Problem Chosen	2025 MCM/ICM Summary Sheet	Team Control Number
B		**2518438**

Managing Sustainable Tourism

Summary

We have developed a **sustainable development model** applicable to the city of Juneau and other tourist destinations to address the challenges posed by the rapid growth of tourism on infrastructure, the natural environment, and resident satisfaction.

For Task 1, we have developed a **multi-objective optimization model** based on the **Non-dominated Sorting Genetic Algorithm III (NSGA-III)**, where the dual optimization objectives are the maximization of economic benefits and the minimization of carbon emissions. Economic benefits are quantified through the number of tourists, while carbon emissions are used as a **key indicator of sustainability**. The model introduces three key constraints: The **maximum capacity of infrastructure**, The **upper limit of tourist numbers**, The **threshold for resident satisfaction**. Resident satisfaction is derived from a weighted analysis of dissatisfaction data in the survey report, with a 15% **dissatisfaction threshold** set (based on the actual dissatisfaction rate of 16% in 2023). Furthermore, this study has established additional expenditure plans covering **Community projects** by these areas: **Environmental protection**, **Infrastructure development**.
For task 2, we classifies different types of tourist destinations through **cluster analysis**, focusing on the differential characteristics of **core indicators** such as tourist numbers, infrastructure carrying capacity, and tourism revenue. Based on the classification results, the constraints and parameters of the model are **dynamically adjusted and optimized** to enhance its universality and flexibility, ensuring its applicability to various types of destinations affected by overtourism.

For task 3, we prepared a **policy recommendation memorandum** building on the empirical findings to provide scientific decision-making support for the sustainable development of tourism in Juneau City. The memorandum highlights **key conclusions** derived from the model's empirical analysis and proposes **actionable policy recommendations**.

All in all, the tasks are **interconnected both theoretically and practically**: Task 1 establishes the **foundational model**, providing methodological support for subsequent research; Task 2 extends and optimizes the model, enhancing its **applicability and scalability**; Task 3 returns to the practical level, offering **empirical evidence** for policy formulation. Innovatively, we have developed a **scalable optimization model** for sustainable tourism development, which not only serves as a decision-making reference for Juneau City but also provides a **replicable analytical framework** and methodological guidance for the sustainable development of similar tourist destinations.

Keywords: Sustainable Development, NSGA-III, Over-tourism, Multi-objective Optimization

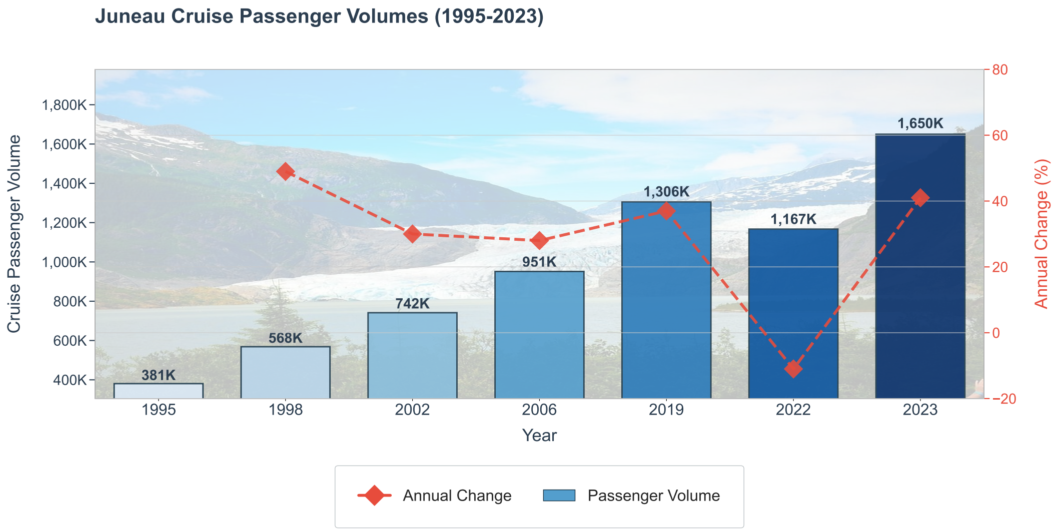
Contents

1 Introduction

1.1 Background

As the capital of Alaska, Juneau has a permanent population of approximately $30,000$. In recent years, tourism has become a key driver of its economic growth, contributing an annual revenue of approximately $\$375$ million. However, the rapid growth of tourism, particularly during peak seasons in Juneau, has led to a daily influx of up to $20,000$ visitors.These phenomena pose significant challenges to the local social, economic, and environmental fabric. Specifically, issues such as the overburdening of urban infrastructure, a decline in residents' quality of life, and environmental degradation have become increasingly evident.

In light of these issues, Juneau urgently requires the development and implementation of a scientifically-based sustainable tourism strategy that balances economic benefits, tourist experience, and environmental conservation. This case study not only highlights the typical challenges faced by Juneau as a climate-sensitive tourism destination but also offers valuable insights for other regions worldwide grappling with similar issues. The research on the sustainable development of Juneau's tourism industry provides both theoretical foundations and practical guidance for reconciling the demands of tourism growth with the need for ecological preservation in diverse contexts.



Noted: Passenger volumes in 2022 decreased due to COVID-19

1.2 Restatement and Analysis of the Problem

We aim to explore key issues in the sustainable development of tourism by constructing a multi-objective optimization model. Our research tasks primarily consist of the following three interrelated aspects:

- **Task 1: Tourism Sustainability Optimization Framework** This task establishes a comprehensive analytical system combining multivariate regression, input-output analysis, and multi-objective optimization to quantify tourism-environment interactions and optimize investment strategies. Based on literature research and existing survey data, we construct a multiple regression model to examine the relationships between carbon emissions, economic benefits of tourism, glacier melting rates, resident satisfaction, and tourist numbers. Building on this basis, an input-output analysis will be used to assess the investment effectiveness in three areas: environmental protection (carbon reduction), infrastructure development, and community programs. Subsequently, a multi-objective optimization model will be developed using the NSGA-III algorithm. This model will integrate the objective functions and constraints, aiming to determine the optimal tourist carrying capacity and investment allocation plan, followed by a sensitivity analysis of key parameters.

