

Hierarchical GLM

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```
library(tidyverse)
library(here)
# for bayes stuff
library(rstan)
library(coda)
library(bayesplot)
library(loo)
library(tidybayes)
library(ggplot2)
library(dplyr)
library(arm)
```

Please hand in Rmd, pdf, and stan files. Due next Wednesday because of delay in lecture.

Lip cancer

Here is the lip cancer data as seen in the lecture.

- `observe.i` is observed deaths in each region
- `expect.i` is expected deaths, based on region-specific age distribution and national-level age-specific mortality rates.

```
observe.i <- c(
  5,13,18,5,10,18,29,10,15,22,4,11,10,22,13,14,17,21,25,6,11,21,13,5,19,18,14,17,3,10,
  7,3,12,11,6,16,13,6,9,10,4,9,11,12,23,18,12,7,13,12,12,13,6,14,7,18,13,9,6,8,7,6,16,4,6,12,5,5,
  17,5,7,2,9,7,6,12,13,17,5,5,6,12,10,16,10,16,15,18,6,12,6,8,33,15,14,18,25,14,2,73,13,14,6,20,8,
  12,10,3,11,3,11,13,11,13,10,5,18,10,23,5,9,2,11,9,11,6,11,5,19,15,4,8,9,6,4,4,2,12,12,11,9,7,7,
  8,12,11,23,7,16,46,9,18,12,13,14,14,3,9,15,6,13,13,12,8,11,5,9,8,22,9,2,10,6,10,12,9,11,32,5,11,
  9,11,11,0,9,3,11,11,11,5,4,8,9,30,110)
expect.i <- c(
  6.17,8.44,7.23,5.62,4.18,29.35,11.79,12.35,7.28,9.40,3.77,3.41,8.70,9.57,8.18,4.35,
  4.91,10.66,16.99,2.94,3.07,5.50,6.47,4.85,9.85,6.95,5.74,5.70,2.22,3.46,4.40,4.05,5.74,6.36,5.13,
  16.99,6.19,5.56,11.69,4.69,6.25,10.84,8.40,13.19,9.25,16.98,8.39,2.86,9.70,12.12,12.94,9.77,
  10.34,5.09,3.29,17.19,5.42,11.39,8.33,4.97,7.14,6.74,17.01,5.80,4.84,12.00,4.50,4.39,16.35,6.02,
  6.42,5.26,4.59,11.86,4.05,5.48,13.13,8.72,2.87,2.13,4.48,5.85,6.67,6.11,5.78,12.31,10.56,10.23,
  2.52,6.22,14.29,5.71,37.93,7.81,9.86,11.61,18.52,12.28,5.41,61.96,8.55,12.07,4.29,19.42,8.25,
  12.90,4.76,5.56,11.11,4.76,10.48,13.13,12.94,14.61,9.26,6.94,16.82,33.49,20.91,5.32,6.77,8.70,
  12.94,16.07,8.87,7.79,14.60,5.10,24.42,17.78,4.04,7.84,9.89,8.45,5.06,4.49,6.25,9.16,12.37,8.40,
  9.57,5.83,9.21,9.64,9.09,12.94,17.42,10.29,7.14,92.50,14.29,15.61,6.00,8.55,15.22,18.42,5.77,
  18.37,13.16,7.69,14.61,15.85,12.77,7.41,14.86,6.94,5.66,9.88,102.16,7.63,5.13,7.58,8.00,12.82,
  18.75,12.33,5.88,64.64,8.62,12.09,11.11,14.10,10.48,7.00,10.23,6.82,15.71,9.65,8.59,8.33,6.06,
  12.31,8.91,50.10,288.00)
```

Question 1

The `expect.i` indicates the expected number of lip cancer deaths for a particular area if the relative risk of that area is at average level.

Question 2

Run three different models in Stan with three different set-up's for estimating θ_i , that is the relative risk of lip cancer in each region:

1. θ_i is same in each region = θ
2. θ_i is different in each region and modeled separately
3. θ_i is different in each region and modeled hierarchically

```
stan_data <- list(N = length(observe.i),
                 offset = expect.i,
                 deaths = observe.i)
mod1 <- stan(data = stan_data,
             file = "lip1.stan",
             iter = 1000,
             seed = 23)
```

