

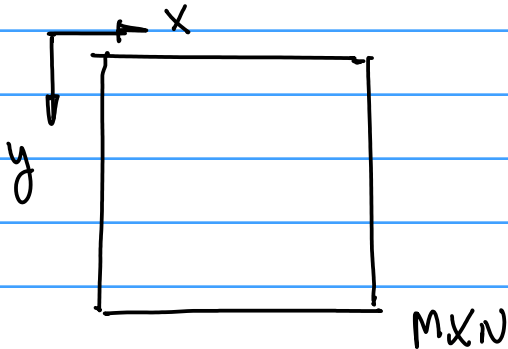
Step 6. Convert the weights in DARKNET to T.F. format.

Step 7. Run the code for either Video or Image(s).

Notations & Mathematical Formulation

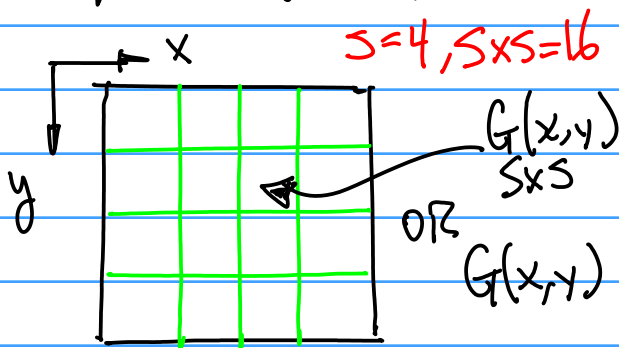
1. Image  $I(x, y)$ ,  $I_{m \times n}(x, y)$

Subscript  $\rightarrow$   $m \times n$  Resolution



2. Divide  $I(x, y)$  into  $S \times S$  ~~pixels~~ <sup>Grids</sup>

Each  $S \times S$  patch/Tile/Subimage is defined as grid  $G(x, y)$



3. Bounding Box, e.g. R.O.I.

(Region of Interests)  $\rightarrow$  Localized ROI  
On a given object  $B(x, y)$

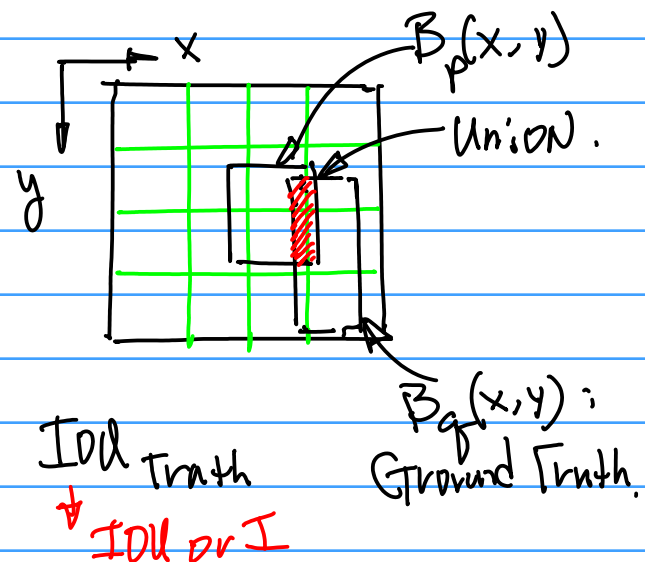
Note: The size of  $B(x, y)$  is different from the size of  $G(x, y)$ , And it is usually bigger than  $G(x, y)$ .

Note: Each Bounding Box is required for Each Object, the objects may belong to Same class or different classes.

$B_j(x, y)$ , for  $j=1, 2, \dots, m$ ;  
4. Object(s)

$O_i(x, y)$  for  $i=1, 2, \dots, K$   
 $\uparrow$  Center of the Object  
 $\uparrow$  total Objects

5. IOU (Intersection <sup>over</sup> Union)



IOU Truth

$\rightarrow$  IOU or I

$B_g(x, y)$ :  
Ground Truth.

$$IDU = \begin{cases} 1 & B_p(x,y) = B_q(x,y) \\ [0,1) & o/w \dots (1) \end{cases}$$

$$\text{Prob}(C_i) \sum_{i=1}^N \text{Prob}(C_i) = 1 \dots (4)$$

6. Five parameters defined for

$$B_j(x,y) \cdot \{ \underbrace{x, y}_{\text{Centroid}}, \underbrace{w, h, f(B_j(x,y))}_{\text{Confidence}} \}$$

a. Define Condition Probability.

$$\text{Prob}(C_i | O_j) \dots (5)$$

Given an object  $O_j$ , find the Probability of  $C_i$ , e.g. ~ that this object belongs to class  $i$

$$\begin{array}{ccc} B_1(x,y) & f(B_1(x,y)) = f(B_1) & \text{Confidence} \\ B_2(x,y) & \dots & f(B_2) \\ \vdots & & \\ B_m(x,y) & & f(B_m) \end{array}$$

