# Multiclass Network Toll Design Problem with Social and Spatial Equity Constraints

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**Abstract:** In congestion pricing, apart from the conventional social equity issue between poor and rich drivers who pay the same toll charge, there exists a spatial equity issue in the sense that the changes of the generalized travel costs of drivers travelling between different origin-destination (O-D) pairs may be significantly different when tolls are charged at some selected links. The former has been debated extensively, whereas the later is blatantly ignored in the literature. In this paper, we propose bilevel programming models for the network toll design problem by explicitly incorporating the social and spatial equity constraints in terms of the maximum relative increase of the generalized equilibrium O-D travel costs between all O-D pairs for various classes of drivers with different values of time. A penalty function approach, using a simulated annealing method, is applied for solving the equity-constrained toll design problem and demonstrated with a simple network example.

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## Introduction

Road pricing has long been recognized as an efficient way to improve the economic efficiency of the transportation system and has been implemented in many metropolises around the world to reduce traffic congestion and pollution. In addition, the revenue from road pricing provides a basis for investment decisions in transportation infrastructure, such as expanding the road capacity, providing better maintenance, and improving public transport. The advanced technology of electronic road pricing mechanisms offers lower cost and new possibilities for road pricing systems. So far, many countries or regions have built pricing systems successfully such as Norway, Singapore, and Hong Kong.

Traditionally, the first-best congestion pricing theory, namely, the theory of marginal cost pricing, is well established and widely advocated by economists. In line with this theory, a toll that is equal to the difference between marginal social cost and marginal private cost is charged on each link so as to achieve a system optimum flow pattern in the network (Beckmann 1965; Dafermos and Sparrow 1971; Smith 1979). Investigations are made into how this classical economic principle would work on a general congested road network with queuing (Yang and Huang 1998) and on a congested network in a stochastic equilibrium (Yang 1999). In spite of its perfect theoretical basis, the principle of marginal cost pricing can be difficult to apply in real situations. Apart from the public and political resistance, a primary reason is due to the high extra cost spent on the equipment for toll collection in the entire network. This has motivated a number of researchers to consider various forms of second-best pricing schemes, where only a subset of links is subjected to toll charges. A typical simple example of the second-best pricing involves the two parallel route problem where an untolled alternative exists. This problem has been investigated for both static and dynamic situations by, for example, Braid (1996), Verhoef et al. (1996), Liu and McDonald (1999), and De Palma and Lindsey (2000). Optimal determination of tolls for a subset of links in a general network is studied by Yang and Lam (1996) for system optimum with fixed demand for traffic restraint (Ferrari 1995; Yang and Bell 1997), for minimization of toll revenue subject to a user equilibrium (Hearn and Ramana 1998; Dial 1999a, 2000), and for private highway modeling (Yang and Meng 2000; Yang and Woo 2000). The second-best pricing for users with discrete or continuous time value distributions is investigated by Dial (1999b,c); Leurent (1993, 1998) and Yang et al. (2002).

Whereas congestion pricing is theoretically and technologically easy to implement, it has long been viewed as a political issue. A common criticism is that road use charge makes unequivocally distributional impacts on users with different incomes. Generally speaking, the equity implications of congestion pricing are complex because of all the different options facing users under a congestion pricing scheme (Richardson and Bae 1998). People who continue to use the highway after the toll are imposed to pay the toll but also have a lower travel time—the toll decreases traffic volume, which decreases travel time. Some users with very high values of time would find that they are made better off (the reduced congestion can more than compensate the users for the extra cost of toll charges). Whereas, those with low values of time and still using the roads are generally made much worse off than before. People who stop using the highway avoid the toll, but forgo the benefits associated with using the highway and experience the inconvenience of switching to another mode of transport. This type of social equity problem between the poor and the rich users has continued to receive attention (Foster 1975; Small

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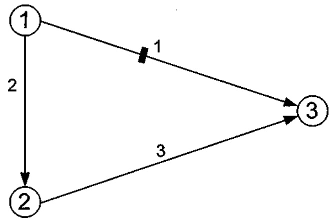


**Fig. 1.** Simple network example showing inequity problem in road pricing

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1983; Hau 1992; Johansson and Mattsson 1995; Button and Verhoef 1998) and has often been used as an argument to justify the political unacceptability of road pricing (Giuliano 1992). Although a direct redistribution of the revenue generated by the congestion charge among users in equal or unequal shares could partially or completely resolve the aforementioned inequity problem, a practicable redistribution mechanism is not established nor adopted.

The aforementioned equity issue among different social classes of users is often the primary focus of road pricing arguments; nevertheless, the spatial equity issue among users traveling between different locations is just ignored in the literature. It is evident that after introducing congestion pricing in a road network, the changes in travel costs (inclusive of toll charges) between different origin-destination (O-D) pairs can be substantially different, depending on the amounts and locations of toll charges. Thus, people traveling between different O-D pairs will receive different effects from a congestion pricing scheme that could result in another kind of unfairness on travelers and become a new obstruct on the implementation of pricing policy due to the public rejection.

This paper addresses the aforementioned two equity issues of road pricing that have spatial and social equity impacts on drivers in a network. Without loss of generality, our study focuses on the practically, meaningful second-best pricing, namely, we consider the case where not all links in the network are subjected to a toll charge. After demonstrating the inequity problem with a simple network, we formally propose mathematical programming models for the network toll design problem by explicitly incorporating the social and spatial equity constraint in terms of the maximum relative increase of the generalized equilibrium O-D travel costs for each class of users between all O-D pairs. A penalty function approach that embodies a simulated annealing method is applied for solving the equity-constrained toll design problem and illustrated with a simple network example. Finally, conclusions and suggestions for future studies are provided.

## Example for Equity Problem

The inequity problems using a simple network are depicted in Fig. 1. This network consists of 3 nodes and 3 links. Let $q_{13} = 5000$ vehicles per hour (vph) and $q_{23} = 500$ vph be the traffic demands from 1 to 3 and 2 to 3, respectively. There are two classes of users (termed "rich" and "poor" users) with equal travel demand, their value-of-times (VOT) are assumed to be 120 Hong Kong dollars per hour (HK\$/h) and 90 (HK\$/h), respectively. Link travel time functions are given as

$$t_1(v_1) = 25 + \frac{v_1}{400}; \quad t_2(v_2) = 20 + \frac{v_2}{4{,}000}; \quad t_3(v_3) = 15 + \frac{v_3}{4{,}800}$$

Assuming that the route choice behavior of users follows the principle of deterministic user equilibrium, it is straightforward to obtain the traffic volume and travel time cost of each link in the case without pricing as follows:

$$\tilde{v}_1 = 4190 \ \text{(vph)}, \ \tilde{v}_2 = 810 \ \text{(vph)}, \ \tilde{v}_3 = 1310 \ \text{(vph)}$$

$$\tilde{t}_1 = 35.475 \ \text{(min)}, \ \tilde{t}_2 = 20.202 \ \text{(min)}, \ \tilde{t}_3 = 15.273 \ \text{(min)}$$

The corresponding equilibrium travel cost for each O-D pair is

$$\tilde{\mu}_{13} = 35.475 \ \text{(min)}, \ \tilde{\mu}_{23} = 15.273 \ \text{(min)}$$

Under user equilibrium, the total network travel time cost is 3,083.5 veh h.

