

Drawsgtree: a tool for visualizing properties in the semigroup tree

Maria Bras-Amorós

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In these pages we illustrate examples drawn by the code `drawsgtree`.
The code `drawsgtree` can be downloaded from <https://github.com/mbrasamoros/drawsgtree>.
Please, contact `maria.bras@urv.cat` for any comment or questions.

COMMAND:

```
./drawsgtree -h
```

OUTPUT:

```
./sgroup [options]      generate a latex file with the semigroup tree
-h                      display this help
-g <int>                [mandatory option] maximum genus
-m <int>                multiplicity
-n [option]             node representation
-n list:                list of semigroup elements
-n minimalgenerators:   representation by minimal generator set
-n gapset:              representation by gapsets
                        (S. Eliahou, J. Fromentin: Gapsets and
                        ↪ numerical semigroups, Journal of
                        ↪ Combinatorial Theory, Series A, 2020)
-n gapseedbitstream:    representation with the gap bitstream and the
                        ↪ seed bitstream
                        (M. Bras-Amoros, J. Fernandez-Gonzalez:
                        ↪ Computation of numerical semigroups
                        ↪ by means of seeds, Math of Comput,
                        ↪ 2018)
-n seedstable:          representation by seeds tables
                        (M. Bras-Amoros, J. Fernandez-Gonzalez:
                        ↪ Computation of numerical semigroups
                        ↪ by means of seeds, Math of Comput,
                        ↪ 2018)
-n dyckhook:            representation by augmented Dyck paths and Hook
                        ↪ lengths
                        (M. Bras-Amoros, A. de Mier: Representation
                        ↪ of Numerical Semigroups by Dyck
                        ↪ Paths, Semigroup Forum, 2007)
                        H. Constantin, B. Houston-Edwards, N.
                        ↪ Kaplan: Numerical sets, core
                        ↪ partitions, and integer points in
                        ↪ polytopes, Combinatorial and
                        ↪ Additive Number Theory, 2017)
-n aperykunzposet:      representation by Apéry sets, Kunz coordinates,
                        ↪ and posets
                        (E. Kunz: Uber die Klassifikation
                        ↪ numerischer Halbgruppen, Regensburger
                        ↪ Mathematische Schriften, 1987
                        J.C. Rosales, P.A. Garcia-Sanchez, J.I.
                        ↪ Garcia-Garcia, M.B. Branco: Systems
                        ↪ of inequalities and numerical
                        ↪ semigroups, J. Lond. Math. Soc.,
                        ↪ 2002
                        N. Kaplan, K. O'Neill: Numerical
                        ↪ semigroups, polyhedra, and posets I:
                        ↪ the group cone, Combinatorial
                        ↪ Theory, 2021)
-n infinitichains:      draw the infinite chains in the semigroup tree
```

(M. Bras-Amoros, S. Bulygin: Towards a
↪ better understanding of the semigroup
↪ tree, Semigroup Forum, 2009)

-incremental incremental with genus
-inputfile input file (not compiling without a calling file)
-vertical vertical tree growing down
-plain plain representation of objects using less memory
-blackandwhite graph without colors
-framednodes frame each tree node
-d <float> enlarge distance between generations by the
↪ specified factor
-rotated rotated 90 degrees
0 N[1] N[2] ... N[k] root at the semigroup {0,N[1],N[2],N[k],N[k]+1,N[k]
↪]+2,...}

examples: ./drawsgtree -g5 -n list
./drawsgtree -g7 -n list -incremental
./drawsgtree -g4 -n minimalgenerators -vertical
./drawsgtree -g5 -n gapset -vertical
./drawsgtree -g7 -n gapseedbitstream -n list -plain
./drawsgtree -g25 -n seedstable -vertical 0 8 16 18 19 24 26 27
↪ 30
./drawsgtree -g10 -n aperykunzposet 0 6 7 9
./drawsgtree -g8 -m4 -n dyckhook
./drawsgtree -g11 -n infinitechains
./drawsgtree -g11 -n infinitechains -d 3.
./drawsgtree -m3 -g8 -n list -n gapset -n minimalgenerators -n
↪ gapseedbitstream -n aperykunzposet -framednodes
./drawsgtree -g15 0 7 9 11 14 16 18 20 21 22 23 25 27 -n
↪ aperykunzposet
./drawsgtree -g33 0 12 19 24 28 31 34 36 38 40 42 43 45 -n
↪ dyckhook

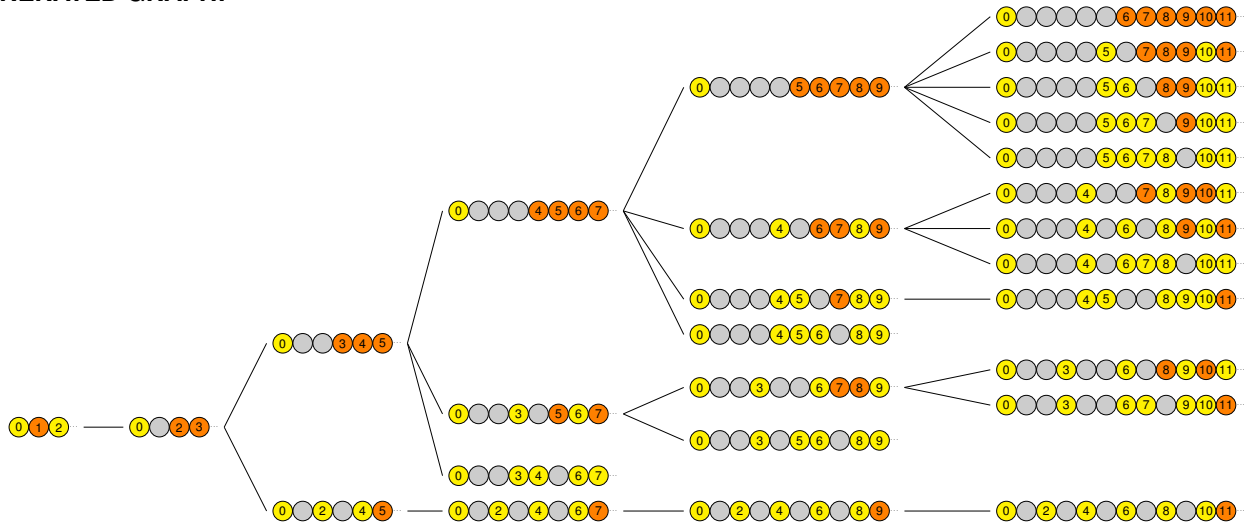
COMMAND:

```
./drawsgtree -g5 -n list -inputfile
```