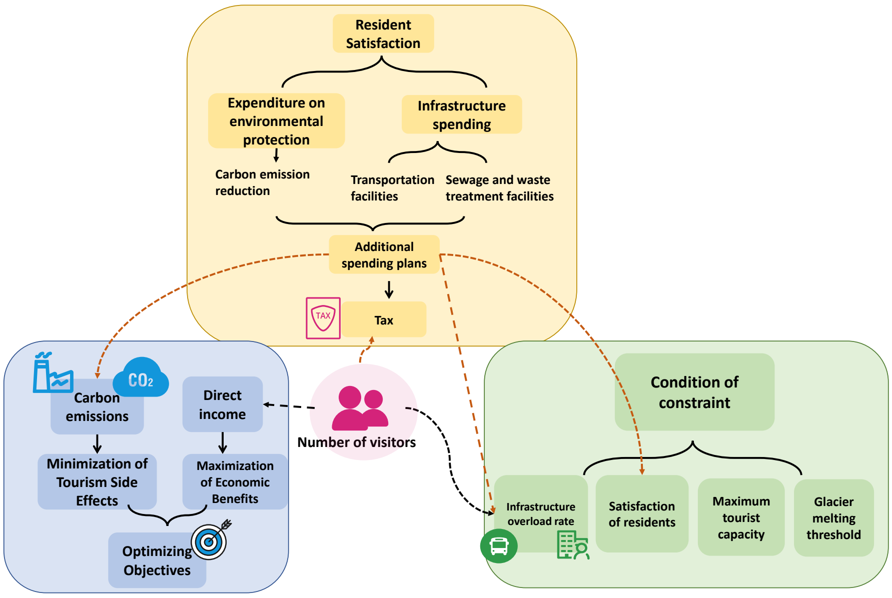


Figure 1: Tourism Sustainability Optimization Framework

- **Task 2: Data-driven adaptability Summary.** This task develops a dynamic clustering methodology to implement differentiated management strategies for diverse tourism destinations. A clustering analysis will be conducted to classify different types of tourist destinations, with a focus on examining the differential characteristics of core indicators such as tourist numbers, infrastructure carrying capacity, and tourism revenue. Based on the classification results, the constraint parameters of the optimization model will be dynamically adjusted to ensure the model's adaptability and flexibility in addressing over-tourism impacts at different types of destinations.

- **Task 3: Evidence-based memo writing.** This task translates quantitative findings into actionable governance tools for sustainable tourism management. Based on the empirical research findings, a policy recommendation memorandum will be drafted to provide scientific decision-making support for the sustainable development of Juneau's tourism industry. The memorandum will focus on the key conclusions derived from the model's empirical analysis and offer actionable policy recommendations.

There is a significant theoretical and practical connection between the tasks outlined above: The foundational model developed in Task 1 provides the methodological support for subsequent research;

2 Assumptions and Justification

To simplify the problem and make it convenient for us to simulate real-life conditions, we make the following basic assumptions, each of which is properly justified.

- **We conceptualize glacial ablation as an environmental baseline parameter.** Glacial ablation constitutes the principal constraint on regional tourism development as a long-term geomorphic process. $2010-2020$ observations demonstrate sustained annual retreat ($5.9km^3$) with limited interannual variability. Our passage posits glacial mass balance systems function independently from tourism dynamics, quantifying their tourism impact as an environmental baseline parameter.

3 List of Notation

Symbol	Meaning
V_i	Visitor Volume
TA^{α}	Tax Allocation in area α
TR_i	Tourism Revenue
S_i	Resident Satisfaction
G_i	Glacier Melt Volume
C_i	Carbon Emissions
P_i	Infrastructure Pressure

Noted: Footmark i means in year i

4 Data Pre-processing

4.1 Data Collection

Since there is no data provided in the question, we consulted relevant survey results and research reports. The data we found about Greenhouse Gas Emissions, Number of Tourists and Total Tourism Revenue from 2011 to 2023 are as Table 1:

- **Comprehensive evaluation indicators of tourism industry**

  Given that businesses serving the tourism industry also typically cater to the residents of Southeast Alaska, such as restaurants and support services for air and water transportation, it is not feasible to attribute employment and wage data solely to the tourism sector. In the Economic Indicators Reports published annually by the City of Juneau, all positions within the leisure, hospitality, and transportation sectors are aggregated to assess the overall health of the tourism industry. Consequently, the total income reported for the Leisure & Hospitality sector is considered representative of the total income generated by the tourism industry.

- **Questionnaire survey on residents' satisfaction**

  We reviewed the tourism satisfaction survey data from the Juneau City Government and plotted the corresponding charts. The results indicate that residents are concerned about tourism-related issues.

Table 1: Environmental Impact and Tourism Development (2011-2023)

Year	Greenhouse Gas Emissions	Number of Tourists	Total Tourism Revenue
	$(tonCO_2e)$		(USD)
2011	$71,727.76$	$1,556,800$	N/A
2012	$113,587.49$	$1,586,000$	$27,713,469$
2013	$95,652.89$	$1,693,800$	$28,374,275$
2014	$76,765.20$	$1,659,600$	$30,711,658$
2015	$51,892.62$	$1,780,000$	$75,068,464$
2016	$56,959.98$	$1,857,500$	$79,294,933$
2017	$49,644.26$	$1,926,300$	$82,318,620$
2018	$49,236.73$	$2,026,300$	$92,094,125$
2019	104,822.60	2,213,000	103,225,389
2022	N/A	N/A	119,520,965
2023	N/A	N/A	134,631,332

Data for 2020-2021 affected by COVID-19 are not included.

Most are dissatisfied with crowding and sidewalk congestion, with scores between 1 and 2. Regarding overcapacity and visitor limits, attitudes are neutral, with scores between 2 and 3, showing some support but not universal agreement. Traffic congestion is also a concern, with scores mostly between 2 and 3. Cruise frequency and emissions also raise concern, as residents worry about their environmental impact. In contrast, there is strong support for clean energy, with scores concentrated between 4 and 5. Flight emissions also received attention, reflecting concern about the environmental impact of aviation.

Overall, residents hope tourism can drive economic growth while prioritizing environmental protection. Detailed survey results are shown in Figure 2.

5 Task1: Tourism Sustainability Optimization Framework

5.1 Precondition description

5.1.1 Relationship between Carbon Emissions and Tourist Numbers

We selected greenhouse gas emission data from 2011 to 2019 as the dependent variable to examine the dynamic impacts of tourist numbers and temporal factors on emissions. The independent variables were chosen based on the following rationale:

- **Quadratic term of tourist numbers:** Reflects the nonlinear relationship between tourism activities and emissions, capturing changes in marginal effects.
- **Year deviation term $(Year - 2011)$:** Quantifies cumulative effects of environmental policy implementation while eliminating baseline year selection bias.
- **Data standardization:** Tourist numbers were converted to millions (V_i^M) to enhance model stability.

