Homework 5
ECE2504 CRN:82729

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Question 1: (8 pts) For the Boolean functions E and F shown in the truth table below

X	Y	Z	E	F	E+F	$E \bullet F$
0	0	0	1	0	1	0
0	0	1	0	1	1	0
0	1	0	0	0	0	0
0	1	1	1	1	1	1
1	0	0	1	0	1	0
1	0	1	1	1	1	1
1	1	0	0	1	1	0
1	1	1	0	0	0	0

a) Express E in sum of minterms form

$$E = X'Y'Z' + X'YZ + XY'Z' + XY'Z$$

b) Express F in sum of minterms form

$$F = X'Y'Z + X'YZ + XY'Z + XYZ'$$

c) Express E in product of maxterms form

$$E = (X + Y + Z')(X + Y' + Z)(X' + Y' + Z)(X' + Y' + Z')$$

d) Express F in product of maxterms form

$$F = (X + Y + Z)(X + Y' + Z)(X' + Y + Z)(X' + Y' + Z')$$

e) Express $(E + F)$ in sum of minterms form

$$(E + F) = X'Y'Z' + X'Y'Z + X'YZ + XY'Z' + XY'Z + XYZ'$$

f) Express $(E \bullet F)$ in product of maxterms form

$$(E \bullet F) = (X + Y + Z)(X + Y + Z')(X + Y' + Z)(X' + Y + Z)(X' + Y' + Z)(X' + Y' + Z')$$

g) Simplify E

X \ YZ	00	01	11	10
0	1	0	1	0
1	1	1	0	0

$$E = Y'Z' + XY' + X'YZ$$

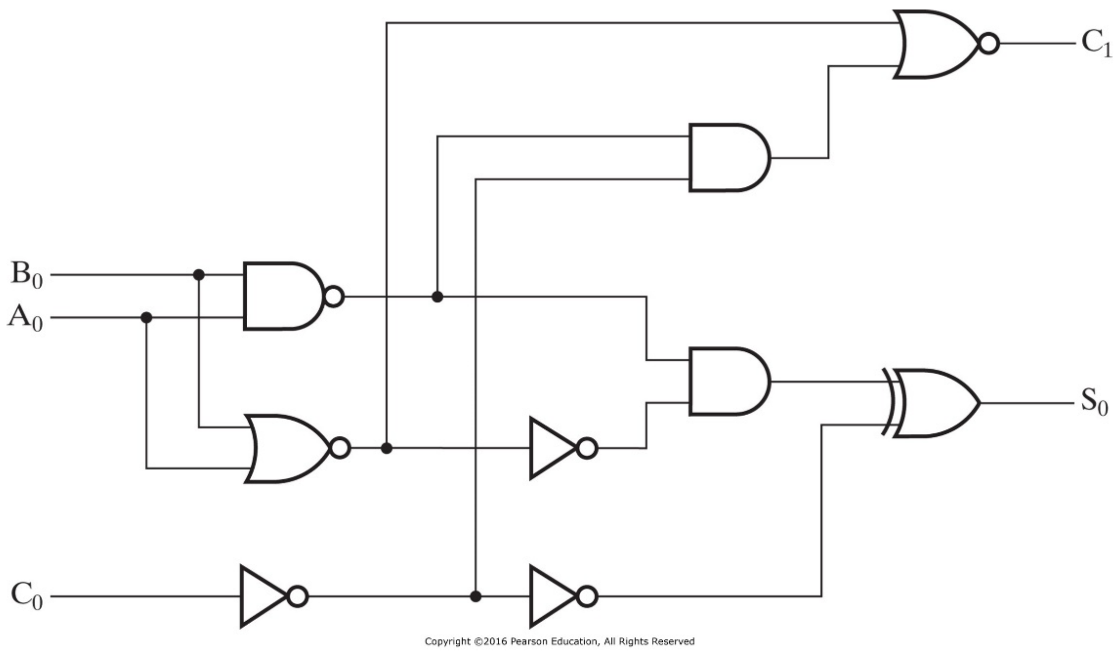
h) Simplify F

X \ YZ	00	01	11	10
0	1	0	1	0
1	1	1	0	0

$$F = (X' + Y')(Y' + Z)(X + Y + Z')$$

Question 2: (3 pts) Assume the gates in the circuit below have the following propagation delays. What is the propagation delay of the longest path through the circuit? Recall that $x \oplus y = x'y + xy'$

