Hierarchical GLM

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```r
library(tidyverse)
library(here)
# for bayes stuff
library(rstan)
library(coda)
library(bayesplot)
library(loo)
library(tidybayes)
library(ggplot2)
library(dplyr)
library(arm)
```

Please hand in Rmd, pdf, and stan files. Due next Wednesday because of delay in lecture.

Lip cancer

Here is the lip cancer data as seen in the lecture.

- `observe.i` is observed deaths in each region
- `expect.i` is expected deaths, based on region-specific age distribution and national-level age-specific mortality rates.

```r
observe.i <- c(
  5,13,18,5,10,18,29,10,15,22,4,11,10,22,13,14,17,21,25,6,11,21,13,5,19,18,14,17,3,10,
  7,3,12,11,6,16,13,6,9,10,4,9,11,12,23,18,12,7,13,12,12,13,6,14,7,18,13,9,6,8,7,6,16,4,6,12,5,5,
  17,5,7,2,9,7,6,12,13,17,5,5,6,12,10,16,10,16,15,18,6,12,6,8,33,15,14,18,25,14,2,73,13,14,6,20,8,
  12,10,3,11,3,11,13,11,13,10,5,18,10,23,5,9,2,11,9,11,6,11,5,19,15,4,8,9,6,4,4,2,12,12,11,9,7,7,
  8,12,11,23,7,16,46,9,18,12,13,14,14,3,9,15,6,13,13,12,8,11,5,9,8,22,9,2,10,6,10,12,9,11,32,5,11,
  9,11,11,0,9,3,11,11,11,5,4,8,9,30,110)
expect.i <- c(
    6.17,8.44,7.23,5.62,4.18,29.35,11.79,12.35,7.28,9.40,3.77,3.41,8.70,9.57,8.18,4.35,
    4.91,10.66,16.99,2.94,3.07,5.50,6.47,4.85,9.85,6.95,5.74,5.70,2.22,3.46,4.40,4.05,5.74,6.36,5.13,
    16.99,6.19,5.56,11.69,4.69,6.25,10.84,8.40,13.19,9.25,16.98,8.39,2.86,9.70,12.12,12.94,9.77,
    10.34,5.09,3.29,17.19,5.42,11.39,8.33,4.97,7.14,6.74,17.01,5.80,4.84,12.00,4.50,4.39,16.35,6.02,
    6.42,5.26,4.59,11.86,4.05,5.48,13.13,8.72,2.87,2.13,4.48,5.85,6.67,6.11,5.78,12.31,10.56,10.23,
    2.52,6.22,14.29,5.71,37.93,7.81,9.86,11.61,18.52,12.28,5.41,61.96,8.55,12.07,4.29,19.42,8.25,
    12.90,4.76,5.56,11.11,4.76,10.48,13.13,12.94,14.61,9.26,6.94,16.82,33.49,20.91,5.32,6.77,8.70,
    12.94,16.07,8.87,7.79,14.60,5.10,24.42,17.78,4.04,7.84,9.89,8.45,5.06,4.49,6.25,9.16,12.37,8.40,
    9.57,5.83,9.21,9.64,9.09,12.94,17.42,10.29,7.14,92.50,14.29,15.61,6.00,8.55,15.22,18.42,5.77,
    18.37,13.16,7.69,14.61,15.85,12.77,7.41,14.86,6.94,5.66,9.88,102.16,7.63,5.13,7.58,8.00,12.82,
    18.75,12.33,5.88,64.64,8.62,12.09,11.11,14.10,10.48,7.00,10.23,6.82,15.71,9.65,8.59,8.33,6.06,
    12.31,8.91,50.10,288.00)
```

Question 1

The `expect.i` indicates the expected number of lip cancer deaths for a particular area if the relative risk of that area is at average level.

Question 2

Run three different models in Stan with three different set-up's for estimating θ_i, that is the relative risk of lip cancer in each region:

1. θ_i is same in each region $= \theta$
2. θ_i is different in each region and modeled separately
3. θ_i is different in each region and modeled hierarchically

```
stan_data <- list(N = length(observe.i),
                  offset = expect.i,
                  deaths = observe.i)
mod1 <- stan(data = stan_data,
             file = "lip1.stan",
             iter = 1000,
             seed = 23)
```