Now, we consider a marginal cost toll charged on each link to achieve the system optimal traffic flow distribution in terms of the total travel time minimization. Note that all the costs used here are in travel time units and, in this case, the first-best pricing requires toll differentiation on each link between the two classes of users according to their respective VOTs multiplied by the common travel time externality. The flow and travel time cost (not including toll charge) of each link are

$$\overline{v}_1 = 2500 \ \text{(vph)}, \ \overline{v}_2 = 2500 \ \text{(vph)}, \ \overline{v}_3 = 3000 \ \text{(vph)};$$

$$\overline{t}_1 = 31.250 \ \text{(min)}, \ \overline{t}_2 = 20.625 \ \text{(min)}, \ \overline{t}_3 = 15.625 \ \text{(min)}$$

The differential tolls to be charged on each link for rich and poor users are

$$\overline{y_1^r} = 12.500 \ \text{(HK\$)}, \ \overline{y_2^r} = 1.250 \ \text{(HK\$)}, \ \overline{y_3^r} = 1.250 \ \text{(HK\$)}$$

$$\overline{y_1^p} = 9.375 \ \text{(HK\$)}, \ \overline{y_2^p} = 0.938 \ \text{(HK\$)}, \ \overline{y_3^p} = 0.938 \ \text{(HK\$)}$$

where $r$ and $p$ denote rich and poor users, respectively.

The generalized travel costs (inclusive of toll) between each O-D pair are identical for both user classes when measured in time unit and are given as

$$\overline{\mu}_{13} = 37.500 \ \text{(min)}, \ \overline{\mu}_{23} = 16.250 \ \text{(min)}$$

Under the first-best pricing scheme, the total network travel time cost decreases to 2,942.7 veh h. To compare the variations in generalized travel costs, the corresponding ratios of the generalized O-D travel costs (in time unit) after and before introduction of the first-best pricing for each O-D pair are calculated by

$$\frac{\overline{\mu}_{13}}{\tilde{\mu}_{13}} = 1.057, \quad \frac{\overline{\mu}_{23}}{\tilde{\mu}_{23}} = 1.064$$

Note that the ratio for each O-D pair is identical for both user classes due to toll differentiation. It is clear that the travel costs for both O-D pairs increase, and their percentage increases are different, so an inequity problem occurs between users traveling between the two O-D pairs.

We next look at a second-best pricing scheme with the same network. Toll charge on each link and toll differentiation among users is impractical in reality; hence, we suppose an anonymous toll is charged on link 1 only. It can be easily checked that the same system-optimal traffic flow pattern with the lowest network travel time can still be achieved by an amount of toll charge $\hat{y}_2 = 8.0\,(\text{HK\$})$. The resulting generalized travel costs for each class and each O-D pair now become

$$\hat{\mu}_{13}^r = 35.250 \ \text{(min)}, \ \hat{\mu}_{13}^p = 36.250 \ \text{(min)},$$

$$\hat{\mu}_{23}^r = 15.625 \ \text{(min)}, \ \hat{\mu}_{23}^p = 15.625 \ \text{(min)}$$

In this case, the corresponding ratios of the generalized O-D travel costs after and before the toll charge are



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$$\frac{\hat{\mu}_{13}^{r}}{\tilde{\mu}_{13}}=0.994,\quad \frac{\hat{\mu}_{13}^{p}}{\tilde{\mu}_{13}}=1.022,\quad \frac{\hat{\mu}_{23}^{r}}{\tilde{\mu}_{23}}=1.023,\quad \frac{\hat{\mu}_{23}^{p}}{\tilde{\mu}_{23}}=1.023$$

It is obvious that the inequity problem becomes more significant than the first-best case. Both rich and poor classes of users traveling from node 2 to node 3 suffer the same 2.3% travel cost increase due to the toll charge. The percentage travel cost change from node 1 to 3 is less than that from node 2 to 3 but different between the two user classes. The rich users benefit 0.6% cost reduction, but the poor users suffer a 2.2% cost increase. Thus, the proposed second-best toll charge scheme brings negative and positive effects on the four groups of users with different VOTs and traveling between different O-D pairs.

This simple example shows that a proposed toll scheme may have both spatial (traveling between different O-D pairs) and social (with different VOTs) inequity impacts on road users. To mitigate such inequity impacts, toll levels and toll locations for a pricing scheme in a road network should be selected cautiously.

## Model Formulation

### *User Heterogeneity and Multiclass Network Equilibrium*

Value of time is a very important concept in transportation modeling. It is well known that each user has a different VOT, depending on his or her level of income. Recent advanced network equilibrium models acknowledge this fact by incorporating user heterogeneity in route choice. These models simulate the way users select a route from among the competing paths which are differentiated on the basis of two cost criteria—journey time and monetary cost. There are, in general, two lines of approach to deal with the tradeoffs between money and time in simulating users' response to toll charge. A first line of approach consists of differentiating several discrete classes of users, each one with a VOT belonging to some interval (Dafermos 1973; Daganzo 1983; Florian 1998). The second line of approach assumes a continuously distributed VOT across the users (Dial 1996, 1997; Leurent 1993, 1996, 1998). The VOT statistical distribution could be inferred from the income distribution of the population.

This paper adopts the discrete approach for VOT distribution, which is considered to be particularly suitable for congestion pricing modeling with elastic demand. The whole society is divided into some classes, each class is assumed to have an average VOT belonging to some interval, and can be characterized by class-specific demand function. This treatment, in certain cases, can be regarded as a discrete approximation of the model using continuously distributed VOT and has some practical advantages. It is consistent with the conventional market segmentation approach for travel demand modeling and, thus, greatly facilitates model calibration with actual data. Traditionally, different socioeconomic groups are dealt with by developing an entirely distinct model for each subgroup (e.g., Ben-Akiva and Lerman 1985). There are numerous empirical studies of demand models that employed segmentations into socioeconomic subgroups. These market segment models incorporate the different tastes in the specifications of the utilities, so that it may be better to proceed with separate market segment models. With those considerations, we will now provide a discrete multiclass network equilibrium approach.

