Finite Automaton Based Computer Game Model for Occupancy of Niujiu Card

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Abstract

Computer game is a carrier of artificial intelligence research. Niujiu Card as a highly complex card of computer game, the research of its game model can establish a sound theoretical foundation for the research of non-zero-sum card games with imperfect information. First, the components of card game model is given in this paper. Second, we propose a game model based on the finite Moore automaton for occupancy of Niujiu Card, and then present an IMP-minimax based search algorithm. Finally, we design and implement the game algorithm, which simulates the process of minimal occupancy and optimal occupancy of the first player. Experimental results show that the proposed model and algorithm are feasible and effective.

Keywords: Finite automaton; Niujiu card; computer game; IMP-minimax algorithm; evaluation function

1. Introduction*

Computer game is an important branch of artificial intelligence. Its main domain targets at how to enable computer to simulate the thinking of human brains. Through studying computer game, people have obtained a large number of achievements. These achievements have had extensive influence to other fields [1-5]. It is therefore believed that computer game, like Drosophila that is known as an optimal carrier of the research of genetics, is now known as a carrier of the research of artificial intelligence [6-8].

However, the current study on computer game, either at home or abroad, is mostly focused on search algorithm and evaluation function [9-12]. In addition, the researchers are much focused on two-player zero-sum games with perfect information [12-17], whereas very few work on multi-player non-zero-sum games with imperfect information. Most of card games are belongs to multi-player non-zero-sum games with imperfect information. Relative research of card game is less optimal and deeper than board game. It is therefore very meaningful to conduct the research of multi-player non-zero-sum games with imperfect information.

Finite automaton is a dynamically and systematically mathematical model of discrete events. At present, automaton based computer game has got a large number of achievements [18-20]. These achievements provide a good reference for modelling of automaton in computer game. Niujiu card is an ancient card game derived from northwestern China. Up to now, Niujiu card has a history of about 1000 years. Its logic of occupancy is complex and flexible. Niujiu card belongs to multi-player non-zero-sum games with imperfect information. By studying Niujiu card game model, a sound theoretical foundation could be established for its research, and then a good computer game theoretical model could be presented. This paper first describes a model for the analysis of three-player card game and provides formal definitions of card game model. Secondly, we build a theoretical game model based on Moore finite automaton for occupancy of Niujiu Card. Because IMP-minimax based search algorithm can provide a timely linear strategy of the search tree for non-zero-sum games with imperfect information [20, 21], we present an IMP-minimax algorithm-based search method for special card types. Finally, we realize the gaming process of minimal occupancy and optimal occupancy of the first player and verify the feasibility and effectiveness of the model.

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2. Composition of Card Game Model

Computer game mainly refers to the

everyday board game and card game completed by computer. Thanks to its clear rules and typical process, both board game and card are highly practical to play on computer [23, 24].

For games such as chess, Gobang, Chinese chess, they are all belongs to the two-player zero-sum game with perfect information. A systematic computer game model for two-player board game is given in [23]. On the other hand, bridge and poker games belong to the multi-player games with imperfect information. This paper has mainly established the computer game modelling system for analysis of three-player card games, which represent all the modelling systems quoted hereafter.

Definition 1 (Card game model) A card game model is defined as 7-tuple *{Ha, Hb, Ri, C, S, y, W}*, where

(1)*Ha* represents the holding card type that means the sum of the holding card types of all players in the game. Ha is invisible to other players, describing a state of unplaying cards.

(2)Where *Hb* represents the inholding card type that means the sum of the un-holding card types of all players in the game. Hb is visible to other players, describing a state of playing cards.

(3) *Ri (i= 1, 2, 3)* represents the game rules composed by 4-tuple {d, p, t, I}, in which d represents the playing sequence of players, p represents play method. *t* represents time limit, and I represents game information. The symbol => is used to describe derivation process. If formula *R1: p=>d* was workable, formula *R2: d=>t* would be workable. If formula *R2: d=>t* were workable, formula *R3: t=>I* would be workable. These rules ensure gaming process for every player are equal and reasonable. Furthermore, through game rules, program can also generate all available card types.

(4) *y* represents evaluation functions that, combined with searching algorithm, can evaluate current situation in order to select optimal method of play. Currently there exist such methods that genetic algorithms is applied to evaluation function [25].

(5) *S* represents the search technology composed by 4-tuple *{e,s,v,o}*, where e represents card type that needs to be searched, s represents search algorithm. IMP-minimax algorithm or Monte Carlo algorithm is generally used to search [17]. *v* represents verification function used to verify the correctness of the search results, and o represents card types that need output and selects optimal playing strategies at end of the searching by search tree.