```
## Running /opt/R/4.1.2/lib/R/bin/R CMD SHLIB foo.c
## gcc -I"/opt/R/4.1.2/lib/R/include" -DNDEBUG   -I"/cloud/lib/x86_64-pc-linux-gnu-library/4.1/Rcpp/incl
## In file included from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Core:88,
##                  from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Dense:1,
##                  from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/StanHeaders/include/stan/math/prim/
##                  from <command-line>:
## /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:1: err
##   628 | namespace Eigen {
##       | ^~~~~~~~~
## /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:17: er
##   628 | namespace Eigen {
##       |                 ^
## In file included from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Dense:1,
##                  from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/StanHeaders/include/stan/math/prim/
##                  from <command-line>:
## /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Core:96:10: fatal error: complex:
##    96 | #include <complex>
##       |          ^~~~~~~~~
## compilation terminated.
## make: *** [/opt/R/4.1.2/lib/R/etc/Makeconf:168: foo.o] Error 1
##
## SAMPLING FOR MODEL 'lip1' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 2e-05 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.2 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:   1 / 1000 [  0%]  (Warmup)
## Chain 1: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 1: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 1: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 1: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 1: Iteration: 500 / 1000 [ 50%]  (Warmup)
```

```
## Chain 1: Iteration:  501 / 1000 [ 50%]  (Sampling)
## Chain 1: Iteration:  600 / 1000 [ 60%]  (Sampling)
## Chain 1: Iteration:  700 / 1000 [ 70%]  (Sampling)
## Chain 1: Iteration:  800 / 1000 [ 80%]  (Sampling)
## Chain 1: Iteration:  900 / 1000 [ 90%]  (Sampling)
## Chain 1: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0.09845 seconds (Warm-up)
## Chain 1:                0.051984 seconds (Sampling)
## Chain 1:                0.150434 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'lip1' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 1.4e-05 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.14 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:    1 / 1000 [  0%]  (Warmup)
## Chain 2: Iteration:  100 / 1000 [ 10%]  (Warmup)
## Chain 2: Iteration:  200 / 1000 [ 20%]  (Warmup)
## Chain 2: Iteration:  300 / 1000 [ 30%]  (Warmup)
## Chain 2: Iteration:  400 / 1000 [ 40%]  (Warmup)
## Chain 2: Iteration:  500 / 1000 [ 50%]  (Warmup)
## Chain 2: Iteration:  501 / 1000 [ 50%]  (Sampling)
## Chain 2: Iteration:  600 / 1000 [ 60%]  (Sampling)
## Chain 2: Iteration:  700 / 1000 [ 70%]  (Sampling)
## Chain 2: Iteration:  800 / 1000 [ 80%]  (Sampling)
## Chain 2: Iteration:  900 / 1000 [ 90%]  (Sampling)
## Chain 2: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 2:
## Chain 2:  Elapsed Time: 0.102438 seconds (Warm-up)
## Chain 2:                0.053807 seconds (Sampling)
## Chain 2:                0.156245 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'lip1' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 1.3e-05 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.13 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:    1 / 1000 [  0%]  (Warmup)
## Chain 3: Iteration:  100 / 1000 [ 10%]  (Warmup)
## Chain 3: Iteration:  200 / 1000 [ 20%]  (Warmup)
## Chain 3: Iteration:  300 / 1000 [ 30%]  (Warmup)
## Chain 3: Iteration:  400 / 1000 [ 40%]  (Warmup)
## Chain 3: Iteration:  500 / 1000 [ 50%]  (Warmup)
## Chain 3: Iteration:  501 / 1000 [ 50%]  (Sampling)
## Chain 3: Iteration:  600 / 1000 [ 60%]  (Sampling)
## Chain 3: Iteration:  700 / 1000 [ 70%]  (Sampling)
## Chain 3: Iteration:  800 / 1000 [ 80%]  (Sampling)
```

```
## Chain 3: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 3: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 0.105644 seconds (Warm-up)
## Chain 3:                0.058399 seconds (Sampling)
## Chain 3:                0.164043 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'lip1' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 1.3e-05 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.13 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration:    1 / 1000 [  0%]  (Warmup)
## Chain 4: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 4: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 4: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 4: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 4: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 4: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 4: Iteration: 600 / 1000 [ 60%]  (Sampling)
## Chain 4: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 4: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 4: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 4: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 4:
## Chain 4:  Elapsed Time: 0.111969 seconds (Warm-up)
## Chain 4:                0.057385 seconds (Sampling)
## Chain 4:                0.169354 seconds (Total)
## Chain 4:
```

