EE264 Edge Detection Technique
Part II, HL, 2009.1.16

1) Given a digital image $f(x,y)$ derive orientation insensitive edge detector (Invariant)

Sol

From Laplace Operator

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad \cdots (1)$$

we can derive such edge detector.

First,

$$\frac{\partial^2}{\partial x^2} f(x,y) = \frac{\partial}{\partial x}\left[\frac{\partial f(x,y)}{\partial x}\right]$$

Since

$$\frac{\partial f(x,y)}{\partial x} = \frac{1}{2}[f(x+1,y) - f(x-1,y)]$$

$$\frac{\partial^2}{\partial x^2} f(x,y) = \frac{1}{2}\left[\frac{\partial f(x+1,y)}{\partial x} - \frac{\partial f(x-1,y)}{\partial x}\right]$$

$$= \frac{1}{2}\left\{\frac{1}{2}[f(x+2,y) - f(x,y)] - \frac{1}{2}[f(x,y) - f(x-2,y)]\right\}$$

$$= \frac{1}{4}[f(x+2,y) - 2f(x,y) + f(x-2,y)] \quad \cdots (2)$$

Similarly, for y,

$$\frac{\partial^2}{\partial y^2} f(x,y) = \frac{1}{4}[f(x,y+2) - 2f(x,y) + f(x,y-2)] \quad \cdots (3)$$

Therefore $\nabla^2 f(x,y)$ edge detector

Forms 5x5 kernel as follows,

$$\xrightarrow{\;y\;} \qquad x\downarrow \qquad \frac{1}{4}\begin{bmatrix} & & 1 & & \\ & & & & \\ 1 & & -4 & & 1 \\ & & & & \\ & & 1 & & \end{bmatrix}$$

Note: The Operator

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$ is the sum of eqn (2) and eqn (3).

2) Derive LoG operator (laplace of Gauss)

Sol First, 1D Gaussian function

$$G(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad \cdots (4)$$

Then, 2D Gaussian function,

$$G(x,y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu_x)^2 + (y-\mu_y)^2}{2\sigma^2}} \quad \cdots (5)$$

Assume $\mu_x = \mu_y = 0$,

$$\frac{\partial}{\partial x} G(x,y) = -\frac{x}{\sqrt{2\pi}\sigma^3} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

hence

$$\frac{\partial^2}{\partial x^2} G(x,y) = -\frac{1}{\sqrt{2\pi}\sigma^3} e^{-\frac{x^2+y^2}{2\sigma^2}} + \frac{x^2}{\sqrt{2\pi}\sigma^5} e^{-\frac{x^2+y^2}{2\sigma^2}} \quad \cdots (6)$$

Similarly,

$$\frac{\partial^2}{\partial y^2}G(x,y) = -\frac{1}{\sqrt{2\pi}\sigma^3}e^{-\frac{x^2+y^2}{2\sigma^2}} + \frac{y^2}{\sqrt{2\pi}\sigma^5}e^{-\frac{x^2+y^2}{2\sigma^2}} \quad \cdots 7)$$

Therefore,

$$\nabla^2 G(x,y) = \frac{\partial^2}{\partial x^2}G(x,y) + \frac{\partial^2}{\partial y^2}G(x,y)$$

$$= \frac{x^2+y^2-2\sigma^2}{\sqrt{2\pi}\sigma^5} e^{-\frac{x^2+y^2}{2\sigma^2}} \quad \cdots (8)$$

Based on the result of eqn (8), a powerful edge detector LoG can be defined.

Note: Very often the size of the LoG kernel can range from 5x5, 7x7 to 11x11, 13x13 and beyond.

(END)