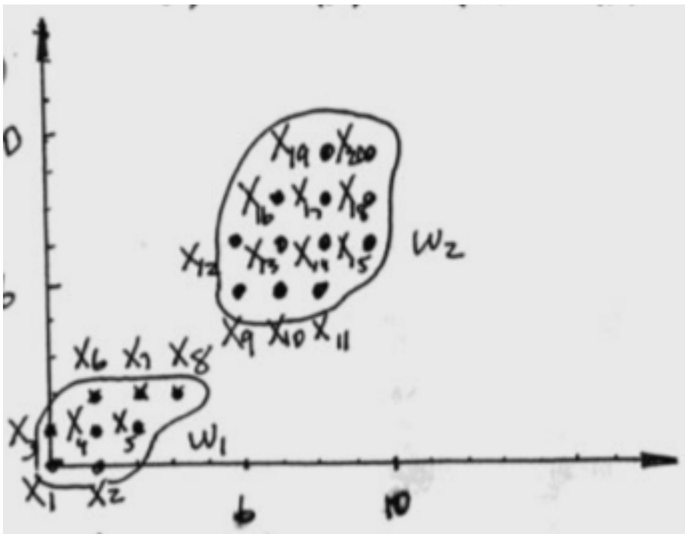
K-mean Cluster Algorithm

개ℒ

Given the following feature vectors use K-mean Algorithm to find the clusters.

$X_1 = \begin{bmatrix}0\\0\end{bmatrix}$	$X_2 = \begin{bmatrix}1\\0\end{bmatrix}$	$X_3 = \begin{bmatrix}0\\1\end{bmatrix}$	$X_4 = \begin{bmatrix}1\\1\end{bmatrix}$
$X_5 = \begin{bmatrix}2\\1\end{bmatrix}$	$X_6 = \begin{bmatrix}1\\2\end{bmatrix}$	$X_7 = \begin{bmatrix}2\\2\end{bmatrix}$	$X_8 = \begin{bmatrix}3\\2\end{bmatrix}$
$X_9 = \begin{bmatrix}6\\6\end{bmatrix}$	$X_{10} = \begin{bmatrix}7\\6\end{bmatrix}$	$X_{11} = \begin{bmatrix}8\\6\end{bmatrix}$	$X_{12} = \begin{bmatrix}6\\7\end{bmatrix}$
$X_{13} = \begin{bmatrix}7\\7\end{bmatrix}$	$X_{14} = \begin{bmatrix}8\\7\end{bmatrix}$	$X_{15} = \begin{bmatrix}9\\7\end{bmatrix}$	$X_{16} = \begin{bmatrix}7\\8\end{bmatrix}$
$X_{17} = \begin{bmatrix}8\\8\end{bmatrix}$	$X_{18} = \begin{bmatrix}9\\8\end{bmatrix}$	$X_{19} = \begin{bmatrix}8\\9\end{bmatrix}$	$X_{20} = \begin{bmatrix}9\\9\end{bmatrix}$

Sol. We can plot these feature vectors below



Step1. Define Number of Cluster K=2 Based on Heuristics (the plot of the feature vectors).

Let K=2

And make initialization by arbitrarily select 2 points $\vec{x}_1$, $\vec{x}_2$ as the cluster Center

Let cluster

$$\vec{m}_1^0 = \vec{x}_1 = \begin{pmatrix}0\\0\end{pmatrix}, \quad \cdots (1)$$

$$\vec{m}_2^0 = \vec{x}_2 = \begin{pmatrix}1\\0\end{pmatrix} \quad \cdots (2)$$

And let Class 1 be

$$C_1 : (\vec{x}_1, \vec{x}_3) \quad \cdots (3)$$

Class 2 be

$$C_2 : (x_2, x_4, x_5, \cdots, x_{20}) \quad \cdots (4)$$

Step2, Use Equation

$$\|\vec{x}_p - \vec{m}_i\| \le \|\vec{x}_p - \vec{m}_j\| \quad \cdots (5)$$

for $\vec{x}_p$ from Class i to regroup feature Vectors in Class i to j if Eqn (5) does not hold. Hence

From Class C_1,

$$\|\vec{x}_1 - \vec{m}_1^0\| = \left\| \begin{pmatrix}x_{11}\\x_{12}\end{pmatrix} - \begin{pmatrix}m_{11}\\m_{12}\end{pmatrix} \right\| = \quad \begin{matrix} m_{11} = x_{11} \\ m_{12} = x_{12} \end{matrix}$$

$$= \sqrt{(x_{11} - x_{11})^2 + (x_{12} - x_{12})^2} = 0$$

And

$$\|\vec{x}_1 - \vec{m}_2\| = \left\| \begin{pmatrix}x_{11}\\x_{12}\end{pmatrix} - \begin{pmatrix}m_{21}\\m_{22}\end{pmatrix} \right\| = \quad \begin{matrix} m_{21} = x_{21} \\ m_{22} = x_{22} \end{matrix}$$

$$= \sqrt{(x_{11} - x_{21})^2 + (x_{12} - x_{22})^2}$$

$$= \sqrt{1^2 + 0^2} = 1$$

$\because \|\vec{X_1} - \vec{m_1^0}\| \le \|\vec{X_1} - \vec{m_2^0}\|$

so $\vec{X_1}$ stays in C_1.

