Design & Simulate 7 Ex2.1

ECE2204 CRN:82929

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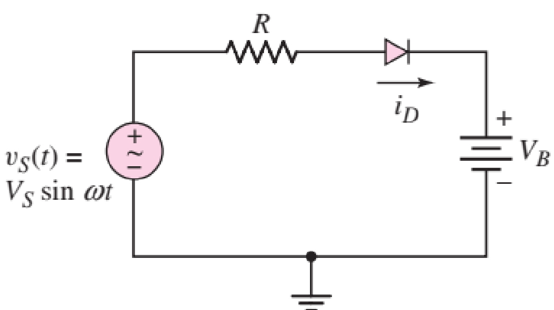
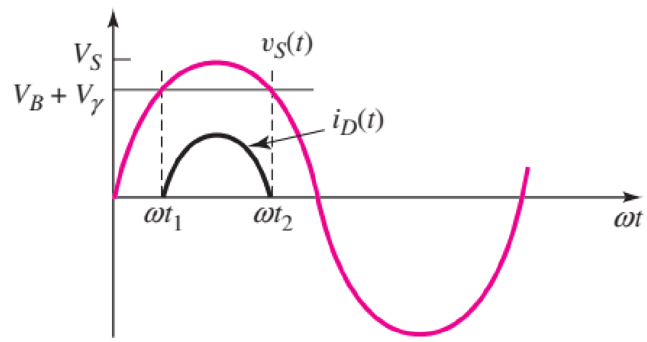
Problem 7.2-1.a.1:

Design

Determine the currents and voltages in a half-wave rectifier circuit. Consider the circuit shown below. Assume $V_B = 5V$, $R = 200\Omega$, $V_\gamma = 0.73V$, and, $v_S(t) = 12\sin\omega t$. Determine the peak diode current, maximum reverse-bias diode voltage, and the fraction of the cycle over which the diode is conducting.





$$i_D(\text{peak}) = \frac{V_S - V_B - V_\gamma}{R} = \frac{12V - 5V - 0.73V}{200\Omega} = 31.35mA \tag{1}$$

$$i_D = i_D(\text{peak})\sin\omega t = 31.35mA\sin\omega t \tag{2}$$

$$v_R(\max) = V_S + V_B = 12V + 5V = 17V \tag{3}$$

$$\omega t_1 = v_S^{-1}(V_B + V_\gamma) = \arcsin(\frac{V_B + V_\gamma}{v_S}) = \arcsin(\frac{5V + 0.73V}{12}) = 28.52° \tag{4}$$

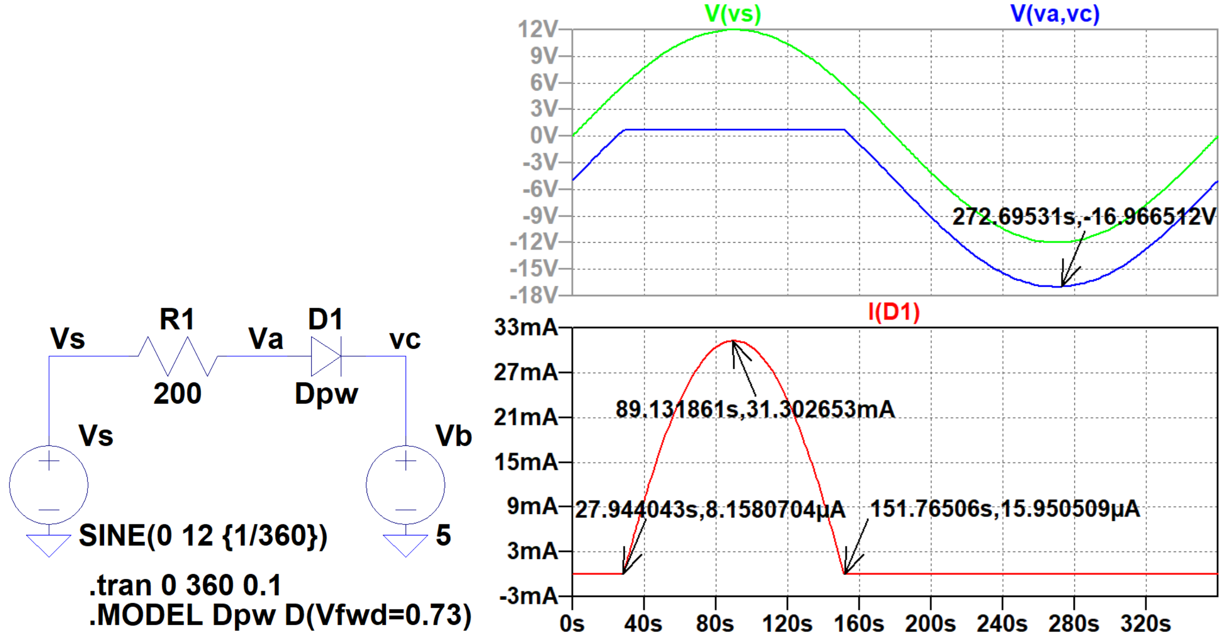
$$\omega t_2 = v_S^{-1}(V_B + V_\gamma) = \text{supplement}(\omega t_1) = 180° - \omega t_1 = 180° - 28.52° = 151.84° \tag{5}$$

