Drawsgtree: a tool for visualizing properties in the semigroup tree

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In these pages we illustrate examples drawn by the code `drawsgtree`. It can be downloaded from `https://github.com/mbrasamoros/drawsgtree`.

COMMAND:

```
./drawsgtree -h
```

OUTPUT:

```
./sgroup [options]      generate a latex file with the semigroup tree
  -h                    display this help
  -g <int>              [mandatory option] maximum genus
  -m <int>              multiplicity
  -n [option]           node representation
     -n list:              list of semigroup elements
     -n minimalgenerators: representation by minimal generator set
     -n gapset:            representation by gapsets
                               (S. Eliahou, J. Fromentin: Gapsets and
                                  ↪ numerical semigroups, Journal of
                                  ↪ Combinatorial Theory, Series A, 2020)
     -n gapseedbitstream:  representation with the gap bitstream and the
        ↪ seed bitstream
                               (M. Bras-Amoros, J. Fernandez-Gonzalez:
                                  ↪ Computation of numerical semigroups
                                  ↪ by means of seeds, Math of Comput,
                                  ↪ 2018)
     -n seedstable:        representation by seeds tables
                               (M. Bras-Amoros, J. Fernandez-Gonzalez:
                                  ↪ Computation of numerical semigroups
                                  ↪ by means of seeds, Math of Comput,
                                  ↪ 2018)
     -n dyckhook:          representation by augmented Dyck paths and Hook
        ↪  lengths
                               (M. Bras-Amoros, A. de Mier: Representation
                                  ↪  of Numerical Semigroups by Dyck
                                  ↪ Paths, Semigroup Forum, 2007)
                                H. Constantin, B. Houston-Edwards, N.
                                   ↪ Kaplan: Numerical sets, core
                                   ↪ partitions, and integer points in
                                   ↪ polytopes, Combinatorial and
                                   ↪ Additive Number Theory, 2017)
     -n aperykunzposet:    representation by Apery sets, Kunz coordinates,
        ↪  and posets
                               (E. Kunz: Uber die Klassifikation
                                  ↪ numerischer Halbgruppen, Regensburger
                                  ↪  Mathematische Schriften, 1987
                                J.C. Rosales, P.A. Garcia-Sanchez, J.I.
                                   ↪ Garcia-Garcia, M.B. Branco: Systems
                                   ↪ of inequalities and numerical
                                   ↪ semigroups, J. Lond. Math. Soc.,
                                   ↪ 2002
                                N. Kaplan, K. O'Neill: Numerical
                                   ↪ semigroups, polyhedra, and posets I:
                                   ↪  the group cone, Combinatorial
                                   ↪ Theory, 2021)
     -n infinitechainst:   draw the infinite chains in the semigroup tree
```

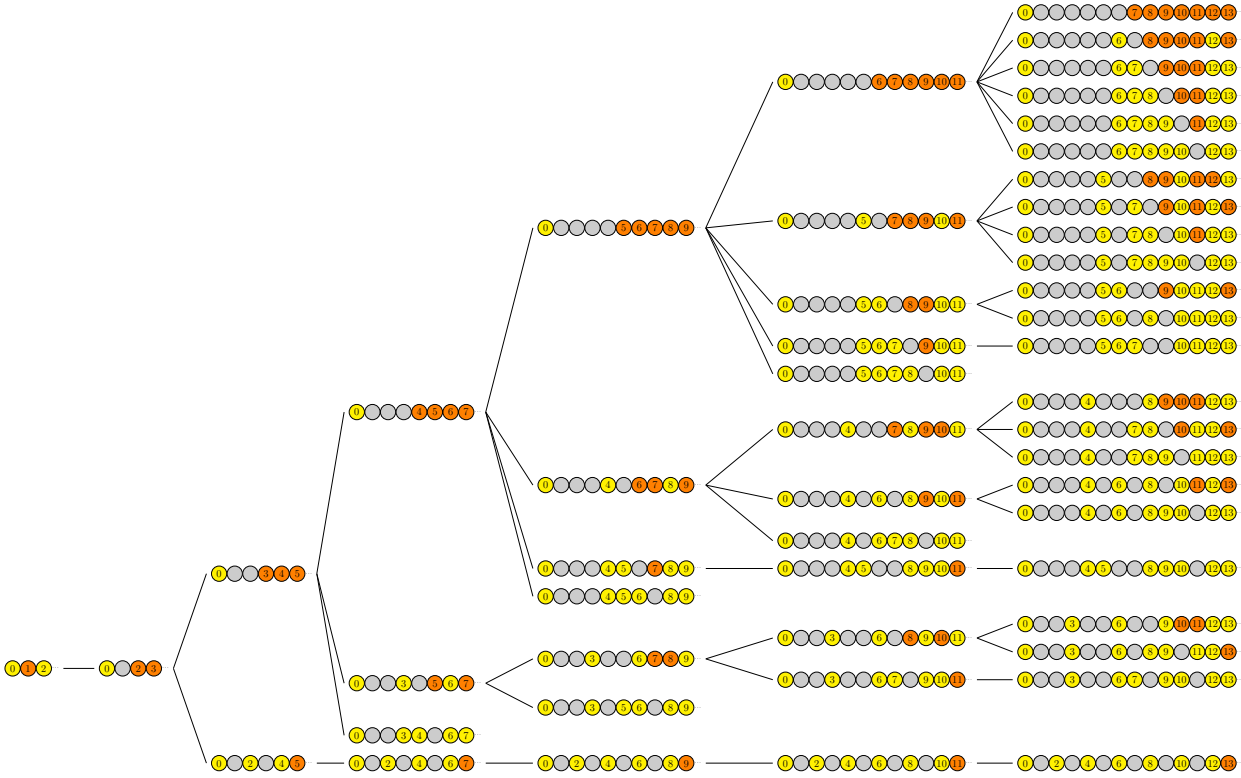
```
                                (M. Bras-Amoros, S. Bulygin: Towards a
                                   ↪ better understanding of the semigroup
                                   ↪  tree, Semigroup Forum, 2009)
  -incremental                  incremental with genus
  -inputfile                    input file (not compiling without a calling file)
  -vertical                     vertical tree growing down
  -plain                        plain representation of objects using less memory
  -blackandwhite                graph without colors
  -framednodes                  frame each tree node
  -d <float>                    enlarge distance between generations by the
     ↪ specified factor
  -rotated                      rotated 90 degrees
  0 N[1] N[2] ... N[k]          root at the semigroup {0,N[1],N[2],N[k],N[k]+1,N[k
     ↪ ]+2,...}

examples:  ./drawsgtree -g6 -n list
           ./drawsgtree -g7 -n list -incremental
           ./drawsgtree -g4 -n minimalgenerators -vertical
           ./drawsgtree -g5 -n gapset -vertical
           ./drawsgtree -g7 -n gapseedbitstream -n list -plain
           ./drawsgtree -g25 -n seedstable -vertical 0 8 16 18 19 24 26 27
              ↪  30
           ./drawsgtree -g8 -m6 -n aperykunzposet
           ./drawsgtree -g8 -m4 -n dyckhook
           ./drawsgtree -g11 -n infinitechains
           ./drawsgtree -g11 -n infinitechains -d 3.
           ./drawsgtree -m3 -g8 -n list -n gapset -n minimalgenerators -n
              ↪ gapseedbitstream -n aperykunzposet -d 2. -framednodes
           ./drawsgtree -g15 0 7 9 11 14 16 18 20 21 22 23 25 27 -n
              ↪ aperykunzposet
           ./drawsgtree -g33 0 12 19 24 28 31 34 36 38 40 42 43 45 -n
              ↪ dyckhook
```