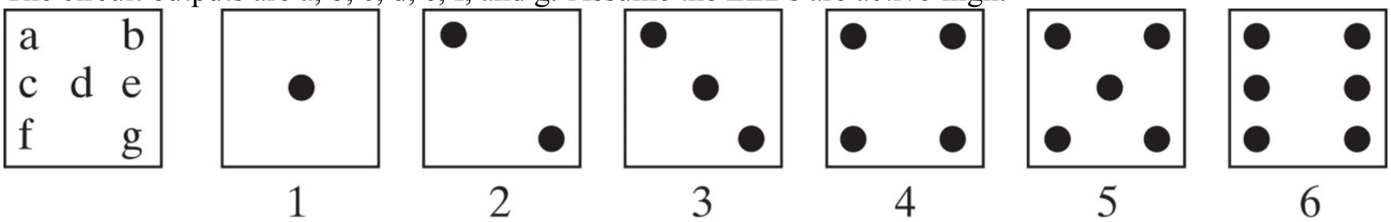
Inverter $t_{pd} = 0.05ns$
NAND gate $t_{pd} = 0.07ns$
NOR gate $t_{pd} = 0.07ns$
AND gate $t_{pd} = 0.10ns$
OR gate $t_{pd} = 0.10ns$



The longest path is
NOR -> NOT -> AND -> XOR.
This path can be rendered into
NOR -> NOT -> AND -> NOT -> AND -> OR
by converting the xor gate into primitive gates.

$$\begin{aligned} t_{pd\ max} =& 0.07ns + 0.05ns + 0.10ns \\ & + 0.05ns + 0.10ns + 0.10ns \\ t_{pd\ max} =& 0.47ns \end{aligned}$$

Question 3: Assume an electronic game uses an array of seven LEDs to display the results of a random roll of a die. The value of each roll is represented by a 3-bit binary number $X = X_2X_1X_0$. Design a logic circuit to illuminate the appropriate LEDs to display each of the possible six die values, as shown below. The circuit outputs are a, b, c, d, e, f, and g. Assume the LEDs are active-high.



a) (3.5 pts) Derive the truth table for a-g. Assume that invalid input combinations (000 and 111) should result in all the LEDs remaining dark.

X_2	X_1	X_0	a	b	c	d	e	f	g
0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0	0	0
0	1	0	1	0	0	0	0	0	1
0	1	1	1	0	0	1	0	0	1
1	0	0	1	1	0	0	0	1	1
1	0	1	1	1	0	1	0	1	1
1	1	0	1	1	1	0	1	1	1
1	1	1	0	0	0	0	0	0	0

b) (3.5 pts) Determine simplified Boolean equations for a-g.