If Confidence  $f(B_j(x,y))$  ~~represents~~ Probability value

$$\sum_{i=1}^N \text{Prob}(C_i | O_j) = 1 \dots (5b)$$

$$\sum_{j=1}^m f(B_j(x,y)) = 1 \dots (2)$$

7. Define probability for Each object as follows (on A Grid) S=4, SxS=16

$$\text{Prob}(O_i(x,y)) = \text{Prob}(O_i) \dots (3)$$

8. Denote classes as

$$C_i \text{ for } i=1, 2, \dots, N$$

Hence, the probability for  $C_i$  is

↓ IDU or I

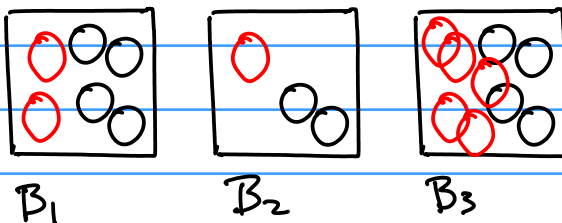
# Appendix : Review on Bayes Theorem

21

$$\Pr(\text{Class}_i | \text{Object}) * \Pr(\text{Object}) * \text{IOU}_{\text{pred}}^{\text{truth}} = \Pr(\text{Class}_i) * \text{IOU}_{\text{pred}}^{\text{truth}} \quad (1)$$

## Review, Bayesian Theorem

Example: 3 Boxes:  $B_1, B_2, B_3$ , Drawing A Red Ball



$$\text{Prob}(B_1 \cap O) = P(B_1) P(O|B_1) \dots (3a)$$

$$\text{Prob}(B_2 \cap O) = P(B_2) P(O|B_2) \dots (3b)$$

$$\text{Prob}(B_3 \cap O) = P(B_3) P(O|B_3) \dots (3c)$$

$$\text{Prob}(R) = \text{Prob}(B_1 \cap R) + \text{Prob}(B_2 \cap R) + \text{Prob}(B_3 \cap R) \dots (1)$$

$$B_1 \cap R + \text{Prob}(B_3 \cap R) \dots (1)$$

where  $R: B_1 \cap R + B_2 \cap R + B_3 \cap R$

Re-arrange Boxes to form an Image  $I(x,y)$  as follows

$$\text{Prob}(B_1 \cap R) = P(B_1) P(R|B_1) \dots (2a)$$

$$\text{Prob}(B_2 \cap R) = P(B_2) P(R|B_2) \dots (2b)$$

$$\text{Prob}(B_3 \cap R) = P(B_3) P(R|B_3) \dots (2c)$$

So, Assume  $\text{Prob}(B_1) = \text{Prob}(B_2) = \text{Prob}(B_3)$

$$\text{Prob}(B_1 \cap R) = P(B_1) P(R|B_1) = \left(\frac{1}{3}\right) \left(\frac{2}{6}\right)$$

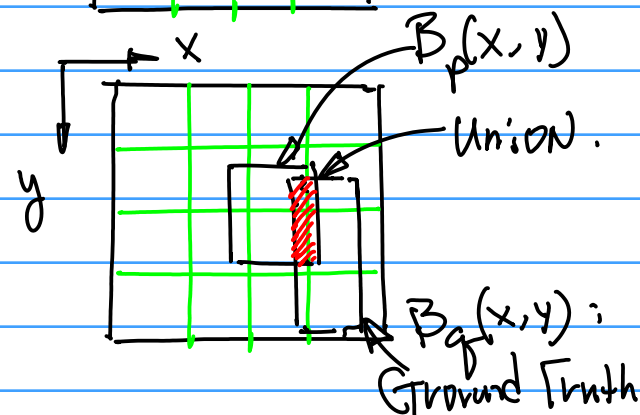
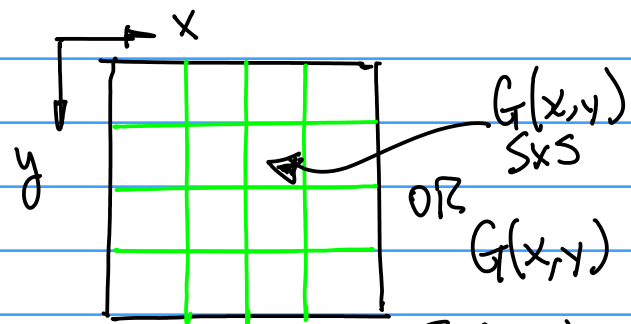
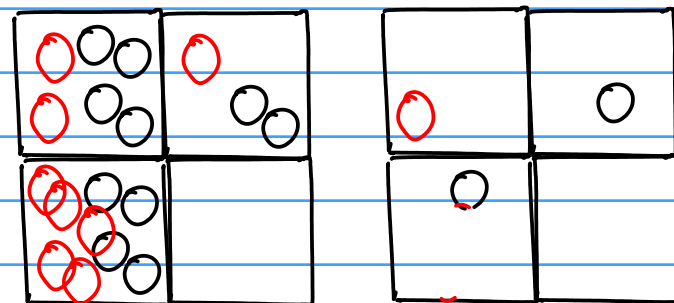
$$\text{Prob}(B_2 \cap R) = P(B_2) P(R|B_2) = \left(\frac{1}{3}\right) \left(\frac{1}{3}\right)$$

$$\text{Prob}(B_3 \cap R) = P(B_3) P(R|B_3) = \left(\frac{1}{3}\right) \left(\frac{5}{9}\right)$$

Now, Change Red Ball "R" to Object Detection "O"

$$\text{Prob}(O) = \text{Prob}(B_1 \cap O) + \text{Prob}(B_2 \cap O) + \text{Prob}(B_3 \cap O) \dots (3)$$

Hence,



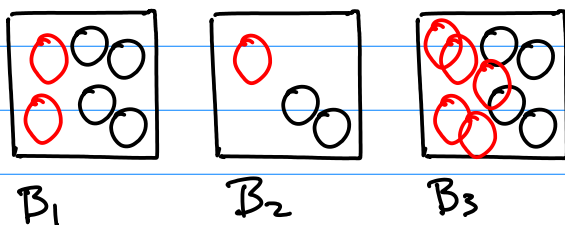
# Appendix : Review on Bayes Theorem

21

$$\Pr(\text{Class}_i | \text{Object}) * \Pr(\text{Object}) * \text{IOU}_{\text{pred}}^{\text{truth}} = \Pr(\text{Class}_i) * \text{IOU}_{\text{pred}}^{\text{truth}} \quad (1)$$