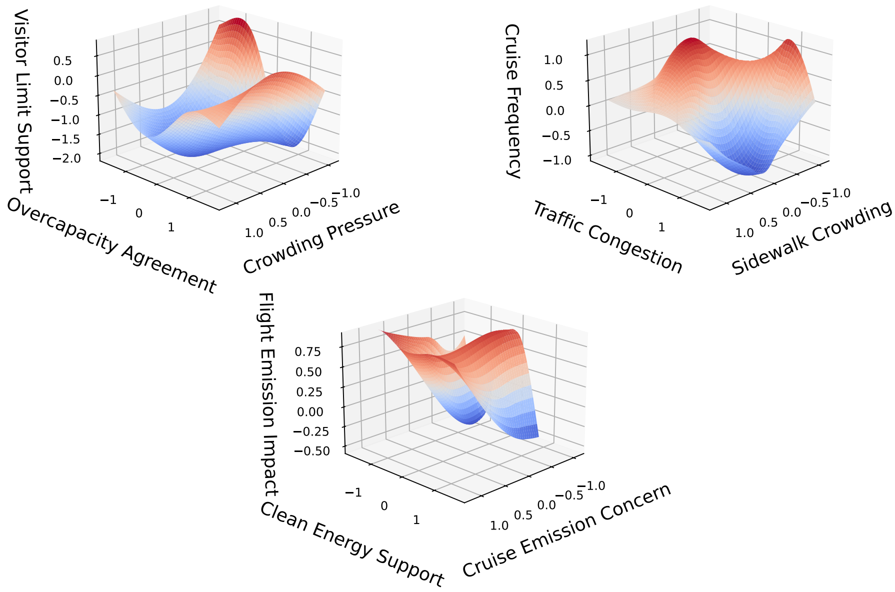


Figure 2: Result of Satisfaction Questionnaire

Based on this analysis, we established a quadratic regression model with temporal trend:

$$C_i = \beta_0 + \beta_1(\text{Year-2011}) + \beta_2 V_i^M + \beta_3 \left(V_i^M\right)^2 + \epsilon \tag{1}$$

Through solving the equation, we derived results as shown in the Figure 3

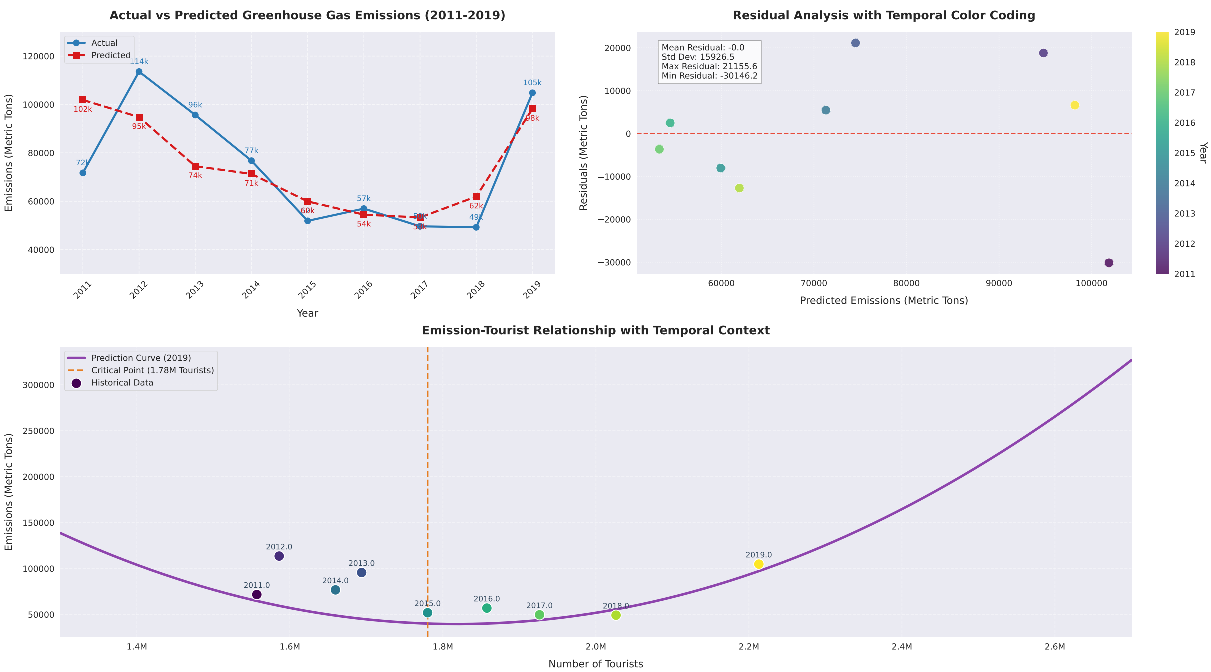


Figure 3: Relationship between Carbon Emissions and Tourist Numbers

- $\beta_0 = 1,151,001$ represents the baseline intercept for the reference year.
- $\beta_1 = -6,723$ quantifies the annual emission reduction effect (unit: metric tons per year).

- $\beta_2 = -1,198,962$ and $\beta_3 = 337,702$ jointly capture the nonlinear effects of tourism activities on carbon emissions.

By deriving the first derivative of this function:

$$\frac{\partial C}{\partial V} = \beta_2 + 2\beta_3 V = 0 \quad \Rightarrow \quad V_c = -\frac{\beta_2}{2\beta_3} \tag{2}$$

Substituting estimated parameters yields:

$$V_c = \frac{1,198,962}{2 \times 337,702} \approx 1.77 \text{ million} \tag{3}$$

This demonstrates that when Juneau's tourist numbers exceed 1.77 million visitors per year, subsequent growth in tourism will lead to a significant increase in local CO_2 emissions, causing environmental degradation through accelerated glacier melting and other ecological consequences.

5.1.2 Tourism revenue regression model

We employed regression analysis to investigate the quantitative relationship between tourist volume and tourism revenue. The linear regression model constructed based on the least squares method is as follows:

$$TR_i = \beta_0 + \beta_1 V_i + \epsilon \tag{4}$$

Through regression analysis, the following empirical results were obtained

- **The regression coefficient** ($\beta_1 = 136.581$) indicates that for each additional tourist, tourism revenue increases by an average of $\$136.581$. This coefficient quantifies the direct marginal effect of tourist numbers on tourism revenue.
- **The intercept term** ($\beta_0 = -186,843,122$) suggests that when tourist numbers equal zero, the predicted tourism revenue would be $\$-186,843,122$, reflecting fixed costs and other unobserved factors affecting the tourism industry in Juneau.
- **The coefficient of determination** ($R^2 = 0.844$) demonstrates that the model explains 84.4% of the variance in tourism revenue, indicating a strong goodness-of-fit to the observed data.
- Statistical significance tests reveal that the regression coefficient exhibits a **t-statistic of** 5.701 with a corresponding **p-value of** 0.001, which is statistically significant at the $\alpha = 0.05$ level. These results confirm a significant linear relationship between tourist numbers and tourism revenue, rejecting the null hypothesis of no association.