```r
summary(mod1)$summary[c("theta"),]
```

```
##         mean      se_mean           sd         2.5%          25%          50%
## 9.555786e-01 7.451330e-04 1.884632e-02 9.184766e-01 9.424488e-01 9.563095e-01
##          75%        97.5%        n_eff         Rhat
## 9.683908e-01 9.911235e-01 6.397135e+02 1.005249e+00
```

```r
stan_data <- list(N = length(observe.i),
                  offset = expect.i,
                  deaths = observe.i)
mod2 <- stan(data = stan_data,
             file = "lip2.stan",
             iter = 1000,
             seed = 23)
```

```
## Running /opt/R/4.1.2/lib/R/bin/R CMD SHLIB foo.c
## gcc -I"/opt/R/4.1.2/lib/R/include" -DNDEBUG   -I"/cloud/lib/x86_64-pc-linux-gnu-library/4.1/Rcpp/incl
## In file included from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Core:88,
##                  from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Dense:1,
##                  from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/StanHeaders/include/stan/math/prim/r
##                  from <command-line>:
## /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:1: err
```

```
##   628 | namespace Eigen {
##       | ^~~~~~~~~
## /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:17: er:
##   628 | namespace Eigen {
##       |                 ^
## In file included from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Dense:1,
##                  from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/StanHeaders/include/stan/math/prim/
##                  from <command-line>:
## /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Core:96:10: fatal error: complex:
##    96 | #include <complex>
##       |          ^~~~~~~~~
## compilation terminated.
## make: *** [/opt/R/4.1.2/lib/R/etc/Makeconf:168: foo.o] Error 1
##
## SAMPLING FOR MODEL 'lip2' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 4.2e-05 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.42 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:    1 / 1000 [  0%]  (Warmup)
## Chain 1: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 1: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 1: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 1: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 1: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 1: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 1: Iteration: 600 / 1000 [ 60%]  (Sampling)
## Chain 1: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 1: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 1: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 1: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 1.01619 seconds (Warm-up)
## Chain 1:                0.839561 seconds (Sampling)
## Chain 1:                1.85575 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'lip2' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 2.2e-05 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.22 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:    1 / 1000 [  0%]  (Warmup)
## Chain 2: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 2: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 2: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 2: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 2: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 2: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 2: Iteration: 600 / 1000 [ 60%]  (Sampling)
```

```
## Chain 2: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 2: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 2: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 2: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 2:
## Chain 2:  Elapsed Time: 1.31185 seconds (Warm-up)
## Chain 2:                1.62417 seconds (Sampling)
## Chain 2:                2.93602 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'lip2' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 2.3e-05 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.23 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:    1 / 1000 [  0%]  (Warmup)
## Chain 3: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 3: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 3: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 3: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 3: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 3: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 3: Iteration: 600 / 1000 [ 60%]  (Sampling)
## Chain 3: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 3: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 3: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 3: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 1.58005 seconds (Warm-up)
## Chain 3:                1.8693 seconds (Sampling)
## Chain 3:                3.44936 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'lip2' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 2.3e-05 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.23 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration:    1 / 1000 [  0%]  (Warmup)
## Chain 4: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 4: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 4: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 4: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 4: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 4: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 4: Iteration: 600 / 1000 [ 60%]  (Sampling)
## Chain 4: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 4: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 4: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 4: Iteration: 1000 / 1000 [100%]  (Sampling)
```

```
## Chain 4:
## Chain 4:   Elapsed Time: 0.94959 seconds (Warm-up)
## Chain 4:                 0.820446 seconds (Sampling)
## Chain 4:                 1.77004 seconds (Total)
## Chain 4:
```

```r
stan_data <- list(N = length(observe.i),
                  offset = expect.i,
                  deaths = observe.i)
mod3 <- stan(data = stan_data,
             file = "lip3.stan",
             iter = 1000,
             seed = 23)
```