## Review, Bayesian Theorem

Example: 3 Boxes:  $B_1, B_2, B_3$ , Drawing A Red Ball



$$\text{Prob}(B_1 \cap O) = P(B_1) P(O|B_1) \dots (3a)$$

$$\text{Prob}(B_2 \cap O) = P(B_2) P(O|B_2) \dots (3b)$$

$$\text{Prob}(B_3 \cap O) = P(B_3) P(O|B_3) \dots (3c)$$

$$\text{Prob}(R) = \text{Prob}(B_1 \cap R) + \text{Prob}(B_2 \cap R) + \text{Prob}(B_3 \cap R) \dots (1)$$

$$B_i \cap R$$

where  $R: B_1 \cap R + B_2 \cap R + B_3 \cap R$

Re-arrange Boxes to form an Image  $I(x,y)$  as follows

$$\text{Prob}(B_1 \cap R) = P(B_1) P(R|B_1) \dots (2a)$$

$$\text{Prob}(B_2 \cap R) = P(B_2) P(R|B_2) \dots (2b)$$

$$\text{Prob}(B_3 \cap R) = P(B_3) P(R|B_3) \dots (2c)$$

So, Assume  $\text{Prob}(B_1) = \text{Prob}(B_2) = \text{Prob}(B_3)$

$$\text{Prob}(B_1 \cap R) = P(B_1) P(R|B_1) = \left(\frac{1}{3}\right) \left(\frac{2}{6}\right)$$

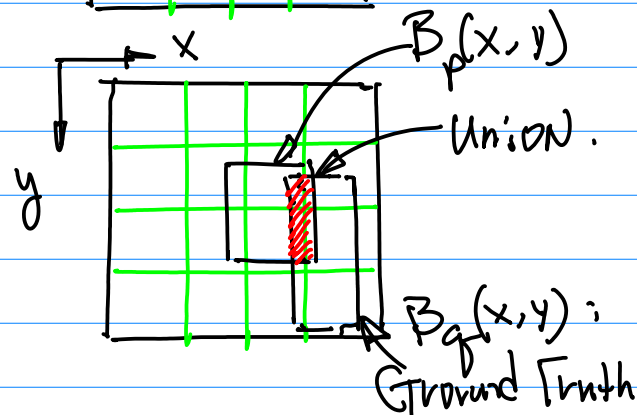
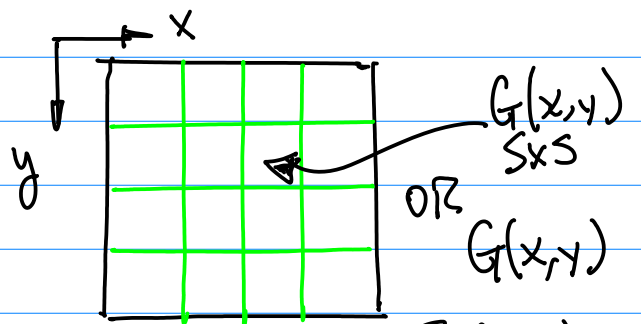
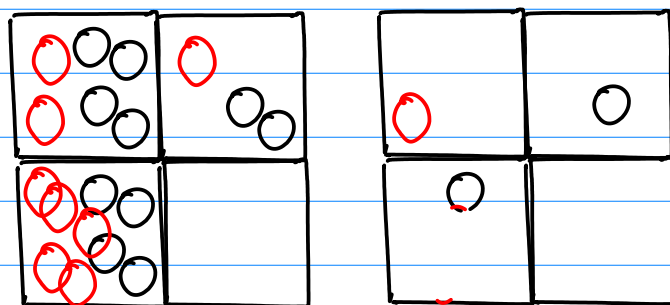
$$\text{Prob}(B_2 \cap R) = P(B_2) P(R|B_2) = \left(\frac{1}{3}\right) \left(\frac{1}{3}\right)$$

$$\text{Prob}(B_3 \cap R) = P(B_3) P(R|B_3) = \left(\frac{1}{3}\right) \left(\frac{5}{9}\right)$$

Now, Change Red Ball "R" to Object Detection "O"

$$\text{Prob}(O) = \text{Prob}(B_1 \cap O) + \text{Prob}(B_2 \cap O) + \text{Prob}(B_3 \cap O) \dots (3)$$

Hence,



From Ref. Paper, we have

$$\Pr(\text{Class}_i) = \Pr(\text{Class}_i | \text{Object}) * \Pr(\text{Object})$$

$$\text{Prob}(C) = \text{Prob}(O_1 \cap C) + \dots \quad (4-0)$$

$$\text{where } \text{Prob}(O_2 \cap C) + \text{Prob}(O_3 \cap C) \dots (4)$$

$$\text{Prob}(O_1 \cap C) = P(O_1) P(C | O_1) \dots (4a)$$

$$\text{Prob}(O_2 \cap C) = P(O_2) P(C | O_2) \dots (4a)$$

$$\text{Prob}(O_3 \cap C) = P(O_3) P(C | O_3) \dots (4a)$$

So,

$$\begin{aligned} \text{Prob}(C) &= P(O_1) P(C | O_1) + P(O_2) P(C | O_2) \\ &\quad + P(O_3) P(C | O_3) \end{aligned}$$

which matches to Eqn (40).

