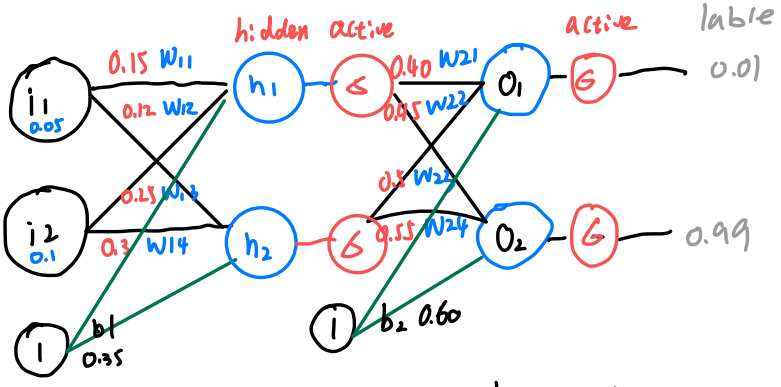

Forward:

$h_1 = i_1 \cdot W_{11} + i_2 \times W_{12} + b_1 \times 1$

$= 0.15 \times 0.05 + 0.1 \times 0.12 + 0.35$

$= 0.3775$

$a(h_1) = \frac{1}{1+e^{-h_1}} = 0.593$

$h_2 = i_1 \cdot w_{13} + i_2 \times w_{14} + b_1 \times 1$

$a(h_2) = \frac{1}{1+e^{-h_2}} = 0.596$

output layer:

$O_1 = a h_1 \cdot w_{21} + a h_2 \cdot w_{22} + b_2 \times 1$

$= 0.593 \times 0.4 + 0.596 \times 0.45 + 0.60$

$= 1.106$

$a(O_1) = \frac{1}{1+e^{-O_1}} = 0.75$ (0.01)

$O_2 = a h_1 \cdot w_{23} + a h_2 \cdot w_{24} + b_2 \times 1$

$a(O_2) = \frac{1}{1+e^{-O_2}} = 0.77$ (0.99)

backward:

1. cal loss: $loss = \sum \frac{1}{2}(target - output)^2$ (MSE)

$loss(O_1) = \frac{1}{2}(0.01 - 0.75)^2 = 0.275$

$loss(O_2) = \frac{1}{2}(0.77 - 0.99)^2 = 0.236$

$Loss = 0.275 + 0.236 = 0.298$

2. hidden → output:

$W_{21} \rightarrow O_1 \rightarrow a(O_1) \rightarrow Loss$

$Loss = L_1 + L_2$

$L_1 = \frac{1}{2}(targe - out)^2$

$\frac{\partial Loss}{\partial W_{21}} = \frac{\partial O_1}{\partial W_{21}} \cdot \frac{\partial a(O_1)}{\partial O_1} \cdot \frac{\partial Loss}{\partial a(O_1)}$

2.1 get $\frac{\partial loss}{\partial a(O_1)}$:

$Loss = \frac{1}{2}(target\,o_1 - a_{O_1})^2 + \frac{1}{2}(target\,o_2 - a_{O_2})^2$

$\frac{\partial Loss}{\partial a_{O_1}} = (target\,o_1 - a_{O_1}) \times (-1)$

$= (0.01 - 0.751) \times (-1) = 0.741$

2.2 get $\frac{\partial a(o_1)}{\partial o_1}$

$$a_{o_1} = \frac{1}{1+e^{-o_1}}$$

$$\frac{\partial a_{o_1}}{\partial o_1} = a_{o_1} \cdot (1 - a_{o_1}) = 0.751 \times (1 - 0.751) = 0.187$$

2.3 get $\frac{\partial o_1}{\partial w_{21}}$

$$o_1 = a(h_1) \times w_{21} + a(h_2) \times w_{22} + b_2 \times 1$$

$$\frac{\partial o_1}{\partial w_{21}} = a(h_1) = 0.593$$

2.4 get $\frac{\partial loss}{\partial w_{21}}$

$$\frac{\partial Loss}{\partial w_{21}} = 0.741 \times 0.187 \times 0.593 = 0.082$$

$$\frac{\partial loss}{\partial w_4} = -(target_{o_1} - a_{o_1}) \cdot a_{o_1} \cdot (1 - a_{o_1}) \cdot a h_1$$

$$\delta_{o_1} = \frac{\partial Loss}{\partial a_{o_1}} \cdot \frac{\partial a_{o_1}}{\partial o_1} = \frac{\partial Loss}{\partial o_1}$$

$$= -(target_{o_1} - a_{o_1}) \cdot a_{o_1} \cdot (1 - a_{o_1})$$

$$\frac{\partial Loss}{w_{21}} = \delta_{o_1} \times a_{h_1}$$

2.5 update w_{21}

$$w_{21}' = w_{21} - \eta \cdot \frac{\partial Loss}{\partial w_{21}} = 0.4 \times 0.5 \cdot 0.082 = 0.359$$

as same: $w_{22}' = 0.410$

$w_{23}' = 0.511$

$w_{24}' = 0.561$

3. input → hidden update:

$$\frac{\partial Loss}{\partial w_{11}} = \frac{\partial Loss}{\partial a_{h_1}} \times \frac{\partial a_{h_1}}{\partial h_1} \times \frac{\partial h_1}{\partial w_{11}}$$

$$\downarrow$$

$$\frac{\partial Loss}{\partial a_{h_1}} = \frac{\partial Loss_1}{\partial a_{h_1}} + \frac{\partial Loss_2}{\partial a_{h_2}}$$

3.1 get $\frac{\partial Loss}{\partial ah_1}$

$$\frac{\partial Loss}{\partial ah_1} = \frac{\partial loss1}{\partial ah_1} + \frac{\partial loss2}{\partial ah_2}$$

$$\frac{\partial loss1}{\partial ah_1} = \frac{\partial loss1}{\partial ao_1} \times \frac{\partial ao_1}{\partial o_1} \times \frac{\partial o_1}{\partial ah_1}$$

$$= 0.554$$

$o_1 = a(h_1) \times w21 + a(h_2) \times w22 + b2 \times 1$

$\frac{\partial o}{\partial ah_1} = w21$

$$\frac{\partial loss2}{\partial ah_1} = -0.019$$

$$\frac{\partial loss}{\partial ah_1} = 0.554 - 0.019 = 0.036$$

3.2 get $\frac{\partial ah_1}{\partial h_1} = ah_1 \cdot (1 - ah_1) = 0.241$

3.3 $\frac{\partial h_1}{\partial w11} = 0.05$

$h_1 = i1 \times w11 + i2 \times w21 + b1 \times 1$

$$\frac{\partial loss}{\partial w11} = 0.036 \times 0.241 \times 0.05 = 0.00043$$

3.4 $\frac{\partial Loss}{\partial w11} = \left(\sum_i \frac{Loss}{\partial ao_i} \times \frac{\partial ao_i}{\partial o_i} \times \frac{\partial o_i}{\partial ah_i}\right) \times \frac{\partial ah_i}{\partial h_i} \times \frac{\partial h_i}{\partial w11}$

$= \left(\sum_i \delta_i \times w_{hi}\right) \times ah_i \times (1 - ah_i) \times i_i$

$= \delta_{h1} \times i_1$

3.5 update w11 $w_{11}' = w_{11} - \eta \times \frac{\partial Loss}{\partial w11} = 0.15 - 0.5 \times 0.0004$

$= 0.15$

Same:

$w12' = 0.1995$

$w13' = 0.25$

$w14' = 0.2995$