Homework 10

MATH2534 CRN:15708

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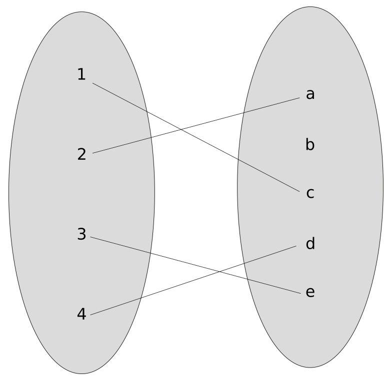
1. Let $X = \{1, 2, 3, 4\}$ and $Y = \{a, b, c, d, e\}$. Define $f : X \mapsto Y$ as follows:

$$f(1) = c,\ f(2) = a,\ f(3) = e, \text{ and } f(4) = d$$

   (a) Draw an arrow diagram and determine if f is a function.



   (b) Find the domain, codomain and range of f.
   Domain: X, Codomain: Y, Range: $\{a, c, d, e\}$

   (c) Let $A = \{b\}$, $B = \{a, d\}$, and $C = \{b, e\}$. The inverse image of a set S is the set of all preimages of each element of S. Find the inverse images of A, B, and C.
   $f^{-1}(A) = \emptyset$, $f^{-1}(B) = \{2, 4\}$, $f^{-1}(C) = \{3\}$

2. Let $f : \mathbb{Q} \mapsto \mathbb{Q}$ be given by $f(\frac{m}{n}) = m + n$ for all integers m and n with $n \neq 0$. Is f well-defined? Why or why not?

   For $n = 0$, f is undefined. $n = 0$ is outside of the defined boundaries of f. Therefore f is well defined for all integers m and n with $n \neq 0$.

3. Let $A = \{1, 2, 3, 4, 5\}$ and define $S : \mathcal{P}(A) \mapsto Z$ as follows: For all sets X in $\mathcal{P}(A)$,

$$S(X) = \begin{cases} 1 & \text{if } A - X \text{ has an even number of elements} \\ 0 & \text{if } A - X \text{ has an odd number of elements} \end{cases}$$

   (a) Find $S(\emptyset)$, $S(\{1, 4\})$, and $S(\{1, 2, 3, 4, 5\})$.
   $S(\emptyset) = 1$
   $S(\{1, 4\}) = 1$
   $S(\{1, 2, 3, 4, 5\}) = 0$

   (b) Is S a function? Why or why not?
   S is a function as it always cleanly maps from any X into some value Z.

4. Let $A = \{a_1, a_2, a_3\}$ and $\{0,1\}^3 = \{0,1\} \times \{0,1\} \times \{0,1\}$ be the set of all ordered 3-tuples of 0's and 1's. Define a function $f : P(A) \mapsto \{0,1\}^3$ by $f(X) = (s_1, s_2, s_3)$ for $X \in P(A)$ where

$$s_i = \begin{cases} 1 & \text{if}\, a_i \in X \\ 0 & \text{if}\, a_i \notin X \end{cases}$$

Draw an arrow diagram for f.

Ran out of time. Did not complete.

5. Determine whether the relation given is one-to-one, onto. If not, explain why not.

   (a) Let $f : R \mapsto Z$ be defined by $f(x) = \lceil x \rceil$.

   It is onto as every rational number can be rounded up to some integer.

   It is not one-to-one as multiple rational numbers can round up to an integer.

   (b) Let $Y = \{0,1,2,3,4,5\}$ where $f : Z^+ \mapsto Y$ is defined by $f(x) = 5x\text{mod}6$.

   It is onto as every positive integer x mod 6 will map onto some value in Y. This is not changed what that integer x is multiplied by 5.

   It is not one-to-one as every 1 out of 6 positive integers maps to each value in Y.

   (c) Let $M : Z^+ \times Z^+ \mapsto Z^+$ be given by $M(m,n) = m + n$.

   It is onto as any addition between 2 positive integers will always result in another positive integer.

   It is not one-to-one as a variety of different integers can be added together to get the same output integer. i.e. $1 + 3 = 4, 2 + 2 = 4$

6. Determine whether the following mapping is one-to-one, onto, bijective? Find an inverse function if it exists. Justify your answer.

   (a) Let $f : R \mapsto R$ be defined by $f(x) = -2x + 6$.

   The function is bijective as every input has 1 and only 1 output and vice versa. (1 Variable, Linear function)

   The inverse, $f^{-1} : R \mapsto R$ is defined as $f^{-1}(x) = 3 - \frac{y}{2}$

   (b) Let $f : R - \{3\} \mapsto R$ be defined by $f(x) = \frac{2x}{x-3}$.

   This function is bijective as every value in $R - \{3\}$ maps directly and individually onto some value in R and vice versa.

   The inverse, $f^{-1} : R - \{2\} \mapsto R$ is defined as $f^{-1}(x) = \frac{3y}{y-2}$.

7. Prove the statement:

   If $f : X \mapsto Y$ and $g : Y \mapsto Z$, are both one-to-one functions, then $g \circ f : X \mapsto Z$ is one-to-one.

   Ran out of time. Did not complete.