We divide the total number of users into $M$ classes according to their respective VOTs that are well related to their income levels. Classes are ordered according to increasing VOT, namely, $\beta_1<\beta_2<\cdots<\beta_M$. Let $d_w^m$ $(m=1,2,\ldots,M)$ be the travel demand for class $m$ between O-D pair $w\in W$, which is assumed to be a continuously decreasing function of the full price of travel incurred, including travel time plus toll if toll roads are used. The multiclass network equilibrium model with elastic demand is formulated as

$$\underset{\mathbf{d},\mathbf{v}}{\text{minimize}}\sum_{a\in A}\int_0^{v_a}t_a(\omega)\,d\omega+\sum_{a\in A^*}\sum_{m=1}^{M}\frac{1}{\beta_m}v_a^m y_a-\sum_{w\in W}\sum_{m=1}^{M}\int_0^{d_w^m}D_w^{m^{-1}}(\omega)\,d\omega \tag{1}$$

subject to

$$\sum_{r\in R_w}f_{rw}^m=d_w^m,\quad w\in W,\quad m=1,2,\ldots,M \tag{2}$$

$$v_a=\sum_{w\in W}\sum_{m=1}^{M}\sum_{r\in R_w}f_{rw}^m\delta_{ar}^w,\quad a\in A \tag{3}$$

$$v_a^m=\sum_{w\in W}\sum_{r\in R_w}f_{rw}^m\delta_{ar}^w,\quad a\in A,\quad m=1,2,\ldots,M \tag{4}$$

$$f_{rw}^m\geqslant 0,\quad r\in R_w,\quad w\in W,\quad m=1,2,\ldots,M \tag{5}$$

Clearly, the problem is a convex minimization problem, and its optimal solution satisfies the Kuhn-Tucker conditions characterized as

$$c_{rw}^m-\mu_w^m\geqslant 0,\quad \text{if } f_{rw}^m=0,\quad w\in W,\quad m=1,2,\ldots,M \tag{6}$$

$$c_{rw}^m-\mu_w^m=0,\quad \text{if } f_{rw}^m>0,\quad w\in W,\quad m=1,2,\ldots,M \tag{7}$$

$$d_w^m=D_w^m(\mu_w^m),\quad w\in W,\quad m=1,2,\ldots,M \tag{8}$$

where $c_{rw}^m$=travel cost (inclusive of equivalent time of toll charge) on route $r$ between O-D pair $w$ by users of class $m$

$$c_{rw}^m=\sum_{a\in A}t_a(v_a)\delta_{ar}^w+\frac{1}{\beta_m}\sum_{a\in A^*}y_a\delta_{ar}^w,\quad r\in R_w,\quad w\in W,\quad m=1,2,\ldots,M \tag{9}$$

where $A^*$=subset of links with toll charge. These are the user-equilibrium conditions for multiclass network users on the basis of generalized travel cost in terms of equivalent travel time. Note that the conventional Frank-Wolf algorithm can be easily applied to solve the above multiclass network equilibrium problem. For a detailed description, the readers may refer to (Sheffi 1985; Yang et al. 2002).

### *Bilevel Network Toll Design Model without Considering Equity Impacts*

The bilevel network toll design problem proposed by Yang and Lam (1996) can be easily extended to incorporate demand elasticity. The upper level program here aims to maximize the total social welfare and the lower level program is a multiclass network equilibrium model in terms of generalized travel cost (including monetary toll, if any). Note that the travel cost used here in characterizing the route choice behavior is measured in travel time unit where toll is converted into corresponding travel time by a given VOT of each class. Then the second best congestion pricing problem with elastic demand is formulated as



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$$\text{maximize } SW(\mathbf{y}) = \sum_{m=1}^{M}\left[\sum_{w\in W}\int_0^{d_w^m(y)} D_w^{m^{-1}}(\omega)\,d\omega - \sum_{a\in A} v_a^m(\mathbf{y})t_a(v_a)\right] \quad (10)$$

subject to

$$y_a^{\min} \leqslant y_a \leqslant y_a^{\max}, \quad a \in A^* \quad (11)$$

Here $v_a^m(\mathbf{y})$, $a\in A$, $m=1,2,\ldots,M$ and $d_w^m(\mathbf{y})$, $w\in W$, $m=1,2,\ldots,M$ are the solutions of the lower-level multiclass network equilibrium program (1)–(5).

### Specification of Equity Constraint

In reality, it is very difficult, if not impossible, to design a toll pattern that brings completely equitable impact on all users traveling between different O-D pairs. A simple yet practicable method is to prevent the percentage increase of travel cost of all network users from exceeding a certain threshold in designing a toll scheme. In this case, we can say that the relative inequity impact from a toll scheme is limited to a certain level. With this in mind, we deal with the equity issue by incorporating an equity constraint in the upper-level problem. The equity is measured as the relative change of the generalized O-D travel cost (inclusive of toll charge) and thus the equity constraint can be specified as follows:

$$\frac{\mu_w^m(\mathbf{y})}{\tilde{\mu}_w} \leqslant \phi_w, \quad w\in W, \quad m=1,2,\ldots,M \quad (12)$$

The term $\tilde{\mu}_w$=original user equilibrium O-D travel time without pricing; $\mu_w^m(\mathbf{y})$=generalized equilibrium O-D travel cost in time unit (inclusive of toll charge) for class $m$ after introducing a second-best pricing scheme; and $\phi_w$ is designated here as an equity index that dictates the degree of tolerance of the inequity associated with the pricing scheme. A meaningful selection of the value of this index is given by the following equation:

$$\phi_w = \begin{cases} 1+\varphi\left(\dfrac{\overline{\mu}_w}{\tilde{\mu}_w}-1\right) & \text{if } \dfrac{\overline{\mu}_w}{\tilde{\mu}_w}>1 \\ 1 & \text{otherwise} \end{cases} \quad (13)$$

where $\overline{\mu}_w$=O-D travel cost after the first-best or marginal-cost pricing scheme is implemented; and $\varphi$=decision variable satisfying $0\leqslant\varphi\leqslant 1$. Clearly, $1\leqslant\phi_w\leqslant\overline{\mu}_w/\tilde{\mu}_w$, $w\in W$ for $0\leqslant\varphi\leqslant 1$ if $\overline{\mu}_w>\tilde{\mu}_w$. It should be mentioned that for the marginal-cost pricing scheme used here, we assume that the same marginal time externality is internalized by differentiated tolls across user classes according to their respective VOTs. Therefore, for a given O-D pair $w\in W$, different classes have equal travel time cost denoted by $\overline{\mu}_w$ in time unit.