Now, Add Bounding Box and Grid into the formulation.

$$\text{IOU} = \frac{\text{Area of ROI (Bounding Box)}}{\text{Area of Grid}}$$

Now, Consider Loss function

# 5-Fold Loss Function

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$$\lambda_{\text{coord}} \sum_{i=0}^{S^2} \sum_{j=0}^B \mathbb{1}_{ij}^{\text{obj}} \left[ (x_i - \hat{x}_i)^2 + (y_i - \hat{y}_i)^2 \right]$$

Position Loss  $\sim$

$$+ \lambda_{\text{coord}} \sum_{i=0}^{S^2} \sum_{j=0}^B \mathbb{1}_{ij}^{\text{obj}} \left[ (\sqrt{w_i} - \sqrt{\hat{w}_i})^2 + (\sqrt{h_i} - \sqrt{\hat{h}_i})^2 \right]$$

Geometry Loss  $\sim$

$$+ \sum_{i=0}^{S^2} \sum_{j=0}^B \mathbb{1}_{ij}^{\text{obj}} (C_i - \hat{C}_i)^2$$

Class Loss with objects

$$+ \lambda_{\text{noobj}} \sum_{i=0}^{S^2} \sum_{j=0}^B \mathbb{1}_{ij}^{\text{noobj}} (C_i - \hat{C}_i)^2$$

Class Loss without objects

$$+ \sum_{i=0}^{S^2} \mathbb{1}_i^{\text{obj}} \sum_{c \in \text{classes}} (p_i(c) - \hat{p}_i(c))^2$$

Notation

1. Function  $\mathbb{1}(x) = \begin{cases} K & \text{for } x \text{ exists} \\ 0 & \text{o/w. ... (1)} \end{cases}$

K is some Constant

So,  $\mathbb{1}(0) = \begin{cases} K & \text{if 0 object 0 exist} \\ 0 & \text{o/w. ... (1b)} \end{cases}$

2.  $\mathbb{1}_i(0)$  for "0" in cell (Grid) i

$\mathbb{1}_{ij}(0)$  for "0" in cell i & Bounding Box j

3. (x, y, w, h) are centroid (x, y), width, height of the Bounding Box.

$p_i(c)$  for class "c"

$$4. \sum_{i=0}^{S^2} \sum_{j=0}^B \mathbb{1}_{ij}^{\text{noobj}} (C_i - \hat{C}_i)^2$$

$j = 0, 1, \dots, B$   
B+1  
Bounding Boxes

$$\mathbb{1}_i^{\text{obj}}, \mathbb{1}_{ij}^{\text{obj}}, \mathbb{1}_i^{\text{noobj}}, \mathbb{1}_{ij}^{\text{noobj}}$$

$i = 0, 1, \dots, S^2$   
 $S \times S + 1$   
Classes OR grids cells

for all the Cells, all the Bounding Boxes

Coordinates Related

April 15 (Th).

From Eqn (1), Appendix, PP21.

$$\Pr(\text{Class}_i | \text{Object}) * \Pr(\text{Object}) * \text{IOU}_{\text{pred}}^{\text{truth}} = \Pr(\text{Class}_i) * \text{IOU}_{\text{pred}}^{\text{truth}} \dots (1)$$

$$\Pr(C_i) = \Pr(O) \Pr(C_i | O)$$

$$\Pr(C_i) = ? \text{ Starting point}$$

Find probability of an Object  $O$  belongs to Class  $C_i$

Probability, Score  $\rightarrow$  make decision for classification.

$$\textcircled{1} \Pr(C_i) \xrightarrow{\text{Step 1}} \textcircled{2} \Pr(O) * \Pr(C_i | O)$$

Bayesian Theorem

Step 1: Probability of an Object  $O$  Appears on a grid  $G$

Step 2: Conditional ~ Given Object  $O$ , this  $O$  belongs to Class  $C_i$

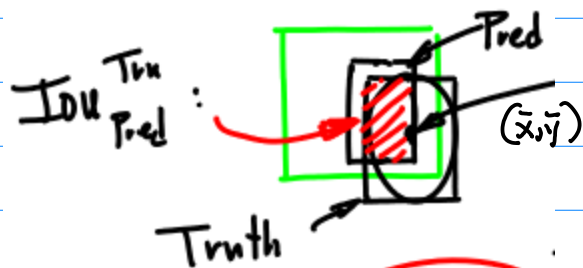
Prior Knowledge

Conversion to a given image  
Region<sup>a</sup>, Cell, Grid

$$\text{IOU}_{\text{pred}}^{\text{truth}} \rightarrow \text{IOU} \rightarrow I(\text{Index}) \in [0, 1]$$

Example: Computation of IOU

Define  $\text{IOU} = (\text{Ratio of Areas})$



$$\text{IOU} = \frac{\text{Red Area}}{(\text{Area of } B\text{-Box}) + (\text{Area of } G\text{-Truth}) - (\text{Area of } \text{Common})} \dots (1)$$

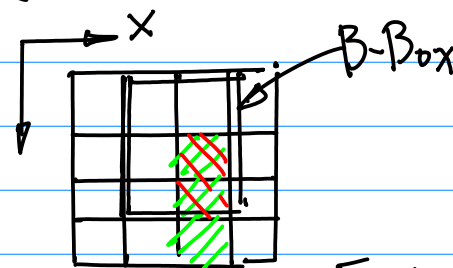


Fig 1.

