Homework 6

MATH2534 CRN:15708

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Prove by mathematical induction.

(a) $\sum_{i=1}^{n}(3i-1) = \frac{n(3n+1)}{2}$

Let $\forall n \in \mathbb{Z}, n \geq 1$

$\sum_{i=1}^{1}(3i-1) = \frac{n(3n+1)}{2}$

$3-1 = \frac{3+1}{2} = 2$

The base case $P(1)$ when $n = 1$ is true.

$P(k)$ is $\sum_{i=1}^{k}(3i-1) = \frac{(k)(3(k)+1)}{2}$

$\forall k \in \mathbb{Z}, k \geq 1$, if $P(k)$ is true, then $P(k+1)$ is true.

$\sum_{i=1}^{k+1}(3i-1) = \frac{(k+1)(3(k+1)+1)}{2}$

$(3k+2) + \frac{3k^2+k}{2} = \frac{3k^2+k+6k+4}{2} = \frac{3k^2+7k+4}{2}$

By mathematical induction, the statement $\sum_{i=1}^{n}(3i-1) = \frac{n(3n+1)}{2}$ is true for all n.

(b) $1 + \frac{1}{4} + \frac{1}{9} + \ldots + \frac{1}{n^2} \leq 2 - \frac{1}{n}$

Let $\forall n \in \mathbb{Z}, n \geq 1$

$\frac{1}{1^2} = 1 \leq 2 - \frac{1}{1} = 1$

The base case $P(1)$ when $n = 1$ is true.

$P(k)$ is $1 + \frac{1}{4} + \frac{1}{9} + \ldots + \frac{1}{k^2} \leq 2 - \frac{1}{k}$

$\forall k \in \mathbb{Z}, k \geq 1$, if $P(k)$ is true, then $P(k+1)$ is true.

$1 + \frac{1}{4} + \frac{1}{9} + \ldots + \frac{1}{k^2} + \frac{1}{(k+1)^2} \leq 2 - \frac{1}{k+1}$

$2 - \frac{1}{k} + \frac{1}{k^2+2k+1} \leq 2 - \frac{1}{k+1}$

$-\frac{1+k+k^2}{k(1+k)^2} \leq -\frac{1}{k+1}$

By mathematical induction, the statement $1 + \frac{1}{4} + \frac{1}{9} + \ldots + \frac{1}{n^2} \leq 2 - \frac{1}{n}$ is true for all n.

(c) $(1 - \frac{1}{2})(1 - \frac{1}{4}) \ldots (1 - \frac{1}{2^n}) \geq \frac{1}{4} + \frac{1}{2^{n+1}}$

—Did not complete—

(d) For any integers $n \geq 7, n! > 3^n$.

Let $\forall n \in \mathbb{Z}, n \geq 7$

$7! = 5040 > 3^7 = 2187$

The base case $P(7)$ when $n = 7$ is true.

$P(k)$ is $k! > 3^k$

$\forall k \in \mathbb{Z}, k \geq 1$, if $P(k)$ is true, then $P(k+1)$ is true.

$(k+1)! = (k+1) \times k! > 3^{k+1} = (3) \times 3^k$

—Did not complete—

(e) For any integer $n \geq 1, 17^n - 12^n$ is divisible by 5.

Let $\forall n \in \mathbb{Z}, n \geq 1$

$17^1 - 12^1 = 17 - 12 = 5 \implies 5|5$

The base case $P(1)$ when $n = 1$ is true.

$P(k)$ is $5|(17^n - 12^n)$

$\forall k \in \mathbb{Z}, k \geq 1$, if $P(k)$ is true, then $P(k+1)$ is true.

$17^{k+1} - 12^{k+1} = 17 \times 17^k - 12 \times 12^k$

$17^k - 12^k$ is divisible by 5.

$17 \times 17^k - 12 \times 12^k - 2(17^k - 12^k) = 15 \times 17^k - 10 \times 12^k$

$15 \times 17^k - 10 \times 12^k$ is divisible by 5.

$17 \times 17^k - 12 \times 12^k = 2(17^k - 12^k) + (15 \times 17^k - 10 \times 12^k)$

The sum of two terms divisible by 5 is also divisible by 5.

Therefore $17^{k+1} - 12^{k+1}$ is divisible by 5.

By mathematical induction, the statement $1 + \frac{1}{4} + \frac{1}{9} + \ldots + \frac{1}{n^2} \leq 2 - \frac{1}{n}$ is true for all n.