In the general case of $\overline{\mu}_w>\tilde{\mu}_w$, we have $\phi_w=\overline{\mu}_w/\tilde{\mu}_w$ when $\varphi$ is equal to 1. This means that $\phi_w$ takes its largest value equal to the ratio of the equilibrium O-D travel cost after introducing marginal-cost pricing to the original O-D travel time cost without pricing. In other words, the degree of inequity in a second-best pricing scheme is bounded by the first-best pricing case, or the generalized travel cost between each O-D pair under a second-best pricing scheme cannot exceed that under the first-best pricing scheme. This specification of the upper bound of the equity parameter is quite justifiable and meaningful. The theoretical foundation of congestion pricing stems from the fundamental economic principle of marginal cost pricing, which states that road users using congested roads should pay a toll equal to the difference between the marginal social cost and the marginal private cost, so as to maximize economic benefit. Thus, by requiring paying a marginal-cost toll, an individual driver will bear the full social cost generated by him or her in using a congested road. A toll charge higher than this amount seems to be economically irrational and unfair to users.

When $\varphi$ is equal to 0, $\phi_w$ becomes 1. This means that the generalized O-D travel cost cannot exceed the equilibrium O-D travel time cost before introducing pricing. Namely, all the O-D travel costs decrease or remain unchanged. Because in a real network, a congestion pricing scheme is very unlikely to lead to a decline in all generalized O-D travel costs inclusive of toll [note that this is not always impossible, for example, in Braess paradox network (Braess 1968), introducing a toll charge on the paradoxical link may reduce the average O-D travel cost]. The zero toll charge (do-nothing alternative) may become the only feasible solution when $\varphi=0$.

To summarize, $\varphi$ is an appropriate decision variable that can be used by the system manager to adjust the level of social and spatial equity for consideration in establishing a fair and reasonable pricing scheme. A higher value of $\varphi$ permits a greater negative inequity impact on users, and a lower value of $\varphi$ places a stricter inequity constraint.

From the equity constraint, Eq. (12), there are totally $M$ constraints associated with $M$ distinct classes of users for each given O-D pair $w\in W$. In fact, only the equity constraint corresponding to class 1 with the lowest VOT is needed and the remaining $(M-1)$ constraints for each O-D pair are redundant. Namely, imposition of the equity constraint for the lowest VOT user class automatically guarantees that the same equity constraints are satisfied for higher VOT user classes. This is due to the fact that the deterministic equilibrium O-D travel costs (in time unit) by class always satisfy $\mu_w^1(\mathbf{y})\geqslant\mu_w^2(\mathbf{y})\geqslant\cdots\geqslant\mu_w^M(\mathbf{y})$, $w\in W$ for any anonymous congestion-pricing scheme with nonnegative tolls, where user classes are ordered according to increasing VOT.

Consider any two classes of users, class $m_1$ and class $m_2$ with $\beta_{m_1}<\beta_{m_2}$, traveling between a given O-D pair $w\in W$. Let $\eta_w^r$ and $\tau_w^r$ $(\tau_w^r\geqslant 0)$ be the travel time and total amount of toll charge along route $r\in R_w$. Clearly, we have

$$c_{rw}^{m_1}=\eta_w^r+\tau_w^r/\beta_{m_1}\geqslant c_{rw}^{m_2}=\eta_w^r+\tau_w^r/\beta_{m_2}$$
$$\text{for any } r\in R_w, \quad w\in W \quad (14)$$

Because travel cost of class $m_1$ is larger than or equal to that of class $m_2$ on any route (used or not used by either class), we can easily conclude that $\mu_w^{m_1}\geqslant\mu_w^{m_2}$, $w\in W$ as long as $\beta_{m_1}<\beta_{m_2}$. We thus conclude that $\mu_w^1(\mathbf{y})\geqslant\mu_w^2(\mathbf{y})\geqslant\cdots\geqslant\mu_w^M(\mathbf{y})$, $w\in W$ for $\beta_1<\beta_2<\cdots<\beta_M$ in deterministic multiclass user equilibrium. From this observation, the set of constraints, Eq. (12), can now be replaced by

$$\frac{\mu_w^1(\mathbf{y})}{\tilde{\mu}_w} \leqslant \phi_w, \quad w\in W \quad (15)$$

Note that if the travel cost is measured in monetary unit such that $c_{rw}^m=\beta_m\eta_w^r+\tau_w^r$, $r\in R_w$, $w\in W$, $m=1,2,\ldots,M$, then we must have $\mu_w^1(\mathbf{y})\leqslant\mu_w^2(\mathbf{y})\leqslant\cdots\leqslant\mu_w^M(\mathbf{y})$, $w\in W$ for $\beta_1<\beta_2<\cdots<\beta_M$ in a deterministic multiclass user equilibrium.



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## *Network Toll Design Models with Equity Constraints*

With the aforementioned social and spatial equity consideration, the network toll design problem with an equity constraint can now be formulated as the following bilevel programming problem.

Model M1

$$\max F_1[\mathbf{y},v_a^m(\mathbf{y}),d_w^m(\mathbf{y})]=\sum_{m=1}^{M}\left[\sum_{w\in W}\int_0^{d_w^m(\mathbf{y})}D_w^{m^{-1}}(\omega)d\omega - \sum_{a\in A} v_a^m(\mathbf{y})t_a(v_a)\right] \quad (16)$$

subject to

$$\frac{\mu_w^1(\mathbf{y})}{\tilde{\mu}_w}\leqslant\phi_w,\quad w\in W \quad (17)$$

$$y_a^{\min}\leqslant y_a\leqslant y_a^{\max},\quad a\in A^* \quad (18)$$

Here $\phi_w=1+\varphi(\overline{\mu}_w/\tilde{\mu}_w-1)$ if $\overline{\mu}_w/\tilde{\mu}_w>1$ and $\phi_w=1$ otherwise, parameter $\varphi$ is a given appropriate constant satisfying $0\leqslant\varphi\leqslant1$. Link flow $v_a^m(\mathbf{y})$, $a\in A$, $m=1,2,\ldots,M$, O-D demand of each class $d_w^m(\mathbf{y})$, $w\in W$, $m=1,2,\ldots,M$ and O-D travel cost of the lowest VOT class $\mu_w^1(\mathbf{y})$, $w\in W$ are obtained from the multiclass network equilibrium model Eqs. (1)–(5). Note that parameter $\varphi$ introduced before reflects the allowable degree of inequity in terms of the increase in the equilibrium O-D travel cost after and before implementing a pricing scheme. This parameter is selected by the decision maker and can be, in fact, treated as a decision variable in the programming model. For each given $\varphi$, let MSW($\varphi$) be the maximum social welfare obtained by the bilevel model M1, and MSW($\varphi$) can be regarded as an implicit function of parameter $\varphi$. By incorporating this equity decision parameter $\varphi$, we have the following trilevel programming model with dual top-level objective functions:

$$\min_{\varphi}F_2(\varphi)=\begin{bmatrix}F_2^1=\dfrac{\mathrm{MSW}(0)-\mathrm{MSW}(\varphi)}{\mathrm{MSW}(1)-\mathrm{MSW}(0)}\\ F_2^2=\varphi\end{bmatrix} \quad (19)$$

subject to

$$0\leqslant\varphi\leqslant1 \quad (20)$$

where MSW($\varphi$)=optimal objective value of the bilevel model M1 for given $\varphi$.