5.1.3 Structural equation model of residents' satisfaction

Through Structural Equation Modeling (SEM), this study systematically investigates the complex relationships among tourist volume, carbon emissions, glacier status, and satisfaction as Figure 4.

The model comprises four latent variables with corresponding observed indicators:

- **Tourist:** Measured by $Tourist_{Q1}$ (traffic congestion level) and $Tourist_{Q2}$ (cruise frequency), reflecting tourism activity intensity.

- **Glacier (glacial status):** Evaluated through $Glacier_{Q1}$ (residents' concern about tourism-induced glacial degradation), representing glacial environment protection awareness.
- **Carbon (carbon emissions):** Assessed by $Carbon_{Q1}$ (residents' concern about tourism-related carbon emissions), measuring carbon emission cognition and behavioral patterns.
- **Satisfaction:** Integrated with $Satisfy_{Q1}$ (resident satisfaction), reflecting the interaction between environmental policies and public satisfaction.

Significance Analysis of Critical Pathways:

The model results demonstrate a statistically significant positive impact of tourist volume on satisfaction (path coefficient: 0.524, $p < 0.05$). This suggests that increased tourist numbers may enhance overall resident/visitor satisfaction through stimulating local economic vitality or improving infrastructure investment.

A significant feedback effect emerges from satisfaction to glacial status (path coefficient: 0.587,$**p < 0.01$), indicating that heightened satisfaction may drive stricter glacial protection policies (e.g., tourist capacity restrictions), thereby alleviating glacial pressure.

Notably, while the negative impact of carbon emissions on satisfaction lacks statistical significance (path coefficient: -0.204, $p > 0.05$), its directional tendency implies potential indirect erosion of public satisfaction through environmental quality deterioration.

Comprehensive Analysis:

The integrated findings reveal a U-shaped correlation between tourist volume and resident satisfaction. Short-term tourism growth enhances satisfaction through economic benefits, whereas excessive long-term tourism may reduce satisfaction through **Intensified greenhouse gas emissions** and **Infrastructure pressure**

5.1.4 To be continue

Since a relatively large number of concepts are introduced in our model, we first identify and clarify the meaning of the introduced concepts here.

- **Short-term Motivation** refers to a brief period of above-average performance by a player following a motivating event.
- **Confidence Boost** is when a player, drawing upon their self-awareness developed through multiple games in their professional career and an assessment of the current situation, generates positive expectations regarding the outcome of the match.
- **Serve Success Rate** is probability of scoring in a service game.
- **Winning Probability** is the likelihood of the player winning the game/set/match.
- **Leverage** measures how a ball's outcome impacts the margin of victory.
- **Momentum** aims to describe which player is in control at any given point in the match—based on who is currently winning more points and who is prevailing in crucial (high-leverage) moments. It is defined as a player's exponentially weighted leverage average.

  At each point in the match, both competing players will have their own momentum values.

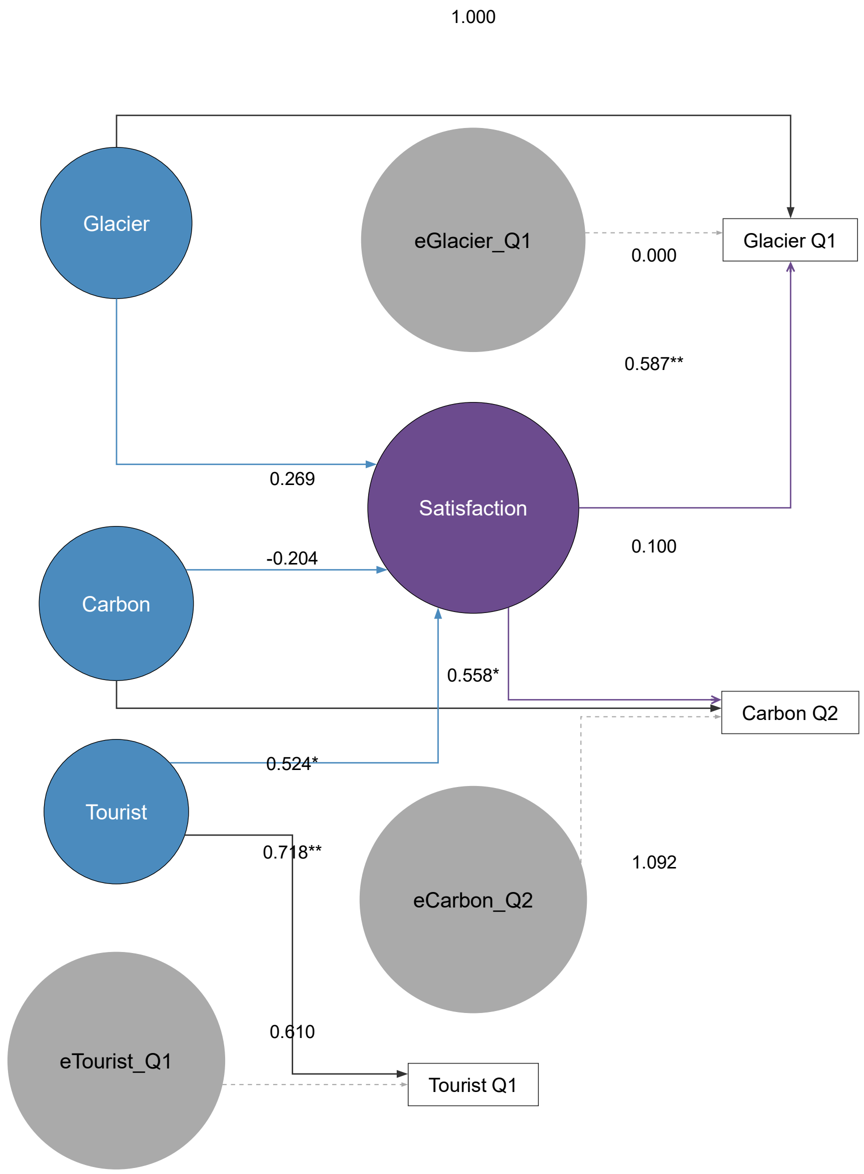


Figure 4: SEM for Resident Satisfaction Surveys

- We define the fluctuations in momentum as the **Flow of Play.**

The relationship between the above concepts, i.e., the framework of the model, is shown schematically below, and in the following we show the model building process in full.

