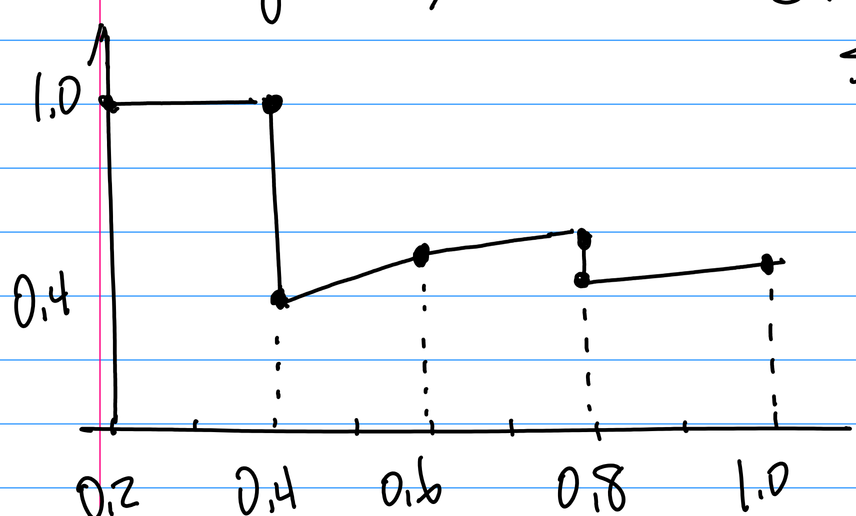
MAP Example
Mean Average Precision

HL

Example: Given the Following Data Set, find MAP.

Rank	P (Precision)	R (Recall)
0	1.0	0.2
1	1.0	0.4
2	0.67	0.4
3	0.5	0.4
4	0.4	0.4
5	0.5	0.6
6	0.57	0.8
7	0.44	0.8
8	0.5	1.0

Step 1. Plot R-P Chart as x-y chart, N=9



Note: for (0.4, 1.0), (0.4, 0.67) (0.4, 0.5), and (0.4, 0.4), Choose (0.4, 1.0) the max, and (0.4, 0.4), the min, in the plot, Similarly for (0.8, 0.57), (0.8, 0.44), Choos the first as max, and the 2nd as min in the plot.

Note: Insert $r_i = 0.3$ between [0.2, 0.4], and $r_i = 0.5$ between [0.4, 0.6], and $r_i = 0.7$ between [0.6, 0.8], as well as 0.9 between [0.8, 1.0].

Step 2.

Since $AP = \int_0^1 P(r)\,dr$

$$\simeq \frac{1}{N} \sum_{i=0}^{N-1} P(r_i) \quad \text{for } N = 9$$

$$AP \simeq \frac{1}{9} \sum_{i=0}^{8} P(r_i) \quad \dots (1)$$

$$= \frac{1}{9}\left(P(r_0) + P(r_1) + P(r_2) + \dots + P(r_8)\right)$$

$$= \frac{1}{9}\left(P(0.2) + P(0.3) + P(0.4) + P(0.5) + P(0.6) + P(0.7) + P(0.8) + P(0.9)\right) \quad \dots (2)$$

From the plot,

$P(0.2)=1.0$, $P(0.3)=1.0$, $P(0.4)=1.0$

and for Sloped line of P value we use

$$\tilde{P}(r_i) = \underset{i \le K \le u}{\mathrm{Max}}\ \hat{P}(r_K) \quad \ldots(3)$$

where $i=3$, $r_3=0.4$, $u=7$

$$\tilde{P}(r_i) = \tilde{P}(r_3) = \underset{3 \le K \le 7}{\mathrm{Max}}\ \hat{P}(r_K)$$

Note the upper bound u

$$\underset{i \le K \le u}{\mathrm{Max}} \{\tilde{P}(r_K)\}$$

Should be Selected from the Same Slopped line Segment

$$\tilde{P}(r_3) = \mathrm{Max}\{\tilde{P}(r_3), \tilde{P}(r_4), \tilde{P}(r_5), \tilde{P}(r_6), \tilde{P}(r_7)\}$$

$$= \mathrm{Max}\{0.4, \tilde{P}(r_4), \tilde{P}(r_5), 0.5, \tilde{P}(r_7), 0.44\} \quad \ldots(4)$$

Where $\tilde{P}(r_4), \tilde{P}(r_5) \le 0.5$, and $\tilde{P}(r_7) \le 0.57$ So

$$\tilde{P}(r_3) = \mathrm{Max}\{0.4, \tilde{P}(r_4), \tilde{P}(r_5), 0.5, \tilde{P}(r_7), 0.44\} = \tilde{P}(r_7)$$

$$= 0.57 \quad \ldots(5)$$

Similarly, On the Same Slopped line Segment

$$\tilde{P}(r_4) = \tilde{P}(r_7)$$

$$\tilde{P}(r_5) = \tilde{P}(r_7)$$

$$\tilde{P}(r_6) = \tilde{P}(r_7)$$

Now, for $\tilde{P}(r_8)$, $\tilde{P}(r_9)$, we are on A New Slopped Line Segment, from Eqn(3),

$$\tilde{P}(r_8) = \underset{8 \le K \le 9}{\max}\ \hat{P}(r_K)$$

$$= \max\{\tilde{P}(r_8), \tilde{P}(r_9)\}$$

from the line Segment, $\tilde{P}(r_8) \le \tilde{P}(r_9)$

So, $\tilde{P}(r_8) = \tilde{P}(r_9) = 0.5$

Now, Substitute the Above

From Eqn (2),

$$AP = \frac{1}{9}\left(P(r_0)+P(r_1)+P(r_2)+\cdots+P(r_8)\right)$$

$$\cong \frac{1}{9}\left(\tilde{P}(r_0)+\tilde{P}(r_1)+\cdots\tilde{P}(r_8)\right)$$

$$= \frac{1}{9}\left(\tilde{P}(r_0)+\tilde{P}(r_0)+\tilde{P}(r_0)+\tilde{P}(r_7)+\tilde{P}(r_7)+\tilde{P}(r_7)+\tilde{P}(r_7)+\tilde{P}(r_7)+\tilde{P}(r_9)+\tilde{P}(r_9)\right)$$

$$= \frac{1}{9}\left(3*\tilde{P}(r_0)+5*\tilde{P}(r_7)+2*\tilde{P}(r_9)\right)$$

$$= \frac{1}{9}\left(3*1.0+5\times 0.57+2*0.5\right)$$

$$= \frac{1}{9}\left(3+2.85+1.\right) = 6.85/9 = 0.7611$$

Note, if we have the second set of P and R data from Class II, we calculate AP_2 the same way, suppose this gives $AP_2 = 0.6300$, then add AP_2 to the previous AP_1, divided by 2, we have

$$mAP = \frac{1}{2}\left(AP_1+AP_2\right)$$

$$= \frac{1}{2}\left(0.7611+0.6300\right)$$

$$= 0.6956$$

We can compute mAP for 3 classes,

$$mAP = \frac{1}{3}\left(AP_1+AP_2+AP_3\right)$$

And for M classes,

$$mAP = \frac{1}{M}\left(AP_1+AP_2+\cdots+AP_M\right)$$

$$= \frac{1}{M}\sum_{i=1}^{M} AP_i$$

(END)