incidence)

度 (degree/valency) : 与同一顶点关联的边数

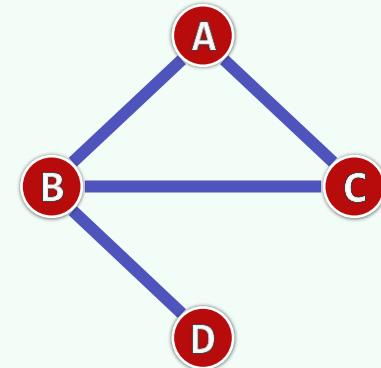


无向图 + 有向图

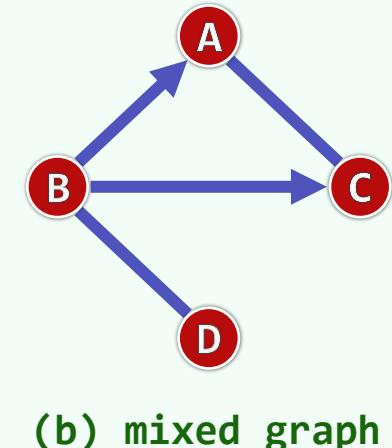
❖ 若邻接顶点 u 和 v 的次序无所谓

则 (u, v) 为无向边 (undirected edge)

❖ 所有边均无方向的图，即无向图 (undigraph)



(a) undigraph



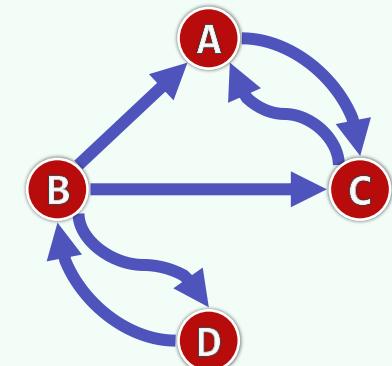
(b) mixed graph

❖ 反之，有向图 (digraph) 中均为有向边 (directed edge)

u, v 分别称作边 (u, v) 的尾 (tail)、头 (head)

❖ 无向边、有向边并存的图，称作混合图 (mixed graph)

❖ 有向图通用性更强，故本章主要针对有向图介绍相关结构及算法

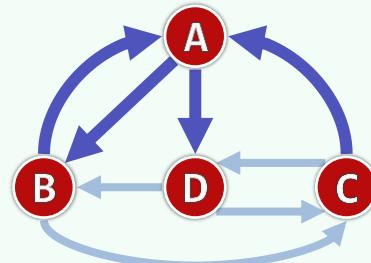


(c) digraph

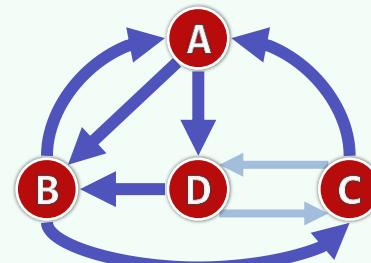
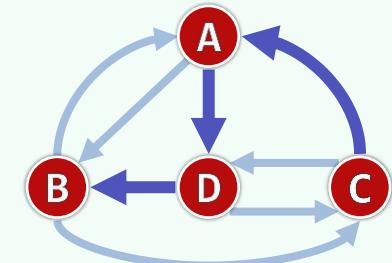
路径 + 环路

❖ 路径 $\pi = \langle v_0, v_1, \dots, v_k \rangle$

长度 $|\pi| = k$



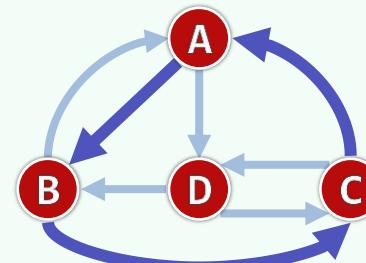
❖ 简单路径 : $v_i \neq v_j$ 除非 $i = j$



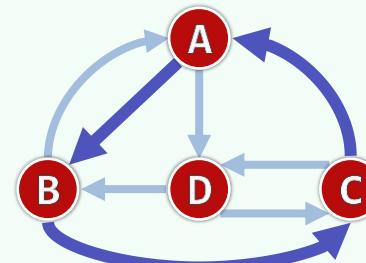
(i) path

(ii) simple path

❖ 环/环路 : $v_0 = v_k$



❖ 有向无环图 (DAG)