OUTPUT:

```
[g=5] count=12 ng=12 [0 seconds]
```

```
GENERATED FILE: inputfile-list-semigrouptree-5.tex
```

GENERATED GRAPH:

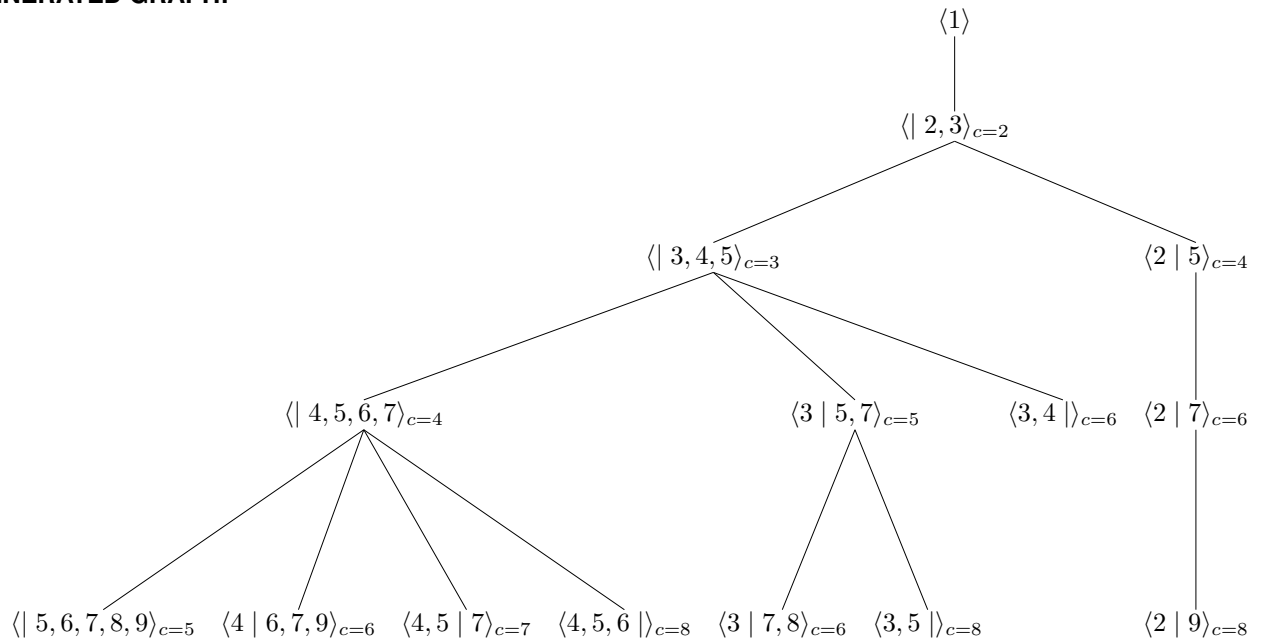
COMMAND:

```
./drawsgtree -g4 -n minimalgenerators -vertical -inputfile
```

OUTPUT:

```
[g=4] count=7 ng=7 [0 seconds]
```

```
GENERATED FILE: inputfile-minimalgenerators-semigroupptree-4.tex
```

GENERATED GRAPH:

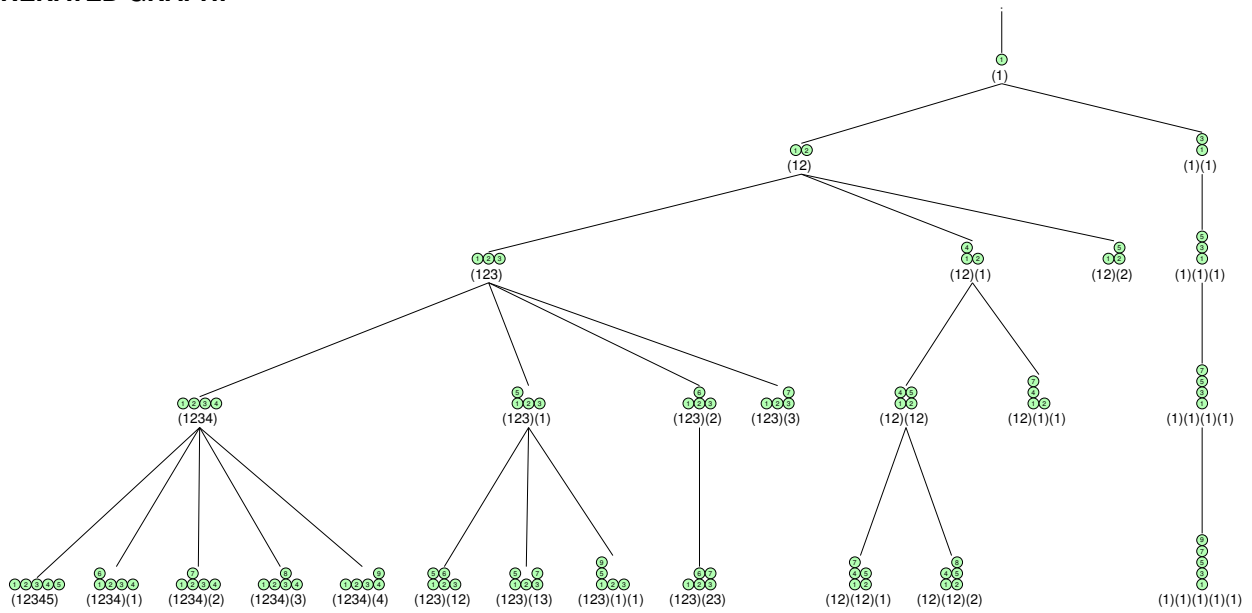
COMMAND:

```
./drawsgtree -g5 -n gapset -vertical -inputfile
```