5.2 Leverage Quantitative Momentum Capture Model

5.2.1 Impact of Short-term Motivation and Confidence Boost on Serve Success Rate

Short-term Motivation refers to a brief period of above-average performance by a player following a motivating event (e.g., a breaking serve) during a match, a phenomenon known in sports as "hot hands". We developed a probabilistic model that quantifies Short-term Motivation as an increase in serve success rate.

The following two assumptions are necessary:

- When a player wins a point, he experiences a short-term ”motivational” effect that increases the probability of winning the next point.
- Lifting effects are cumulative. A string of successes may have a positive psychological and emotional effect on a player, so that a player winning more points in a row will result in a greater probability of winning the next point on serve.

Here i denotes the number of consecutive points scored by a player A or his opponent B; and m_i denotes the increase in the rate of serving points on a player's next serve after winning $i$points consecutively due to the short-term motivation gained. Then, the probability of winning the next serve under short-term motivation p_s versus q_s is calculated by (5).

$$\begin{aligned} p_s &= p_f(1 + m_i^A); \\ q_s &= q_f(1 + m_i^B), \end{aligned} \tag{5}$$

where p_f and q_f are the scoring probabilities before player A and player B are motivated, respectively.

The short-term motivation that a player receives for scoring a game n times in a row is referred to as n-order short-term motivation. Using the dataset given in the title, we statistically obtained a plot of the total number of short-term motivations occurring in a match versus order, and the results are shown in Figure 5.

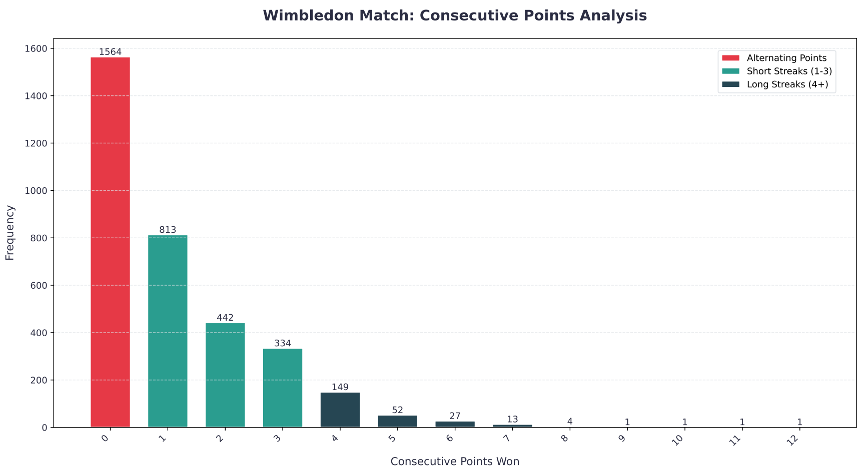


Figure 5: The number of short-term motivations of different orders in a match.

It is clear to see that short-term motivations of order four and above rarely occur in the dataset used in this study. Therefore, only short-term motivations of order three and below are considered in this paper.

Short-termmotivation focuses on short-termstatus enhancement during matches and ignores sources of motivation on a longer time scale in the player's lifetime, so we continue to introduce a **confidence-boosting factor**.

A new tuning parameter λ is introduced, which represents the effect of the confidence boosting factor on the success rate of the next serve, the probabilities after the effect are expressed as

p_s and q_s – see equation (6)

$$\begin{aligned} p_c &= \frac{T_{AB}}{\lambda} p_t + \left(1 - \frac{T_{AB}}{\lambda}\right) p_0; \\ q_c &= \frac{T_{AB}}{\lambda} q_t + \left(1 - \frac{T_{AB}}{\lambda}\right) q_0, \end{aligned} \tag{6}$$

where T_{AB} denotes the cumulative total score of the opposing players since the start of the match, and p_t and q_t denote the probability of scoring for player A and player B, respectively, since the start of the match up to the present. p_0 and q_0 denote the historical data on the success rate of the opposing teams' serves on the grass court surface prior to the current match (source: `https://www.infosys.com/`)

Eq. (2) describes the phenomenon that a player's historical results always have the greatest impact on player status at the start of a match, and then diminish as the match progresses - the performance of the current match begins to influence player play more.

We then performed a linear interpolation of the obtained probabilities. The interpolation coefficient is the cumulative total score of the current pair divided by the coefficient λ (i.e.$\frac{T_{AB}}{\lambda}$).

The short-term motivation is fluctuating as the match progresses, while the confidence boost takes into account the player's self-perception established over the course of his career as well as the current situation in the match, which is the "undertone" of the motivated state, therefore, we use pc and qc in Eq. (6) instead of p_f and q_f in Eq. (5), we can get the success rate of the next serve after considering the short-term motivation and confidence improvement:

$$\begin{aligned} p &= \left[\frac{T_{AB}}{\lambda} \cdot p_c + \left(1 - \frac{T_{AB}}{\lambda}\right) p_t\right] (1 + m_i^A); \\ q &= \left[\frac{T_{AB}}{\lambda} \cdot q_c + \left(1 - \frac{T_{AB}}{\lambda}\right) q_t\right] (1 + m_i^B). \end{aligned}$$

The above equation shows the effect of short-term motivation and confidence enhancement on serve percentage.

In tennis matches, players typically have an advantage when serving. According to the data we have collected, the serving success rate of professional players exceeds 50%. Based on this understanding, **players gain confidence in serving games.** Due to this "confidence boost," we have incorporated the player's performance in serving games into consideration in our model.

5.2.2 Monte Carlo Simulation for Winning Probability

As a sports competition, there is a great deal of uncertainty in tennis, for which Monte Carlo algorithms can provide probabilistic estimates of the outcome.

Based on the serve success rate obtained in the previous section, we use the Monte Carlo algorithm to simulate each ball, and ultimately obtain the winners and losers of each game, set and even the whole match. Then in this process, the randomness and diversity of the simulation are ensured by generating a large number of random number seeds.

Here is the pseudo-code for the algorithm we used:

Symbols	Description	Unit
h	Convection heat transfer coefficient	W/(m^2· K)
k	Thermal conductivity	W/(m · K)
c_p	Specific heat	J/(kg · K)
ρ	Density	kg/m^2
δ	Thickness	m
t	Temperature	°C, K
τ	Time	s, min, h
q_m	Mass flow	kg/s
Φ	Heat transfer power	W
T	A period of time	s, min, h
V	Volume	m^3, L
M, m	Mass	kg
A	Aera	m^2
a, b, c	The size of a bathtub	m^3

where we define the main parameters while specific value of those parameters will be given later.

6 Model Overview

In our basic model, we aim at three goals: keeping the temperature as even as possible, making it close to the initial temperature and decreasing the water consumption.

