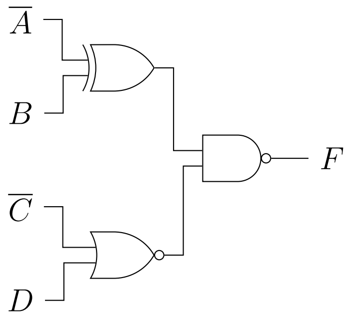
Homework 3

ECE2504 CRN:82729

Jacob Abel

September 17, 2017

Question 1: (6 pts)



a) Write the logic equation for the variable F in the circuit below, as implemented.

$F = \overline{(\overline{A} \oplus B)(\overline{\overline{C} + D})}$

b) Complete the truth table.

A	B	C	D	$\overline{A} \oplus B$	$\overline{\overline{A} + B}$	F
0	0	0	0	1	0	1
0	0	0	1	1	0	1
0	0	1	0	1	1	0
0	0	1	1	1	0	1
0	1	0	0	0	0	1
0	1	0	1	0	0	1
0	1	1	0	0	1	1
0	1	1	1	0	0	1
1	0	0	0	0	0	1
1	0	0	1	0	0	1
1	0	1	0	0	1	1
1	0	1	1	0	0	1
1	1	0	0	1	0	1
1	1	0	1	1	0	1
1	1	1	0	1	1	0
1	1	1	1	1	0	1

A	B	$\overline{A} \oplus B$
0	0	1
0	1	0
1	0	0
1	1	1

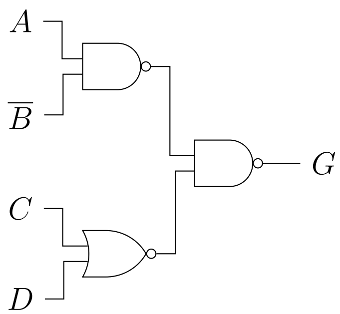
A	B	$\overline{A} \oplus B$
0	0	1
0	1	1
1	0	1
1	1	0

A	B	$\overline{\overline{A} + B}$
0	0	0
0	1	0
1	0	1
1	1	0

c) Find an equivalent expression in sum-of-products form. (Hint: you can check your result by verifying that the truth table remains the same.)

$$
\begin{aligned}
F(A,B,C,D) &= \Sigma m(0,1,3,4,5,6,7,8,9,10,11,12,13,15) \\
F(A,B,C,D) &= \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}\,D + \overline{A}\,\overline{B}\,C\,D + \overline{A}\,B\,\overline{C}\,\overline{D} + \overline{A}\,B\,\overline{C}\,D \\
&\quad + \overline{A}\,B\,C\,\overline{D} + \overline{A}\,B\,C\,D + A\,\overline{B}\,\overline{C}\,\overline{D} + A\,\overline{B}\,\overline{C}\,D + A\,\overline{B}\,C\,\overline{D} + A\,\overline{B}\,C\,D \\
&\quad + A\,B\,\overline{C}\,\overline{D} + A\,B\,\overline{C}\,D + A\,B\,C\,D \\
F(A,B,C,D) &= \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + A\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}\,D + A\,\overline{B}\,\overline{C}\,D + \overline{A}\,\overline{B}\,C\,D + A\,\overline{B}\,C\,D \\
&\quad + \overline{A}\,B\,\overline{C}\,\overline{D} + A\,B\,\overline{C}\,\overline{D} + \overline{A}\,B\,\overline{C}\,D + A\,B\,\overline{C}\,D + \overline{A}\,B\,C\,D \\
&\quad + A\,B\,C\,D + A\,\overline{B}\,C\,\overline{D} + \overline{A}\,B\,C\,\overline{D} \\
F(A,B,C,D) &= (\overline{A} + A)(\overline{B}\,\overline{C}\,\overline{D} + \overline{B}\,\overline{C}\,D + \overline{B}\,C\,D + B\,\overline{C}\,\overline{D} + B\,\overline{C}\,D + B\,C\,D) \\
&\quad + A\,\overline{B}\,C\,\overline{D} + \overline{A}\,B\,C\,\overline{D} \\
F(A,B,C,D) &= \overline{B}\,\overline{C}\,\overline{D} + B\,\overline{C}\,\overline{D} + \overline{B}\,\overline{C}\,D + B\,\overline{C}\,D + \overline{B}\,C\,D + B\,C\,D \\
&\quad + A\,\overline{B}\,C\,\overline{D} + \overline{A}\,B\,C\,\overline{D} \\
F(A,B,C,D) &= (\overline{B} + B)(+\,\overline{C}\,\overline{D} + \ \overline{C}\,D + \ C\,D) + A\,\overline{B}\,C\,\overline{D} + \overline{A}\,B\,C\,\overline{D} \\
F(A,B,C,D) &= \overline{C}\,D + C\,D + \overline{C}\,\overline{D} + A\,\overline{B}\,C\,\overline{D} + \overline{A}\,B\,C\,\overline{D} \\
F(A,B,C,D) &= (\overline{C} + C)(+D) + \overline{C}\,\overline{D} + A\,\overline{B}\,C\,\overline{D} + \overline{A}\,B\,C\,\overline{D} \\
F(A,B,C,D) &= D + \overline{C}\,\overline{D} + A\,\overline{B}\,C\,\overline{D} + \overline{A}\,B\,C\,\overline{D}
\end{aligned}
$$