COMMAND:

```
./drawsgtree -g6 -n list -inputfile
```

OUTPUT:

```
[g=6] count=23 ng=23 [0 seconds]
GENERATED FILE: inputfile-list-semigrouptree-6.tex
```

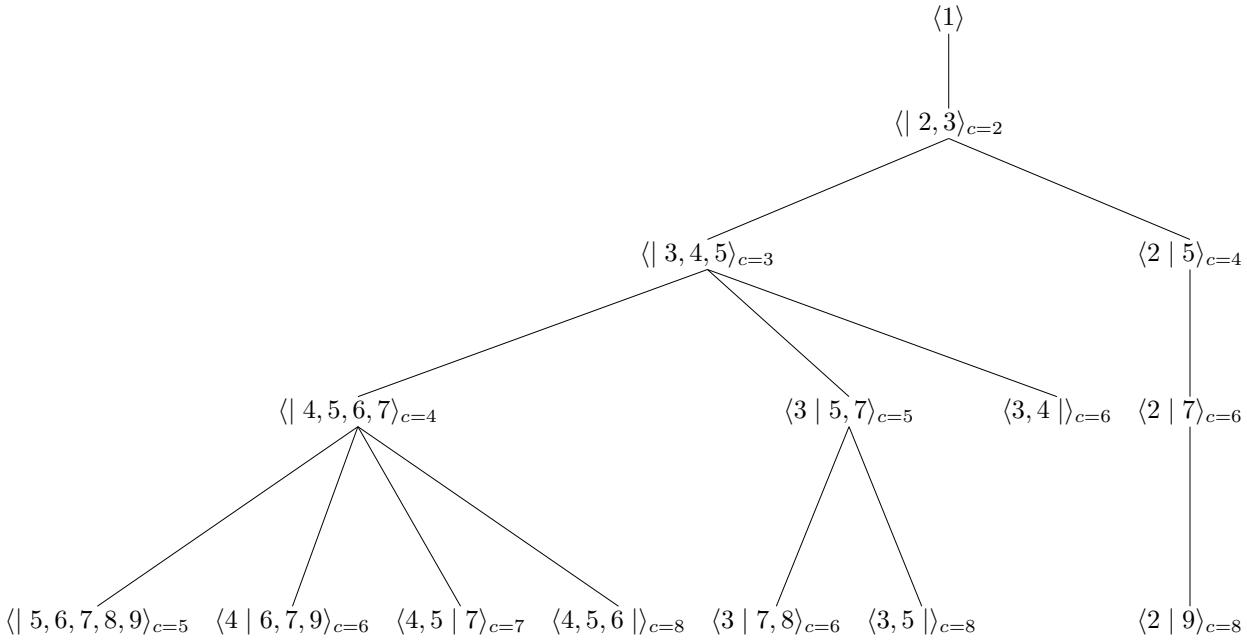


COMMAND:

```
./drawsgtree -g4 -n minimalgenerators -vertical -inputfile
```

OUTPUT:

```
[g=4] count=7 ng=7 [0 seconds]
GENERATED FILE: inputfile-minimalgenerators-semigrouptree-4.tex
```