Ground Truth: Green

Find  $\text{IOU} = ?$

Sol:

Step 1: Area of B-Box

$$A_{B^2} = 6 = \sum_y \sum_x B(x, y)$$

$$A_G = 3 = \sum_y \sum_x B_G(x, y)$$

$$\text{Step 2: } A_{\text{Com}} = 2$$

$$\therefore \text{IOU} = \frac{2}{6+3-2} = 2/7$$



$$\text{Pr}(C_i) = \text{Prob}(D) \text{Pr}(C_i|D)$$

Prior Distribution.

"K-mean" Clustering Technique  
 (2) K of them  
 (1) mean Value  
 Centered by mean

Given 2 values,  $x_1 = 1, x_2 = 1.5$

Find mean

$$\bar{X} = \left( \sum_{i=1}^N X_i \right) \frac{1}{N} \dots (2)$$

$$\bar{X} = \frac{1}{2} \sum_{i=1}^2 X_i = \frac{1}{2} (X_1 + X_2) = \frac{1}{2} (1 + 1.5)$$

$$= \frac{2.5}{2} = 1.25$$

Feature Vector  $\bar{X}$  (Simply write  $X$ )

$$\bar{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \text{ 2 Dimensional vector}$$

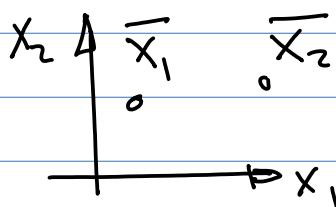


Fig 2.

$$\bar{X}_1 = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix}, \quad \bar{X}_2 = \begin{pmatrix} 1.5 \\ 2.3 \end{pmatrix} = \begin{pmatrix} X_{21} \\ X_{22} \end{pmatrix}$$

$$= \begin{pmatrix} X_{11} \\ X_{12} \end{pmatrix}$$

1st 2nd  
Point Dimension

$$\bar{M} = \left( \sum_{i=1}^N \bar{X}_i \right) \frac{1}{N} \dots (3)$$

$$\bar{m} = \frac{1}{2} \left( \sum_{i=1}^2 \bar{X}_i \right)$$

$$= \frac{1}{2} (\bar{X}_1 + \bar{X}_2)$$

$$= \frac{1}{2} \left( \begin{pmatrix} 1 \\ 0.5 \end{pmatrix} + \begin{pmatrix} 1.5 \\ 2.3 \end{pmatrix} \right)$$

$$= \frac{1}{2} \begin{pmatrix} 1 + 1.5 \\ 0.5 + 2.3 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 2.5 \\ 2.8 \end{pmatrix}$$

$$= \begin{pmatrix} 1.25 \\ 1.4 \end{pmatrix}$$

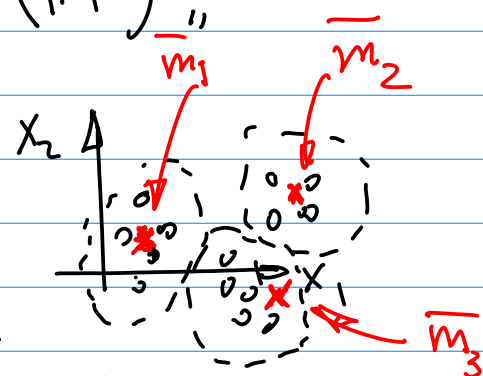


Fig 2.

Given a set of data

$\{\bar{X}_1, \bar{X}_2, \dots, \bar{X}_N\}$  is denoted as  $S$   
 find means to cluster  $S$   
 the data, e.g.

$$S = \{\bar{X}_1, \bar{X}_2, \dots, \bar{X}_N\} \dots (4)$$

Find  $\bar{m}_i$   
 (means, K of them)



Project II 10 pts  
 Yolo v4 Due 2 weeks  
 from April 15, Due May 2nd  
 Sunday, 11:59 PM.

- 1° Build / Run Yolo v4
- 2° Record your Video 10-15  
 seconds, Run Optimized  
 Yolo v4 By Reducing Classes  
 to be detected;
- 3° Generate README.txt  
 Submit

- a Your Code
- b README
- c Video (Processed)

Example: K-mean Cluster  
 Computation.

Given a set of Data (in Vector  
 form, feature vectors)

$$(\vec{X}_1, \vec{X}_2, \dots, \vec{X}_n)$$

then K Groups or Clusters  
 denoted as

$$\{S_1, S_2, \dots, S_K\}$$

$$S_i = (\vec{X}_1, \vec{X}_2, \vec{X}_n \dots)$$

distance  $\|\vec{X} - \vec{m}_i\|^2$  for <sup>26</sup>  
 Group (Cluster) i

$$\text{argmin} (\|\vec{X} - \vec{m}_i\|^2)$$

minimize the distance  
 $\text{Min} (\|\vec{X} - \vec{m}_i\|^2) \dots (5)$   
 for one data  $\vec{X}$ , one  
 Cluster i,

All Data in Group i

$$\sum_{\vec{X} \in S_i} \|\vec{X} - \vec{m}_i\|^2 \dots (6)$$

For all groups

$$\sum_{i=1}^K \left( \sum_{\vec{X} \in S_i} \|\vec{X} - \vec{m}_i\|^2 \right) \dots (7)$$