$$\%t = \frac{\omega t_2 - \omega t_1}{360°} = \frac{151.48° - 28.52°}{360°} = 0.341\overline{5} = 34.16\% \tag{6}$$

The peak diode current is $i_D(\text{peak}) = 31.35mA$. The maximum reverse-bias diode voltage is $v_R = 17V$, and the fraction of each cycle that the diode is conducting is $\%t = 34.16\%$

Validation

LTSpice Implementation (accurate with $< 1\%$ deviation from design result)



$$Err_{V_R} = \frac{|17-16.966|}{17} = 0.002 = 0.2\% \qquad Err_{i_D(\text{peak})} = \frac{|31.35-31.30|}{31.35} = 0.0015 = 0.15\%$$

$$Err_{\%t} = \frac{|0.3416-\frac{151.7-27.9}{360}|}{34.16} = 0.000007 = 0.00\% \qquad Err_{Avg} = \frac{0.20+0.15+0.00}{3} = 0.12\%$$

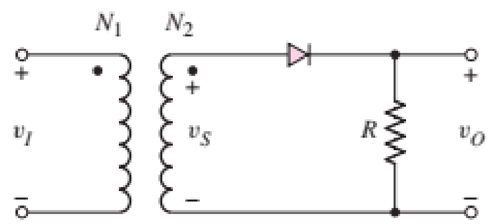
Problem 7.2-1.b.1:

The problem is derived from problem 2.3 on page 112 of the textbook by changing the values, making R dependent, and fixing the peak diode current.

Design

A half-wave rectifier such as shown below has a peak diode current of $I_D = 2mA$. The input is $V_I = 120V_{rms}$, $f = 60$Hz signal and the transformer is a $10:1$ step down transformer. The diode has a cut-in voltage of $V_\gamma = 1.2V$. Assume $r_f = 0$.

Determine the value of the resistor R, the peak output voltage, the fraction (percent) of a cycle that $v_O > 0$, the average output voltage, and the average current in the load.



$$\omega = 120\pi \tag{7}$$

$$v_I = 120V_{rms} = 120\sqrt{2}V\sin(\omega t) \tag{8}$$

$$v_S = \frac{1}{10}120\sqrt{2}V\sin(\omega t) = 12\sqrt{2}V\sin(120\pi t) \tag{9}$$

$$I_D = \frac{v_S - v_\gamma}{R} \tag{10}$$

$$\implies R = \frac{v_S - v_\gamma}{I_D} = \frac{12\sqrt{2} - 1.2V}{2mA} = 7885\Omega \tag{11}$$

$$\omega t_1 = v_S^{-1}(V_\gamma) = \arcsin(\frac{V_\gamma}{v_S}) = \arcsin(\frac{1.2V}{12\sqrt{2}V}) = 4.055° \tag{12}$$

$$\omega t_2 = v_S^{-1}(V_\gamma) = \text{supplement}(\omega t_1) = 180° - \omega t_1 = 180° - 4.055° = 175.945° \tag{13}$$