X_2 \ X_1X_0	00	01	11	10
0	0	0	1	1
1	1	1	0	1

$$a = X_2X_1' + X_2'X_1 + X_1X_0'$$

X_2 \ X_1X_0	00	01	11	10
0	0	0	0	0
1	1	1	0	1

$$b = X_2X_1' + X_2X_0'$$

X_2 \ X_1X_0	00	01	11	10
0	0	0	0	0
1	0	0	0	1

$$c = X_2X_1X_0'$$

X_2 \ X_1X_0	00	01	11	10
0	0	1	1	0
1	0	1	0	0

$$d = X_1'X_0 + X_2'X_0$$

X_2 \ X_1X_0	00	01	11	10
0	0	0	0	0
1	0	0	0	1

$$e = X_2X_1X_0'$$

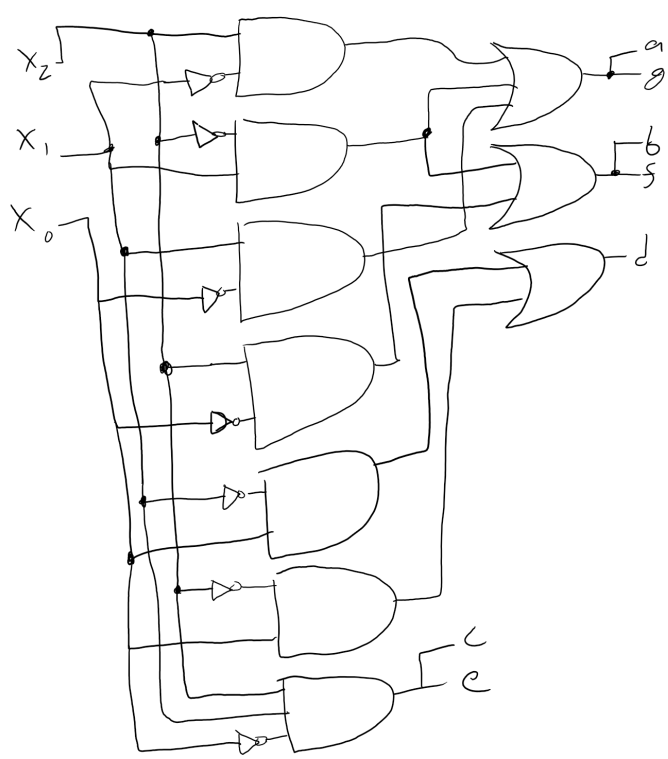
X_2 \ X_1X_0	00	01	11	10
0	0	0	0	0
1	1	1	0	1

$$f = X_2X_1' + X_2X_0'$$

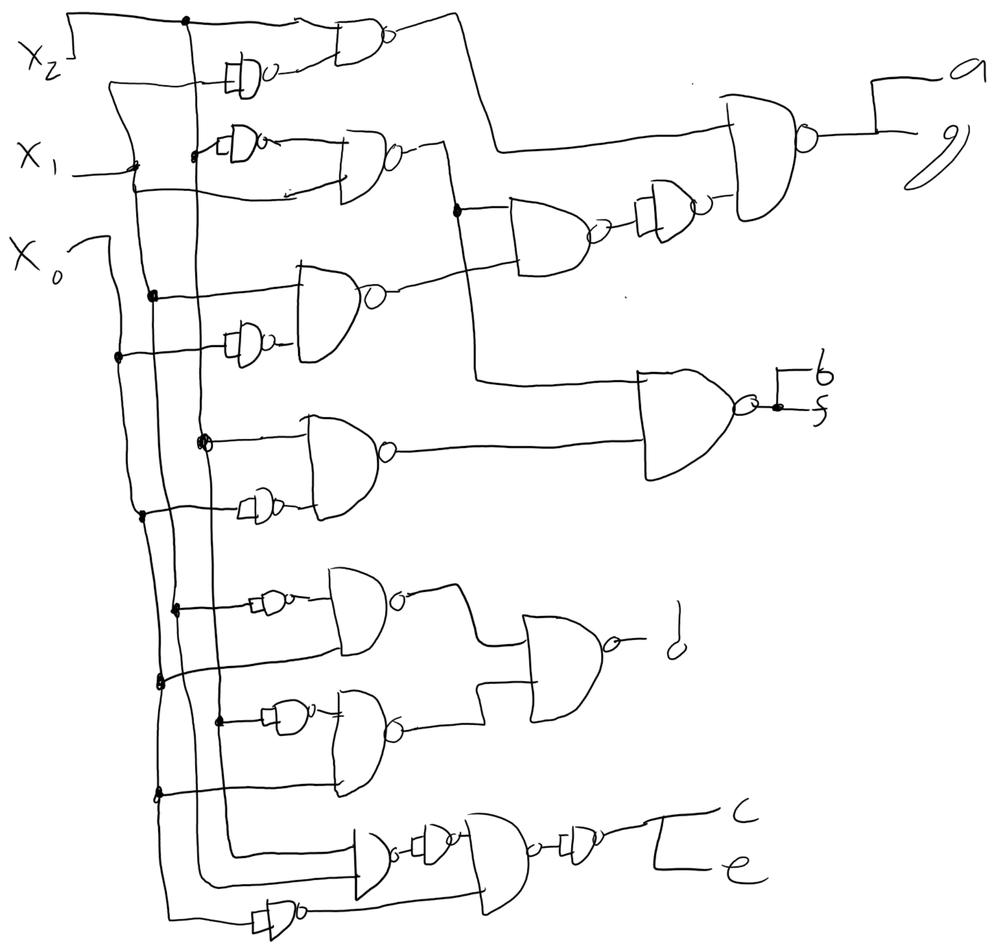
X_2 \ X_1X_0	00	01	11	10
0	0	0	1	1
1	1	1	0	1

$$g = X_2X_1' + X_2'X_1 + X_1X_0'$$

c) (2 pts) Implement a-d using AND gates, OR gates, and inverters. (Assume a maximum of 4 inputs for each gate.)



d) (2 pts) Implement a-d using only 2-input NAND gates.