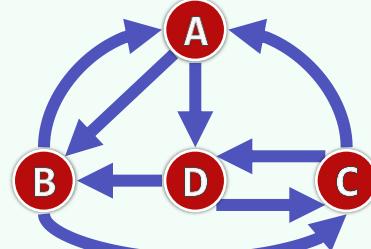


❖ 欧拉环路 : $|\pi| = |E|$

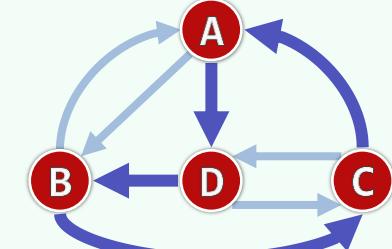
(i) cycle

(ii) simple cycle

各边恰好出现一次



(i) Eulerian tour



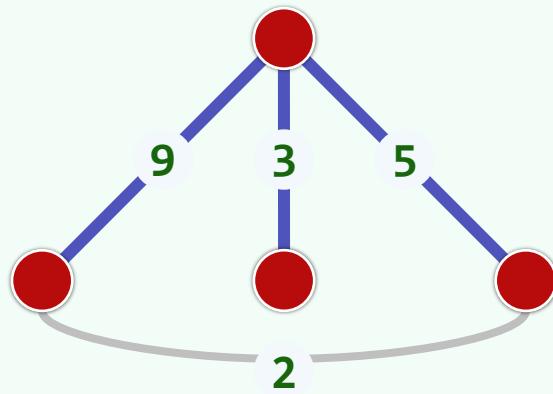
(ii) Hamiltonian tour

各顶点恰好出现一次

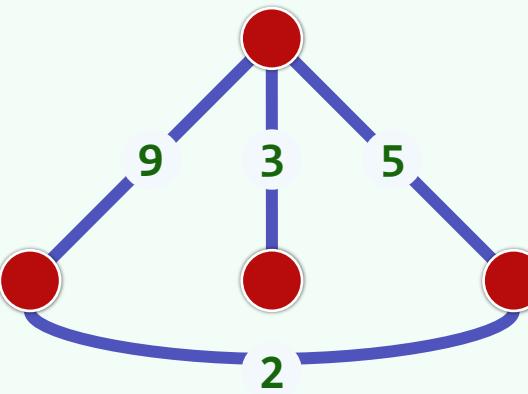
支撑树 + 带权网络 + 最小支撑树

❖ 图 $G = (V; E)$ 的子图 $T = (V; F)$ 若是树，即为其支撑树（ **spanning tree** ）

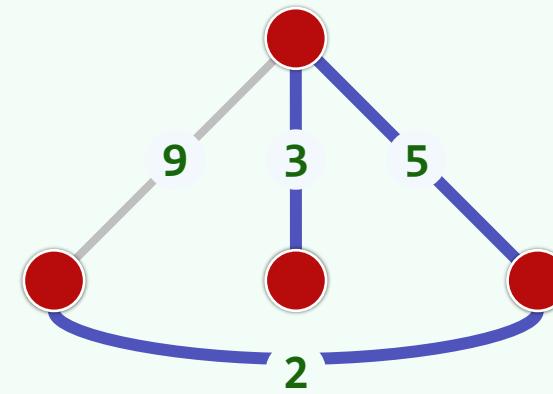
同一图的支撑树，通常并不唯一



spanning tree



weighted network
(triangle inequality?)



minimum spanning tree

❖ 各边 e 均有对应的权值 $\text{wt}(e)$ ，则为带权网络（ **weighted network** ）

❖ 同一网络的支撑树中，总权重最小者为最小支撑树（ **MST** ）