(6)Where *C* represents controller composed by 3-tuple *{C_1, C_2, C_3}*, where C_1 represents initial stage that is mainly about initializing required functions and parameters. C_2 represents implementation stage that is mainly about selecting all available card types and C_3 represents authentication stage. If C_2 were workable, C_3 would pass available card types to evaluation function.

(7) *W* represents game interface.

The following is the analysis on the computer modelling of section (1)-(6).

3. Finite Automaton Based Card Game Model

Finite automaton is a mathematic model with discrete I/O system. It has any limited number of internal states in order to record information about input last. According to current information input, finite automaton can be set for next state and behaviour. In card game model, the state of play is finite and enumerated. Therefore, finite automaton can be used to model.

In computer game, every player needs to change his or her strategies of play. In order to adapt to this change, Moore automaton-based computer game is chosen as the core to model. The model as shown in Figure 1 is therefore proposed.

As shown in Figure 1. C represents a controller of all player's card type creator that is used to identify all card types of players. *x1* represents output of available card type detected by response set. *y1* represents the detection function as output of card type to adjust corresponding card type. *x2* represents card type input from M1 to evaluation function, and *y2* represents feedback information output from evaluation to M1.

Evaluation function, as the core of entire model, on the one hand, accepts the output of card type from M1 and transfers evaluated information to M1 and on the other hand, transfers evaluated results x3 to Moore named *M'* that transfers the search results y3 to evaluation function.

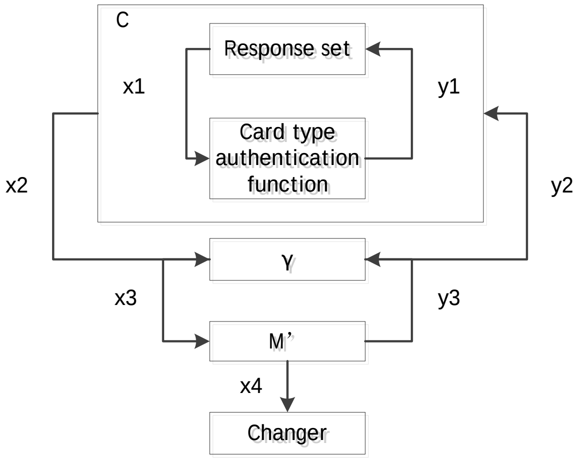

Figure 1. Moore automaton-based computer game model

M' represents Moore automaton of card type of Player p whose structure is shown in Figure2.

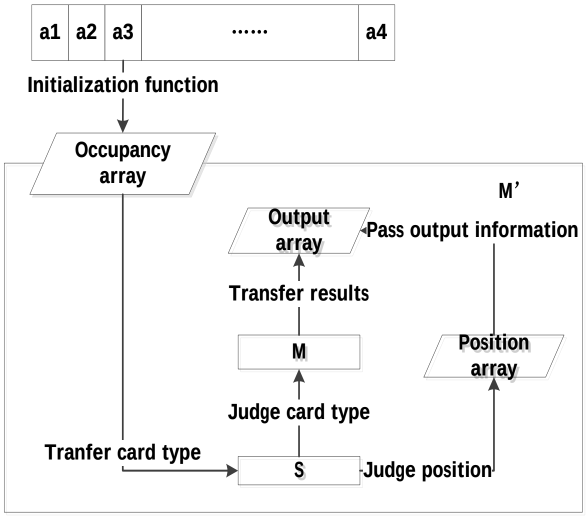

Figure 2. Structure of Moore automaton

As shown in figure 2, M is a finite automaton. *{a1,a2,a3,……,an}* represents a read-only tape acquires available card type from evaluation function. When finished initializing, occupancy array saves every card number in order. When finished saving, card number information is transferred to algorithm of card type recognition. M' utilizes search technology Sto recognize card type. Afterward, M' gets available card type. If M' recognized card types, which needs to be playing, M' transfer position of card number to occupancy array. Meanwhile, recognized card types are transferred to finite automaton. After finite automaton transits states, finite automaton transfers results to output array. Finally, results are outputted on interface.

Finite automation *M* of computer game player P can be expressed by a 5-tuple $M=\{A,\Sigma,f,B,Z\}$, where

A represents a finite set that a player can achieve and $A=\{A_1,A_2,A_3,A_4……A_n\},A_i \in \mathbb{A}$.

B is a non-null initial state, Z is a terminal state set, both of which contain $B \in \mathbb{A}, Z \in \mathbb{A}$.

Σ is a finite elemental set and f is a state transition function. $f:A_i \times \Sigma \rightarrow A_j$.