We start with the simple sub-model where hot water is added constantly. At first we introduce convection heat transfer control equations in rectangular coordinate system. Then we define the mean temperature of bath water.

Afterwards, we introduce Newton cooling formula to determine heat transfer capacity. After deriving the value of parameters, we get calculating results via formula deduction and simulating results via CFD.

Secondly, we present the complicated sub-model in which hot water is added discontinuously. We define an iteration consisting of two process: heating and standby. As for heating process, we derive control equations and boundary conditions. As for standby process, considering energy conservation law, we deduce the relationship of total heat dissipating capacity and time.

Then we determine the time and amount of added hot water. After deriving the value of parameters, we get calculating results via formula deduction and simulating results via CFD.

At last, we define two criteria to evaluate those two ways of adding hot water. Then we propose optimal strategy for the user in a bathtub. The whole modeling process can be shown as follows.

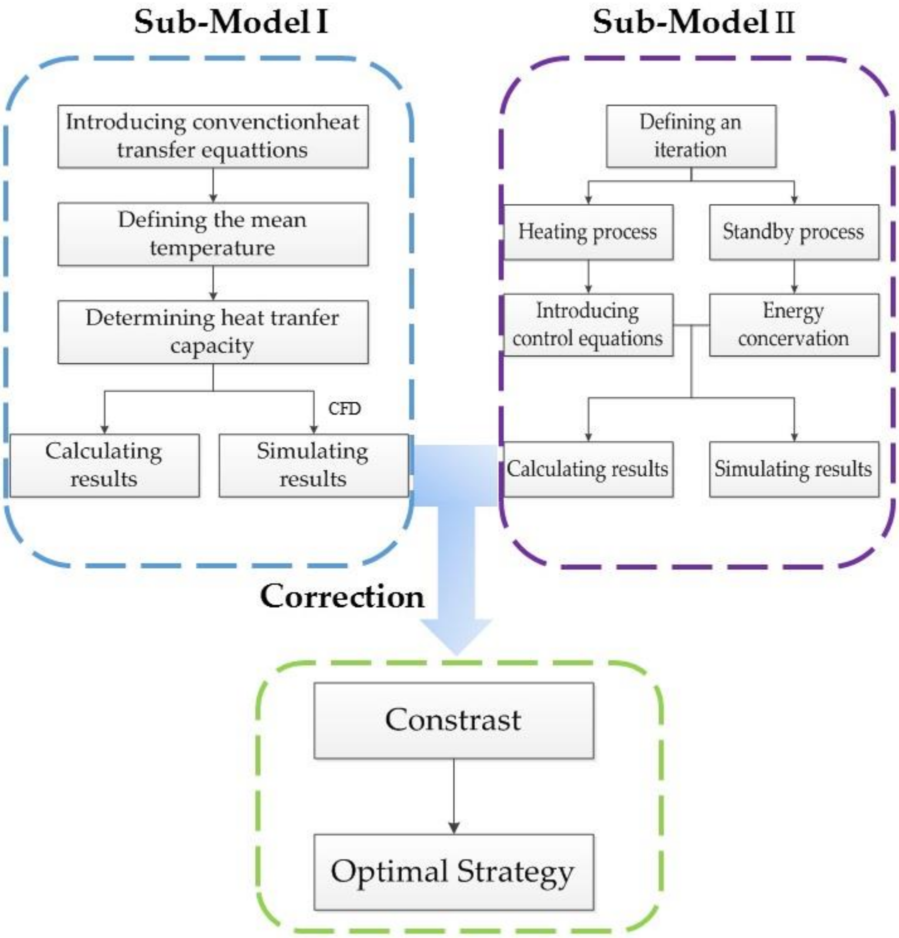


Figure 6: Modeling process

7 Sub-model I : Adding Water Continuously

We first establish the sub-model based on the condition that a person add water continuously to reheat the bathing water. Then we use Computational Fluid Dynamics (CFD) to simulate the change of water temperature in the bathtub. At last, we evaluate the model with the criteria which have been defined before.

7.1 Model Establishment

Since we try to keep the temperature of the hot water in bathtub to be even, we have to derive the amount of inflow water and the energy dissipated by the hot water into the air.

We derive the basic convection heat transfer control equations based on the former scientists' achievement. Then, we define the mean temperature of bath water. Afterwards, we determine two types of heat transfer: the boundary heat transfer and the evaporation heat transfer. Combining thermodynamic formulas, we derive calculating results. Via Fluent software, we get simulation results.

7.1.1 Control Equations and Boundary Conditions

According to thermodynamics knowledge, we recall on basic convection heat transfer control equations in rectangular coordinate system. Those equations show the relationship of the temperature of the bathtub water in space.

We assume the hot water in the bathtub as a cube. Then we put it into a rectangular coordinate system. The length, width, and height of it is a, b and c.

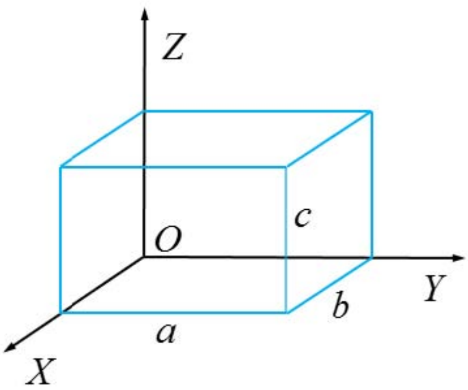


Figure 7: Modeling process

In the basis of this, we introduce the following equations [5]:

- **Continuity equation:**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{7}$$

where the first component is the change of fluid mass along the X-ray. The second component is the change of fluid mass along the Y-ray. And the third component is the change of fluid mass along the Z-ray. The sum of the change in mass along those three directions is zero.

- **Moment differential equation (N-S equations):**

$$\begin{cases} \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \eta\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \\ \rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \eta\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) \\ \rho\left(u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -g - \frac{\partial p}{\partial z} + \eta\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) \end{cases} \tag{8}$$

- **Energy differential equation:**

$$\rho c_p\left(u\frac{\partial t}{\partial x} + v\frac{\partial t}{\partial y} + w\frac{\partial t}{\partial z}\right) = \lambda\left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}\right) \tag{9}$$

where the left three components are convection terms while the right three components are conduction terms.

By Equation (9), we have ......

......

On the right surface in Fig. 7, the water also transfers heat firstly with bathtub inner surfaces and then the heat comes into air. The boundary condition here is ......