$$\%t = \frac{\omega t_2 - \omega t_1}{360°} = \frac{175.945° - 4.055°}{360°} = 0.47747\overline{2} = 47.75\% \tag{14}$$

$$V_O = V_S - V_\gamma = 12\sqrt{2}V - 1.2V = 15.77V \tag{15}$$

$$v_O = V_O \sin(\omega t) = 15.77V \sin(\omega t) \tag{16}$$

$$v_{O\text{avg}} = \frac{V_O}{\omega} \int_{\omega t_1}^{\omega t_2} \sin(\omega t) d\omega t = \frac{15.77V}{2\pi} \int_{4.055°}^{175.945°} \sin(\omega t) d\omega t = 5.0075V \tag{17}$$

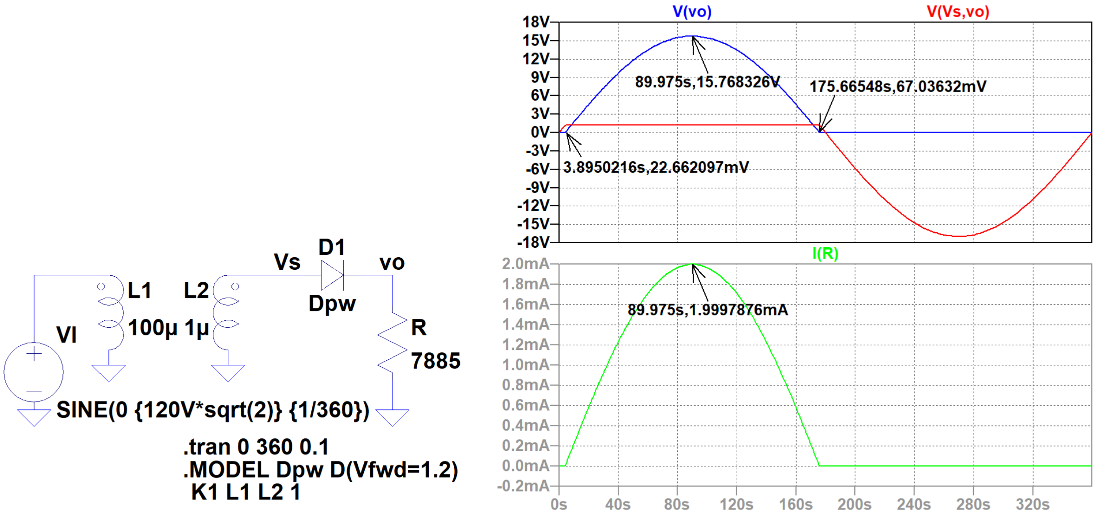
$$i_L = i_D = 2mA \sin(120\pi t) \tag{18}$$

$$i_{L\text{avg}} = \frac{I_L}{\omega} \int_{\omega t_1}^{\omega t_2} \sin(\omega t) d\omega t = \frac{2mA}{2\pi} \int_{4.055°}^{175.945°} \sin(\omega t) d\omega t = 635.1\mu A \tag{19}$$

The the value of the resistor is $R = 7885\Omega$, the peak output voltage is $V_O = 15.77V$, the fraction (percent) of a cycle that $v_O > 0$ is $\%t = 47.75$, the average output voltage is $V_{O\text{avg}} = 5.0075V$, and the average current in the load is $I_{L\text{avg}} = 635.1\mu A$.

Validation

LTSpice Implementation



$$Err_{V_O} = \frac{|15.77 - 15.768|}{15.77} = 0.01\%$$

$$Err_{V_{Oavg}} = \frac{|5.0075 - 4.817|}{5.0075} = 3.80\%$$

$$Err_{I_{Lavg}} = \frac{|635.1 - 610.91|}{635.1} = 3.80\%$$

$$Err_{\%t} = \frac{|0.4775 - \frac{175.665 - 3.895}{360}|}{0.4775} = 0.08\%$$

$$Err_{Avg} = \frac{0.01 + 3.80 + 3.80 + 0.08}{4} = 1.92\%$$

Error above 1% is due to measurement error.

This assignment should demonstrate a basic understanding of manipulating basic half wave diode rectification circuits.

I have neither given nor received unauthorized assistance on this assignment.