And

$$\|\vec{X_3} - \vec{m_1^0}\| = \sqrt{(X_{31}-X_{11})^2+(X_{32}-X_{12})^2}$$

$$= \sqrt{0^2+1^2} = 1$$

$$\|\vec{X_3} - \vec{m_2^0}\| = \sqrt{(X_{31}-X_{21})^2+(X_{32}-X_{22})^2}$$

$$= \sqrt{1^2+1^2} = \sqrt{2}$$

Hence,

$$\|\vec{X_3} - \vec{m_1}\| \le \|\vec{X_3} - \vec{m_2}\|$$

so $\vec{X_3}$ stays in C_1.

Therefore $\{\vec{X_1}, \vec{X_3}\} \in C_1$, and

$\{\vec{X_2}, \vec{X_4}, \vec{X_5}, \cdots, \vec{X_{20}}\} \in C_2$

Step 3. Update Clusters

$$\vec{m_1^1} = \frac{1}{N}\sum_{i=1}^{N} \vec{X_{1i}} = \frac{1}{2}(\vec{X_1}+\vec{X_3})$$

$$= \begin{pmatrix} 0 \\ 0.5 \end{pmatrix}$$

$$\vec{m_2^1} = \frac{1}{18}(\vec{X_2}+\vec{X_4}+\vec{X_5}+\cdots+\vec{X_{20}})$$

$$= \begin{pmatrix} 5.67 \\ 5.33 \end{pmatrix}$$

Step 4. Check the clusters for Convergence verification.

$\vec{m_1^0} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\vec{m_1^1} = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix}$,

so $\vec{m_1^0} \ne \vec{m_1^1}$,

And $\vec{m_2^0} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\vec{m_2^1} = \begin{pmatrix} 5.67 \\ 5.33 \end{pmatrix}$

$\vec{m_2^0} \ne \vec{m_2^1}$

Therefore, continue the cluster seeking Computation.

Step 5, with New updated Cluster, Compute

$$\|\vec{X_l} - \vec{m_1^1}\| \le \|\vec{X_l} - \vec{m_2^1}\|$$

for $l = 1, 2, \cdots 8$; (For Simplicity, the steps of these Computation were not listed here, they are similar to what we did), and

$$\|\vec{X_l} - \vec{m_2^1}\| \le \|\vec{X_l} - \vec{m_1^1}\|$$

for $l = 9, 10, \cdots, 20$;

So, Regroup $\{\vec{X_1}, \vec{X_2}, \cdots, \vec{X_8}\} \in C_1$

$\{\vec{X_9}, \vec{X_{10}}, \cdots, \vec{X_{20}}\} \in C_2$

Then, Compute the Clusters

$$\vec{m_1^2} = \frac{1}{8}(\vec{X_1}+\vec{X_2}+\cdots+\vec{X_8}) = \begin{pmatrix} 1.25 \\ 1.13 \end{pmatrix}$$

$$\vec{m_2^2} = \frac{1}{12}(\vec{X_9}+\vec{X_{10}}+\cdots+\vec{X_{20}}) = \begin{pmatrix} 7.67 \\ 7.33 \end{pmatrix}$$

Step 6. Check Clusters for Convergence

$\vec{m}_1^1 \neq \vec{m}_1^2$, $\vec{m}_2^1 \neq \vec{m}_2^2$

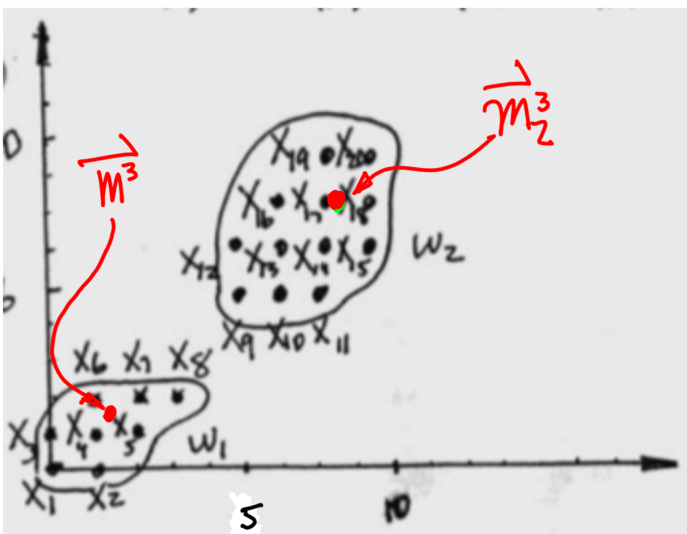
Therefore, we will Continue the Computation. We go through the Same process, which Leads $\{\vec{x}_1, \vec{x}_2, \cdots, \vec{x}_8\}$ Same as the previous class, And $\{\vec{x}_9, \vec{x}_{10}, \cdots, \vec{x}_{20}\}$ is also the Same as the previous Class. which gives

$$\vec{m}_1^3 = \vec{m}_1^2 \quad \text{and}$$

$$\vec{m}_2^3 = \vec{m}_2^2$$

Now, the clusters Converged the Result is

$$\vec{m}_1^3 = \begin{pmatrix} 1.25 \\ 1.13 \end{pmatrix}, \quad \vec{m}_2^3 = \begin{pmatrix} 7.67 \\ 7.33 \end{pmatrix}.$$ marked below



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