OUTPUT:

```
[g=5] count=12 ng=12 [0 seconds]
```

```
GENERATED FILE: inputfile-gapset-semigroup-tree-5.tex
```

GENERATED GRAPH:

COMMAND:

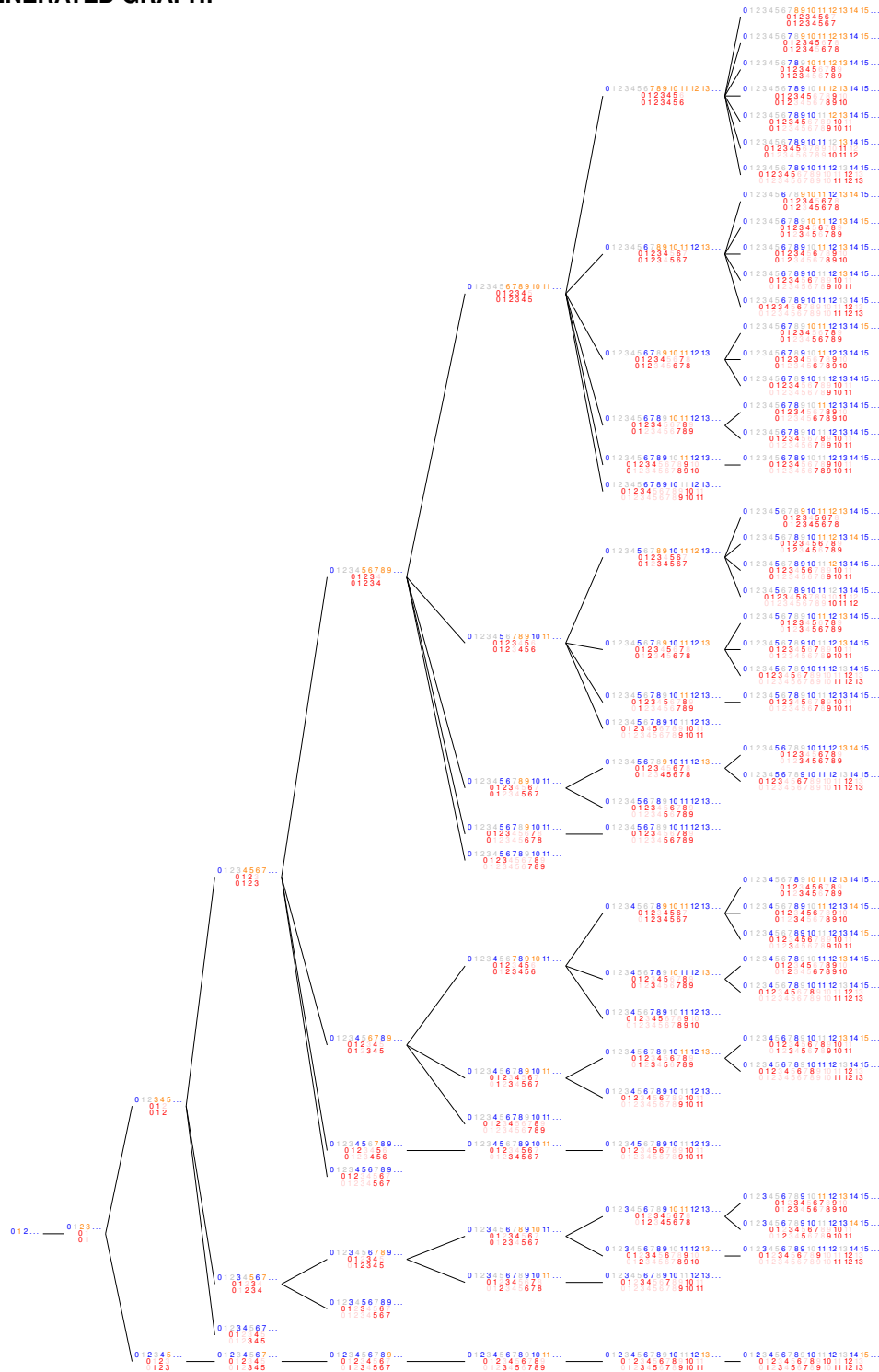
```
./drawsgtree -g7 -n gapseedbitstream -n list -plain -inputfile
```

OUTPUT:

```
[g=7] count=39 ng=39 [0 seconds]
```

GENERATED FILE: inputfile-plain-gapseedbitstream-list-semigrouptree-7.tex

GENERATED GRAPH:



COMMAND:

```
./drawsgtree -g25 -n seedstable -vertical 0 8 16 18 19 24 26 27 30 -  
↪ inputfile
```

OUTPUT:

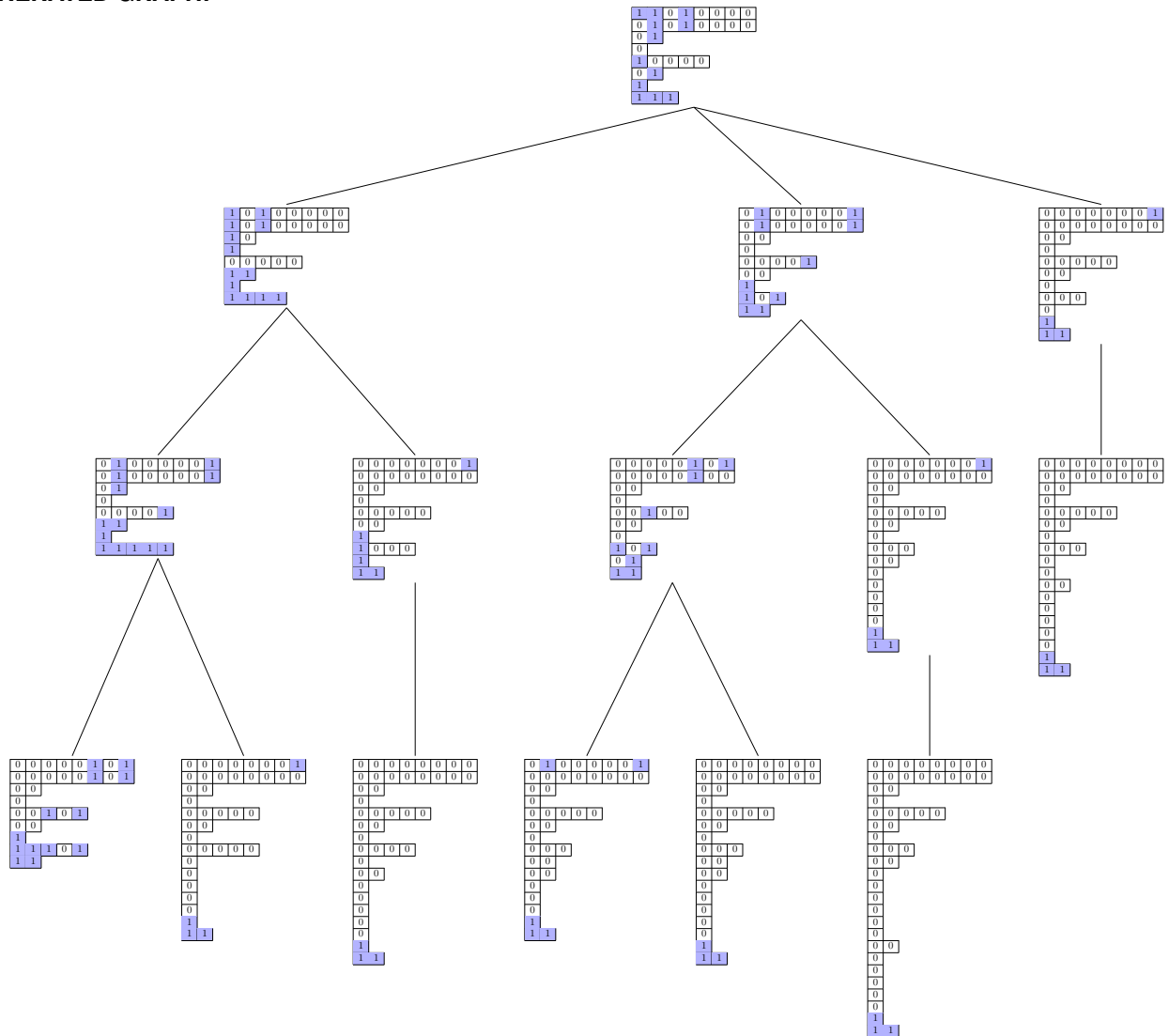
N[0]=0
N[1]=8
N[2]=16
N[3]=18
N[4]=19
N[5]=24
N[6]=26
N[7]=27
N[8]=30

[g=25] count=6 ng=467224 [0 seconds]

GENERATED FILE: inputfile-seedstable-semigrouptree-25-root0816181924262730

↪ .tex

GENERATED GRAPH:



COMMAND:

```
./drawsgtree -g10 -n aperykunzposet 0 6 7 9 -inputfile
```

OUTPUT:

$$N[0] = 0$$
$$N[1] = 6$$

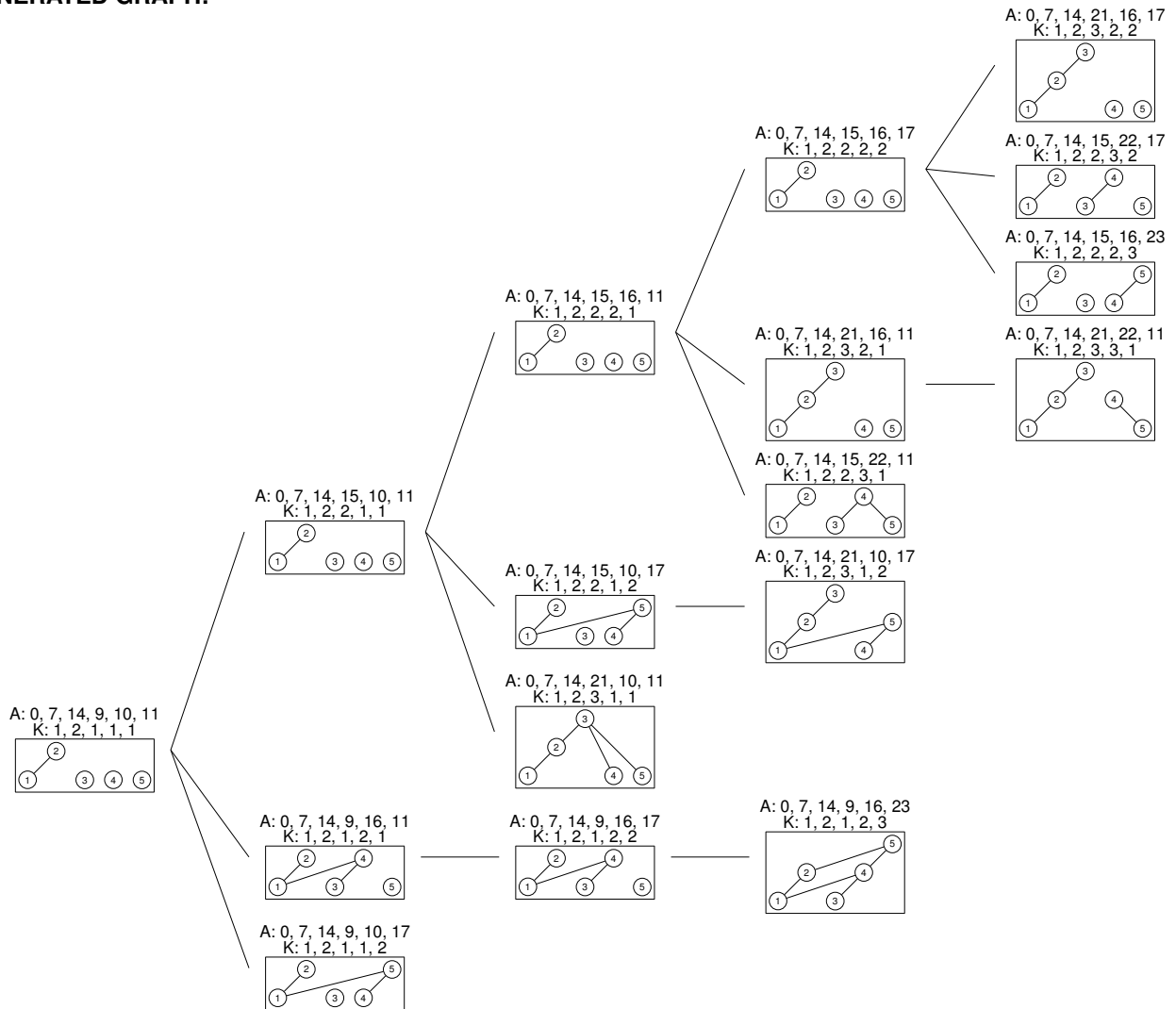
N [2] = 7

N [3] = 9

```
[g=10] count=4 ng=204 [0 seconds]
```

GENERATED FILE: inputfile-aperykunuzposet-semigrouptree-10-root0679.tex

GENERATED GRAPH:



COMMAND:

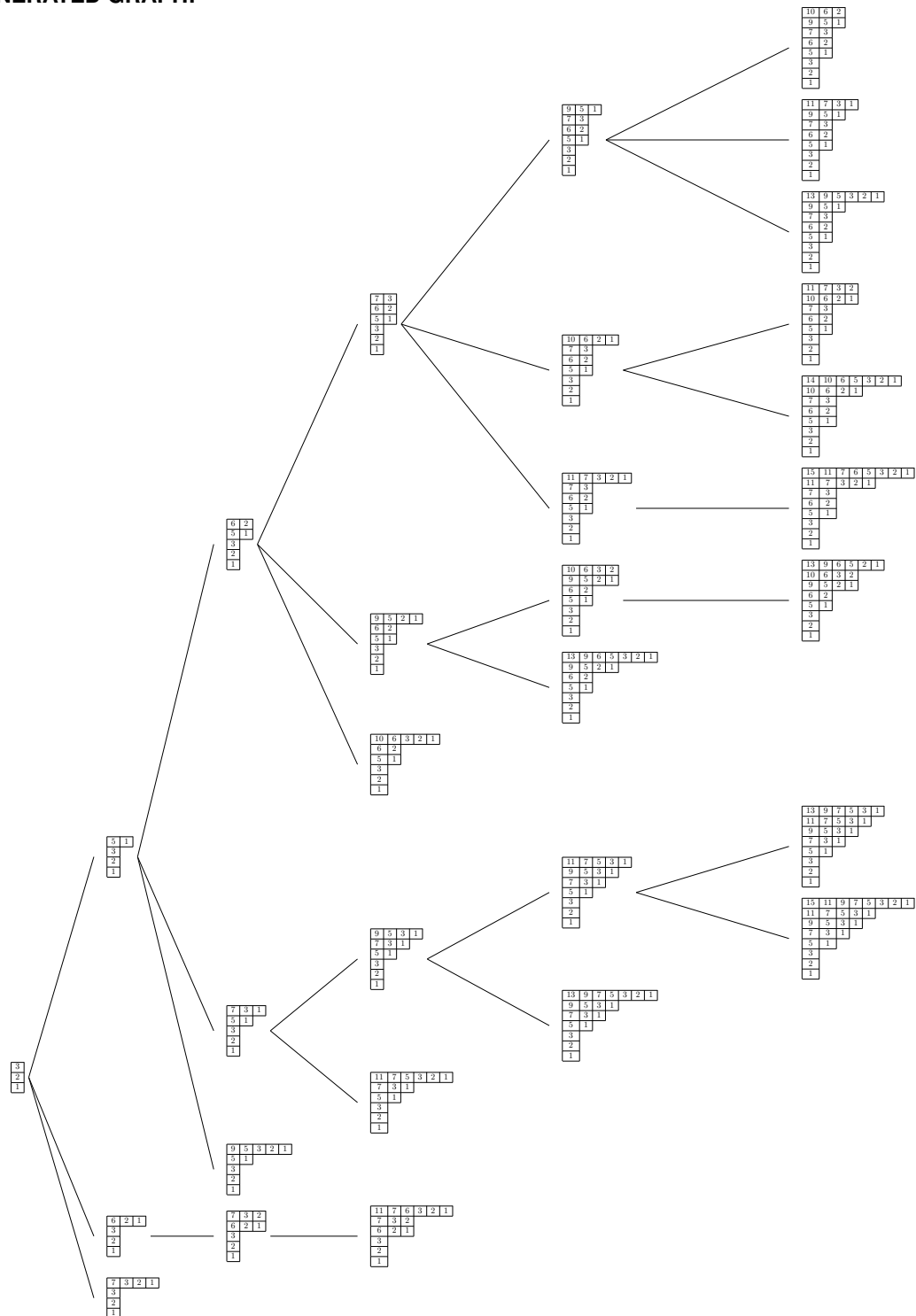
```
./drawsgtree -g8 -m4 -n dyckhook -inputfile
```

OUTPUT:

```
[g=8] count=9 ng=67 [0 seconds]
```

GENERATED FILE: inputfile-dyckhook-semigrouptree-8-root04.tex

GENERATED GRAPH:



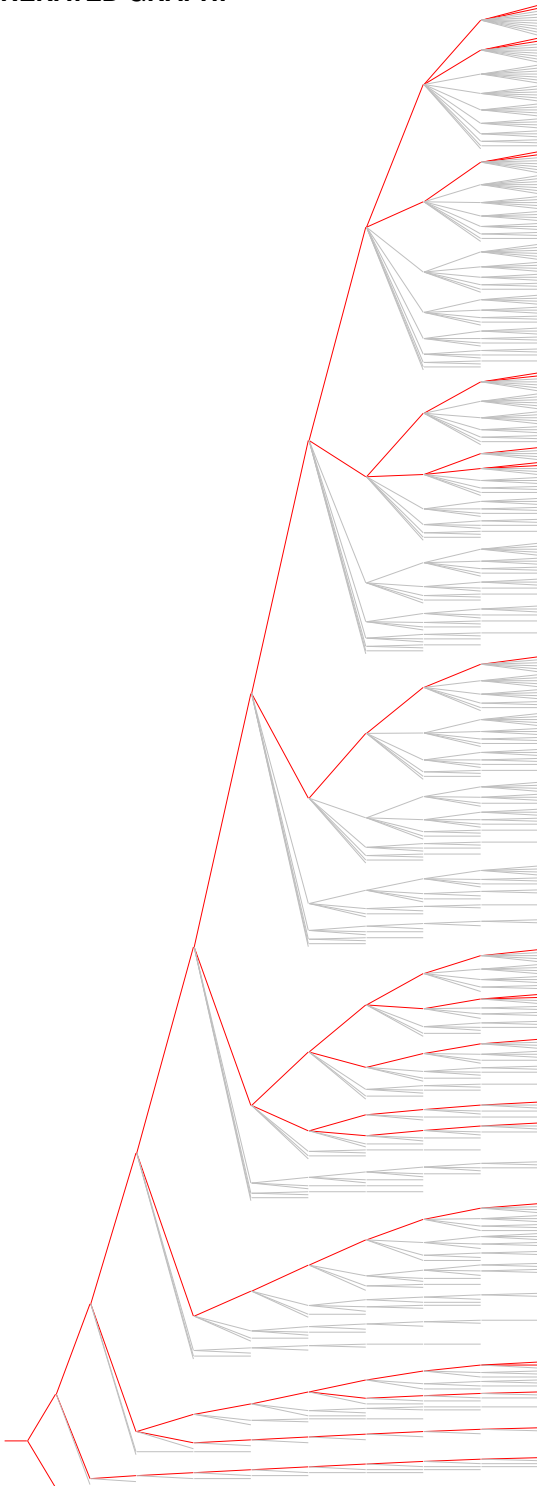
COMMAND:

```
./drawsgtree -g11 -n infinitechains -inputfile
```

OUTPUT:

```
[g=11] count=343 ng=343 [0 seconds]
```

```
GENERATED FILE: inputfile-infinitechains-semigrouptree-11.tex
```

GENERATED GRAPH:

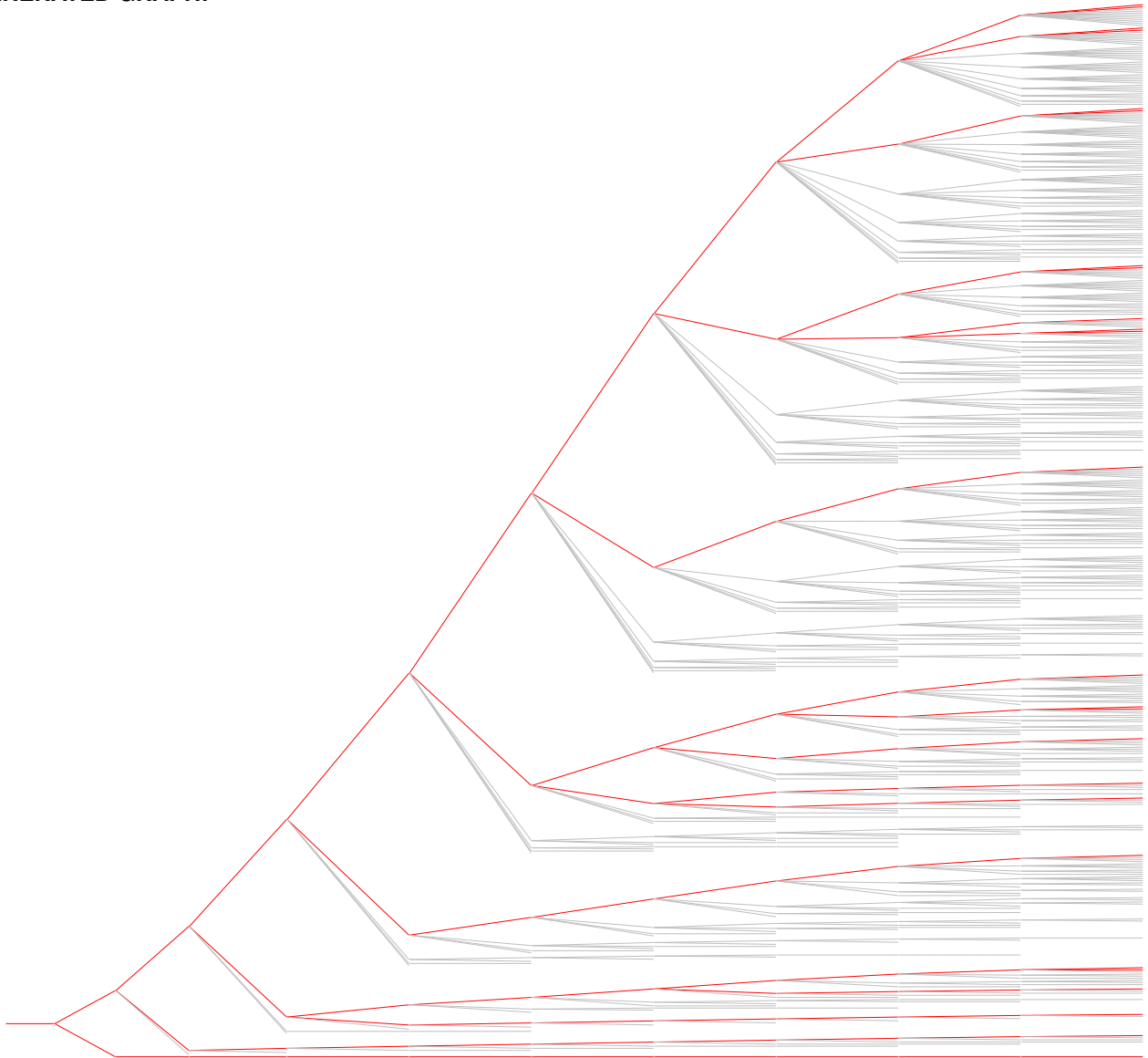
COMMAND:

```
./drawsgtree -g11 -n infinitechains -d 3. -inputfile
```

OUTPUT:

```
[g=11] count=343 ng=343 [0 seconds]
```

```
GENERATED FILE: inputfile-infinitechains-semigrouptree-11.tex
```