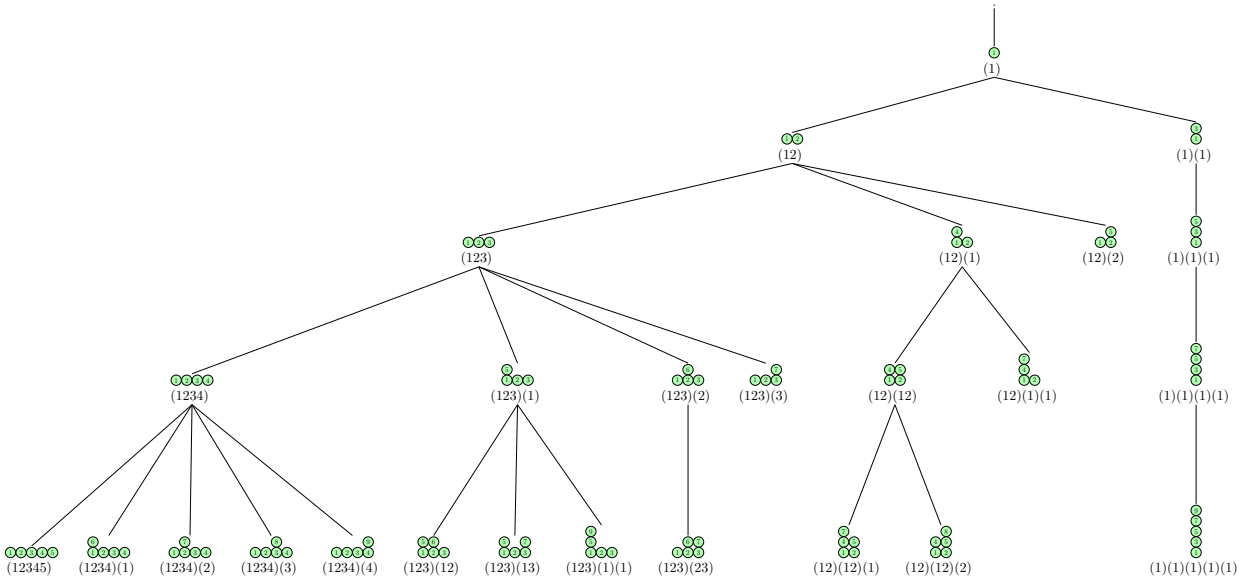


COMMAND:

```
./drawsgtree -g5 -n gapset -vertical -inputfile
```

OUTPUT:

```
[g=5] count=12 ng=12 [0 seconds]
GENERATED FILE: inputfile-gapset-semigrouptree-5.tex
```

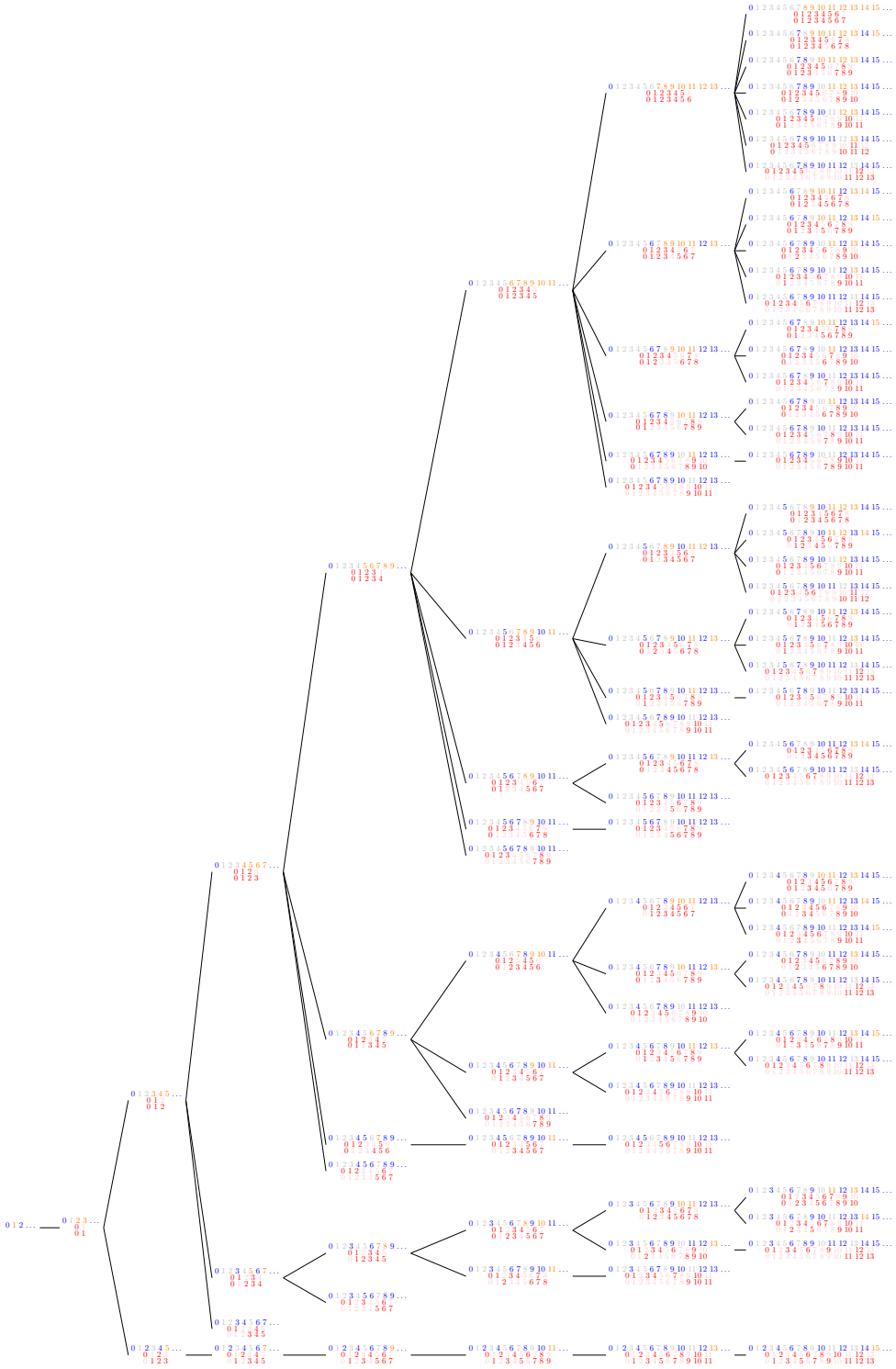


COMMAND:

```
./drawsgtree -g7 -n gapseedbitstream -n list -plain -inputfile
```

OUTPUT:

```
[g=7] count=39 ng=39 [0 seconds]
GENERATED FILE: inputfile-plain-gapseedbitstream-list-semigrouptree-7.tex
```