```
## Running /opt/R/4.1.2/lib/R/bin/R CMD SHLIB foo.c
## gcc -I"/opt/R/4.1.2/lib/R/include" -DNDEBUG   -I"/cloud/lib/x86_64-pc-linux-gnu-library/4.1/Rcpp/incl
## In file included from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Core:88,
##                  from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Dense:1,
##                  from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/StanHeaders/include/stan/math/prim/r
##                  from <command-line>:
## /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:1: erro
##   628 | namespace Eigen {
##       | ^~~~~~~~
## /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:17: er
##   628 | namespace Eigen {
##       |                 ^
## In file included from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Dense:1,
##                  from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/StanHeaders/include/stan/math/prim/r
##                  from <command-line>:
## /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Core:96:10: fatal error: complex:
##    96 | #include <complex>
##       |          ^~~~~~~~~
## compilation terminated.
## make: *** [/opt/R/4.1.2/lib/R/etc/Makeconf:168: foo.o] Error 1
##
## SAMPLING FOR MODEL 'lip3' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 4e-05 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.4 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:    1 / 1000 [  0%]  (Warmup)
## Chain 1: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 1: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 1: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 1: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 1: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 1: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 1: Iteration: 600 / 1000 [ 60%]  (Sampling)
## Chain 1: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 1: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 1: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 1: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 1:
```

```
## Chain 1:   Elapsed Time: 0.382148 seconds (Warm-up)
## Chain 1:                 0.342066 seconds (Sampling)
## Chain 1:                 0.724214 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'lip3' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 2.5e-05 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.25 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:    1 / 1000 [  0%]  (Warmup)
## Chain 2: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 2: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 2: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 2: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 2: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 2: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 2: Iteration: 600 / 1000 [ 60%]  (Sampling)
## Chain 2: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 2: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 2: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 2: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 2:
## Chain 2:   Elapsed Time: 0.386676 seconds (Warm-up)
## Chain 2:                 0.352383 seconds (Sampling)
## Chain 2:                 0.739059 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'lip3' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 2.4e-05 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.24 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:    1 / 1000 [  0%]  (Warmup)
## Chain 3: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 3: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 3: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 3: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 3: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 3: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 3: Iteration: 600 / 1000 [ 60%]  (Sampling)
## Chain 3: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 3: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 3: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 3: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 3:
## Chain 3:   Elapsed Time: 0.366436 seconds (Warm-up)
## Chain 3:                 0.351243 seconds (Sampling)
## Chain 3:                 0.717679 seconds (Total)
## Chain 3:
```

```
##
## SAMPLING FOR MODEL 'lip3' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 2.3e-05 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.23 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration:    1 / 1000 [  0%]  (Warmup)
## Chain 4: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 4: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 4: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 4: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 4: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 4: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 4: Iteration: 600 / 1000 [ 60%]  (Sampling)
## Chain 4: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 4: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 4: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 4: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 4:
## Chain 4:  Elapsed Time: 0.385155 seconds (Warm-up)
## Chain 4:                0.350899 seconds (Sampling)
## Chain 4:                0.736054 seconds (Total)
## Chain 4:
summary(mod3)$summary[c("theta[2]","mu","sigma_mu"),]
```

```
##                mean      se_mean         sd        2.5%         25%        50%
## theta[2]  1.4060763 0.005404523 0.35745985   0.8013954   1.1570848 1.3807698
## mu        0.1514147 0.279829010 0.54880511  -0.7091834  -0.2971713 0.0283373
## sigma_mu  0.4418978 0.001097276 0.03355925   0.3784728   0.4189658 0.4402686
##                75%     97.5%         n_eff      Rhat
## theta[2]  1.6125207 2.2193123 4374.614909 0.9990430
## mu        0.6611669 1.1739234    3.846368 2.5847882
## sigma_mu  0.4631634 0.5132205  935.389717 0.9989132
```

Question 3

Make three plots (appropriately labeled and described) that illustrate the differences in estimated θ_i's across regions and the differences in θs across models.