After sending information to M and that information is valid, evaluation function will then transfers the results x4 to the converter that will convert the results into user-friendly and identifiable language and finally send the language to interface as feedback to players.

4. Terms of Niujiu card

4.1 Formulas and definition

Niujiu card is composed by one pack of poker excluding the two jokers and the card of No. 4 and. To decribe the terms of Niujiu cards , the following definitions are given.

Definition 2: (Card mark x∏) In a pack of poker, card mark is used to distinguish big and small of card. x is a notation on top left corner and lower right corner ($x \in \{1,2,3,5,6,7,8,9,10,J,Q,K\}$), ∏ is card number mark.

Definition 3 (Card number symbol) Card number symbol used to determine the number of card mark in a pair of poker whose left part is number variable, represented by *n, m and r* and the right part of which is marked card number.

Definition 4 (Card number expression) Card number expression is an expression with number variable and a complete expression of marked card number and card number. For example, card number expression $n \rightarrow 1\Pi$ represents player holds n pieces of cards marked as 1∏.

Definition 5 (Suit symbol) If suit needs to be distinguished, suit will add – or + to the front of card number expression. Suit symbols represent the number of a certain card suite, of either red or black. For example, the card number expression $-n \rightarrow 1\Pi$ represents the number of black cards marked with 1∏is n.

Definition 6 (Card type φ) A set composed by designated same or different card number that has different number denoted as φ.

4.2Terms of Niujiu card

According to above definitions , here are the terms of Niujiu card.

(1)Niu card: card mark is 9Π, If formula $\varphi_1=\{9\Pi \mid n\rightarrow 9\Pi \wedge n=1\}$ were workable, then it would be called Niu card. A Niu card is equal of a 9 card. However, if formula $\varphi_{1a}=\{9\Pi \mid (-n\rightarrow 9\Pi \wedge n=1) \wedge (+m\rightarrow 9\Pi \wedge m=1)\}$ were workable, then it would be called Two-Niu Card. Two-Niu Card can defeat any pair except Tian Card and Xi Card. If formula $\varphi_{1b}=\{9\Pi \mid n\rightarrow 9\Pi \wedge n=3\}$is workable, then it were called Three-Niu Card. Three-Niu Card can defeat any triple except Tian Card and Xi Card.

(2)Xi card: card mark is 9Π. If formula $\varphi_2=\{5\Pi \mid n\rightarrow 5\Pi \wedge n=1\}$ were workable, then it were called Xi card. A Xi card is equal of a 5 card. However, if formula $\varphi_{2a}=\{5\Pi \mid (-n\rightarrow 5\Pi \wedge n=1) \wedge (+m\rightarrow 5\Pi \wedge m=1)\}$ were workable, then it were called Two-Xi Card. If formula $\varphi_{2b}=\{5\Pi \mid n\rightarrow 5\Pi \wedge n=3\}$is workable, then it is called Three-Xi Card. Two-Xi Card and Three-Xi Card cannot be defeated and defeat.

(3)Liangzi: card mark is $x\Pi$. If formula $\varphi_3=\{x\Pi \mid n\rightarrow x\Pi \wedge n=4\}$ were workable, then it were called Liangzi.

(4)Fish card: card mark are $1\Pi, 2\Pi$ and 3Π. If formula $\varphi_4=\{1\Pi, 2\Pi, 3\Pi \mid n\rightarrow 1\Pi \wedge m\rightarrow 2\Pi \wedge r\rightarrow 3\Pi \wedge n=m=r=1\}$ were workable, then it would be called Fish card. Fish card only can be defeated by Bai card. In a same way, $2\varphi_4$ were called Two-Fish card. $3\varphi_4$ were called Three-Fish card. $4\varphi_4$ were called Four-Fish card. If formulas $\varphi_{4a}=\{1\Pi, 2\Pi, 3\Pi \mid n\rightarrow 1\Pi \wedge m\rightarrow 2\Pi \wedge r\rightarrow 3\Pi \wedge n=m=1 \wedge r=4\}$, $\varphi_{4b}=\{1\Pi, 2\Pi, 3\Pi n\rightarrow 1\Pi \wedge m\rightarrow 2\Pi \wedge r\rightarrow 3\Pi \wedge n=r=1 \wedge m=4\}$, $\varphi_{4c}=\{1\Pi, 2\Pi, 3\Pi \mid n\rightarrow 1\Pi \wedge m\rightarrow 2\Pi \wedge r\rightarrow 3\Pi \wedge r=m=1 \wedge n=4\}$ were workable, then $_{4a} \vee \varphi_{4b} \vee \varphi_{4c}$ would be called Yuliang card. Yuliang card only can be defeated by Bailiang card.