7.1.2 Definition of the Mean Temperature

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7.1.3 Determination of Heat Transfer Capacity

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8 Sub-model II: Adding Water Discontinuously

In order to establish the unsteady sub-model, we recall on the working principle of air conditioners. The heating performance of air conditions consist of two processes: heating and standby. After the user set a temperature, the air conditioner will begin to heat until the expected temperature is reached. Then it will go standby. When the temperature get below the expected temperature, the air conditioner begin to work again. As it works in this circle, the temperature remains the expected one.

Inspired by this, we divide the bathtub working into two processes: adding hot water until the expected temperature is reached, then keeping this condition for a while unless the temperature is lower than a specific value. Iterating this circle ceaselessly will ensure the temperature kept relatively stable.

8.1 Heating Model

8.1.1 Control Equations and Boundary Conditions

8.1.2 Determination of Inflow Time and Amount

8.2 Standby Model

8.3 Results

We first give the value of parameters based on others' studies. Then we get the calculation results and simulating results via those data.

8.3.1 Determination of Parameters

After establishing the model, we have to determine the value of some important parameters.

As scholar Beum Kim points out, the optimal temperature for bath is between 41 and 45°C [1]. Meanwhile, according to Shimodozono's study, 41°C warm water bath is the perfect choice for individual health [2]. So it is reasonable for us to focus on $41°\text{C} \sim 45°\text{C}$. Because adding hot water continuously is a steady process, so the mean temperature of bath water is supposed to be constant. We value the temperature of inflow and outflow water with the maximum and minimum temperature respectively.

The values of all parameters needed are shown as follows:

.....

8.3.2 Calculating Results

Putting the above value of parameters into the equations we derived before, we can get the some data as follows:

Table 2: The calculating results

Variables	Values	Unit
A_1	1.05	m^2
A_2	2.24	m^2
Φ_1	189.00	W
Φ_2	43.47	W
Φ	232.47	W
q_m	0.014	g/s

From Table 2, ...

......

9 Correction and Contrast of Sub-Models

After establishing two basic sub-models, we have to correct them in consideration of evaporation heat transfer. Then we define two evaluation criteria to compare the two sub-models in order to determine the optimal bath strategy.

9.1 Correction with Evaporation Heat Transfer

Someone may confuse about the above results: why the mass flow in the first sub-model is so small? Why the standby time is so long? Actually, the above two sub-models are based on ideal conditions without consideration of the change of boundary conditions, the motions made by the person in bathtub and the evaporation of bath water, etc. The influence of personal motions will be discussed later. Here we introducing the evaporation of bath water to correct sub-models.

9.2 Contrast of Two Sub-Models

Firstly we define two evaluation criteria. Then we contrast the two submodels via these two criteria. Thus we can derive the best strategy for the person in the bathtub to adopt.

10 Model Analysis and Sensitivity Analysis

10.1 The Influence of Different Bathtubs

Definitely, the difference in shape and volume of the tub affects the convection heat transfer. Examining the relationship between them can help people choose optimal bathtubs.

10.1.1 Different Volumes of Bathtubs

In reality, a cup of water will be cooled down rapidly. However, it takes quite long time for a bucket of water to become cool. That is because their volume is different and the specific heat of water is very large. So that the decrease of temperature is not obvious if the volume of water is huge. That also explains why it takes 45 min for 320 L water to be cooled by 1°C.

In order to examine the influence of volume, we analyze our sub-models by conducting sensitivity Analysis to them.

We assume the initial volume to be 280 L and change it by $\pm 5\%$, $\pm 8\%$, $\pm 12\%$ and $\pm 15\%$. With the aid of sub-models we established before, the variation of some parameters turns out to be as follows

Table 3: Variation of some parameters

V	A_1	A_2	T_2	q_{m1}	q_{m2}	Φ_q
-15.00%	-5.06%	-9.31%	-12.67%	-2.67%	-14.14%	-5.80%
-12.00%	-4.04%	-7.43%	-10.09%	-2.13%	-11.31%	-4.63%
-8.00%	-2.68%	-4.94%	-6.68%	-1.41%	-7.54%	-3.07%
-8.00%	-2.68%	-4.94%	-6.68%	-1.41%	-7.54%	-3.07%
-8.00%	-2.68%	-4.94%	-6.68%	-1.41%	-7.54%	-3.07%
-8.00%	-2.68%	-4.94%	-6.68%	-1.41%	-7.54%	-3.07%
-8.00%	-2.68%	-4.94%	-6.68%	-1.41%	-7.54%	-3.07%
-8.00%	-2.68%	-4.94%	-6.68%	-1.41%	-7.54%	-3.07%
-8.00%	-2.68%	-4.94%	-6.68%	-1.41%	-7.54%	-3.07%
-8.00%	-2.68%	-4.94%	-6.68%	-1.41%	-7.54%	-3.07%
-8.00%	-2.68%	-4.94%	-6.68%	-1.41%	-7.54%	-3.07%

11 Strength and Weakness

11.1 Strength

- We analyze the problem based on thermodynamic formulas and laws, so that the model we established is of great validity.
- Our model is fairly robust due to our careful corrections in consideration of real-life situations and detailed sensitivity analysis.
- Via Fluent software, we simulate the time field of different areas throughout the bathtub. The outcome is vivid for us to understand the changing process.
- We come up with various criteria to compare different situations, like water consumption and the time of adding hot water. Hence an overall comparison can be made according to these criteria.
- Besides common factors, we still consider other factors, such as evaporation and radiation heat transfer. The evaporation turns out to be the main reason of heat loss, which corresponds with other scientist's experimental outcome.

11.2 Weakness

- Having knowing the range of some parameters from others' essays, we choose a value from them to apply in our model. Those values may not be reasonable in reality.
- Although we investigate a lot in the influence of personal motions, they are so complicated that need to be studied further.
- Limited to time, we do not conduct sensitivity analysis for the influence of personal surface area.