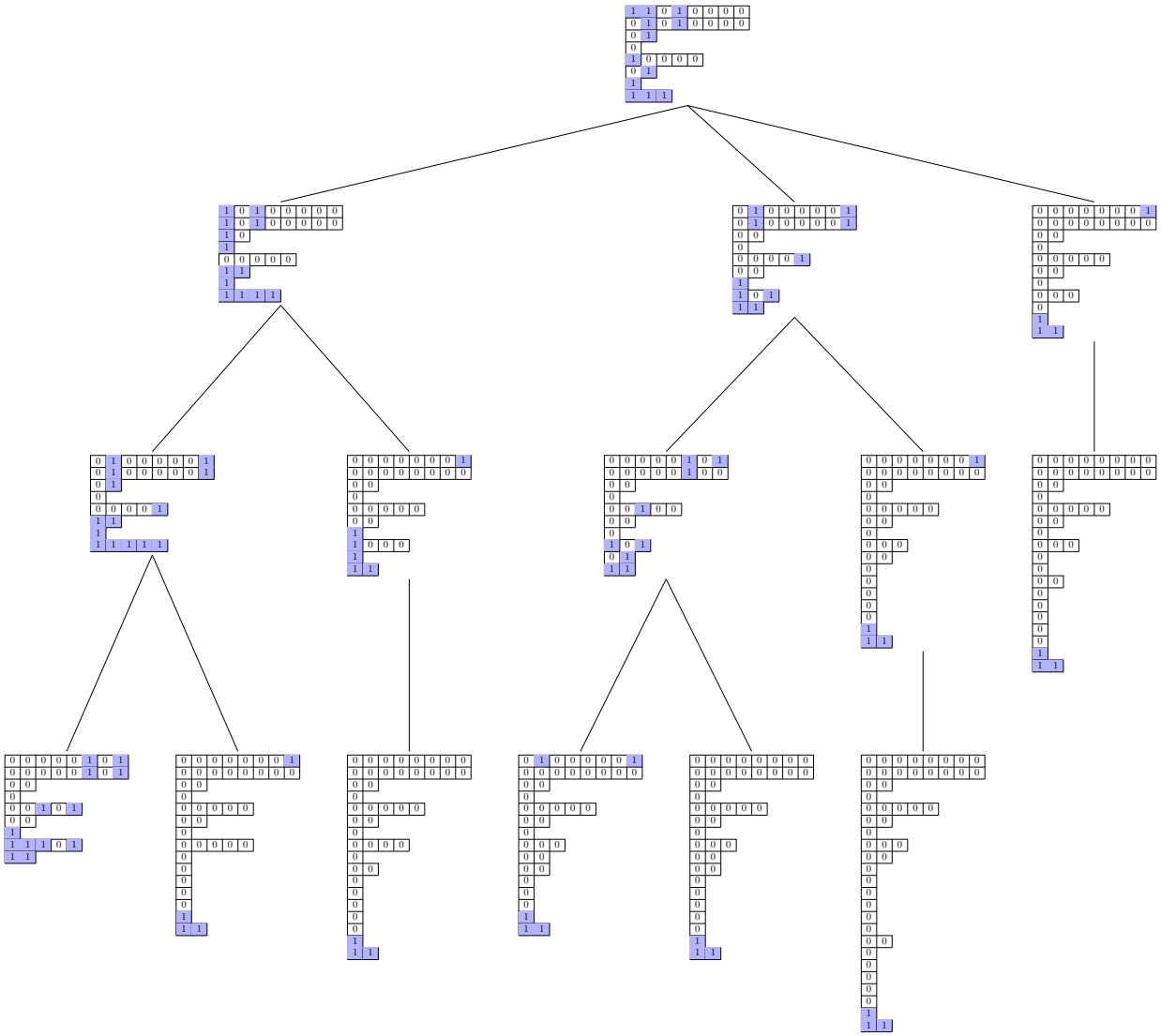


COMMAND:

```
./drawsgtree -g25 -n seedstable -vertical 0 8 16 18 19 24 26 27 30 -
    ↪ inputfile
```

OUTPUT:

```
N[0]=0
N[1]=8
N[2]=16
N[3]=18
N[4]=19
N[5]=24
N[6]=26
N[7]=27
N[8]=30
[g=25] count=6 ng=467224 [0 seconds]
GENERATED FILE: inputfile-seedstable-semigrouptree-25-root0816181924262730
    ↪ .tex
```

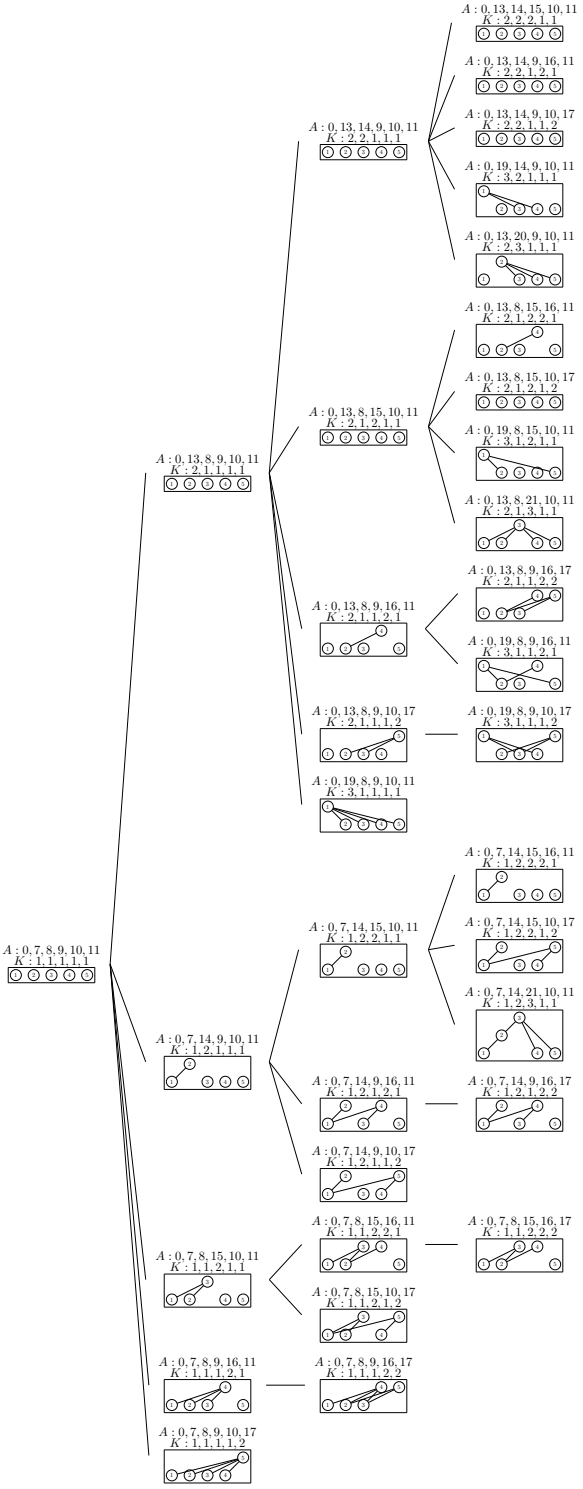


COMMAND:

```
./drawsgtree -g8 -m6 -n aperykunzposet -inputfile
```

OUTPUT:

```
[g=8] count=17 ng=67 [0 seconds]
GENERATED FILE: inputfile-aperykunzposet-semigrouptree-8-root06.tex
```