Question 2: (6 pts)



a) Write the logic equation for the variable F in the circuit below, as implemented.
$G = \overline{\overline{(A\overline{B})}\ \overline{(C+D)}}$

b) Complete the truth table.

A	B	C	D	$\overline{(A\overline{B})}$	$\overline{(C+D)}$	G
0	0	0	0	1	1	0
0	0	0	1	1	0	1
0	0	1	0	1	0	1
0	0	1	1	1	0	1
0	1	0	0	1	1	0
0	1	0	1	1	0	1
0	1	1	0	1	0	1
0	1	1	1	1	0	1
1	0	0	0	0	1	1
1	0	0	1	0	0	1
1	0	1	0	0	0	1
1	0	1	1	0	0	1
1	1	0	0	1	1	0
1	1	0	1	1	0	1
1	1	1	0	1	0	1
1	1	1	1	1	0	1

A	B	$\overline{(A\overline{B})}$
0	0	1
0	1	1
1	0	0
1	1	1

A	B	$\overline{A\ B}$
0	0	1
0	1	1
1	0	1
1	1	0

A	B	$\overline{(A+B)}$
0	0	1
0	1	0
1	0	0
1	1	0

c) Find an equivalent expression in product-of-sums form. (Hint: you can check your result by verifying that the truth table remains the same.)

$$G(A,B,C,D) = \Pi m(0,4,12)$$
$$G(A,B,C,D) = (A+B+C+D)(A+\overline{B}+C+D)(\overline{A}+\overline{B}+C+D)$$
$$G(A,B,C,D) = (AB'+AC+AD+BA'+BC+C+CD+D)(A+\overline{B}+C+D)$$
$$G(A,B,C,D) = (AB'+AC+AD+ACD+B'D+A'BC+C+CD+A'BD+D)$$

Question 3: (3 pts) Use Boolean algebra to prove that $wxy' + xy'z' + xy + y'z = x + y'z$

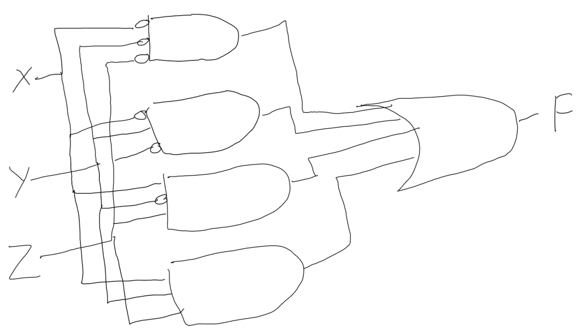
$$\begin{aligned} wx\overline{y} + x\overline{y}\,\overline{z} + xy + \overline{y}z &= x + \overline{y}z \\ x\overline{y}(w + \overline{z}) + xy + \overline{y}z &= x + \overline{y}z \\ (\overline{y} + y)(x(w + \overline{z}) + 1) + \overline{y}z &= x + \overline{y}z \\ x + \overline{y}z &= x + \overline{y}z \end{aligned}$$

Question 4: (10 pts) Given the Boolean function $F = x'y'z' + x'yz' + xy'z + xyz$

a) List the truth table.

x	y	z	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

b) Draw the logic diagram using the original Boolean expression.