A mathematically well-defined optimal solution does not exist for this multiobjective programming model. Therefore, a considerate trade off is necessary to balance the need of increasing the total social welfare and the requirement of avoiding the great negative inequity impact. The problem thus becomes how to select a nondominated efficient solution using an appropriate method. A widely used utility function approach is adopted here to transfer the two top-level objectives into a single objective function. Thus objective vector, Eq. (19), now becomes

$$F_3(\varphi)=\omega F_2^1+(1-\omega)F_2^2=\omega\left[\frac{\mathrm{MSW}(0)-\mathrm{MSW}(\varphi)}{\mathrm{MSW}(1)-\mathrm{MSW}(0)}\right]+(1-\omega)\varphi \quad (21)$$

Here, $\omega$=weighting parameter satisfying $0<\omega<1$. After transformation, the model with equity constraints becomes the following standard trilevel programming problem:

$$\min_{\varphi}F_3(\varphi)=\omega\left[\frac{\mathrm{MSW}(0)-\mathrm{MSW}(\varphi)}{\mathrm{MSW}(1)-\mathrm{MSW}(0)}\right]+(1-\omega)\varphi \quad (22)$$

subject to

$$0\leqslant\varphi\leqslant1 \quad (23)$$

where MSW($\varphi$) is again the optimal objective value of the bilevel model M1 for given $\varphi$.

## Solution Methods

We have proposed the bilevel programming model M1 to characterize the congestion-pricing problem with social and spatial equity constraints, and the trilevel programming model M2 to treat the equity reference parameter as an endogenous decision variable. The proposed bilevel programming models, just like any other form of bilevel mathematical programming problems, are intrinsically nonconvex, and hence might be difficult to solve for a global optimum (Friesz et al., 1990; Meng and Yang 2002). The difficulty comes from the fact that the equilibrium link flow $v_a^m(\mathbf{y})$, $a\in A$, $m=1,2,\ldots,M$ and the O-D demand $d_w^m(\mathbf{y})$, $w\in W$, $m=1,2,\ldots,M$ generally are nonconvex and nondifferentiable functions in $\mathbf{y}$.

In view of the difficulty in applying the standard algorithmic approaches for search of the global optimum, we adopt the Simulated Annealing (SA) method (Dekkers and Aarts 1991; Romeijin and Smith 1994), which is particularly suitable for the models proposed here. The SA method has been successfully applied to solve the continuous network design problems (Friesz et al. 1992; Meng and Yang 2002). It has the ability to obtain the global optimal solution without the requirement for differentiability.

Now we consider how to solve the model M1 (solution for M2 is discussed later). The nonlinear, implicit constraint, Eq. (17), is incorporated into the objective function by using an inner penalty function approach in applying the SA method. For the sake of clarity, let

$$\Omega=(\mathbf{y}|y_a^{\min}\leqslant y_a\leqslant y_a^{\max},a\in A^*) \quad (24)$$

denote the feasible set of $\mathbf{y}$ dictated by its lower and upper bound.

### *Solution for Model M1*

The procedure of the penalty function approach in conjunction with the SA method for solving M1 is presented as follows.

#### Inner Penalty Function Approach

***Step 0.*** *Initialization*: Set a stop tolerance $\epsilon$, an initial penalty multiplier $\lambda_0$, a scale parameter $\rho>1$ and an initial point $\mathbf{y}^{(0)}\in\Omega$. Let $k=0$.

***Step 1.*** *Finding an optimal solution*: Starting with $\mathbf{y}^{(k)}$, solving the following problem through the SA method and let $\mathbf{y}^{(k+1)}$ be the optimal solution

$$\min_{y\in\Omega}\hat{F}(\mathbf{y})=F_1(\mathbf{y})+\lambda_k\alpha(\mathbf{y}) \quad (25)$$

where

$$\alpha(\mathbf{y})=\sum_{w\in W}\max\left[\frac{\mu_w^1(\mathbf{y})}{\tilde{\mu}_w}-\phi_w,0\right] \quad (26)$$

***Step 2.*** *Verifying the stop criterion*: If $\lambda_k\alpha(\mathbf{y}^{(k+1)})<\epsilon$, stop. Otherwise, set $\lambda_{k+1}=\rho\lambda_k$ and $k:=k+1$, go to Step 1.



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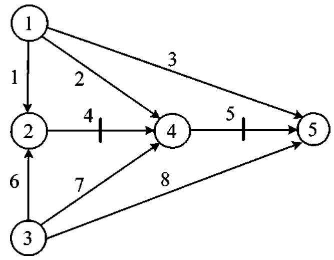


**Fig. 2.** Network used in numerical example

For any given vector $\mathbf{y}$ of the toll pattern, the equilibrium traffic assignment procedure is used to find the link flow $v_a^m(\mathbf{y})$, the O-D demand for each class $d_w^m(\mathbf{y})$ and the generalized O-D travel cost $\mu_w^m(\mathbf{y})$; and so, it is straightforward to calculate the objective function value $\hat{F}(\mathbf{y})$. With this in mind, the procedure of simulated annealing method for solving problem M1 with any fixed penalty parameter $\lambda_k$ is given as follows.

### Simulated Annealing Method

***Step 0.*** *Initialization*: Given an initial point $\mathbf{y}^{(0)} \in \Omega$ and the parameter $0 < \chi_0 < 1.0$, $0 < \delta < 1.0$, $0 < t < 1.0$ integer $L_0$, $m_0$ and $T_s$ (stop tolerance of temperature). Set $k = k_1 = 0$.

***Step 1.*** *Finding an initial temperature*: Uniformly generate at random $m_0$ points denoted by $\mathbf{z}^{(i)}$ $(i = 1, \ldots, m_0)$ over the feasible set $\Omega$. For each $\mathbf{z}^{(i)}$ use equilibrium traffic assignment to obtain the equilibrium link flow and O-D travel cost associated with $\mathbf{z}^{(i)}$ and then calculate the corresponding function value $\hat{F}(\mathbf{z}^{(i)})$. Let $m_2$ denote the number of points $\mathbf{z}^{(i)}$ with $\hat{F}(\mathbf{z}^{(i)}) - \hat{F}(\mathbf{y}^{(0)}) \geqslant 0$ and $\overline{\Delta \hat{F}^+}$ the average value of those $\hat{F}(\mathbf{z}^{(i)}) - \hat{F}(\mathbf{y}^{(0)})$, for which $\hat{F}(\mathbf{z}^{(i)}) - \hat{F}(\mathbf{y}^{(0)}) \geqslant 0$. Then the initial temperature $T_0$ is calculated as

$$T^{(0)} = \begin{cases} \overline{\Delta \hat{F}^+} & \text{if } m_2 \leqslant m_0/2 \\ \left( \ln \dfrac{m_2}{m_2 \chi_0 + (1-\chi_0)(m_0 - m_2)} \right)^{-1} & \text{if } m_2 > m_0/2 \\ +\infty & \text{if } m_2 = 0 \end{cases} \tag{27}$$

***Step 2.*** *Verifying the termination*: If $T^{(k)} < T_s$, then stop. Otherwise, go to Step 3.

***Step 3.*** *Checking the termination of a Markov chain*: If $k_1 > L_0 N$ ($N$ represents the number of decision variables), then go to Step 6. Otherwise, go to Step 4.

