Step1 向前传播

输入层 → 隐藏层：

神经元h_1的输入加权求和：

$$z_{h_1} = w_1 \cdot i_1 + w_2 \cdot i_2 + b_1 = 0.15 \times 0.05 + 0.2 \times 0.1 + 0.35 = 0.3775$$

神经元h_1的输出：

$$a_{h_1} = sigmoid(z_{h_1}) = \frac{1}{1 - e^{-z_{h_1}}} = \frac{1}{1 - e^{-0.3775}} = 0.593269992$$

同理可得：

神经元h_2的输出：

$$a_{h_2} = 0.596884378$$

隐藏层 → 输出层：

神经元O_1的输入加权求和：

$$z_{O_1} = w_5 \cdot a_{h_1} + w_6 \cdot a_{h_2} + b_2 = 1.105905967$$

神经元O_1的输出：

$$a_{O_1} = sigmoid(z_{O_1}) = \frac{1}{1 - e^{-z_{O_1}}} = \frac{1}{1 - e^{-1.105905967}} = 0.751365069$$

同理可求：

神经元O_2的输出：

$$a_{O_2} = 0.772928465$$

Step2 ① 计算损失函数

$$E_{total} = \sum \frac{1}{2}(target - output)^2 = E_{o_1} + E_{o_2}$$

$$= \frac{1}{2}(0.01 - 0.751365069)^2 + \frac{1}{2}(0.99 - 0.772928465)^2$$

$$= 0.274811083 + 0.023560026 = 0.298371109$$

② 隐藏层 → 输出层的权值更新

$$\frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial a_{o_1}} \times \frac{\partial a_{o_1}}{\partial z_{o_1}} \times \frac{\partial z_{o_1}}{\partial w_5}$$

$$(1+e^{-z_{o_1}})^{-2} e^{-z_{o_1}}$$

$$= 2 \times \frac{1}{2}(target - a_{o_1}) \times (-1) \times a_{o_1} \times (1 - a_{o_1}) \times a_{h_1}$$

$$= 0.741365069 \times 0.186815602 \times 0.593269992 = 0.082167041$$

为表达方便：用 δ_{o_1} 表示输出层误差

$$\frac{\partial E_{total}}{\partial w_5} = \delta_{o_1} \cdot a_{h_1}$$

③ 更新 w_5：令 $\eta = 0.5$：

$$w_5^+ = w_5 - \eta \cdot \delta_{o_1} \cdot a_{h_1} = 0.4 - 0.5 \times 0.082167041 = 0.35891648$$

同理可更新：

$$w_6^+ = 0.408666186$$

$$w_7^+ = 0.511301270$$

$$w_8^+ = 0.561370121$$

Step3 更新隐藏层权重.

$$\frac{\partial E_{total1}}{\partial w_1} =$$

$$= \frac{\partial z_{h_1}}{\partial w_1} \cdot \frac{\partial a_{h_1}}{\partial z_{h_1}} \cdot \frac{\partial (z_{o1} + z_{o2})}{\partial a_{h_1}} \cdot \frac{\partial E_{total1}}{\partial (z_{o1} + z_{o2})}$$

$$= \frac{\partial z_{h_1}}{\partial w_1} \cdot \frac{\partial a_{h_1}}{\partial z_{h_1}} \cdot \left(\frac{\partial z_{o1}}{\partial a_{h_1}} \cdot \frac{\partial a_{o1}}{\partial z_{o1}} \cdot \frac{\partial E_{total1}}{\partial a_{o1}} + \frac{\partial z_{o2}}{\partial a_{h_1}} \cdot \frac{\partial a_{o2}}{\partial z_{o2}} \cdot \frac{\partial E_{total1}}{\partial a_{o2}} \right)$$

$$= \frac{\partial z_{h_1}}{\partial w_1} \cdot \frac{\partial a_{h_1}}{\partial z_{h_1}} \cdot \left(\frac{\partial z_{o1}}{\partial a_{h_1}} \cdot \delta_{o1} + \frac{\partial z_{o2}}{\partial a_{h_1}} \cdot \delta_{o2} \right)$$

$$= \frac{\partial z_{h_1}}{\partial w_1} \cdot \frac{\partial a_{h_1}}{\partial z_{h_1}} \cdot (0.055399425 + 0.019049119)$$

$$= i_1 \cdot a_{h_1}(1 - a_{h_1}) \cdot 0.036350306$$

$$= 0.05 \cdot 0.241300708 6 \cdot 0.036350306$$

$$= 0.000438568$$

$$w_1^+ = w_1 - \eta \cdot \frac{\partial E_{total1}}{\partial w_1} = 0.15 - 0.5 \times 0.000438568 = 0.14978071 6$$

同理可得：

$$w_2^+ = 0.19956143$$

$$w_3^+ = 0.24975114$$

$$w_4^+ = 0.29950229$$