

Homework 3
MATH2534 CRN:15708

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1. Consider $D = \mathbb{Q}$ as the domain of the predicate variables x and y . Discern which of the following statements are true and which are false. Give counterexamples for the statements which are false.

(a) $x > 0$ and $y < 0 \implies x - y^2 > 0$

False in the case of $|x| \geq |y|$

(b) $xy = 0 \iff x = 0$

True

(c) $\forall x, y \geq 2, 2xy > x + y$

False in the case of $x \leq 0$

2. Put the following statements into symbolic logic using multiple quantifiers. Negate each statement.

- (a) Some colors are loved by everyone.

$\exists x \in \text{Colours}, \forall y \in \text{People} \mid x \text{ is loved by } y$

Negate: $\forall x \in \text{Colours}, \exists y \in \text{People} \mid x \text{ is not loved by } y$

- (b) There are some students who get an A on all classes.

$\exists x \in \text{Students}, \forall y \in \text{Classes} \mid x \text{ get an A in } y$

Negate: $\forall x \in \text{Students}, \exists y \in \text{Classes} \mid x \text{ does not get an A in } y$

- (c) Everyone is loved by someone.

$\forall x \in \text{People}, \exists y \in \text{People} \mid x \text{ is loved by } y$

Negate: $\exists x \in \text{People}, \forall y \in \text{People} \mid x \text{ is not loved by } y$

- (d) No movie will please everyone.

$\exists x \in \text{Movies}, \forall y \in \text{People} \mid x \text{ will not please } y$

Negate: $\forall x \in \text{Movies}, \exists y \in \text{People} \mid x \text{ will please } y$

3. Write the negation, contrapositive, converse and inverse of the following statements.

- (a) For all sport cars x , x is expensive.

Negation: There exists a sports car x , x is not expensive.

Contrapositive: For all non-expensive cars x , x is not a sports car.

Converse: For all expensive cars x , x is a sports car.

Inverse: For all non-sport cars x , x is not expensive.

- (b) $\forall x \in \mathbb{R}^+$, if $x^2 < 1$, then $x < 1$.

Negation: $\exists x \in \mathbb{R}^+, x^2 < 1 \wedge x \geq 1$.

Contrapositive: $\forall x \in \mathbb{R}^+, \text{if } x \geq 1, \text{ then } x^2 \geq 1$

Converse: $\forall x \in \mathbb{R}^+, \text{if } x < 1, \text{ then } x^2 < 1$

Inverse: $\forall x \in \mathbb{R}^+, \text{if } x^2 \geq 1, \text{ then } x \geq 1$

4. Write the following statements as quantified conditional statements; if-then form.

(a) Driving over 70 miles per hour is a sufficient condition for getting a ticket.

$\forall x \in \text{Vehicles} \mid \text{if } x \text{ is moving faster than 70 mph, then } x \text{ receives a ticket.}$

(b) Being responsible is a necessary condition for being a president for the club.

$\forall x \in \text{People} \mid \text{if } x \text{ is not responsible, then } x \text{ cannot be president for the club.}$

5. Let $A = B = \{-5, -2, 2, 4, 6, 8, 12, 15\}$. Determine whether the given statement is true and write a negation for each statement.

(a) $\forall x \in A, \exists y \in B$ such that $x + y = 10$.

True

Negation: $\exists x \in A, \forall y \in B$ such that $x + y \neq 10$.

(b) $\exists x \in A$ such that $\forall y \in B, x \geq y^2$.

False

Negation: $\forall x \in A$ such that $\exists y \in B, x < y^2$.

6. Domain D: VT students

Domain E: VT football games

Predicate: $P(x, y) = x$ watches y .

Write the following symbolic statements and negations in conventional English:

(a) $\forall y \in E, \exists x \in D$ such that $P(x, y)$.

All VT football games have VT students watching them.

(b) $\exists x \in D$ such that $\forall y \in E, P(x, y)$.

There are VT students that watch all VT football games.

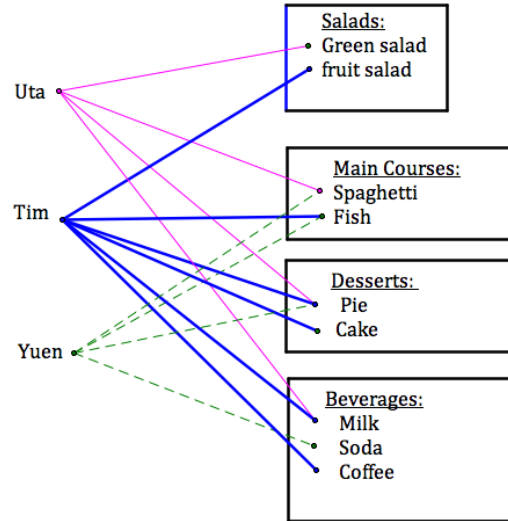
(c) $\exists y \in E$ such that $\forall x \in D, P(x, y)$.

There are VT football games that all VT students watch.

(d) $\forall x \in D, \exists y \in E$ such that $P(x, y)$.

All VT students watch some VT football game.

7. This problem refers to Example 3.3.3 in the textbook or Example 2, 3.3 in lecture notes.



Write each of following statements informally and find its truth value.

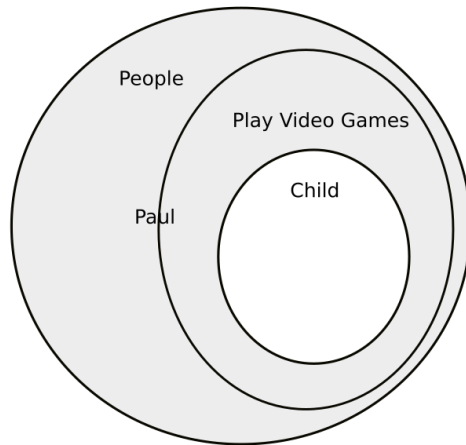
- (a) \forall students S , \exists a salad I such that S chose I .
All students chose a salad.
True
- (b) \forall items I , *exists* a student S such that S did not choose I .
All items were not chosen by some student.
False
- (c) \exists a student S such that \exists a station Z such that S chose items I in Z .
There is a student that chose all items at a station.
True
- (d) \exists a station Z such that \forall students S , an item I such that S chose I in Z .
There is a station that all students chose an item from.
True

8. Use a diagram and argument forms from class, to discern whether or not the following argument is valid.

(a) All children play a video game.

Paul is not a child.

\therefore Paul does not play a video game.



Child \implies plays a video game.

\neg Child

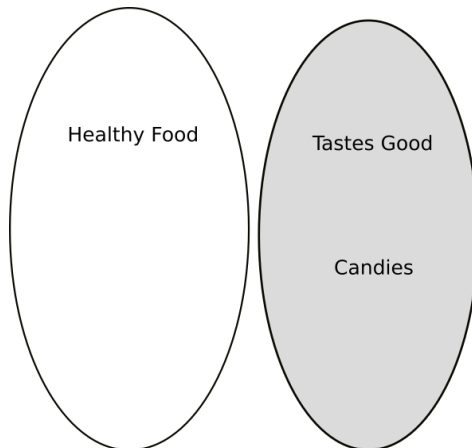
$\therefore \neg$ Plays Video Games

False by Inverse Error

(b) No healthy food tastes good.

Candies taste good.

\therefore Candies are not healthy food.



Healthy $\implies \neg$ Tastes good

Taste Good

\therefore Not healthy

True by Modus Tollens