Ref: Github  
 - qa  
 - ab

April 22nd (Last Day of Instruction)  
 17th, May

1. Presentation May 6th.
2. Review & Last Class May 13
3. Final Example: May 24th (Monday)

Appendix A (April 22) This page is To Be discussed Next Lecture 27

$$\begin{aligned} & \lambda_{\text{coord}} \sum_{i=0}^{S^2} \sum_{j=0}^B \mathbb{1}_{ij}^{\text{obj}} \left[ (x_i - \hat{x}_i)^2 + (y_i - \hat{y}_i)^2 \right] \dots (1) \\ & + \lambda_{\text{coord}} \sum_{i=0}^{S^2} \sum_{j=0}^B \mathbb{1}_{ij}^{\text{obj}} \left[ (\sqrt{w_i} - \sqrt{\hat{w}_i})^2 + (\sqrt{h_i} - \sqrt{\hat{h}_i})^2 \right] \\ & + \sum_{i=0}^{S^2} \sum_{j=0}^B \mathbb{1}_{ij}^{\text{obj}} (C_i - \hat{C}_i)^2 \\ & + \lambda_{\text{noobj}} \sum_{i=0}^{S^2} \sum_{j=0}^B \mathbb{1}_{ij}^{\text{noobj}} (C_i - \hat{C}_i)^2 \\ & + \sum_{i=0}^{S^2} \mathbb{1}_i^{\text{obj}} \sum_{c \in \text{classes}} (p_i(c) - \hat{p}_i(c))^2 \end{aligned}$$

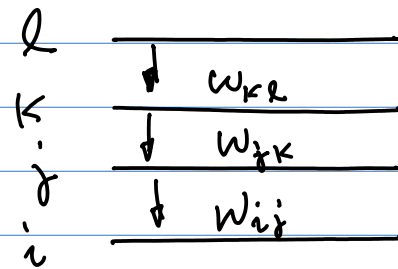
Simplification with Condition Object(s) Appears in the Grid.  
Removal of  $\mathbb{1}(\cdot)$  function.

So, (1) Becomes 
$$\sum_{i=0}^{S^2-1} \sum_{j=0}^{B-1} \left[ (x_i - \hat{x}_i)^2 + (y_i - \hat{y}_i)^2 \right] \dots (1a)$$

Where  $\hat{x}_i = \hat{x}_i(w_{hl}, b)$

then Apply Chain Rule accordingly.

Note: mAP (mean Average Precision)



Input

Output

Fig 1.

Output Input  
(1st) (2nd)

Example:

$$w_{ij} w_{jk} w_{kl}$$

Note: See my Lecture Note 8

$$\arg \min_S \sum_{i=1}^k \sum_{x \in S_i} \|x - \mu_i\|^2 \quad \text{(Cluster)}$$

minimization  $\rightarrow$   $S$   
 for all classes  $\rightarrow$   $i=1$  to  $k$   
 for all pts. within a given class  $\rightarrow$   $x \in S_i$   
 feature Vector / Data Point  $\rightarrow$   $x$   
 mean for the class  $i$   $\rightarrow$   $\mu_i$   
 ... (1)

Group Class  $i$

for "Any"

$$S_i^{(t)} = \{x_p : \|x_p - m_i^{(t)}\|^2 \leq \|x_p - m_j^{(t)}\|^2 \quad \forall j, 1 \leq j \leq k\}$$

data Point  $\rightarrow$   $x_p$   
 Distance (Difference) of class  $i$  is smaller or equal to the distance for class  $j$   
 for Class  $i$   $\rightarrow$   $m_i^{(t)}$   
 for Class  $j$   $\rightarrow$   $m_j^{(t)}$   
 A collection (A Set) of Data Points  $\rightarrow$   $S_i^{(t)}$   
 ... (2)

As long as  $j$  belongs / or covers entire classes, Note  $j \neq i$

Example (Hand Calculation)

1. Assume  $K=2$  from prior Knowledge.
2. Assumption for the cluster initial value

$$\vec{m}_1 = \begin{pmatrix} m_{11} \\ m_{12} \end{pmatrix}, \quad \vec{m}_2 = \begin{pmatrix} m_{21} \\ m_{22} \end{pmatrix}$$

3. Computation

$$\bar{X}_1 - \bar{m}_1 = \begin{pmatrix} x_{11} \\ x_{12} \end{pmatrix} - \begin{pmatrix} m_{11} \\ m_{12} \end{pmatrix} =$$

$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\text{then } \|\bar{X}_1 - \bar{m}_1\| = \sqrt{(x_{11} - m_{11})^2 + (x_{12} - m_{12})^2}$$

$$= \sqrt{0^2 + 0^2} = 0$$

$$\text{And } \bar{X}_1 - \bar{m}_2 = \begin{pmatrix} x_{11} \\ x_{12} \end{pmatrix} - \begin{pmatrix} m_{21} \\ m_{22} \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \|\bar{X}_1 - \bar{m}_2\| = \sqrt{(X_{11} - m_{21})^2 + (X_{12} - m_{22})^2} = \sqrt{(0-1)^2 + 0^2} = 1$$

$\|\bar{X}_1 - \bar{m}_1\| \leq \|\bar{X}_1 - \bar{m}_2\|$   
 Holds good.  $\rightarrow \bar{X}_1 \in C_1$   
 Class  $C_1$  with mean  $\bar{m}_1$ .

Continue the Computation, to group Each of Every Points, if  $\|\bar{X}_k - \bar{m}_1\|$  does not satisfy the Condition of  $\|\bar{X}_k - \bar{m}_1\| \leq \|\bar{X}_k - \bar{m}_2\|$ , then re-group  $\bar{X}_k$  to Class 2.