***Step 4.*** *Generation of points*: Uniformly generate at random a number denoted by $t_{\text{random}}$ from the internal $[0,1)$. If $t_{\text{random}} > t$ then use the method of Hooke and Jeeves with discrete step (Bazaraa et al. 1993, p. 288) from the point $\mathbf{y}^{(k_1)}$ as a local search procedure to find a local solution for the problem denoted by $\mathbf{x}$. If $t_{\text{random}} \leqslant t$, then uniformly generate at random a point denoted by $x$ over $\Omega$.

***Step 5.*** *Metropolis' rule*: If $\hat{F}(\mathbf{x}) \leqslant \hat{F}[\mathbf{y}^{(k_1)}]$ or if $\exp\{-[\hat{F}(\mathbf{x}) - \hat{F}(\mathbf{y}^{(k_1)})]/T^{(k)}\} > \text{random}[0,1]$ when $\hat{F}(\mathbf{x}) > \hat{F}[\mathbf{y}^{(k_1)}]$, set $\mathbf{y}^{(k_1+1)} = \mathbf{x}$, $k_1 = k_1 + 1$ and go to Step 3. Otherwise, $\mathbf{y}^{(k_1+1)} = \mathbf{y}^{(k_1)}$, $k_1 := k_1 + 1$ and go to Step 3.

***Step 6.*** *Cooling schedules*: Calculate the standard derivation of the values of the objective function $\hat{F}(\mathbf{y}^{(k_1)})$ $(k_1 = 0, \ldots, L_0 N)$, denoted by $\sigma(T^{(k)})$. Set the temperature as

$$T^{(k+1)} = T^{(k)} \left[ 1 + \frac{T^{(k)} \ln(1+\delta)}{3\sigma(T^{(k)})} \right]^{-1} \tag{28}$$

$k := k+1$, $\mathbf{y}^{(0)} = \mathbf{y}^{(L_0 N)}$, $k_1 = 0$ and go to Step 2.

The convergence property of this SA algorithm is proved by Dekkers and Aarts (1991). Note that the original Hooke-Jeeves method is designed for the unconstrained optimization problem. In the problem examined here, there are simple bound constraints, $y_a^{\min} \leqslant y_a \leqslant y_a^{\max}$, $a \in A^*$ only, we can slightly modify the Hooke-Jeeves method to deal with this situation by projecting the trial point in the method onto the region $\Omega$ defined by the bound constraints. Furthermore, the objective function evaluation is required in the Hooke-Jeeves method. This means that we need to perform a user equilibrium traffic assignment procedure at each trial point in this local search method.

## Solution for Model M2

We first analyze how the two components $F_2^1$ and $F_2^2$ of the objective function $F_3$ vary with $\varphi$. When $\varphi$ is equal to 0, or when no travel cost increase is allowed between any O-D pair, the tolls have to be set equal to 0, and obviously $F_2^1 = 0.0$. When $\varphi$ increases to a value greater than 0, nonzero tolls are charged on certain links, and the social welfare gain in the network must increase, otherwise, the solution is not optimal. In other words, the component $F_2^1$ will decrease to a number smaller than 0.0. When $\varphi$ comes to 1.0, $F_2^1$ reaches the minimum value $-1.0$. Of course, the term $(1-\omega)F_2^2$ or the term $(1-\omega)\varphi$ linearly increases with $\varphi$. In summary, one component $F_2^1$ of the objective function $F_3$ decreases with $\varphi$, the other component $F_2^2$ of $F_3$ increases with $\varphi$.

Furthermore, the value of variable $\varphi$ is restricted within the limited interval $[0,1]$. For any given value of $\varphi$ in the interval $0 \leqslant \varphi \leqslant 1$, constraint, Eq. (23), disappears, in view of the fact that MSW(0), MSW(1), $\omega$ together with $\varphi$ are constant, minimization of $F_3$ is equivalent to maximization of $F_1$. In other words, model M2 reduces to M1 for a given value of $\varphi$. Based on this observation, we can use a 1D search method in conjunction with the

**Table 1.** Input Data for Test Network in Fig. 2

| Link | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $t_a^0$ (min) | 10 | 23 | 42 | 10 | 15 | 10 | 23 | 42 |
| $C_a$ (vph) | 1,000 | 2,000 | 3,000 | 4,000 | 6,000 | 1,000 | 2,000 | 3,000 |

**Table 2.** Change in Equilibrium O-D Travel Costs Before and After Road Pricing

| O-D pair | 1→5 | 2→5 | 3→5 | 4→5 |
|---|---|---|---|---|
| Travel cost without pricing (min) | 42.631 | 30.587 | 42.779 | 19.544 |
| Travel cost with marginal-cost pricing (min) | 48.651 | 37.120 | 49.084 | 25.408 |
| Ratio of the equilibrium O-D travel costs | 1.141 | 1.214 | 1.147 | 1.300 |



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**Table 3.** Numerical Results by Model M1 for VOT=75,100,150 (HK$/h)

| $\varphi$ | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $y_1$ (HK$) | 0.00 | 0.14 | 0.30 | 0.60 | 1.35 | 2.08 | 2.91 | 3.49 | 4.22 | 5.21 | 5.21 |
| $y_2$ (HK$) | 0.00 | 1.49 | 2.90 | 3.95 | 4.35 | 4.75 | 5.01 | 5.41 | 5.68 | 5.81 | 5.81 |
| Social welfare ($\times 10^4$ veh h) | 1.7701 | 1.7767 | 1.7815 | 1.7843 | 1.7866 | 1.7883 | 1.7898 | 1.7906 | 1.7908 | 1.7910 | 1.7910 |
| Revenue ($\times 10^4$ HK$) | 0.0000 | 1.0775 | 2.0317 | 2.7542 | 3.2232 | 3.6679 | 4.0264 | 4.3531 | 4.6898 | 5.0274 | 5.0274 |
| System cost ($\times 10^3$ veh h) | 6.6269 | 6.5471 | 6.4860 | 6.4411 | 6.4153 | 6.3905 | 6.3707 | 6.3529 | 6.3428 | 6.3281 | 6.3281 |
| Binding O-D pair | All | 2→5 | 2→5 | 2→5 | 2→5 | 2→5 | 2→5 | 2→5 | 2→5 | No | No |