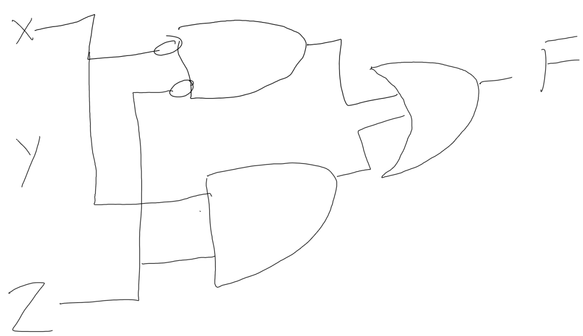
c) Simplify using Boolean algebra.

$$F = x'y'z' + x'yz' + xy'z + xyz$$
$$F = (y' + y)(x'z' + xz)$$
$$F = (x'z' + xz)$$

d) List the truth table of the simplified expression and show it is equivalent to the original.

x	y	z	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

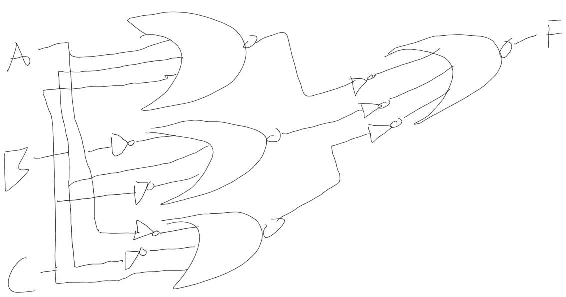
e) Draw the logic diagram of the simplified function and compare the total number of gates to part (b).
3 Gates vs 5 Gates



Question 5: (9 pts) Consider the following Boolean function: $F = A'B'C' + AB'C + ABC'$ For this question, do not use gates with inverted inputs. If you need an inverter, show it explicitly.

a) Implement it using only NOR gates and inverters? Draw the logic circuit. Assume your NOR gates can have up to four inputs.

$$
\begin{aligned}
F &= \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C + AB\overline{C} \\
&= \overline{\overline{\overline{A+B+C} + \overline{\overline{A}+B+\overline{C}} + \overline{\overline{A}+\overline{B}+C}}}
\end{aligned}
$$

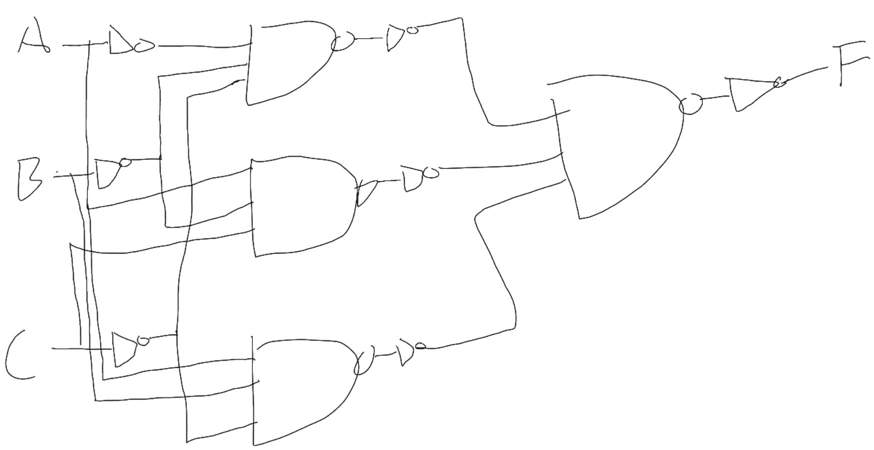


b) Redraw the circuit using only 2-input NOR gates.
Hint: First, convert to 2-input gates. Then convert to NORs.