$k = 1, 2, \dots, 20$  (Entire Data Set)

$$\{X_1, X_3\} \in W_1, \{X_2, X_4, X_5, \dots, X_{20}\} \in W_2$$

Now, Newly formed "Grouping" will need update Clusters, e.g. means,

$$\vec{m}_1(z) = \frac{1}{N} \sum_{i=1}^z \bar{X}_i$$

(All pts in the Class)

$$\frac{1}{2}(\bar{X}_1 + \bar{X}_3) = \frac{1}{2} \left[ \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right] = \frac{1}{2} \begin{pmatrix} 0+0 \\ 0+1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix}$$

Similarly, Compute  $\vec{m}_2(z) = \begin{pmatrix} 5.67 \\ 5.33 \end{pmatrix}$

Compare  $\vec{m}_1(z), \vec{m}_2(z)$  with  $\vec{m}_1(1), \vec{m}_2(1)$  Accordingly.

$$\text{if } \vec{m}_1(z) \stackrel{?}{=} \vec{m}_1(1) \\ \vec{m}_2(z) \stackrel{?}{=} \vec{m}_2(1)$$

Not Equal, then Continue the process. Find

$$\|\vec{X}_k - \vec{m}_1(z)\| \stackrel{?}{\leq} \|\vec{X}_k - \vec{m}_2(z)\|$$

To make sure the Above holds good, if it does, then no re-group. Otherwise, move the pt. to Class 2.

Continue the process,

if there is stable mean

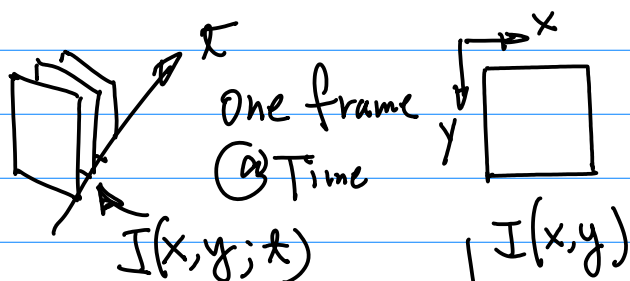
$$\vec{m}_1(t) = \vec{m}_1(t+1)$$

and

$$\vec{m}_2(t) = \vec{m}_2(t+1)$$

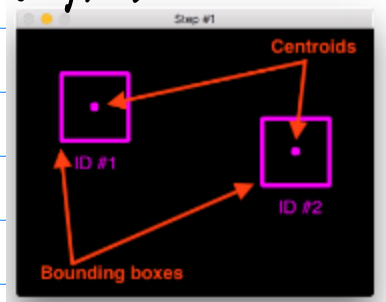
Tracking is Based on "Smoothness"  
e.g. Shortest distance.

"Consider Object Tracker" Technique



Pre-Processing.  
find Object's  $(\bar{x}, \bar{y})$

at  $t$ ,  $I(x, y; t)$

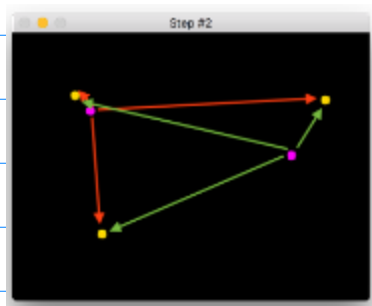


2 pts.  
Pink  
Object 1  
Object 2

At  $t + \Delta t$ , Next frame  $I(x, y; t + \Delta t)$

Same Process, with 3 New Points

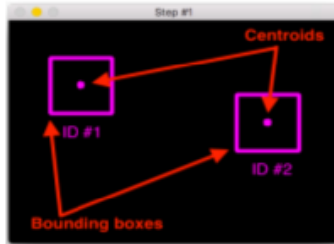
(yellow)



Track Object 1, 2  
at this frame.  
find matching pt?

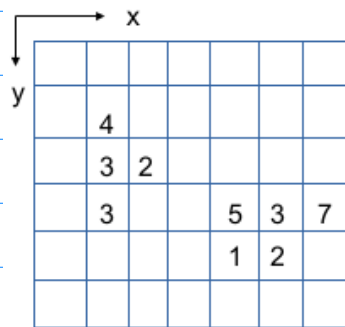


## Example Calculation Part 1



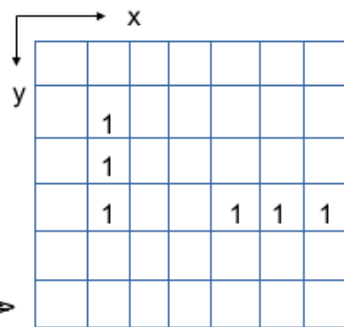
Example:  
Step 0: detection of the object using intensity thresholding technique, set Threshold  $T = 3$ , for  $I(x,y) \geq T$ , set it to 1, o/w to 0.  
Step 1. Compute centroids,

$x1\_bar = 1, y1\_bar = 2$ . Assign ID as O1, similarly,  $x2\_bar = 5, y2\_bar = 3$ . Assign ID as O2. Then create (update the tracking table)