(5)Bai card: card mark are $8\Pi, 10\Pi$ and $K\Pi$. If formula $\varphi_5=\{8\Pi, 10\Pi, K\Pi \mid n\rightarrow 8\Pi \wedge m\rightarrow 10\Pi \wedge r\rightarrow K\Pi \wedge n=m=r=1\}$ were workable, then it would be called Bai card. Bai card only can defeat Fish card. Likewise, $2\varphi_5$ would be called Two-Bai card. $3\varphi_5$ would be called Three-Bai card. $4\varphi_5$ would be called Four-Bai card. If formulas $\varphi_{5a}=\{8\Pi, 10\Pi, K\Pi \mid n\rightarrow 8\Pi \wedge m\rightarrow 10\Pi \wedge r\rightarrow K\Pi \wedge n=m=1 \wedge r=4\}$, $\varphi_{5b}=\{8\Pi, 10\Pi, K\Pi \mid n\rightarrow 8\Pi \wedge m\rightarrow 10\Pi \wedge r\rightarrow K\Pi \wedge n=r=1 \wedge m=4\}$, $\varphi_{5c}=\{8\Pi, 10\Pi, K\Pi \mid n\rightarrow 8\Pi \wedge m\rightarrow 10\Pi \wedge r\rightarrow K\Pi \wedge r=m=1 \wedge n=4\}$ were workable, then $\varphi_{5a} \vee \varphi_{5b} \vee \varphi_{5c}$ would be called Bailiang card. Bailiang card only can defeat Yuliang card. Specially, if formula $\varphi_{5d}=\{8\Pi, 10\Pi, K\Pi \mid n\rightarrow 8\Pi \wedge m\rightarrow 10\Pi \wedge r\rightarrow K\Pi \wedge n=m=1 \wedge (r=1 \vee r=2 \vee r=3 \vee r=4)\}$ were workable, all card type of this formula would be available card type.

(6)Xibaoliang card: If formula $\varphi_6=\{\varphi_2, \varphi_3 \mid (\varphi_2 \wedge \varphi_3) \vee (\varphi_3 \wedge \varphi_2)\}$ were workable, then it would be called Xibaoliang card. Xibaoliang Card only can be defeated by Xibaoliang card.

(7)Tian card: card mark is $K\Pi$. If formula $\varphi_7=\{K\Pi \mid n\rightarrow K\Pi \wedge n=1\}$ were workable, then it were called Tian card. Tian card cannot be defeated. In a same way, $2\varphi_7$ were called Two-Tian card. $3\varphi_7$ were called Three-Tian card. $4\varphi_7$ were called Four-Tian card. Specially, whatever Four-Tian card is played or not played, Four-Tian card scored as Liangzi.

(8) Hu card: card mark is $Q\Pi$. If formula $\varphi_8=\{Q\Pi \mid n\rightarrow Q\Pi \wedge n=1\}$ were workable, then it were called Hu card. Hu card only can be defeated by Tian card and Niu card. In a same way, $2\varphi_8$ were called Two-Hu card. $3\varphi_8$ were called Three-Hu card. $4\varphi_8$ were called Four-Hu card.

(9)Third card is equal to Sixth card: card mark are $3\Pi$and 6Π, If formulas $\varphi_{9a}=\{3\Pi \mid n\rightarrow 3\Pi \wedge n=1\}$ and $\varphi_{9b}=\{6\Pi \mid n\rightarrow 6\Pi \wedge n=1\}$ were workable, then $\varphi_{9a}=\varphi_{9b}$ is workable. This formula also applies to $2\varphi_{9a}=2\varphi_{9b}, 3\varphi_{9a}=3\varphi_{9b}, 4\varphi_{9a}=4\varphi_{9b}$.

(10)Hua-tenth card is equal to Mo-tenth card: card mark are 10Π and $J\Pi$, If formulas $\varphi_{10a}=\{10\Pi \mid n\rightarrow 10\Pi \wedge n=1\}$ and $\varphi_{10b}=\{J\Pi \mid n\rightarrow J\Pi \in n \wedge n=1\}$ were workable, then $\varphi_{10a}=\varphi_{10b}$ is workable. This formula also applies to $2\varphi_{10a}=2\varphi_{10b}, 3\varphi_{10a}=3\varphi_{10b}, 4\varphi_{10a}=4\varphi_{10b}$.