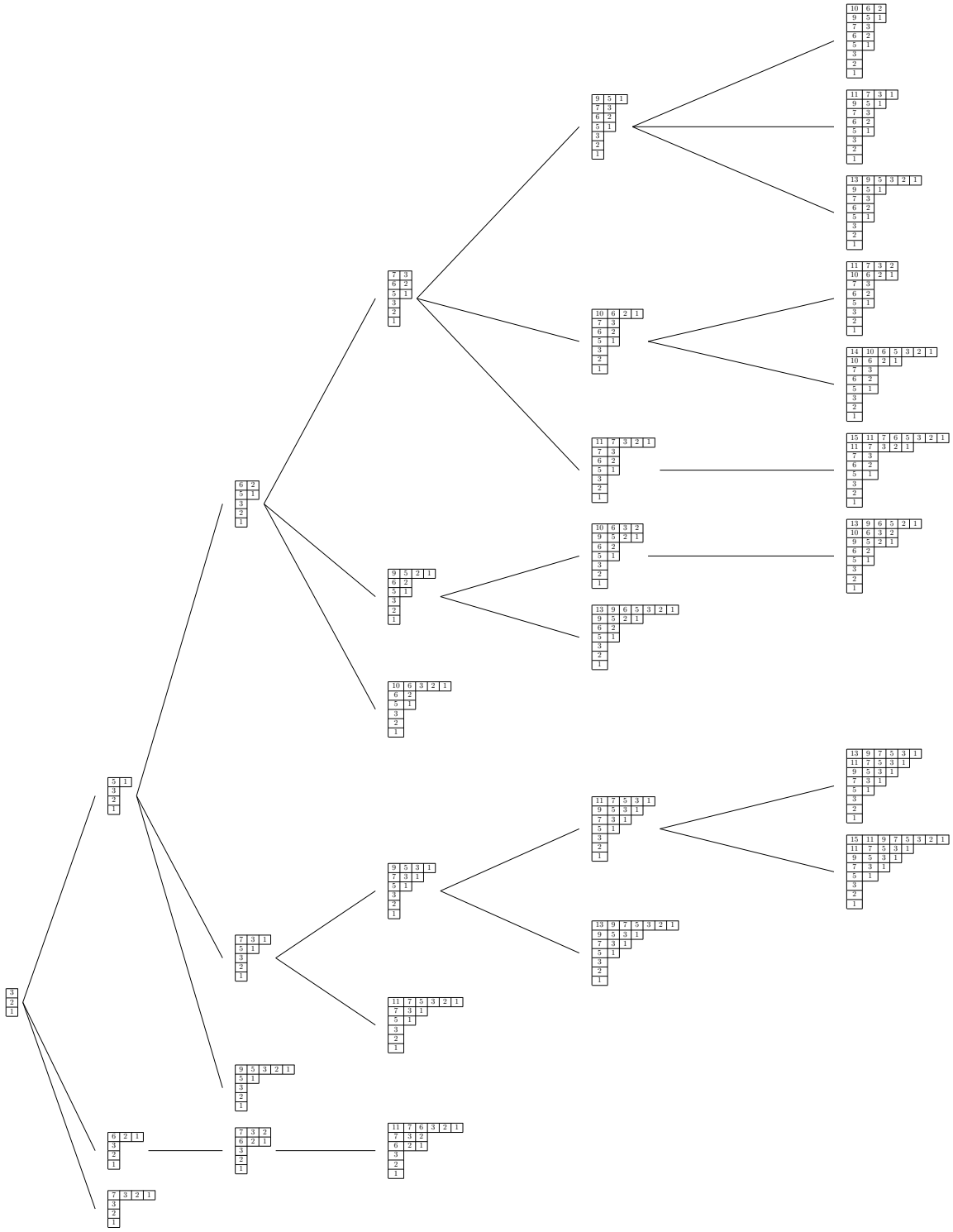


COMMAND:

```
./drawsgtree -g8 -m4 -n dyckhook -inputfile
```

OUTPUT:

```
[g=8] count=9 ng=67 [0 seconds]
GENERATED FILE: inputfile-dyckhook-semigrouptree-8-root04.tex
```

