



$$\text{sigmoid} = \frac{1}{1 + e^{-x}}$$

一. 正向传播.

$$zh_1 = i_1 \times W_1 + i_2 \times W_2 + b_1 \times 1$$

$$= 0.05 \times 0.15 + 0.1 \times 0.2 + 0.35 \times 1$$

$$= 0.3775$$

经过激活函数:

$$h_1 = \frac{1}{1 + e^{-0.3775}} = 0.5932$$

$$zh_2 = i_1 \times W_3 + i_2 \times W_4 + b_1 \times 1$$

$$0.0125$$

$$= 0.05 \times 0.25 + 0.1 \times 0.3 + 0.35 \times 1$$

$$= 0.3925$$

经过激活函数:

$$h_2 = \frac{1}{1 + e^{-0.3925}} = 0.5968$$

$$zo_1 = h_1 \times W_5 + h_2 \times W_6 + 1 \times 0.6$$

$$= 0.5932 \times 0.4 + 0.5968 \times 0.45 + 0.6$$

$$= 1.1059$$

经过激活函数:

$$o_1 = \frac{1}{1 + e^{-1.1059}} = 0.7513$$

下同:  $o_2 = 0.7729$

二. 计算损失函数.  $\sum \frac{1}{2} (\text{target} - \text{output})^2$

$$E_{O_1} = \frac{1}{2} (0.01 - 0.7513)^2 = 0.2748.$$

$$E_{O_2} = \frac{1}{2} (0.99 - 0.7729)^2 = 0.0235.$$

$$E = E_{O_1} + E_{O_2} = 0.2983.$$

三. 反向传播 (更新权重) 求偏导

①  $W_5$  的影响:

$$\frac{\partial E}{\partial W_5} = \frac{\partial E}{\partial O_1} \times \frac{\partial O_1}{\partial W_5} \times \frac{\partial W_5}{\partial W_5}$$

$$E = \frac{1}{2} (0.01 - O_1)^2 + \frac{1}{2} (0.99 - O_2)^2$$

$$\frac{\partial E}{\partial O_1} = -(0.01 - O_1) = O_1 - 0.01 = 0.7413.$$

$$O_1 = \frac{1}{1 + e^{-2O_1}} = (1 + e^{-(h_1 \times W_5 + h_2 \times W_6 + 0.6)})^{-1}$$

$$\frac{\partial O_1}{\partial W_5} = O_1 \times (1 - O_1) = 0.7513 \times (1 - 0.7513) = 0.1868.$$

$$\frac{\partial E}{\partial 2O_1} = h_1 \times W_5 + h_2 \times W_6 + 0.6$$

$$\frac{\partial 2O_1}{\partial W_5} = h_1 = 0.5932. \quad \text{即 } \frac{\partial E}{\partial W_5} = 0.7413 \times 0.1868 \times 0.5932 = 0.0821.$$

$$W_5^+ = W_5 - \eta \times \frac{2E}{2w_5} = 0.4 - 0.5 \times 0.0821 = 0.3589$$

同理  $W_6^+ = 0.4086$

$$w_7^+ = 0.5113$$

$$w_8^+ = 0.5613$$

②  $W_1$  的影响

$$\frac{2E}{2w_1} = \frac{2E}{2h_1} \times \frac{2h_1}{2z_{h_1}} \times \frac{2z_{h_1}}{2w_1}$$

$$\downarrow$$

$$\frac{2E}{2h_1} = \frac{2E_{01}}{2\alpha_1} + \frac{2E_{02}}{2h_1}$$

$$\frac{2E_{01}}{2h_1} = \frac{2E_{01}}{2\alpha_1} \times \frac{2\alpha_1}{2z_{\alpha_1}} \times \frac{2z_{\alpha_1}}{2h_1} = 0.0553$$

$$\frac{2E_{02}}{2h_1} = \frac{2E_{02}}{2\alpha_2} \times \frac{2\alpha_2}{2z_{\alpha_2}} \times \frac{2z_{\alpha_2}}{2h_1} = -0.0190$$

$$\frac{2E}{2h_1} = 0.0363$$

$$W_1^+ = w_1 - \eta \times \frac{2E}{2w_1}$$

$$= 0.1497$$

$$\frac{2h_1}{2z_{h_1}} = \frac{2h_1}{2h_1} \times (1 - \alpha_1) = 0.2413 \text{ 同理得}$$

$$\frac{2z_{h_1}}{2w_1} = 21 = 0.05$$

$$w_2^+ = 0.1995$$

$$w_3^+ = 0.2497$$

$$w_4^+ = 0.2995$$

$$\text{即 } \frac{2E}{2w_1} = 0.0363 \times 0.2413 + 0.05 = 0.0004$$