**Table 4.** Numerical Results by Model M1 for VOT=50,100,200 (HK$/h)

| $\varphi$ | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $y_1$ (HK$) | 0.00 | 0.11 | 0.44 | 0.86 | 1.14 | 1.49 | 1.69 | 2.00 | 2.10 | 2.36 | 2.65 |
| $y_2$ (HK$) | 0.00 | 0.70 | 1.11 | 1.37 | 1.84 | 2.21 | 2.68 | 3.07 | 3.84 | 4.23 | 4.66 |
| Social welfare ($\times 10^4$ veh h) | 1.7701 | 1.7737 | 1.7760 | 1.7779 | 1.7801 | 1.7821 | 1.7839 | 1.7855 | 1.7871 | 1.7881 | 1.7895 |
| Revenue ($\times 10^4$ HK$) | 0.0000 | 0.5316 | 0.9298 | 1.2500 | 1.6530 | 2.0034 | 2.3587 | 2.6879 | 3.1705 | 3.4637 | 3.8562 |
| System cost ($\times 10^3$ veh h) | 6.6269 | 6.5837 | 6.5578 | 6.5366 | 6.5097 | 6.4857 | 6.4616 | 6.4391 | 6.4170 | 6.4016 | 6.3804 |
| Binding O-D pair | All | 2→5 | 2→5 | 2→5 | 2→5 | 2→5 | 2→5 | 2→5 | 2→5 | 2→5 | 2→5 |

**Table 5.** Numerical Results by Model M2 for VOT=75,100,150 (HK$/h)

| $\omega$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|---|---|---|---|---|---|---|---|---|---|
| $\varphi$ | 0.00 | 0.02 | 0.14 | 0.25 | 0.40 | 0.55 | 0.63 | 0.70 | 0.74 |
| $y_1$ (HK$) | 0.00 | 0.00 | 0.22 | 0.32 | 1.35 | 2.49 | 3.04 | 3.49 | 3.89 |
| $y_2$ (HK$) | 0.00 | 0.37 | 2.08 | 3.03 | 4.35 | 4.89 | 5.29 | 5.41 | 5.59 |
| $F_3$ | 0.000 | −0.001 | −0.029 | −0.098 | −0.195 | −0.328 | −0.482 | −0.646 | −0.817 |
| Social welfare ($\times 10^4$ veh h) | 1.7701 | 1.7718 | 1.7789 | 1.7818 | 1.7866 | 1.7892 | 1.7901 | 1.7906 | 1.7908 |
| Revenue ($\times 10^4$ HK$) | 0.0000 | 0.2622 | 1.4905 | 2.1183 | 3.2232 | 3.8541 | 4.2024 | 4.3531 | 4.5504 |
| System cost ($\times 10^3$ veh h) | 6.6269 | 6.6050 | 6.5198 | 6.4811 | 6.4153 | 6.3788 | 6.3628 | 6.3529 | 6.3468 |
| Binding O-D pair | All | 2→5 | 2→5 | 2→5 | 2→5 | 2→5 | 2→5 | 2→5 | 2→5 |

algorithm for M1 to solve M2. At each new point of the 1D search for $\varphi$, a subprogram M1 is solved to find the objective function value $F_3$, and the search is continued when both optimal $\varphi$ and $\mathbf{y}$ are identified.

## Numerical Example

The road network shown in Fig. 2 consists of 5 nodes and 8 links, of which link 4 and link 5 are subjected to toll charges (locations are predetermined). The link travel time function is

$$t_a(v_a)=t_a^0\left[1.0+0.15\left(\frac{v_a}{C_a}\right)^4\right] \tag{29}$$

The values of $t_a^0$ and $C_a$ for each link are given in Table 1. There are 4 O-D pairs with the following negative exponential demand function:

$$d_w^m=D_w^m(\mu_w^m)=\overline{d}_w^m\exp(-\gamma\mu_w^m),\quad w\in W,\quad m=1,2,\ldots,M \tag{30}$$

The potential demands are respectively $\overline{d}_{15}=5{,}500$, $\overline{d}_{25}=2{,}000$, $\overline{d}_{35}=6{,}000$, $\overline{d}_{45}=2{,}000$ vph for the four O-D pairs (1→5, 2→5, 3→5, 4→5). And $\gamma$ is set to be 0.01 for all user classes. Suppose $M=3$ and $m=1$, 2 and 3 represent low, medium and high income user class, the potential market share of the three classes are assumed to be identical for all O-D pairs and given to be 20, 60, and 20%, and their VOTs are 75, 100 and 150 (HK$/h), respectively. Without implementation of a road-pricing scheme, the O-D travel time costs from a user equilibrium assignment are presented in Table 2. The social welfare gain is $1.7701\times 10^4$ veh h. When a marginal time cost toll is charged on each link, the resulting generalized O-D travel costs (in time unit) are also listed in the same table and the social welfare gain increases to a maximum value $1.7940\times 10^4$ veh h. Table 2 also shows the ratios of the generalized O-D travel costs after and before marginal-cost pricing is introduced, which are same among different classes for a given O-D pair. It can be seen from the table that travel costs between all the four O-D pairs increase after the marginal cost tolls are charged. In particular, the travel cost between O-D pair 4→5 increases most.

Now we consider a second-best pricing regime with link 4 and link 5 subjected to toll charge for maximizing social welfare, while considering equity constraints with model M1. Table 3 shows the numerical results for 10 different values of the inequity threshold $\varphi$. The last row of the table lists the O-D pairs, for which the constraint, $\mu_w^1(\mathbf{y})/\tilde{\mu}_w\leqslant\phi_w$, is binding. From the table, we can see that $\varphi=0.0$ means that cost increase is not allowed for all O-D pairs, and in this case no toll can be charged on both links and the equity constraints of all O-D pairs are binding. As $\varphi$ increases, the equity constraint is relaxed gradually, higher tolls can be charged on both links, and more revenue is generated, and as expected, the total travel cost decreases and the total social welfare increases. The binding equity constraint is always associated with O-D pair 2→5 for $\varphi=0.1\sim0.8$; this is clearly due to the fact that free alternative route is not available for traveling from node 2 to 5 and low-income users from 2 to 5 suffer most from



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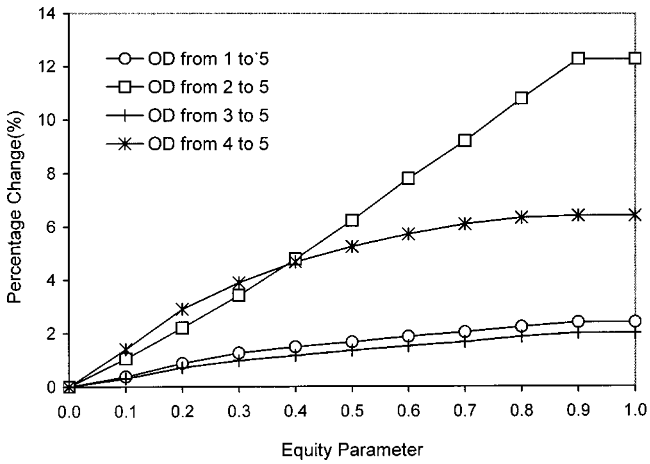