GENERATED GRAPH:

COMMAND:

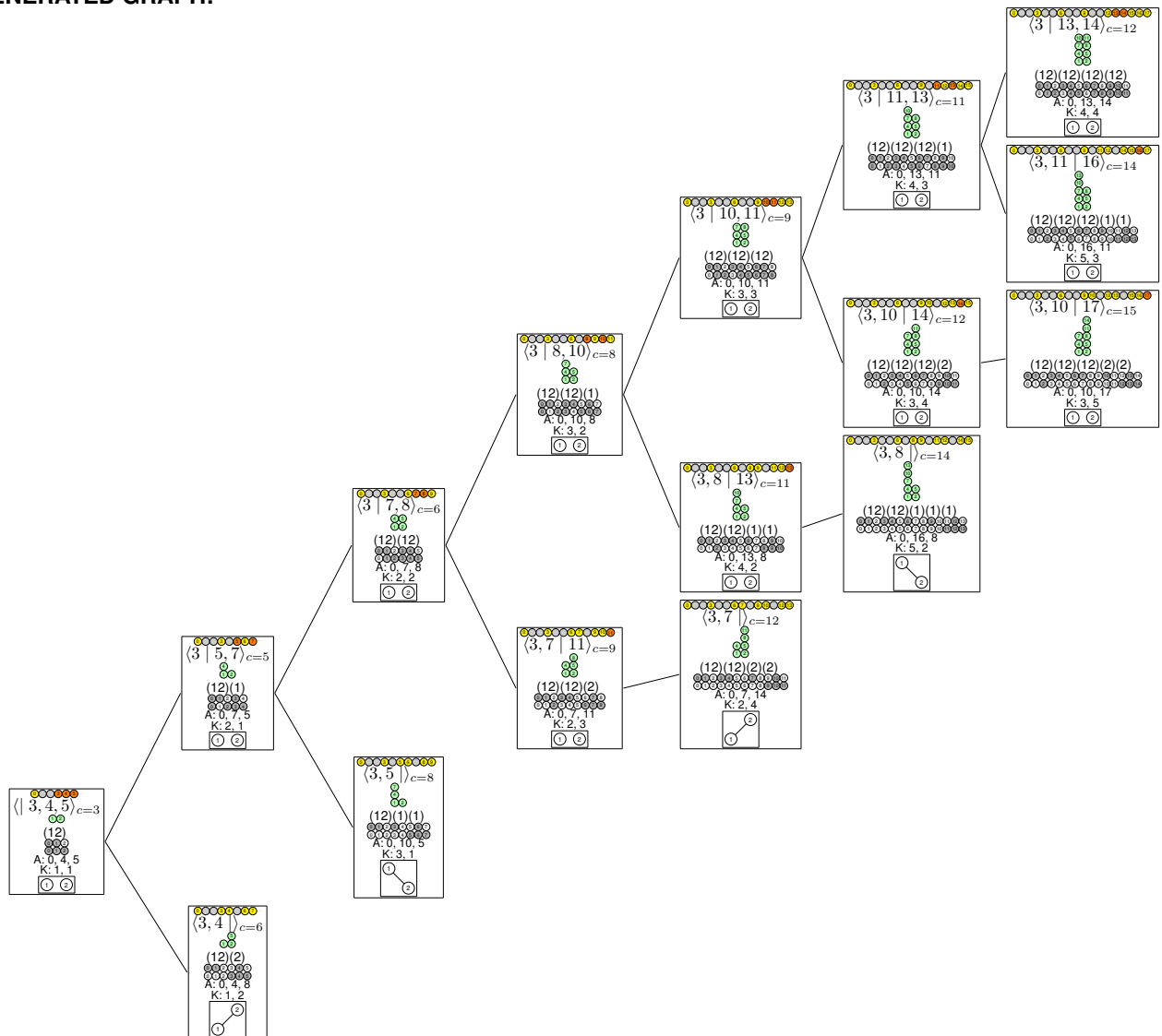
```
./drawsgtree -m3 -g8 -n list -n gapset -n minimalgenerators -n
  ↪ gapseedbitstream -n aperykunzposet -framednodes -inputfile
```

OUTPUT:

```
[g=8] count=3 ng=67 [0 seconds]
```

GENERATED FILE: inputfile-aperykunuzposet-gapseedbitstream-gapset-
 ↪ minimalgenerators-list-semigrouptree-8-root03.tex

GENERATED GRAPH:



COMMAND:

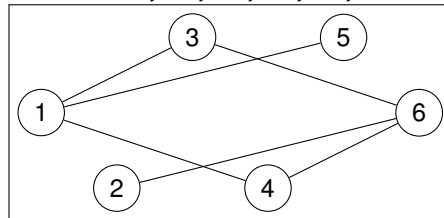
```
./drawsgtree -g15 0 7 9 11 14 16 18 20 21 22 23 25 27 -n aperykunuzposet -  
  ↪ inputfile
```

OUTPUT:

```
N[0]=0  
N[1]=7  
N[2]=9  
N[3]=11  
N[4]=14  
N[5]=16  
N[6]=18  
N[7]=20  
N[8]=21  
N[9]=22  
N[10]=23  
N[11]=25  
N[12]=27  
[g=15] count=1 ng=2857 [0 seconds]  
GENERATED FILE: inputfile-aperykunuzposet-semigroupptree-15-  
  ↪ root07911141618202122232527.tex
```

GENERATED GRAPH:

A: 0, 22, 9, 31, 11, 33, 20
K: 3, 1, 4, 1, 4, 2



COMMAND:

```
./drawsgtree -g33 0 12 19 24 28 31 34 36 38 40 42 43 45 -n dyckhook -  
↪ inputfile
```

OUTPUT:

```
N[0]=0
N[1]=12
N[2]=19
N[3]=24
N[4]=28
N[5]=31
N[6]=34
N[7]=36
N[8]=38
N[9]=40
N[10]=42
N[11]=43
N[12]=45
[g=33] count=1 ng=24896206 [0 seconds]
GENERATED FILE: inputfile-dyckhook-semigrouptree-33-
↳ root0121924283134363840424345.tex
```