(11)Common card: If formula $P=\{x\Pi \mid, n\rightarrow x\Pi \wedge (n=1 \vee n=2 \vee n=3 \vee n=4)\}$is existed, then formula $P\rightarrow Q(Q \not\subset (\varphi_1 \vee \varphi_2 \vee \varphi_3 \vee \varphi_4 \vee \varphi_5 \vee \varphi_6 \vee \varphi_7 \vee \varphi_8), Q=\{K\Pi > Q\Pi > J\Pi = 10\Pi > 9$

*Π>8 Π>7 Π>6 Π>5 Π>3 Π>2 Π>1 Π}*were called Common card.

5. Implementation of Finite Automaton Based Computer Game Model for Occupancy of Niujiu Card

5.1 Building of finite automaton M

When each player finishes dealing, each player holds 16 cards. According to the rules, the final limit of occupancy cards is 1/3 of total cards. It is also 16 cards To maintain the rules that only two players have enough occupancy of cards at most, every player occupy 6 cards is accepted as a final state. Considering cards number for occupancy may vary with different rounds, only optimal occupancy, i.e. a available occupancy of six cards of first player is discussed herein.

Available occupancy of six cards strategy refers to accomplish initial card occupancy with the six cards with absolute biggest numbers to allow players to have maximum room to play further. Computer automaton process is as follows:

(1) According to the number of occupancy, M confirms procession state of first player:

A_0: No valid cards available on play table.

A_1: One first player valid card available on play table.

A_2: Two first player valid cards available on play table.

A_3: Three first player valid cards available on play table.

A_4: Four first player valid cards available on play table.

A_5: Five first player valid cards available on play table.

A_6: Six first player valid cards available on play table.

(2) According to standard card type of occupancy, M confirms finite automaton alphabet. Notation and model of standard card type utilize what be set before. Playing card numbers are listed in brace regardless of sequence.

φ_{1a}:Two-Niu card, formula is {-9Π,+9Π}.

φ_{1b}:Three-Niu card, formula is {-9Π,+9Π,+9Π} or {-9Π,-9Π,+9Π}.

φ_{2a}:Two-Xi card, formula is {-5Π,+5Π}.

φ_{2b}:Three-Xi card, formula is {-5Π,+5Π,+5Π} or {-5Π,-5Π,+5Π}.

φ_{3}:Liangzi card, formula is {xΠ,xΠ,xΠ,xΠ}.

φ_{4}:Fish card: formula is {1Π,2Π,3Π}.

φ_{4abc}:Yuliang card, formula is{1Π,1Π,1Π,1Π,2Π,3Π} or {1Π,2Π,2Π,2Π,2Π,3Π} or {1Π,2Π,3Π,3Π,3Π,3Π}.

φ_{5}:Bai card, formula is {8Π,10Π,KΠ}.

φ_{5abc}:Bailiang card, formula is {8Π,8Π,8Π,8Π,10Π,KΠ} or {8Π,10Π,10Π,10Π,10Π,KΠ} or {8Π,10Π,KΠ,KΠ,KΠ,KΠ}.

φ_{6}:Xibaoliang card, formula is {+5Π,-5Π,xΠ,xΠ,xΠ,xΠ}.

φ_{7}:Tian card, formula is {KΠ}.

φ_{8}:Hu card, formula is {QΠ}.

(3)According to finite automaton and character of alphabet, M designs equivalent substitution as following.

Let a= φ_{7}

Let b= $\varphi_{1a} \mid \varphi_{2a}$

Let c= $\varphi_{1b} \mid \varphi_{2b} \mid \varphi_{5}$

Let d= φ_{3}

Let e= $\varphi_{4abc} \mid \varphi_{5abc} \mid \varphi_{6}$

(4)Finite Automaton M, as shown in figure 3.

State function is used to determine the process of state transformation function. Therefore, automaton enumerates transformation function from A0 to A6. The card type of state transformation function includes Niu card, Bai card and Xi card. If any card type accords with state transformation function, state transformation function will start to skip. If none of card type accords with state transformation function, computer output "Cannot form standard cad type".

In finite automaton M, the process of state tranformation function is as follows:

s1. According to the player's card type, initial state autocalls proper state transformation function, which includes three types named Bai card, Niu card and Xi card. According to different card type, automaton will execute state-jump.

s2. When automaton jumps to next state, automaton will continue to execute state-jump until automaton arrives at terminal.

s3. If none of card types accords with state transformation function, computer will output "Cannot form standard card type".