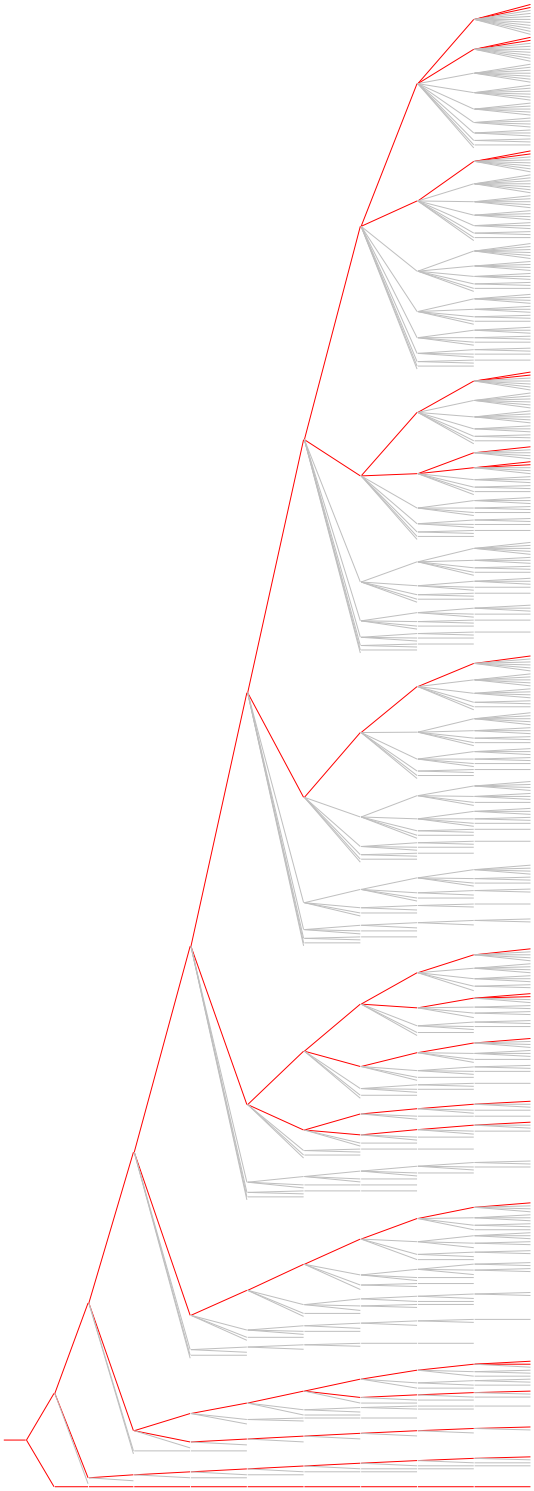


COMMAND:

```
./drawsgtree -g11 -n infinitechains -inputfile
```

OUTPUT:

```
[g=11] count=343 ng=343 [0 seconds]
GENERATED FILE: inputfile-infinitechains-semigrouptree-11.tex
```

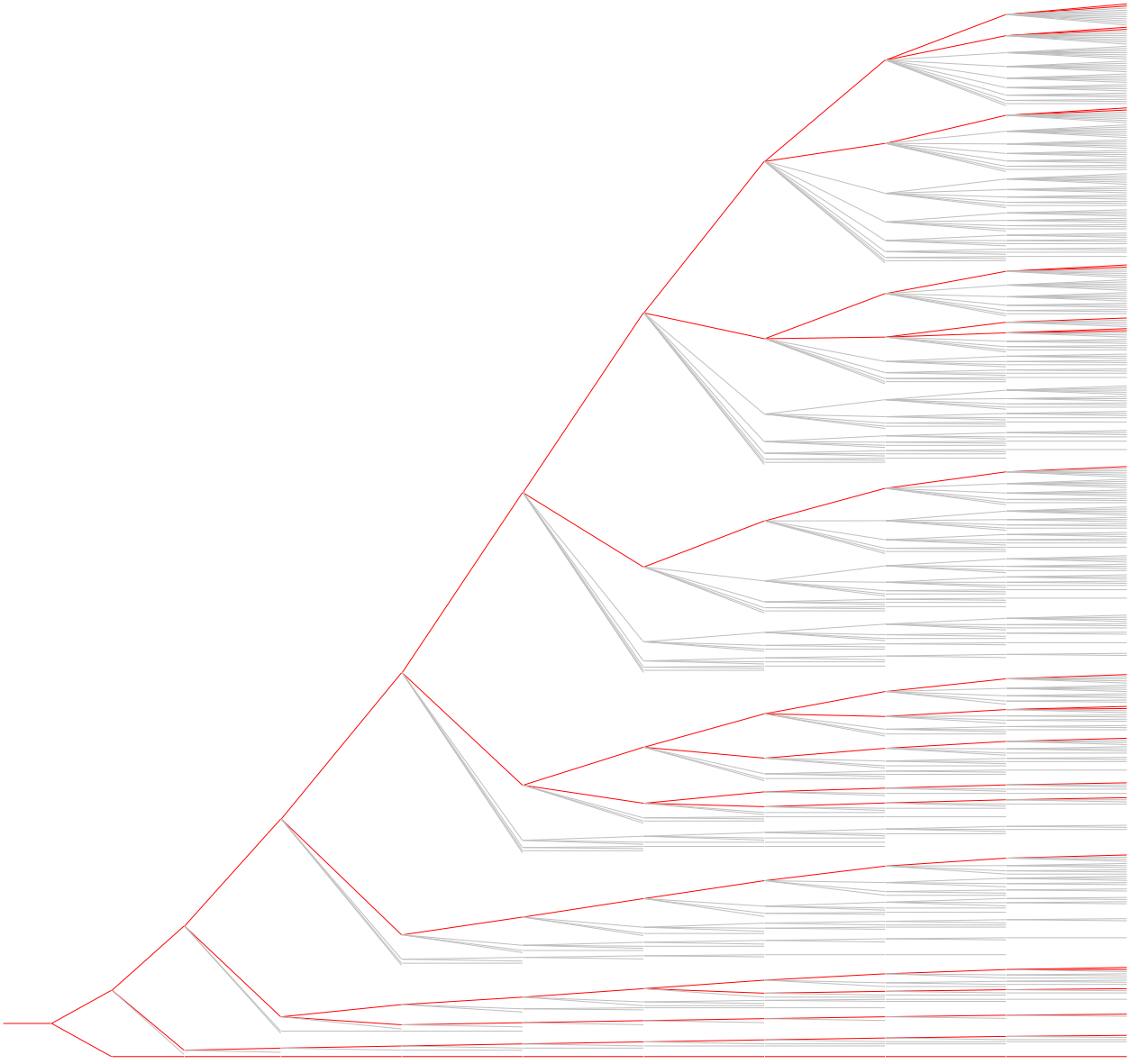


COMMAND:

```
./drawsgtree -g11 -n infinitechains -d 3. -inputfile
```

OUTPUT:

```
[g=11] count=343 ng=343 [0 seconds]
GENERATED FILE: inputfile-infinitechains-semigrouptree-11.tex
```