```
## Running /opt/R/4.1.2/lib/R/bin/R CMD SHLIB foo.c
## gcc -I"/opt/R/4.1.2/lib/R/include" -DNDEBUG -I"/cloud/lib/x86_64-pc-linux-gnu-library/4.1/Rcpp/include"
## In file included from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Core:88,
##                  from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Dense:1,
##                  from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/StanHeaders/include/stan/math/prim/
##                  from <command-line>:
## /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:1: error:
## 628 | namespace Eigen {
##      | ~~~~~
## /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:17: error:
## 628 | namespace Eigen {
##      | ~~~~~
## In file included from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Dense:1,
##                  from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/StanHeaders/include/stan/math/prim/
##                  from <command-line>:
## /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Core:96:10: fatal error: complex:
## 96 | #include <complex>
##      | ~~~~~
## compilation terminated.
## make: *** [/opt/R/4.1.2/lib/R/etc/Makeconf:168: foo.o] Error 1
##
## SAMPLING FOR MODEL 'lip1' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 2.1e-05 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.21 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 1: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 1: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 1: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 1: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 1: Iteration: 500 / 1000 [ 50%] (Warmup)
```

```

## Chain 1: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 1: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 1: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 1: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 1: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 1: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 1:
## Chain 1: Elapsed Time: 0.098891 seconds (Warm-up)
## Chain 1: 0.052204 seconds (Sampling)
## Chain 1: 0.151095 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'lip1' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 1.5e-05 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.15 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 2: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 2: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 2: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 2: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 2: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 2: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 2: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 2: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 2: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 2: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 2: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 2:
## Chain 2: Elapsed Time: 0.101773 seconds (Warm-up)
## Chain 2: 0.053408 seconds (Sampling)
## Chain 2: 0.155181 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'lip1' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 1.3e-05 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.13 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 3: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 3: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 3: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 3: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 3: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 3: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 3: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 3: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 3: Iteration: 800 / 1000 [ 80%] (Sampling)

```

```

## Chain 3: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 3: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 3:
## Chain 3: Elapsed Time: 0.105939 seconds (Warm-up)
## Chain 3: 0.058487 seconds (Sampling)
## Chain 3: 0.164426 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'lip1' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 1.3e-05 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.13 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 4: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 4: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 4: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 4: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 4: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 4: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 4: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 4: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 4: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 4: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 4: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 4:
## Chain 4: Elapsed Time: 0.11183 seconds (Warm-up)
## Chain 4: 0.056464 seconds (Sampling)
## Chain 4: 0.168294 seconds (Total)
## Chain 4:

```

```
summary(mod1)$summary[c("theta"),]
```

```

##          mean      se_mean      sd      2.5%      25%      50%
## 9.555786e-01 7.451330e-04 1.884632e-02 9.184766e-01 9.424488e-01 9.563095e-01
##          75%      97.5%      n_eff      Rhat
## 9.683908e-01 9.911235e-01 6.397135e+02 1.005249e+00

```

```

stan_data <- list(N = length(observe.i),
                 offset = expect.i,
                 deaths = observe.i)
mod2 <- stan(data = stan_data,
             file = "lip2.stan",
             iter = 1000,
             seed = 23)

```

```

## Running /opt/R/4.1.2/lib/R/bin/R CMD SHLIB foo.c
## gcc -I"/opt/R/4.1.2/lib/R/include" -DNDEBUG -I"/cloud/lib/x86_64-pc-linux-gnu-library/4.1/Rcpp/include"
## In file included from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Core:88,
##                  from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Dense:1,
##                  from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/StanHeaders/include/stan/math/prim/
##                  from <command-line>:
## /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:1: error

```

```

## 628 | namespace Eigen {
## | ~~~~~
## /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:17: error:
## 628 | namespace Eigen {
## | ~~~~~
## In file included from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Dense:1,
## from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/StanHeaders/include/stan/math/prim/
## from <command-line>:
## /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Core:96:10: fatal error: complex:
## 96 | #include <complex>
## | ~~~~~
## compilation terminated.
## make: *** [/opt/R/4.1.2/lib/R/etc/Makeconf:168: foo.o] Error 1
##
## SAMPLING FOR MODEL 'lip2' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 4.2e-05 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.42 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 1: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 1: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 1: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 1: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 1: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 1: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 1: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 1: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 1: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 1: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 1: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 1:
## Chain 1: Elapsed Time: 1.05774 seconds (Warm-up)
## Chain 1: 0.861769 seconds (Sampling)
## Chain 1: 1.91951 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'lip2' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 2.3e-05 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.23 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 2: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 2: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 2: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 2: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 2: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 2: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 2: Iteration: 600 / 1000 [ 60%] (Sampling)