5.2 Algorithm design of available card type

Card type recognition algorithm of finite controller is used to judge conditional transfer of available card type. Taking Bai card as example, to mark played card in a pair of card, controller defines an array whose size is 16 and value of every position is 0. If the position of this array are occupied, controller changes value of corresponding position to 1. In the process of judging Bai card, whether 8∏,10∏,K∏ is coexist depends on a number parameter λ that adds 1 when getting one card type mentioned before. When the value of λ is 3, controller saves results to output array. Meanwhile, according to occupied number skip to next state. If the value is not equal 3, controller saves card type K∏ in output array. Meanwhile, according to occupied number skip to next state shown in figure 4.

Niu card and Xi card remainder algorithm utilizes process of number remainder to stimulates different situation. Taking Niu card as example, a parameter μ is defined to receive value. When μ meets +9∏, value equals +7. When μ meets -9∏, value equals 9, which results in the following situations.

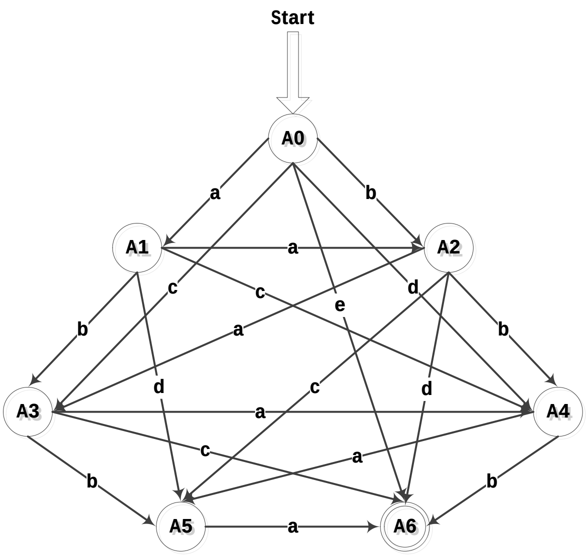

Figure 3. Finite automation in occupancy of six cards state transformation

$\varphi_1=\{9\Pi \mid n\rightarrow 9\Pi \wedge n=1\}$

$\varphi_{1\text{-}1}=\{9\Pi \mid n\rightarrow 9\Pi \wedge n=-1\}:\mu\%10=9$

$\varphi_{1\text{-}2}=\{9\Pi \mid n\rightarrow 9\Pi \wedge n=-2\}:\mu\%10=8$

$\varphi_{1\text{-}3}=\{9\Pi \mid n\rightarrow 9\Pi \wedge n=+1\}:\mu\%10=7$

$\varphi_{1\text{-}4}=\{9\Pi \mid n\rightarrow 9\Pi \wedge n=+2\}:\mu\%10=4$

$\varphi_{1a}=\{9\Pi \mid (-n\rightarrow 9\Pi \wedge n=1) \wedge (+m\rightarrow 9\Pi \wedge m=1)\}:\mu\%10=6$

$\varphi_{1b\text{-}1}=\{9\Pi \mid n\rightarrow 9\Pi \wedge n=2+(-1)\}:\mu\%10=3$

$\varphi_{1b\text{-}2}=\{9\Pi \mid n\rightarrow 9\Pi \wedge n=(-2)+1\}:\mu\%10=5$

$2\varphi_{1a}:\mu\%10=2$

Evidently, no same value appears in all possible sets which make possible easier judgment of Niu card or Xi card. This function can identify all required card types.

Based on above strategy analysis , an acceptable card type notation table of is given as shown in Table 1:

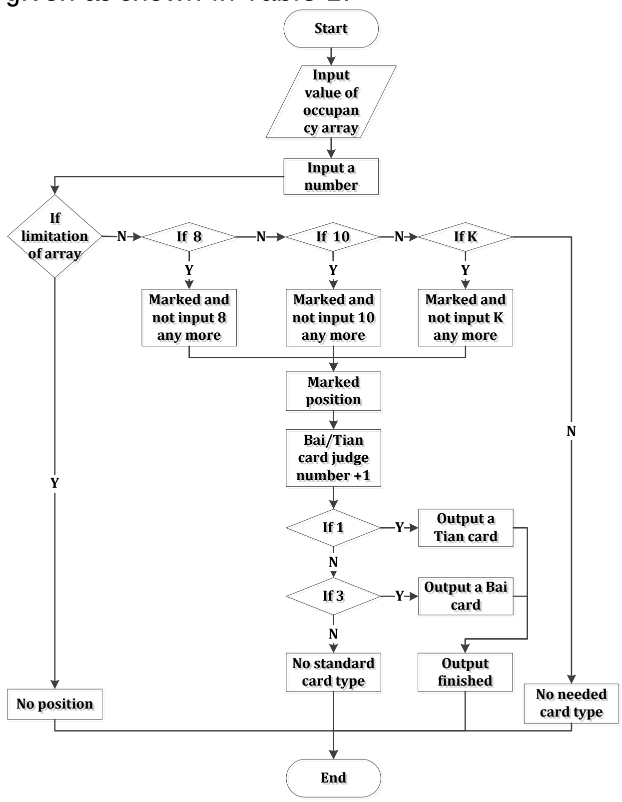

Figure 4. Recognition algorithm of Bai card

5.3 IMP-minimax-based algorithm recognition of special card type

In process of occupancy, except situation of available card type, some special occupancy also is available card type.