```
theta_mod1 <- summary(mod1)$summary[c(paste0('theta')),1]
```

```
theta_mod2 <- c()
for (i in 1:length(expect.i)){
  theta_mod2[i] <- summary(mod2)$summary[c(paste0("theta[",i,']')),1]
}
```

```
theta_mod3 <- c()
for (i in 1:length(expect.i)){
  theta_mod3[i] <- summary(mod3)$summary[c(paste0("theta[",i,']')),1]
}
```
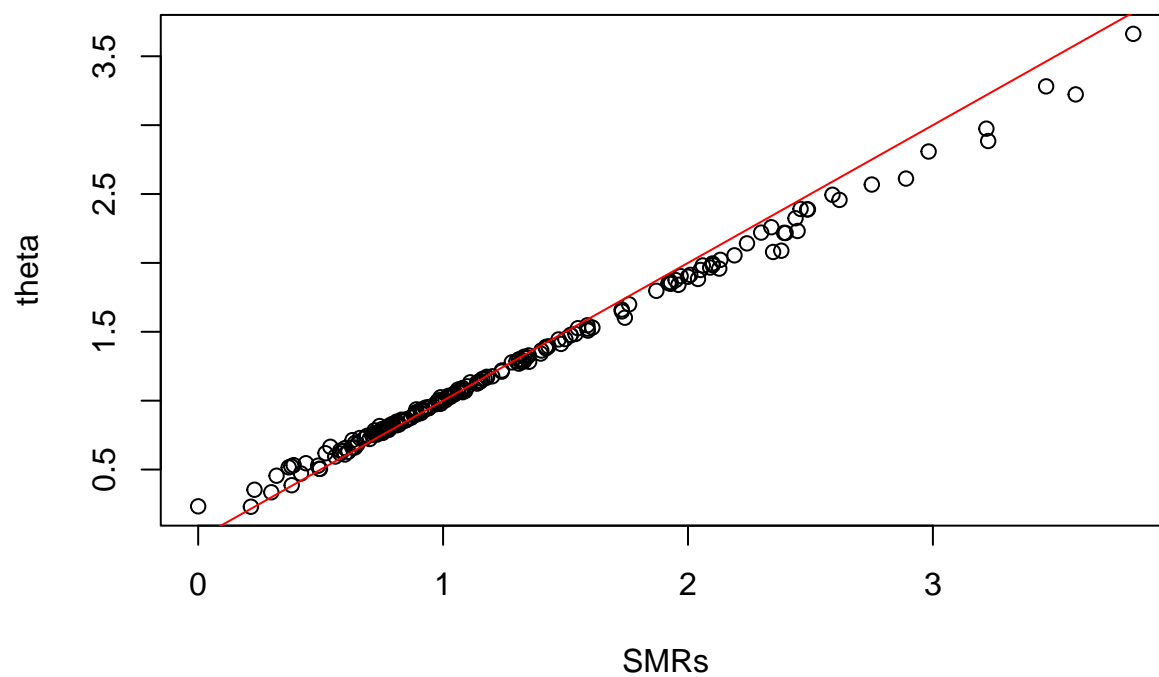
```
plot(density(observe.i/expect.i),main='density of SMRs v.s. estimated theta in model 1',xlab='SMRs')
abline(v=theta_mod1,col='red')
```
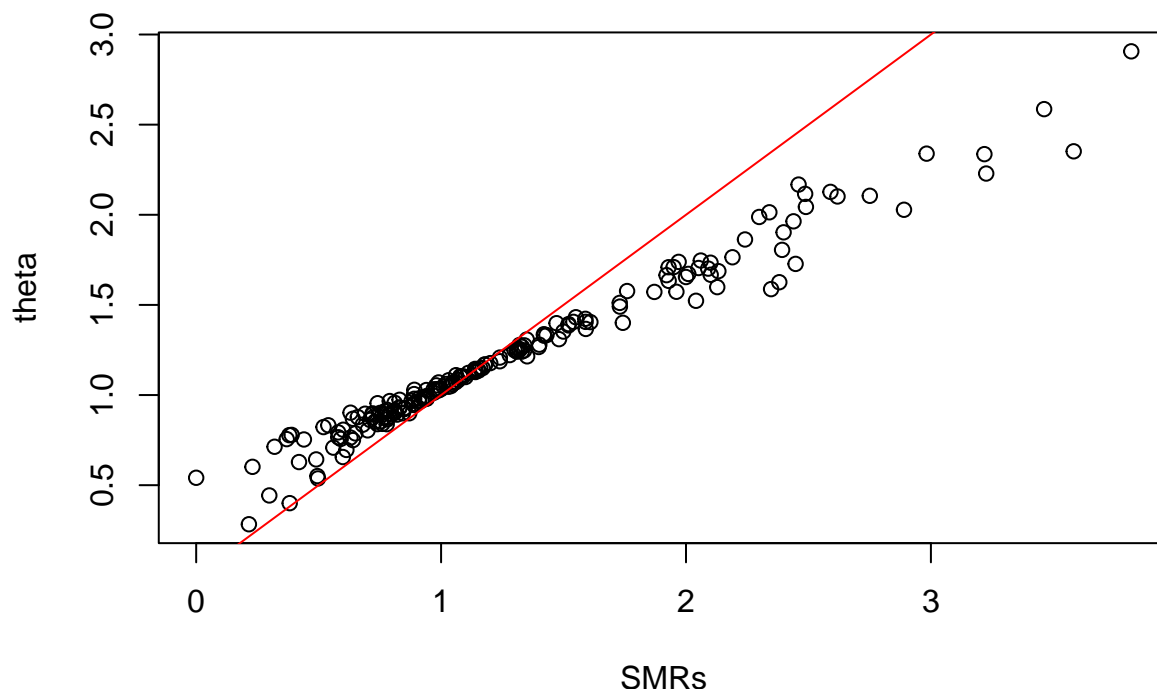
density of SMRs v.s. estimated theta in model 1
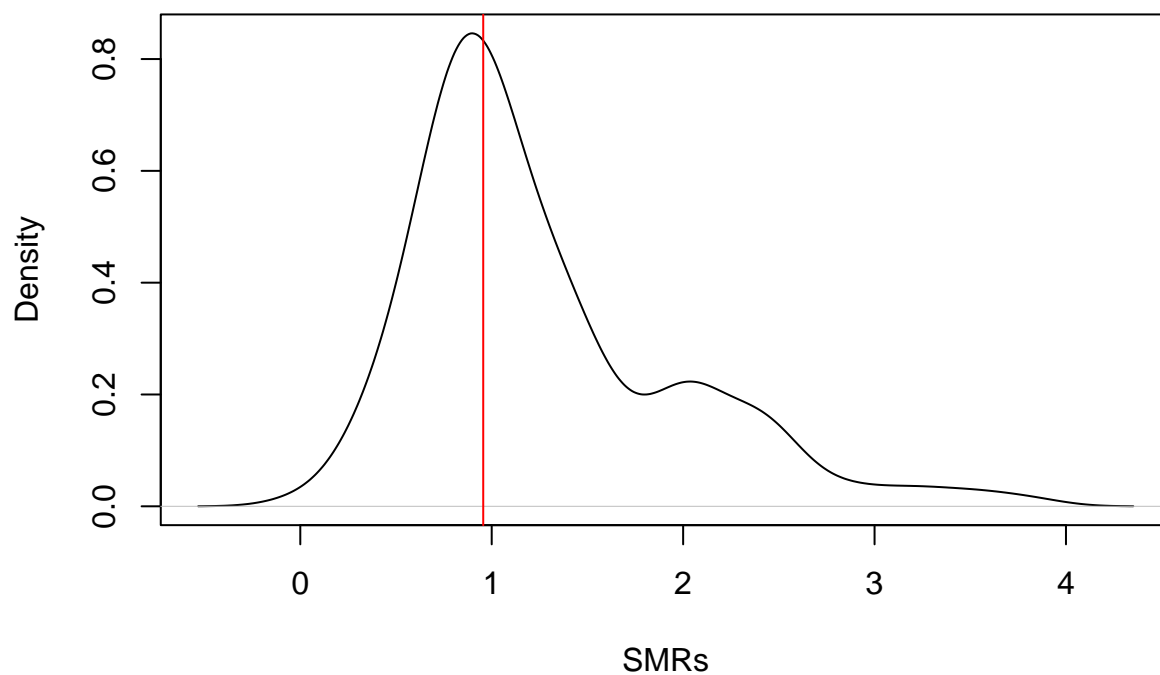