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## Chain 2: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 2: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 2: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 2: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 2:
## Chain 2: Elapsed Time: 1.34315 seconds (Warm-up)
## Chain 2: 1.59938 seconds (Sampling)
## Chain 2: 2.94254 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'lip2' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 2.4e-05 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.24 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 3: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 3: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 3: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 3: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 3: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 3: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 3: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 3: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 3: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 3: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 3: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 3:
## Chain 3: Elapsed Time: 1.64556 seconds (Warm-up)
## Chain 3: 1.93564 seconds (Sampling)
## Chain 3: 3.5812 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'lip2' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 2.3e-05 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.23 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 4: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 4: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 4: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 4: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 4: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 4: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 4: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 4: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 4: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 4: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 4: Iteration: 1000 / 1000 [100%] (Sampling)

```

```

## Chain 4:
## Chain 4: Elapsed Time: 1.00995 seconds (Warm-up)
## Chain 4: 0.857694 seconds (Sampling)
## Chain 4: 1.86765 seconds (Total)
## Chain 4:

stan_data <- list(N = length(observe.i),
                  offset = expect.i,
                  deaths = observe.i)
mod3 <- stan(data = stan_data,
             file = "lip3.stan",
             iter = 1000,
             seed = 23)

## Running /opt/R/4.1.2/lib/R/bin/R CMD SHLIB foo.c
## gcc -I"/opt/R/4.1.2/lib/R/include" -DNDEBUG -I"/cloud/lib/x86_64-pc-linux-gnu-library/4.1/Rcpp/include"
## In file included from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Core:88,
## from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Dense:1,
## from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/StanHeaders/include/stan/math/prim/
## from <command-line>:
## /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:1: error:
## 628 | namespace Eigen {
## | ~~~~~~
## /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:17: error:
## 628 | namespace Eigen {
## | ~~~~~~
## In file included from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Dense:1,
## from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/StanHeaders/include/stan/math/prim/
## from <command-line>:
## /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Core:96:10: fatal error: complex:
## 96 | #include <complex>
## | ~~~~~~
## compilation terminated.
## make: *** [/opt/R/4.1.2/lib/R/etc/Makeconf:168: foo.o] Error 1
##
## SAMPLING FOR MODEL 'lip3' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 3.9e-05 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.39 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 1: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 1: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 1: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 1: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 1: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 1: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 1: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 1: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 1: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 1: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 1: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 1:

```

```

## Chain 1: Elapsed Time: 0.385363 seconds (Warm-up)
## Chain 1:           0.344846 seconds (Sampling)
## Chain 1:           0.730209 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'lip3' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 2.2e-05 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.22 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:   1 / 1000 [  0%] (Warmup)
## Chain 2: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 2: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 2: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 2: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 2: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 2: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 2: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 2: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 2: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 2: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 2: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 2:
## Chain 2: Elapsed Time: 0.390113 seconds (Warm-up)
## Chain 2:           0.355484 seconds (Sampling)
## Chain 2:           0.745597 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'lip3' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 2.1e-05 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.21 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:   1 / 1000 [  0%] (Warmup)
## Chain 3: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 3: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 3: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 3: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 3: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 3: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 3: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 3: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 3: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 3: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 3: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 3:
## Chain 3: Elapsed Time: 0.368189 seconds (Warm-up)
## Chain 3:           0.341909 seconds (Sampling)
## Chain 3:           0.710098 seconds (Total)
## Chain 3:

```



```
##
## SAMPLING FOR MODEL 'lip3' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 2e-05 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.2 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 4: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 4: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 4: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 4: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 4: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 4: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 4: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 4: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 4: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 4: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 4: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 4:
## Chain 4: Elapsed Time: 0.378176 seconds (Warm-up)
## Chain 4: 0.353232 seconds (Sampling)
## Chain 4: 0.731408 seconds (Total)
## Chain 4:
```

