

EE264 Edge Detection Technique
Part II, HL, 2009.1.16

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- 1) Given a digital image $f(x, y)$
derive orientation insensitive edge detector. (Invariant)

Sol

From Laplace Operator

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad \dots (1)$$

we can derive such edge detector.

First,

$$\frac{\partial^2}{\partial x^2} f(x, y) = \frac{\partial}{\partial x} \left[\frac{\partial f(x, y)}{\partial x} \right]$$

Since

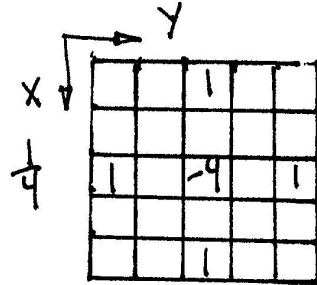
$$\begin{aligned} \frac{\partial f(x, y)}{\partial x} &= \frac{1}{2} [f(x+1, y) - f(x-1, y)] \\ \frac{\partial^2}{\partial x^2} f(x, y) &= \frac{1}{2} \left[\frac{\partial f(x+1, y)}{\partial x} - \frac{\partial f(x-1, y)}{\partial x} \right] \\ &= \frac{1}{2} \left\{ \frac{1}{2} [f(x+2, y) - f(x, y)] - \frac{1}{2} [f(x, y) - f(x-2, y)] \right\} \\ &= \frac{1}{4} [f(x+2, y) - 2f(x, y) + f(x-2, y)] \quad \dots (2) \end{aligned}$$

Similarly, for y ,

$$\begin{aligned} \frac{\partial^2}{\partial y^2} f(x, y) &= \frac{1}{4} [f(x, y+2) - 2f(x, y) \\ &\quad + f(x, y-2)] \quad \dots (3) \end{aligned}$$

Therefore $\nabla^2 f(x, y)$ edge detector

Forms 5x5 kernel. as follows,



Note: The Operator

$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the sum of eqn (2) and eqn (3).

- 2) Derive LoG operator (Laplace of Gauss)

Sol

First, 1D Gaussian function

$$G(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad \dots (4)$$

Then, 2D Gaussian function,

$$G(x, y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu_x)^2 + (y-\mu_y)^2}{2\sigma^2}} \quad \dots (5)$$

Assume $\mu_x = \mu_y = 0$,

$$\frac{\partial}{\partial x} G(x, y) = -\frac{x}{\sqrt{2\pi}\sigma^3} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

hence

$$\begin{aligned} \frac{\partial^2}{\partial x^2} G(x, y) &= -\frac{1}{\sqrt{2\pi}\sigma^3} e^{-\frac{x^2+y^2}{2\sigma^2}} \\ &\quad + \frac{x^2}{\sqrt{2\pi}\sigma^5} e^{-\frac{x^2+y^2}{2\sigma^2}} \quad \dots (6) \end{aligned}$$

Similarly,

$$\frac{\partial^2}{\partial y^2} G(x, y) = -\frac{1}{\sqrt{2\pi} b^3} e^{-\frac{x^2+y^2}{2b^2}} + \frac{y^2}{\sqrt{2\pi} b^5} e^{-\frac{x^2+y^2}{2b^2}} \quad \dots (7)$$

Therefore,

$$\begin{aligned} \nabla^2 G(x, y) &= \frac{\partial^2}{\partial x^2} G(x, y) + \frac{\partial^2}{\partial y^2} G(x, y) \\ &= \frac{x^2+y^2-2b^2}{\sqrt{2\pi} b^3} e^{-\frac{x^2+y^2}{2b^2}} \quad \dots (8) \end{aligned}$$

Based on the result of eqn (8),
a powerful edge detector LoG
can be defined.

Note: Very often the size of the
LoG kernel can range from
5x5, 7x7 to 11x11, 13x13
and beyond.

(END)