12 Further Discussion

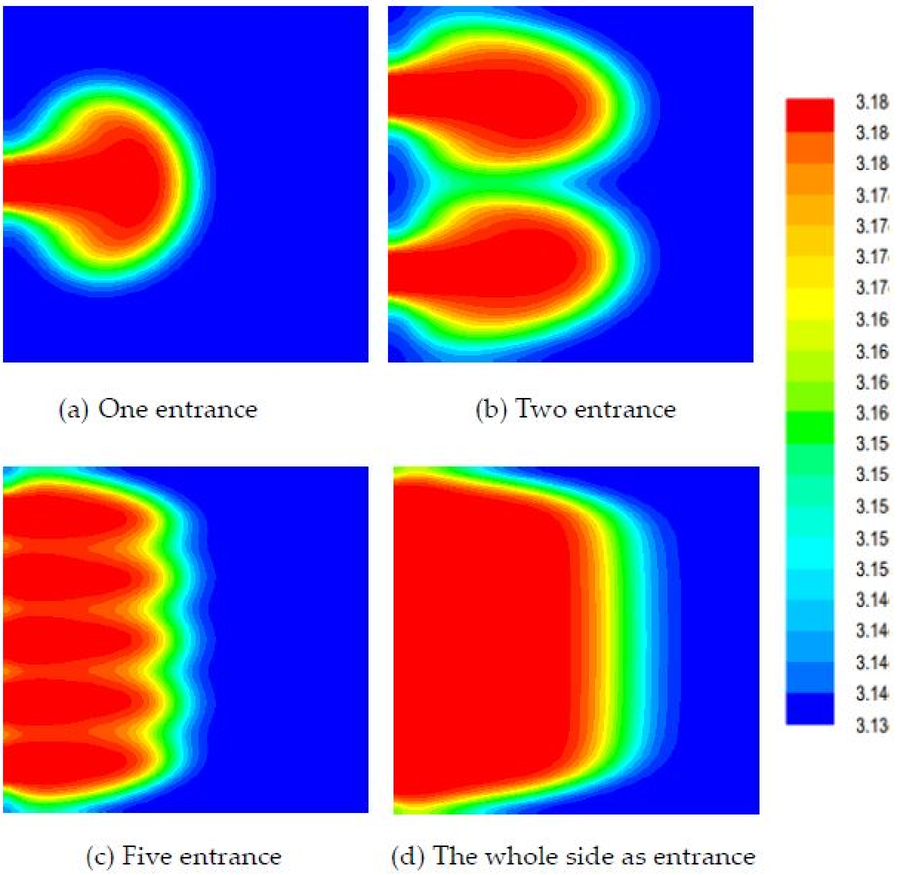
In this part, we will focus on different distribution of inflow faucets. Then we discuss about the real-life application of our model.

- Different Distribution of Inflow Faucets

  In our before discussion, we assume there being just one entrance of inflow.

  From the simulating outcome, we find the temperature of bath water is hardly even. So we come up with the idea of adding more entrances.

  The simulation turns out to be as follows



(a) One entrance (b) Two entrance

(c) Five entrance (d) The whole side as entrance

Figure 8: The simulation results of different ways of arranging entrances

From the above figure, the more the entrances are, the evener the temperature will be. Recalling on the before simulation outcome, when there is only one entrance for inflow, the temperature of corners is quietly lower than the middle area.

In conclusion, if we design more entrances, it will be easier to realize the goal to keep temperature even throughout the bathtub.

- Model Application

  Our before discussion is based on ideal assumptions. In reality, we have to make some corrections and improvement.

  1) Adding hot water continually with the mass flow of 0.16 kg/s. This way can ensure even mean temperature throughout the bathtub and waste less water.
  2) The manufacturers can design an intelligent control system to monitor the temperature so that users can get more enjoyable bath experience.
  3) We recommend users to add bubble additives to slow down the water being cooler and help cleanse. The additives with lower thermal conductivity are optimal.
  4) The study method of our establishing model can be applied in other area relative to convection heat transfer, such as air conditioners.

References

[1] Gi-Beum Kim. Change of the Warm Water Temperature for the Development of Smart Healthecare Bathing System. Hwahak konghak. 2006, 44(3): 270-276.

[2] https://en.wikipedia.org/wiki/Convective_heat_transfer#Newton.27s_law_of_cooling

[3] https://en.wikipedia.org/wiki/Navier%E2%80%93Stokes_equations

[4] https://en.wikipedia.org/wiki/Computational_fluid_dynamics

[5] Holman J P. Heat Transfer (9th ed.), New York: McGraw-Hill, 2002.

[6] Liu Weiguo, Chen Zhaoping, ZhangYin. Matlab program design and application. Beijing: Higher education press, 2002. (In Chinese)

Enjoy Your Bath Time!

From simulation results of real-life situations, we find it takes a period of time for the inflow hot water to spread throughout the bathtub. During this process, the bath water continues transferring heat into air, bathtub and the person in bathtub. The difference between heat transfer capacity makes the temperature of various areas to be different. So that it is difficult to get an evenly maintained temperature throughout the bath water.

In order to enjoy a comfortable bath with even temperature of bath water and without wasting too much water, we propose the following suggestions.

- Adding hot water consistently
- Using smaller bathtub if possible
- Decreasing motions during bath
- Using bubble bath additives
- Arranging more faucets of inflow

Sincerely yours,

Your friends

Appendices

Appendix A First appendix

In addition, your report must include a letter to the Chief Financial Officer (CFO) of the Goodgrant Foundation, Mr. Alpha Chiang, that describes the optimal investment strategy, your modeling approach and major results, and a brief discussion of your proposed concept of a return-on-investment (ROI). This letter should be no more than two pages in length.

Here are simulation programmes we used in our model as follow.

Input matlab source:

```
function [t,seat,aisle]=OI6Sim(n,target,seated)
pab=rand(1,n);
for i=1:n
    if pab(i)<0.4
        aisleTime(i)=0;
    else
        aisleTime(i)=trirnd(3.2,7.1,38.7);
    end
end
```

Appendix B Second appendix

some more text **Input C++ source:**

```
//===================================================================================
// Name        : Sudoku.cpp
// Author      : wzlf11
// Version     : a.0
// Copyright   : Your copyright notice
// Description : Sudoku in C++.
//===================================================================================

#include <iostream>
#include <cstdlib>
#include <ctime>

using namespace std;

int table[9][9];

int main() {

    for(int i = 0; i < 9; i++){
        table[0][i] = i + 1;
    }

    srand((unsigned int)time(NULL));

    shuffle((int *)&table[0], 9);

    while(!put_line(1))
```

```
    {
        shuffle((int *)&table[0], 9);
    }

    for(int x = 0; x < 9; x++){
        for(int y = 0; y < 9; y++){
            cout << table[x][y] << " ";
        }

        cout << endl;
    }

    return 0;
}
```

Report on Use of AI

1. OpenAI ChatGPT (Nov 5, 2023 version, ChatGPT-4,)

   Query1: <insert the exact wording you input into the AI tool>

   Output: <insert the complete output from the AI tool>

2. OpenAI Ernie (Nov 5, 2023 version, Ernie 4.0)

   Query1: <insert the exact wording of any subsequent input into the AI tool>

   Output: <insert the complete output from the second query>

3. Github CoPilot (Feb 3, 2024 version)

   Query1: <insert the exact wording you input into the AI tool>

   Output: <insert the complete output from the AI tool>

4. Google Bard (Feb 2, 2024 version)

   Query1: <insert the exact wording of your query>

   Output: <insert the complete output from the AI tool>