```
summary(mod3)$summary[c("theta[2]", "mu", "sigma_mu"),]
```

```
##              mean      se_mean      sd      2.5%      25%      50%
## theta[2] 1.4060763 0.005404523 0.35745985 0.8013954 1.1570848 1.3807698
## mu       0.1514147 0.279829010 0.54880511 -0.7091834 -0.2971713 0.0283373
## sigma_mu 0.4418978 0.001097276 0.03355925 0.3784728 0.4189658 0.4402686
##              75%      97.5%      n_eff      Rhat
## theta[2] 1.6125207 2.2193123 4374.614909 0.9990430
## mu       0.6611669 1.1739234 3.846368 2.5847882
## sigma_mu 0.4631634 0.5132205 935.389717 0.9989132
```

Question 3

Make three plots (appropriately labeled and described) that illustrate the differences in estimated θ_i 's across regions and the differences in θ s across models.

Question 4

Rerun model 3 (the hierarchical model), but also including an overdispersion parameter. Compare the two models and decide which is more appropriate.

```
stan_data <- list(N = length(observe.i),
                 offset = expect.i,
                 deaths = observe.i)
mod4 <- stan(data = stan_data,
             file = "lip4.stan",
             iter = 1000,
             seed = 23)
```

```
## Running /opt/R/4.1.2/lib/R/bin/R CMD SHLIB foo.c
```

```

## gcc -I"/opt/R/4.1.2/lib/R/include" -DNDEBUG -I"/cloud/lib/x86_64-pc-linux-gnu-library/4.1/Rcpp/include"
## In file included from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Core:88,
## from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Dense:1,
## from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/StanHeaders/include/stan/math/prim/
## from <command-line>:
## /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:1: error:
## 628 | namespace Eigen {
## | ~~~~~
## /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:17: error:
## 628 | namespace Eigen {
## | ~~~~~
## In file included from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Dense:1,
## from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/StanHeaders/include/stan/math/prim/
## from <command-line>:
## /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Core:96:10: fatal error: complex:
## 96 | #include <complex>
## | ~~~~~
## compilation terminated.
## make: *** [/opt/R/4.1.2/lib/R/etc/Makeconf:168: foo.o] Error 1
##
## SAMPLING FOR MODEL 'lip4' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 5.2e-05 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.52 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 1: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 1: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 1: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 1: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 1: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 1: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 1: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 1: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 1: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 1: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 1: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 1:
## Chain 1: Elapsed Time: 1.35756 seconds (Warm-up)
## Chain 1: 0.873056 seconds (Sampling)
## Chain 1: 2.23061 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'lip4' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 3e-05 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.3 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 2: Iteration: 100 / 1000 [ 10%] (Warmup)

```

```

## Chain 2: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 2: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 2: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 2: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 2: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 2: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 2: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 2: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 2: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 2: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 2:
## Chain 2: Elapsed Time: 1.15771 seconds (Warm-up)
## Chain 2: 1.83145 seconds (Sampling)
## Chain 2: 2.98916 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'lip4' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 4.6e-05 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.46 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 3: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 3: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 3: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 3: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 3: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 3: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 3: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 3: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 3: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 3: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 3: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 3:
## Chain 3: Elapsed Time: 1.42225 seconds (Warm-up)
## Chain 3: 1.99886 seconds (Sampling)
## Chain 3: 3.42111 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'lip4' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 2.9e-05 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.29 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 4: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 4: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 4: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 4: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 4: Iteration: 500 / 1000 [ 50%] (Warmup)

```

```
## Chain 4: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 4: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 4: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 4: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 4: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 4: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 4:
## Chain 4: Elapsed Time: 1.21293 seconds (Warm-up)
## Chain 4: 1.60897 seconds (Sampling)
## Chain 4: 2.8219 seconds (Total)
## Chain 4:
```

```
summary(mod4)$summary[c("theta[2]", "mu", "sigma_mu"),]
```

```
##              mean      se_mean      sd      2.5%      25%      50%
## theta[2]  1.37039981 0.01007992 0.3268993  0.82969585  1.1260799  1.33126138
## mu        -0.09180732 0.18498810 0.5882075 -1.31118751 -0.4700268 -0.06660285
## sigma_mu  0.29922211 0.02974819 0.1095723  0.09876038  0.2130765  0.32203225
##              75%      97.5%      n_eff      Rhat
## theta[2]  1.5824596 2.0825091 1051.75391 1.007459
## mu         0.4046398 0.8786499  10.11052 1.699934
## sigma_mu  0.3902860 0.4616859  13.56690 1.332127
```

I don't think adding a ϵ term is necessary, since it can be absorbed by alpha. SO I think the previous model in Q1 is better.