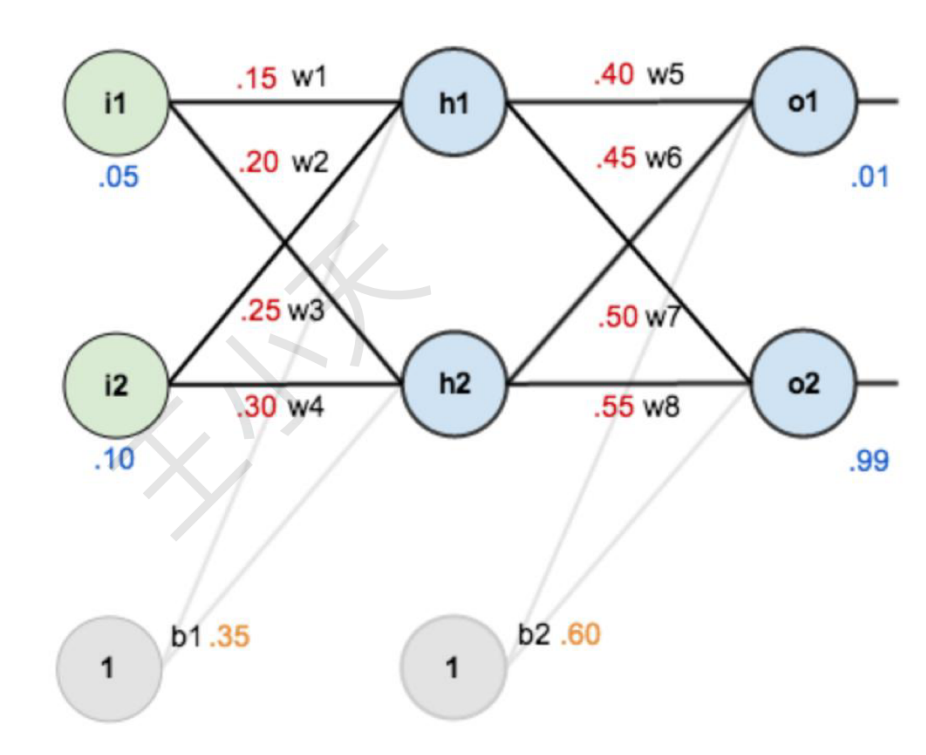


前向传播：

$$z_{o_1} = a_{h_1} \cdot w_5 + a_{h_2} \cdot w_6 + b_2 = 1.105$$

$$z_{h_1} = i_1 \cdot w_1 + i_2 \cdot w_2 + b_1$$

$$z_{h_2} = i_1 \cdot w_3 + i_2 \cdot w_4 + b_1$$

Sigmoid：

$$a_{o_1} = \frac{1}{1 + e^{-z_{o_1}}} \qquad a_{h_1} = \frac{1}{1 + e^{-z_{h_1}}}$$

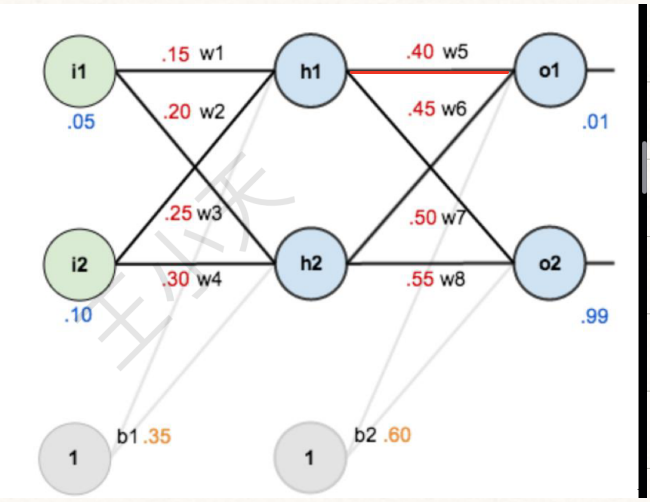
a_{o_2} 同理.

$$\Rightarrow a_{o_1} = 0.7513 \qquad a_{o_2} = 0.7729$$

输出：[0.7513, 0.7729]

实际：[0.01, 0.99]

仅向传播



① 损失函数

$E_{o_1} = \frac{1}{2}(0.01 - 0.7513)^2$

$E_{o_2} = \frac{1}{2}(0.99 - 0.7729)^2$

$E_{total} = E_{o_1} + E_{o_2}$

$= 0.2984$

② 更新 w_5，w_5 对 E_{total} 的影响.