**Fig. 3.** Percentage increase of average travel cost between different O-D pairs

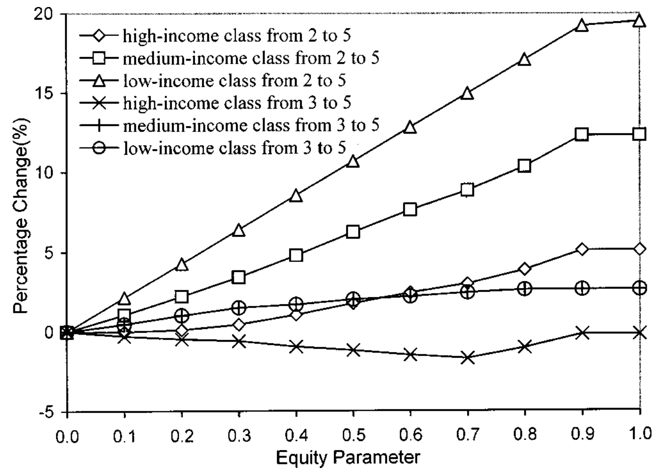


**Fig. 4.** Change in O-D travel cost for different classes of user

the road-pricing scheme. As $\varphi \geqslant 0.9$, relaxing the equity constraint does not lead to further increase in social welfare (objective function), and all equity constraints become nonactive.

In fact, the dispersion of VOT distribution across users has significant impacts on the analysis result. Namely, as the VOT distribution is more dispersed, the inequity effect among users becomes more evident, and thus the equity constraints act more tightly. To verify this effect, Table 4 presents numerical results for another set of more deviated VOT given to be 50, 100, and 200 (HK\$/h) for the three classes. In comparison with the former case, optimal tolls for both links are smaller for the same $\varphi$ value. Even $\varphi$ increases up to its maximal value 1.0, the equity constraint is still binding. Thus, the social welfare for $0.0 \leqslant \varphi \leqslant 1.0$ cannot reach its maximum value for the case without equity constraints. This highlights the fact that both social and spatial equity issues deserve more attention as VOT differential across users becomes more remarkable.

Table 5 provides the numerical results obtained from model M2 for 9 different values of weighting factor $\omega$. As $\omega$ increases, or as more emphasis is placed on maximization of social welfare than on the equity constraint, the value of $\varphi$, as expected, increases, and the toll charge on each link becomes higher, meanwhile, the total revenue increases, the total travel time decreases and the total social welfare increases.

We now demonstrate how the two kinds of inequities examined here occur on the network and how the proposed models work. Fig. 3 plots the percentage variation of the average O-D travel costs (average over 3 classes weighted by their shares) for the 4 O-D pairs. From this figure, we can see that when $\varphi$ is less than 0.4, the average O-D travel cost of 4→5 increases most. As $\varphi$ grows up, travel costs of 2→5 increases most. Fig. 4 depicts the percentage change of O-D travel cost by user class for two selected O-D pairs 2→5 and 3→5. It is observed that for O-D pair 3→5, high-income class users actually benefit from toll charge. As $\varphi$ increases up to about 0.70, they enjoy more benefits, and as $\varphi$ increases further beyond 0.70 they enjoy less benefit. In contrast, all other users suffer from toll charge, and suffer more losses as $\varphi$ increases. Note that for O-D pair 3→5, the medium- and low-income class users have equal travel cost because they are both exclusively concentrated on the nontoll path (link 8). These results reveal that parameter $\varphi$ plays an essential role in controlling the spatial and social equity impacts of a network-pricing scheme on heterogeneous users.

## Conclusions

We have proposed and solved mathematical programming models that explicitly address the social and spatial equity problems in designing a network-pricing scheme. The spatial equity problem is posed in terms of the different absolute and relative changes in the generalized O-D travel cost between different O-D pairs arising from toll charges, whereas the social equity issue is examined in terms of the inequitable impacts of road pricing on users of different social classes. A penalty function method in conjunction with a simulated annealing approach is applied to solve the bilevel programming models with equity constraints. As demonstrated with numerical examples, the proposed models and algorithms allow for selection of a fair and reasonable toll pattern that maximizes social welfare gain, and meanwhile attempt not to bring excessive negative impact on certain groups of users.


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## Notations

*The following symbols are used in this paper:*

$A$ = set of links in network;
$A^*$ = subset of tolled links in network;
$c_{rw}^m$ = travel cost (inclusive of equivalent time of toll charge) on route $r$ between O-D pair $w$ by users of class $m$;
$D_w^m(\mu_w^m)$ = demand for class $m$ between O-D pair $w$ as function of O-D travel time;
$D_w^{m^{-1}}(d_w^m)$ = inverse of the demand function;
$\mathbf{d}$ = vector of all O-D demands for all classes, $\mathbf{d} = (\cdots, d_w^m, \cdots)^T$;
$d_w^m(\mathbf{y})$ = demand of class $m$ between O-D pair $w \in W$;
$f_{rw}^m$ = flow of class $m$ on route $r \in R_w$, $w \in W$;
$m$ = class of network users according to different VOT;
$R_w$ = set of routes between O-D pair $w \in W$;



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$t_a(v_a)$ = travel cost on link $a \in A$, which is function of link flow $v_a$;

$\mathbf{v}$ = vector of all link flows for all classes, $\mathbf{v} = (\cdots, v_a^m, \cdots)^T$;

$v_a$ = link flow on link $a \in A$;

$v_a^m(\mathbf{y})$ = link flow for class $m$ on link $a \in A$;

$W$ = set of O-D pairs;

$\mathbf{y}$ = vector of all link toll charges;

$y_a$ = charge of monetary toll on link $a \in A^*$;

$y_a^{\max}$ = upper bound of toll charge on link $a$;

$y_a^{\min}$ = lower bound of toll charge on link $a$;

$\beta_m$ = average value of time for users of class $m$;

$\delta_{ar}^w$ = 1 if route $r$ between O-D pair $w$ uses link $a$, and 0 otherwise;

$\overline{\mu}_w$ = O-D travel cost after first-best pricing scheme is implemented;

$\tilde{\mu}_w^m$ = generalized equilibrium O-D travel cost without pricing for users of class $m$; and

$\mu_w^m(\mathbf{y})$ = generalized equilibrium O-D travel cost with pricing for users of class $m$.

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