Time = t

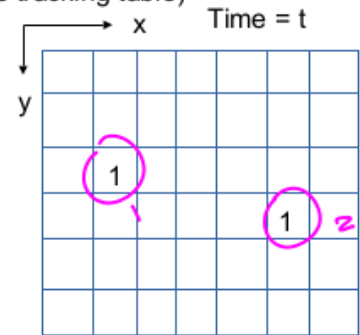
Binarized  
 $T=3$



Time = t

Registration table

$\bar{x}, \bar{y}$



Obj. No.	ID	x-bar, y-bar
Object 1.	1	1, 2
Object 2.	2	5, 3

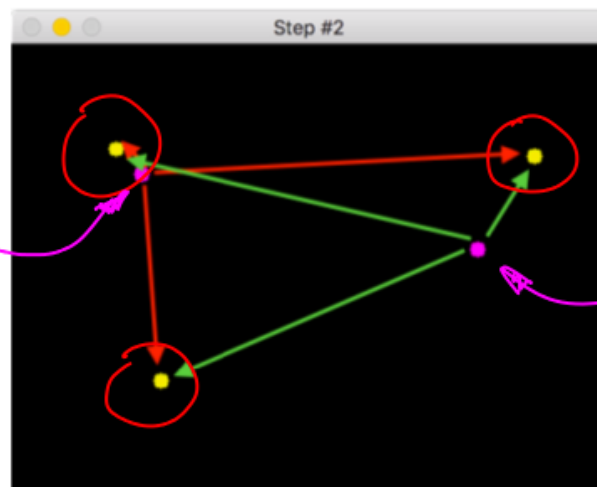
(0,0) origin



## Step 2. Compute Distance Between Boxes

time:  
 $t + \Delta t$   
Next(New)  
Image  
 $I(x,y;t+\Delta t)$  O1

3 New Objects  
(yellow dots)



Original: pink (from previous frame)

New: yellow (the current frame)

Algorithm: compute distance between each pair

ImFE258

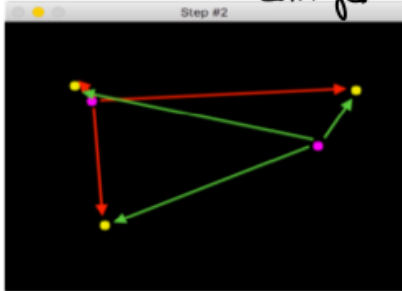
32



$t+dt$  for the New Image

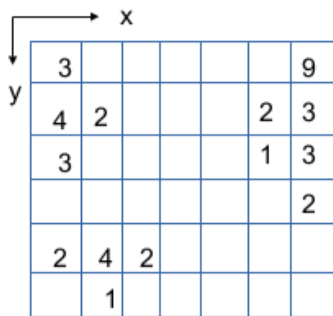
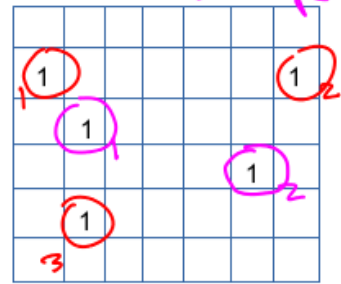
## Example Part 2

Red: Newer Image  
Pink: Previous Image



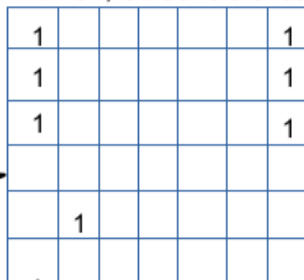
Example:  
Step 2: detection of the object at time  $t+dt$  using the same technique, binarized image  $I(x,y)$ . Compute the centroids, as  $x1\_bar = 0, y1\_bar = 1, x2\_bar = 6, y2\_bar = 2, x3\_bar = 1, y3\_bar = 4$ . Compute distance with O1, O2 as reference points.

$(\bar{x}, \bar{y})$   
for All



Time =  $t+dt$

Binarized  
 $T=3$



Time =  $t+dt$

$D(o1, o1\_new) = \sqrt{2}$ ;  $D(o1, o2\_new) = \sqrt{26}$ ;  $D(o1, o3\_new) = \sqrt{4}$ ;  
 $D(o2, o1\_new) = \sqrt{29}$ ;  $D(o2, o2\_new) = \sqrt{5}$ ;  $D(o1, o3\_new) = \sqrt{17}$ ;

Temporary Registration table time =  $t+dt$

Obj. No.	ID	x-bar, y-bar
Object 1.	1	0, 1
Object 2.	2	6, 1
Object 3.	3	1, 4



## Step 2. Calculation

Minimum distance from o1:

Min  $D = \sqrt{2}$ ,  $D(o1, o1\_new) = \sqrt{2}$ , so the matching is o1\_new (in Yellow circle)

Minimum distance from o2:

Min  $D = \sqrt{5}$ ,  $D(o2, o2\_new) = \sqrt{5}$ ; so the matching is o2\_new (in Yellow circle)

Update the registration table

Note: Continue this Process for  $t+kd\tau, k=2,3,\dots$

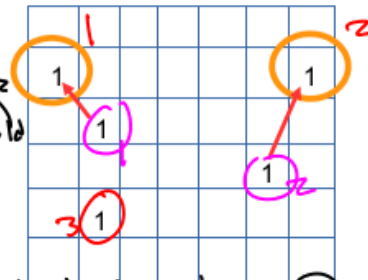
Check Registration Tables at  $t$  and  $t+dt$ ,  $\rightarrow$  Red's must be a new point (Object), keep it for the time frame.

Distance Calculation

$$D = \sqrt{(x_{i,new} - x_{i,old})^2 + (y_{i,new} - y_{i,old})^2}$$

from pink 1. to Red 1, shortest  $D$ .  
 $\therefore$  pink 1  $\rightarrow$  Red 1

pink 2 to Red 2, shortest distance  $D$ .  
 $\therefore$  pink 2  $\rightarrow$  Red 2.



Registration table (time =  $t+dt$ )

Obj. No.	ID	x-bar, y-bar
Object 1.	1	0, 0
Object 2.	2	6, 1
Object 3.	3	1, 4