Due to dealing process contains hidden information, this computer game is an game with imperfect information. Therefore, this playing process can be used to describe special card type by IMP-minimax algorithm.

IMP - minimax algorithm applies a timely linear search strategy for games with imperfect information. Under this algorithm this strategy is the most optimal. Specially, at first player stage and having some natural properties, this paper proposes an IMP – minimax based search algorithm of special card type identification.

In Niujiu card, if a player holds a certain number of 9∏, 8∏, 10∏, K∏, Fish card and Hu card will be available occupancy in some situation. Based on this situation, Let 3 φ

$_{8}$=1, $2\varphi_{8}$=2, φ_{8}=3, φ_{4}=4 as number of all parameter of leaf nodes valued 9∏, 8∏, 10∏, K∏. Values of right nodes are value set above. Two search algorithms of max set in extreme cases are given as follows:

(1) Perfect limitation of max set shown in Figure 5.

Table 1. Identifiable card type notation table of finite automaton for available occupancy of six cards

{K∏,K∏,K∏,K∏,-9∏,+9∏}	{K∏,-9∏,+9∏,-5∏,+5∏,+5∏}
{K∏,K∏,K∏,K∏,-5∏,+5∏}	{K∏,-9∏,+9∏,+5∏,-5∏,-5∏}
{K∏,K∏,K∏,8∏,10∏,K∏}	{K∏,-5∏,+5∏,-9∏,+9∏,+9∏}
{K∏,K∏,K∏,-9∏,+9∏,+9∏}	{K∏,-5∏,+5∏,+9∏,-9∏,-9∏}
{-9∏,+9∏,+9∏,-5∏,+5∏,+5∏}	{K∏,10∏,8∏,-9∏,+9∏,+9∏}
{-9∏,+9∏,+9∏,+5∏,-5∏,-5∏}	{K∏,10∏,8∏,+9∏,-9∏,-9∏}
{+9∏,-9∏,-9∏,-5∏,+5∏,+5∏}	{K∏,10∏,8∏,-5∏,+5∏,+5∏}
{+9∏,-9∏,-9∏,+5∏,-5∏,-5∏}	{K∏,10∏,8∏,+5∏,-5∏,-5∏}
{K∏,K∏,K∏,+9∏,-9∏,-9∏}	{K∏,10∏,8∏,K∏,10∏,8∏}
{K∏,K∏,K∏,-5∏,+5∏,+5∏}	{-9∏,+9∏,-5∏,-5∏,+5∏,+5∏}
{K∏,K∏,K∏,+5∏,-5∏,-5∏}	{-5∏,+5∏,-9∏,-9∏,+9∏,+9∏}
{K∏,K∏,9∏,9∏,5∏,5∏}	{K∏,K∏,10∏,8∏,-5∏,+5∏}
{K∏,K∏,-9∏,-9∏,+9∏,+9∏}	{K∏,K∏,10∏,8∏,-9∏,+9∏}
{K∏,K∏,-5∏,-5∏,+5∏,+5∏}	{K∏,K∏,x∏,x∏,x∏,x∏}
{-5∏,+5∏,x∏,x∏,x∏,x∏}	{1∏,1∏,1∏,1∏,2∏,3∏}
{1∏,2∏,2∏,2∏,2∏,3∏}	{1∏,2∏,3∏,3∏,3∏,3∏}
{8∏,8∏,8∏,8∏,10∏,K∏}	{8∏,10∏,10∏,10∏,10∏,K∏}

In Figure 5, all of left nodes in search tree are the numbers of card type. Let card type 9∏ and K∏ to the number 4. According to IMP-alpha-beta pruning algorithm, Formula 1 shown following is worked out:

$$V(X) = \begin{cases} max\{extend(Y)\} \\ \sum P\pi(\omega)H(\omega) \end{cases} \tag{5.1}$$

Let expectation in search tree to V(X). Used nodes are denoted by ω . $P\pi(\omega)$ represents π set of X. In $max\{extend(Y)\}$, if Y is a child node and X is a π set , formula $\sum P\pi(\omega)H(\omega)$ substitutes $max\{extend(Y)\}$. $P\pi(\omega)$ represents the value of all nodes of π set. $H(\omega)$ represents the value of certain number leaf nodes.