对 w_5 求 E_{total} 的偏导. E_{total} 中与 w_5 有关的：$a_{o_1} \Rightarrow z_{o_1} \Rightarrow z_{o_1} = a_{h_1} \cdot w_5 + a_{h_2} \cdot w_6 + b_2$

$$\frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial a_{o_1}} * \frac{\partial a_{o_1}}{\partial z_{o_1}} * \frac{\partial z_{o_1}}{\partial w_5}$$

$$\frac{\partial E_{total}}{\partial a_{o_1}} = 2 \times \frac{1}{2}(target_{o_1} - a_{o_1}) * -1$$

$$= a_{o_1} - target_{o_1} = 0.7513 - 0.01 = 0.7413$$

$$\frac{\partial a_{o_1}}{\partial z_{o_1}} = a_{o_1} * (1 - a_{o_1}) = 0.7513 \times (1 - 0.7513) = 0.1868$$

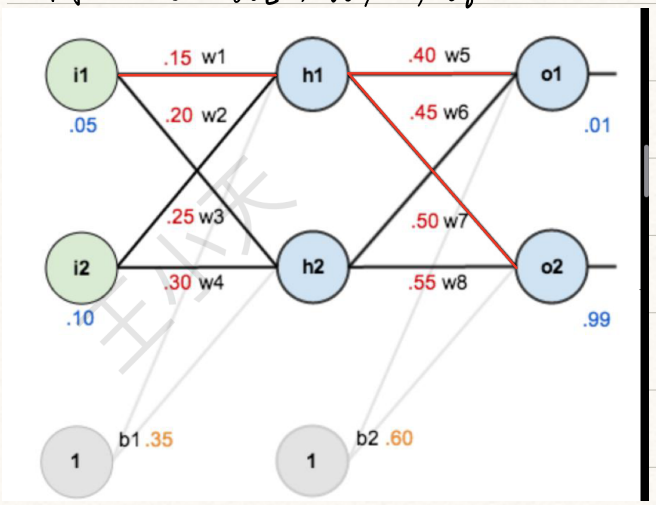
$$\frac{\partial z_{o_1}}{\partial w_5} = a_{h_1} = 0.5932$$

$$\Rightarrow \frac{\partial E_{total}}{\partial w_5} = 0.7413 \times 0.1868 \times 0.5932 = 0.08216$$

③ 更新权重. η取0.5

$$w_5^+ = w_5 - \eta * \frac{\partial E_{total}}{\partial w_5} = 0.4 - 0.5 \times (-0.051) = 0.4255$$

同理可得: w_6^+, w_7^+, w_8^+



更新 w_1: E_{total} 经 $E_{o_2} \to a_{o_2} \to z_{o_2} \to a_{h_1} \to z_{h_1} \to w_1$ 和 $E_{o_1} \to a_{o_1} \to z_{o_1} \to a_{h_1} \to z_{h_1} \to w_1$

$$\Rightarrow \frac{\partial E_{total}}{\partial a_{h_1}} = \frac{\partial E_{o_1}}{\partial a_{h_1}} + \frac{\partial E_{o_2}}{\partial a_{h_1}}$$

$$\Rightarrow \frac{\partial E_{o_1}}{\partial a_{h_1}} = \frac{\partial E_{o_1}}{\partial a_{o_1}} * \frac{\partial a_{o_1}}{\partial z_{o_1}} * \frac{\partial z_{o_1}}{\partial a_{h_1}} = 0.7413 \times 0.1868 \times w_5 = 0.05539$$

同理可得: $\frac{\partial E_{o_2}}{\partial a_{h_1}} = \frac{\partial E_{o_2}}{\partial a_{o_2}} * \frac{\partial a_{o_2}}{\partial z_{o_2}} * \frac{\partial z_{o_2}}{\partial a_{h_1}} = (-0.261) \times 0.1755 \times w_7 = -0.02291$

$\frac{\partial a_{h_1}}{\partial z_{h_1}} = a_{h_1} * (1 - a_{h_1}) = 0.2413$ $\quad \frac{\partial z_{h_1}}{\partial w_1} = i_1 = 0.05$

$$\Rightarrow \frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial a_{h_1}} * \frac{\partial a_{h_1}}{\partial z_{h_1}} * \frac{\partial z_{h_1}}{\partial w_1} = 0.107829 \times 0.2413 \times 0.05 = 0.00009445$$

更新 w_1:

$$w_1^+ = w_1 - \eta * \frac{\partial E_{total}}{\partial w_1} = 0.15 - (0.00009445 \times 0.5) = 0.14952$$

同理可得 w_2^+, w_3^+, w_4^+