GENERATED GRAPH:

[illegible]

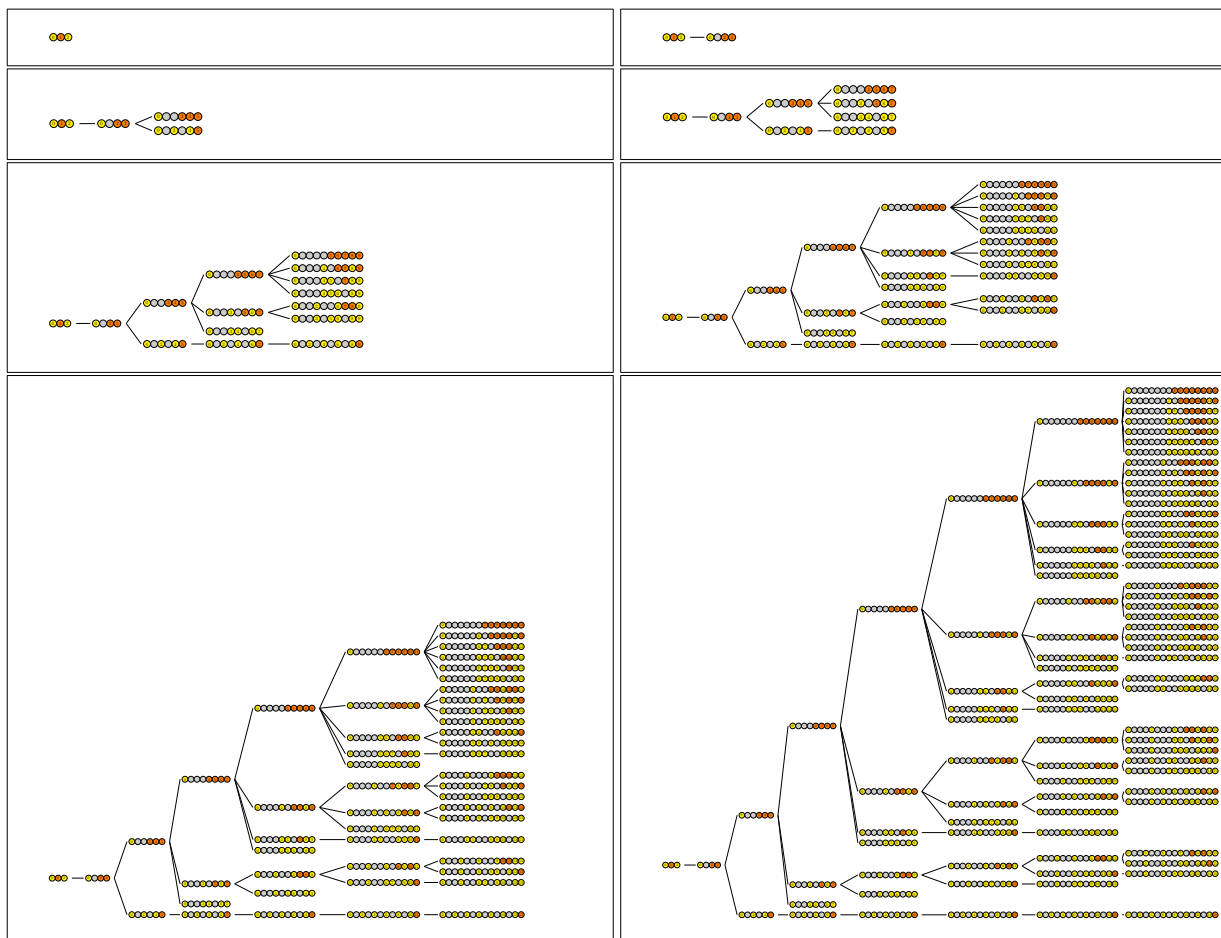
COMMAND:

```
./drawsgtree -g7 -n list -incremental
```

OUTPUT:

```
[g=0] count=1 ng=1 [0 seconds]
[g=1] count=1 ng=1 [0 seconds]
[g=2] count=2 ng=2 [0 seconds]
[g=3] count=4 ng=4 [0 seconds]
[g=4] count=7 ng=7 [0 seconds]
[g=5] count=12 ng=12 [0 seconds]
[g=6] count=23 ng=23 [0 seconds]
[g=7] count=39 ng=39 [0 seconds]
```

GENERATED FILE: incremental-list-semigroup-tree-7.tex

GENERATED GRAPHS:

References

- [1] Maria Bras-Amorós and Stanislav Bulygin. Towards a better understanding of the semigroup tree. *Semigroup Forum*, 79(3):561–574, 2009.
- [2] Maria Bras-Amorós and Anna de Mier. Representation of numerical semigroups by Dyck paths. *Semigroup Forum*, 75(3):677–682, 2007.
- [3] Maria Bras-Amorós and Julio Fernández-González. Computation of numerical semigroups by means of seeds. *Math. Comp.*, 87(313):2539–2550, 2018.
- [4] Hannah Constantin, Ben Houston-Edwards, and Nathan Kaplan. Numerical sets, core partitions, and integer points in polytopes. In *Combinatorial and additive number theory. II*, volume 220 of *Springer Proc. Math. Stat.*, pages 99–127. Springer, Cham, 2017.
- [5] Shalom Eliahou and Jean Fromentin. Gapsets and numerical semigroups. *J. Combin. Theory Ser. A*, 169:105129, 19, 2020.
- [6] Nathan Kaplan and Christopher O'Neill. Numerical semigroups, polyhedra, and posets I: the group cone. *Comb. Theory*, 1:Paper No. 19, 23, 2021.
- [7] Ernst Kunz. *Über die Klassifikation numerischer Halbgruppen*, volume 11 of *Regensburger Mathematische Schriften [Regensburg Mathematical Publications]*. Universität Regensburg, Fachbereich Mathematik, Regensburg, 1987.
- [8] J. C. Rosales, P. A. García-Sánchez, J. I. García-García, and M. B. Branco. Systems of inequalities and numerical semigroups. *J. London Math. Soc. (2)*, 65(3):611–623, 2002.