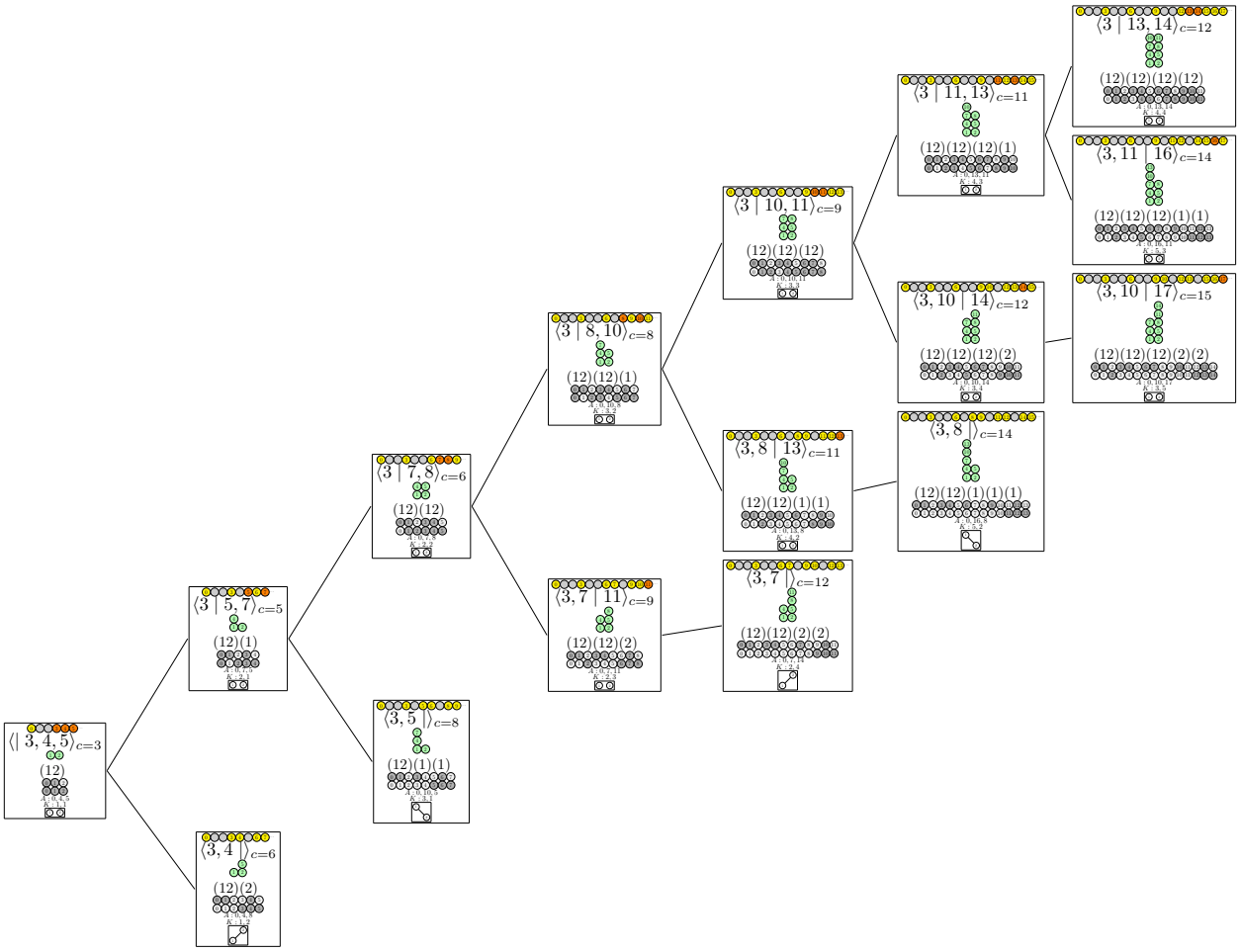


COMMAND:

```
./drawsgtree -m3 -g8 -n list -n gapset -n minimalgenerators -n
    ↪ gapseedbitstream -n aperykunzposet -d 2. -framednodes -inputfile
```

OUTPUT:

```
[g=8] count=3 ng=67 [0 seconds]
GENERATED FILE: inputfile-aperykunzposet-gapseedbitstream-gapset-
    ↪ minimalgenerators-list-semigrouptree-8-root03.tex
```



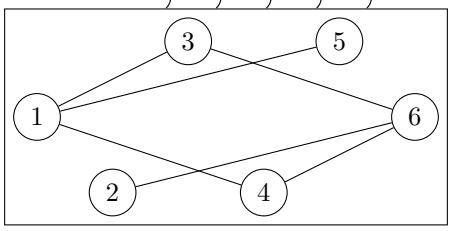
COMMAND:

```
./drawsgtree -g15 0 7 9 11 14 16 18 20 21 22 23 25 27 -n aperykunzposet -
    ↪ inputfile
```

OUTPUT:

```
N[0]=0
N[1]=7
N[2]=9
N[3]=11
N[4]=14
N[5]=16
N[6]=18
N[7]=20
N[8]=21
N[9]=22
N[10]=23
N[11]=25
N[12]=27
[g=15] count=1 ng=2857 [0 seconds]
GENERATED FILE: inputfile-aperykunzposet-semigrouptree-15-
    ↪ root07911141618202122232527.tex
```

$$A : 0, 22, 9, 31, 11, 33, 20$$
$$K : 3, 1, 4, 1, 4, 2$$



COMMAND:

```
./drawsgtree -g33 0 12 19 24 28 31 34 36 38 40 42 43 45 -n dyckhook -
    ↪ inputfile
```

OUTPUT:

```
N[0]=0
N[1]=12
N[2]=19
N[3]=24
N[4]=28
N[5]=31
N[6]=34
N[7]=36
N[8]=38
N[9]=40
N[10]=42
N[11]=43
N[12]=45
[g=33] count=1 ng=24896206 [0 seconds]
GENERATED FILE: inputfile-dyckhook-semigrouptree-33-
    ↪ root012192428313436384042345.tex
```