Question 4: Modify the truth table from Problem 3 by assuming that an output is not specified for invalid input combinations.

a) (2 pts) Derive the modified truth table for a-g.

X_2	X_1	X_0	a	b	c	d	e	f	g
0	0	0	x	x	x	x	x	x	x
0	0	1	0	0	0	1	0	0	0
0	1	0	1	0	0	0	0	0	1
0	1	1	1	0	0	1	0	0	1
1	0	0	1	1	0	0	0	1	1
1	0	1	1	1	0	1	0	1	1
1	1	0	1	1	1	0	1	1	1
1	1	1	x	x	x	x	x	x	x

b) (2 pts) Determine simplified Boolean equations for a-d using the new truth table.

X_1X_0

X_2	00	01	11	10
0	-	0	1	1
1	1	1	-	1

$$a = X_2 + X_1$$

X_1X_0

X_2	00	01	11	10
0	-	0	0	0
1	1	1	-	1

$$b = X_2 + X_1'X_0'$$

X_1X_0

X_2	00	01	11	10
0	-	0	0	0
1	0	0	-	1

$$c = X_2X_1$$

X_1X_0

X_2	00	01	11	10
0	-	1	1	0
1	0	1	-	0

$$d = X_0$$

X_2 \ X_1X_0	00	01	11	10
0	-	0	0	0
1	0	0	-	1

$$e = X_2X_1$$

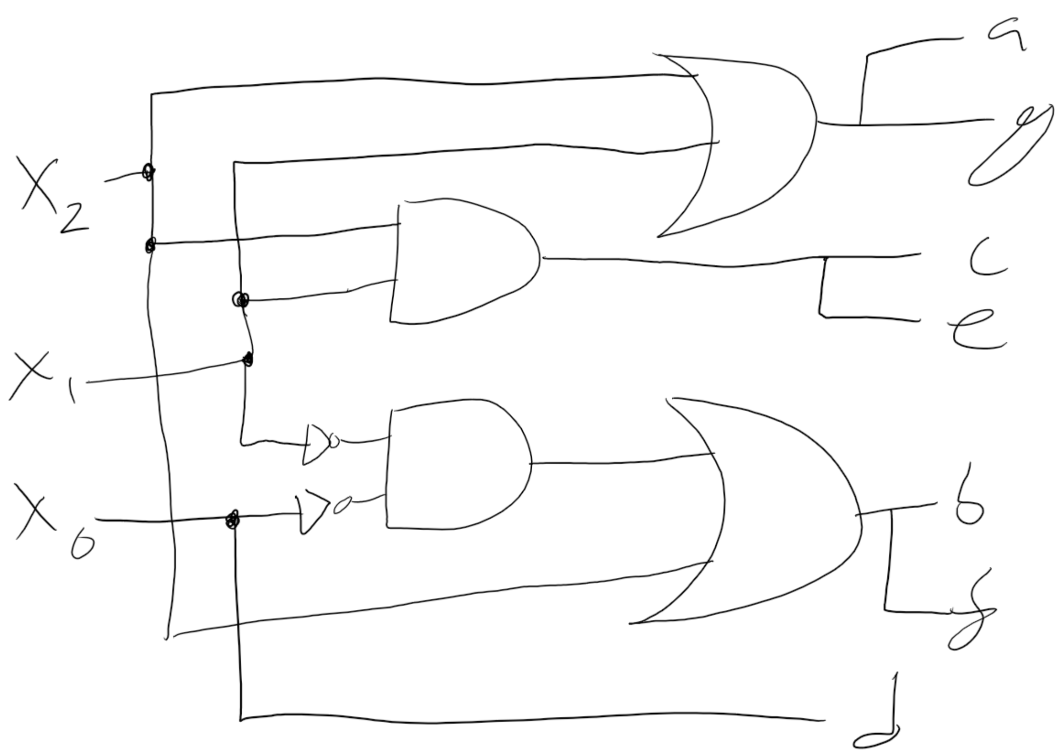
X_2 \ X_1X_0	00	01	11	10
0	-	0	0	0
1	1	1	-	1