```
plot(observe.i/expect.i,theta_mod2,main='SMR v.s. theta in model 2',xlab='SMRs',ylab='theta')
abline(0,1,col='red')
```

SMR v.s. theta in model 2


```
plot(observe.i/expect.i,theta_mod3,main='SMR v.s. theta in model 3',xlab='SMRs',ylab='theta')
abline(0,1,col='red')
```

SMR v.s. theta in model 3

I feel model 2 preforms better as its predictions are closer to the actual values. Model3 can potentially do a better job if we add covariates. However, since we don't have the data of covariates in the assignment, model 3 doesn't have any advantages over model2.

Question 4

Rerun model 3 (the hierarchical model), but also including an overdispersion parameter. Compare the two models and decide which is more appropriate.

```
stan_data <- list(N = length(observe.i),
                  offset = expect.i,
                  deaths = observe.i)
mod4 <- stan(data = stan_data,
             file = "lip4.stan",
             iter = 1000,
             seed = 23)
```

```
## Running /opt/R/4.1.2/lib/R/bin/R CMD SHLIB foo.c
## gcc -I"/opt/R/4.1.2/lib/R/include" -DNDEBUG    -I"/cloud/lib/x86_64-pc-linux-gnu-library/4.1/Rcpp/incl
## In file included from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Core:88,
##                  from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Dense:1,
##                  from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/StanHeaders/include/stan/math/prim/m
##                  from <command-line>:
## /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:1: erro
##   628 | namespace Eigen {
##       | ^~~~~~~~
## /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:17: err
##   628 | namespace Eigen {
##       |                 ^
## In file included from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Dense:1,
##                  from /cloud/lib/x86_64-pc-linux-gnu-library/4.1/StanHeaders/include/stan/math/prim/m
```

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```
##                        from <command-line>:
## /cloud/lib/x86_64-pc-linux-gnu-library/4.1/RcppEigen/include/Eigen/Core:96:10: fatal error: complex:
##     96 | #include <complex>
##        |          ^~~~~~~~~
## compilation terminated.
## make: *** [/opt/R/4.1.2/lib/R/etc/Makeconf:168: foo.o] Error 1
##
## SAMPLING FOR MODEL 'lip4' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 5.5e-05 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.55 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:    1 / 1000 [  0%]  (Warmup)
## Chain 1: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 1: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 1: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 1: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 1: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 1: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 1: Iteration: 600 / 1000 [ 60%]  (Sampling)
## Chain 1: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 1: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 1: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 1: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 1.50307 seconds (Warm-up)
## Chain 1:                0.972209 seconds (Sampling)
## Chain 1:                2.47528 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'lip4' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 2.9e-05 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.29 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:    1 / 1000 [  0%]  (Warmup)
## Chain 2: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 2: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 2: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 2: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 2: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 2: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 2: Iteration: 600 / 1000 [ 60%]  (Sampling)
## Chain 2: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 2: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 2: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 2: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 2:
## Chain 2:  Elapsed Time: 1.29139 seconds (Warm-up)
## Chain 2:                1.99976 seconds (Sampling)
```

```
## Chain 2:                    3.29115 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'lip4' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 3.1e-05 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.31 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:   1 / 1000 [  0%]  (Warmup)
## Chain 3: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 3: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 3: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 3: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 3: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 3: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 3: Iteration: 600 / 1000 [ 60%]  (Sampling)
## Chain 3: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 3: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 3: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 3: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 1.42427 seconds (Warm-up)
## Chain 3:                2.01187 seconds (Sampling)
## Chain 3:                3.43614 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'lip4' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 2.9e-05 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.29 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration:   1 / 1000 [  0%]  (Warmup)
## Chain 4: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 4: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 4: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 4: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 4: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 4: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 4: Iteration: 600 / 1000 [ 60%]  (Sampling)
## Chain 4: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 4: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 4: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 4: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 4:
## Chain 4:  Elapsed Time: 1.26162 seconds (Warm-up)
## Chain 4:                1.64618 seconds (Sampling)
## Chain 4:                2.9078 seconds (Total)
## Chain 4:
```

```
summary(mod4)$summary[c("theta[2]","mu","sigma_mu"),]
```

```
##                   mean     se_mean        sd        2.5%        25%         50%
## theta[2]   1.37039981 0.01007992 0.3268993  0.82969585  1.1260799  1.33126138
## mu        -0.09180732 0.18498810 0.5882075 -1.31118751 -0.4700268 -0.06660285
## sigma_mu   0.29922211 0.02974819 0.1095723  0.09876038  0.2130765  0.32203225
##                  75%      97.5%      n_eff      Rhat
## theta[2]   1.5824596 2.0825091 1051.75391  1.007459
## mu         0.4046398 0.8786499   10.11052  1.699934
## sigma_mu   0.3902860 0.4616859   13.56690  1.332127
```

I don't think adding a ϵ term is necessary, since it can be absorbed by alpha. SO I think the previous model in Q1 is better.