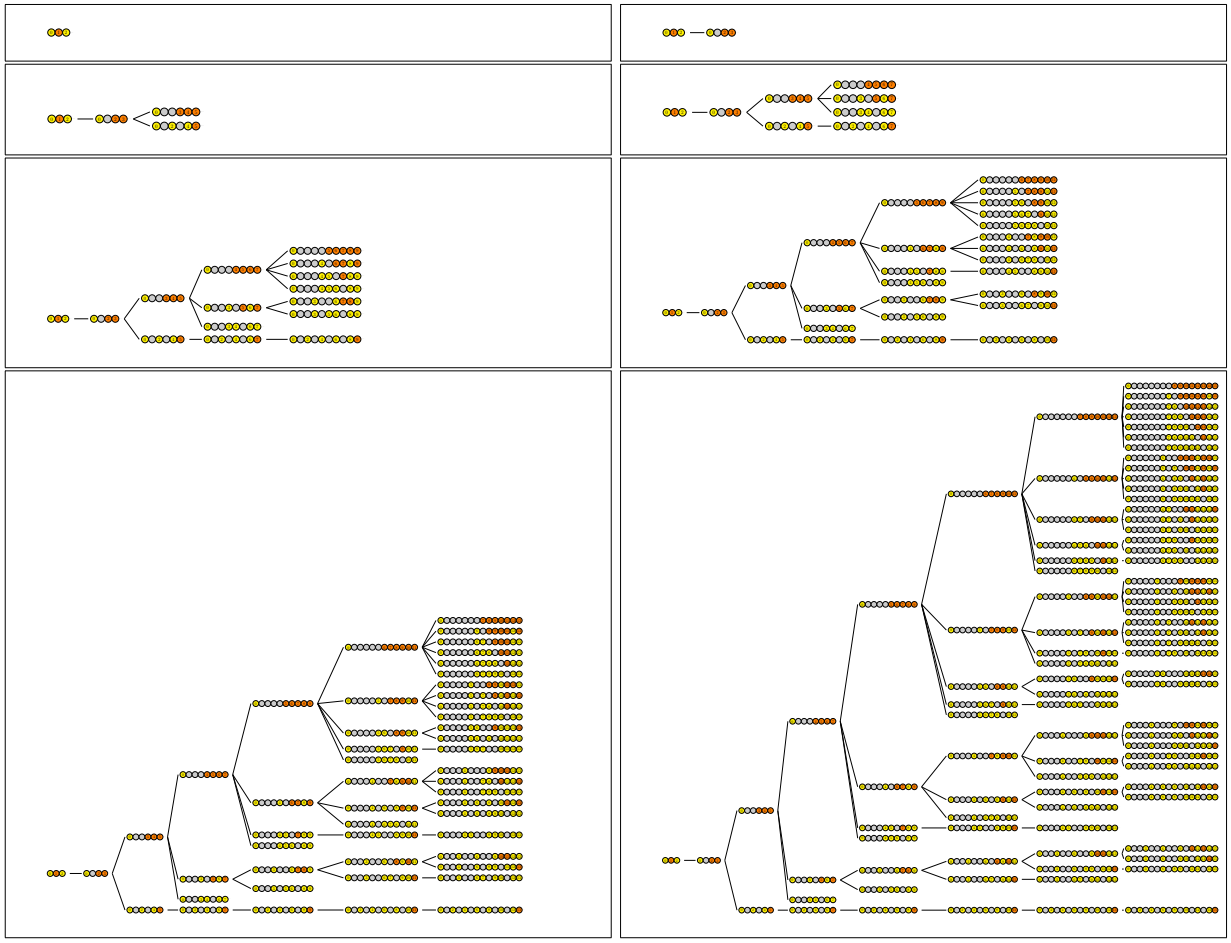
44	32	25	20	16	13	10	8	6	4	2	1
41	29	22	17	13	10	7	5	3	1		
39	27	20	15	11	8	5	3	1			
37	25	18	13	9	6	3	1				
35	23	16	11	7	4	1					
33	21	14	9	5	2						
32	20	13	8	4	1						
30	18	11	6	2							
29	17	10	5	1							
27	15	8	3								
26	14	7	2								
25	13	6	1								
23	11	4									
22	10	3									
21	9	2									
20	8	1									
18	6										
17	5										
16	4										
15	3										
14	2										
13	1										
11											
10											
9											
8											
7											
6											
5											
4											
3											
2											
1											

COMMAND:

```
./drawsgtree -g7 -n list -incremental
```

OUTPUT:

```
[g=0] count=1 ng=1 [0 seconds]
[g=1] count=1 ng=1 [0 seconds]
[g=2] count=2 ng=2 [0 seconds]
[g=3] count=4 ng=4 [0 seconds]
[g=4] count=7 ng=7 [0 seconds]
[g=5] count=12 ng=12 [0 seconds]
[g=6] count=23 ng=23 [0 seconds]
[g=7] count=39 ng=39 [0 seconds]
GENERATED FILE: incremental-list-semigrouptree-7.tex
```



References

[1] Maria Bras-Amorós and Stanislav Bulygin. Towards a better understanding of the semigroup tree. *Semigroup Forum*, 79(3):561–574, 2009.

[2] Maria Bras-Amorós and Anna de Mier. Representation of numerical semigroups by Dyck paths. *Semigroup Forum*, 75(3):677–682, 2007.

[3] Maria Bras-Amorós and Julio Fernández-González. Computation of numerical semigroups by means of seeds. *Math. Comp.*, 87(313):2539–2550, 2018.

[4] Hannah Constantin, Ben Houston-Edwards, and Nathan Kaplan. Numerical sets, core partitions, and integer points in polytopes. In *Combinatorial and additive number theory. II*, volume 220 of *Springer Proc. Math. Stat.*, pages 99–127. Springer, Cham, 2017.

[5] Shalom Eliahou and Jean Fromentin. Gapsets and numerical semigroups. *J. Combin. Theory Ser. A*, 169:105129, 19, 2020.

[6] Nathan Kaplan and Christopher O'Neill. Numerical semigroups, polyhedra, and posets I: the group cone. *Comb. Theory*, 1:Paper No. 19, 23, 2021.

[7] Ernst Kunz. *Über die Klassifikation numerischer Halbgruppen*, volume 11 of *Regensburger Mathematische Schriften [Regensburg Mathematical Publications]*. Universität Regensburg, Fachbereich Mathematik, Regensburg, 1987.

[8] J. C. Rosales, P. A. García-Sánchez, J. I. García-García, and M. B. Branco. Systems of inequalities and numerical semigroups. *J. London Math. Soc. (2)*, 65(3):611–623, 2002.