$$f = X_2 + X_1'X_0'$$

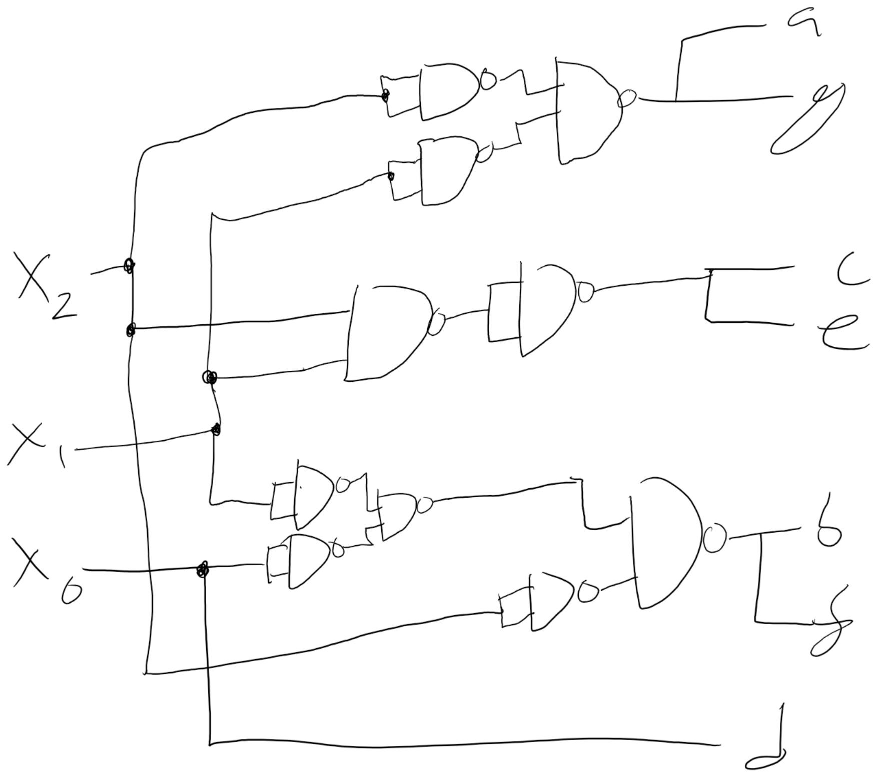
X_2 \ X_1X_0	00	01	11	10
0	-	0	1	1
1	1	1	-	1

$$g = X_2 + X_1$$

c) (2 pts) Implement a-d using AND gates, OR gates, and inverters using the new expressions. (Assume a maximum of 4 inputs for each gate.)



d) (2 pts) Implement a-d using only 2-input NAND gates.



Question 5: (8 pts) Minimize the following five variable function.

$$F(A,B,C,D,E) = \Sigma(0,8,9,10,12,14,15,16,\\ 20,24,27,29,31)$$
$$d(A,B,C,D,E) = \Sigma(4,7,11,13,19,25,28,30)$$

Hint: Consider a 4-variable K-map. It consists of 16 cells, arranged so that adjacent cells differ by only one variable. Suppose a second 4-variable K-map is stacked on top of the first. Differentiate between the top K-map and the bottom K-map using a fifth bit, e.g. 0 for the bottom, 1 for the top. This will be the most significant bit of your 5-variable K-map.

$E=0$

AB \ CD	00	01	11	10
00	1	0	0	0
01	-	0	-	0
11	1	-	1	1
10	1	1	-	1

$E=1$

AB \ CD	00	01	11	10
00	1	0	-	0
01	1	0	0	0
11	-	1	1	-
10	1	-	1	0

$$F = C'D' + AD + AE'$$

GRADING SCALE

Total: 38 pts

Pts	0	4	9	14	19	23	28	33
Letter Grade	D-	D	C-	C	B-	B	A-	A