As shown in figure 5, all $P\pi(\omega)$ values of leaf nodes of left sub trees are $\frac{4+4}{2}$ =4. According to number of limitation of same card type is 4. When value of K∏ is 4, the left sub trees of 8∏ and 10∏ can be cut. Taking the value of four card types set before to leaf nodes of right sub trees, all value of right sub trees are 4 in extreme case. All $P\pi(\omega)$ value of leaf nodes of right sub trees are $\frac{4+4}{2}$ =4≤ 4 in this case. Therefore, all right sub trees are cut. Meanwhile, V(X) is equal to 2.

As shown in deduction above, when a player holds $4\varphi_{1}$ and $4\varphi_{7}$ card types, $3\varphi_{8}$,$2\varphi_{8}$, φ_{8} and φ_{4} are special type of available card type.

(1) Imperfect limitation of max set shown in Figure 6.

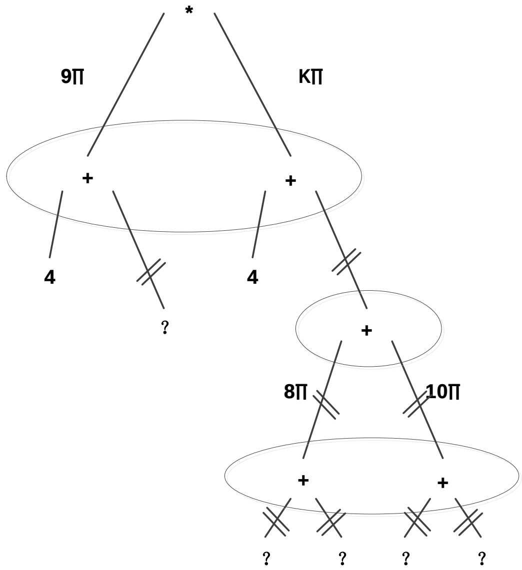

Figure 5. Search tree of perfect limitation of max set

As shown in deduction above, as long as value of $P\pi(\omega)$ of right sub trees' leaf nodes less than or equal to value of $P\pi(\omega)$ of left sub trees' leaf nodes, right sub trees can be cut. Therefore, in extreme case, when "?" in figure 8 is equal to 0, with $P\pi(\omega) = \frac{0+0}{2} = 0\ being\ met$, V(X) is equal to 0.

As shown in deduction above, when a player holds $0\varphi_{1}$ and $0\varphi_{7}$ and number of 8∏ and 10∏ less than or equal to 3, $3\varphi_{8}$,$2\varphi_{8}$, φ_{8} and φ_{4} cannot be special available card type as occupancy.

These are two extreme situations. In the process of playing, with a player holds different cards, V(X) ranges from 0 to 2.

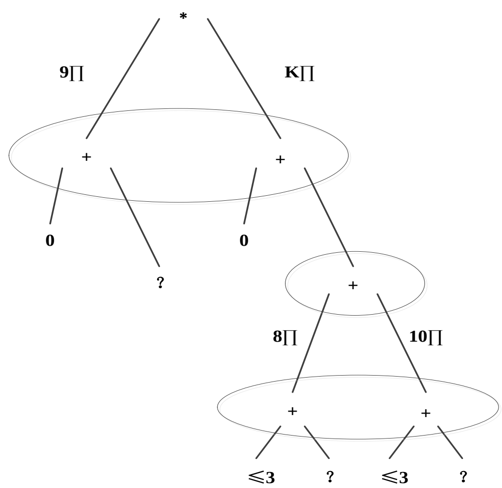

Figure 6. Search tree of imperfect limitation of max set

6. Conclusions

Study of multi-player non-zero-sum games with imperfect information is a difficult issue of computer game. Taking the ancient card game named Niujiu card for example, this paper summarizes the components of computer game model. An automaton based card game model and an IMP-minimax based search algorithm is proposed respectively. Our proposal can get the optimal occupancy of a first player in special situation. This model provides a reference for multi-player non-zero-sum games with imperfect information. Furthermore, this model lays a related research foundation for computer game of Niujiu card. In the future, we will study search algorithm of special card type in computer game system of Niujiu card. In order to improve effectiveness and efficiency in identifying card type, we will also consider using Monte Carlo combined with IMP-minimax algorithm.

7. Acknowledgment

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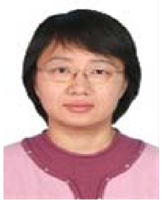

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