$$
\begin{aligned}
F &= \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C + AB\overline{C} \\
&= \overline{A}\,(\overline{B+C}) + A((\overline{B+\overline{C}}) + (\overline{\overline{B}+C})) \\
&= \overline{\overline{\overline{A + (\overline{\overline{B+C}})} + \overline{(\overline{A} + (\overline{(\overline{B+\overline{C}}) + (\overline{\overline{B}+C})}))}}}
\end{aligned}
$$

c) Now implement the function using only NAND gates with up to 4-inputs.

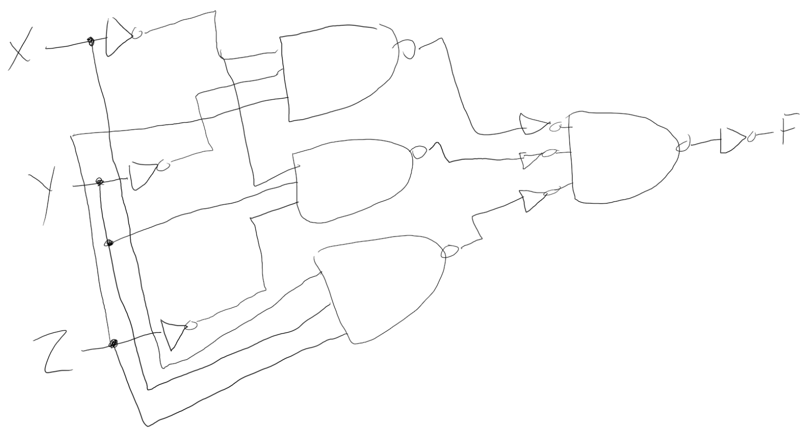
$$F = \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C + AB\overline{C} = (\overline{\overline{\overline{A}\,\overline{B}\,\overline{C}}}) + (\overline{\overline{A\overline{B}C}}) + (\overline{\overline{AB\overline{C}}}) = \overline{((\overline{\overline{A}\,\overline{B}\,\overline{C}})\,(\overline{A\overline{B}C})\,(\overline{AB\overline{C}}))}$$



Question 6: (6 pts) Consider the following Boolean function: $F = (x+y+z')\bullet(x+y'+z)\bullet(x'+y'+z')$For this question, do not use gates with inverted inputs. If you need an inverter, show it explicitly.

a) Implement it using only NAND gates and inverters? Draw the logic circuit. Assume your NAND gates can have up to four inputs.

$$
\begin{aligned}
F =& (x + y + z') \bullet (x + y' + z) \bullet (x' + y' + z') \\
=& \overline{(x'y'z)} \bullet \overline{(x'yz')} \bullet \overline{(xyz)} \\
=& \overline{\overline{(\overline{\overline{\overline{(x'y'z)}}} \bullet \overline{\overline{\overline{(x'yz')}}} \bullet \overline{\overline{\overline{(xyz)}}})}}
\end{aligned}
$$



b) Redraw the circuit using only 2-input NAND gates.
Hint: First, convert to 2-input gates. Then convert to NANDs.

Question 7: (3 pts) Derive a Boolean expression for the complement G' of the function $G(a, b, c) = a'bc' + a'c + ab'c'$. Simplify.

$$
\begin{aligned}
G(a,b,c) &= a'bc' + a'c + ab'c' \\
\overline{G}(a,b,c) &= (a + b' + c)(a + c')(a' + b + c) \\
&= (a + b' + c)(a + c')(a' + b + c) \\
&= (a'b'c' + a'c' + ab + abc + abc' + bc' + ac + ab'c) \\
&= (a'b'c' + a'c' + ab + bc' + ac + ab'c)
\end{aligned}
$$

GRADING SCALE

Total: 43 pts

Pts	0	5	10	15	21	26	32	37
Letter Grade	D-	D	C-	C	B-	B	A-	A