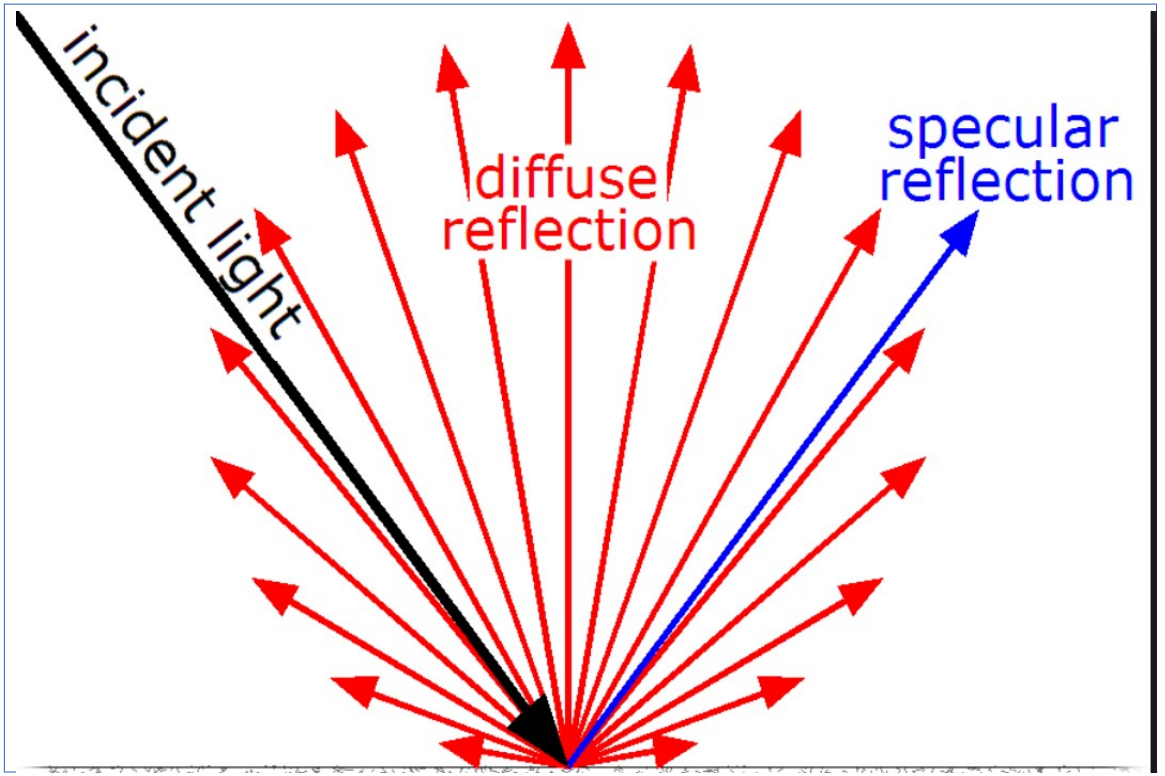
Diffuse Reflection



https://en.wikipedia.org/wiki/Diffuse_reflection

Diffuse Reflection: the reflection of light uniformly in all different directions, the surface of this reflection exhibits Lambert reflection, e.g., equal luminance when viewed from all directions.

Two Key Characteristics:

1. The surface with reflectivity as K_d = (k_r, k_g, k_b), e.g., diffuse coefficients;

2. The decay of incident light is inverse proportional to its distance from the source to the surface point. e.g., 1/(r*r), where r is bing the distance from the light source to the surface.

Specular vs. diffuse reflection

Diffuse Reflection Formulation

$$\vec{I}(x,y,z) = (I_r(x,y,z), I_g(x,y,z), I_b(x,y,z))$$

… (1)

Object I(x,y,z) consists of r, g, b 3 primitive colors, as denoted in (1).

Light source I_s(x,y) consists of r, g, b 3 primitive colors as follows, but let's simplify it as white color, so r, g, b all equal and have the highest value (if in graphics, they are 255)

$$\vec{I_s}(x,y,z) = (I_r(x,y,z), I_g(x,y,z), I_b(x,y,z))$$

… (2)

Object surface consists of reflectivity, e.g., coefficient of reflection

$$\vec{K_d} = (K_r, K_g, K_b)$$

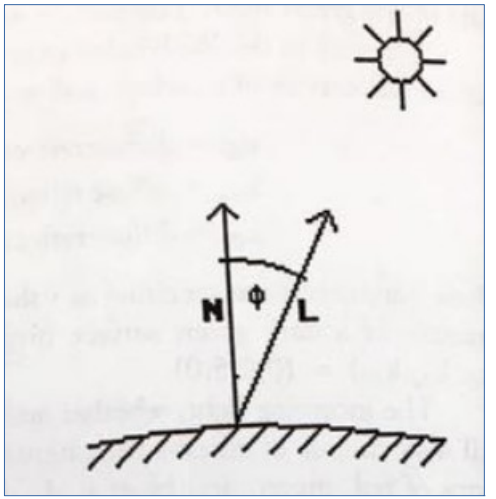
… (3)

r_d vector in Equation (1) is a ray equation, just like I_s(x,y,z) but has no r, g, b primitive color defined in it for the matter of simplicity.

Diffuse Reflection Equation

$$\vec{I}(x,y,z) = (I_r(x,y,z), I_g(x,y,z), I_b(x,y,z))$$

Each primitive olor of the object I(x,y,z) can be written as



$$I_r = K_{dr} \frac{\vec{n} \cdot \vec{r}}{\|\vec{n}\| \|\vec{r}\| \|\vec{r}\|_2^2} \quad \ldots (1.1)$$

where

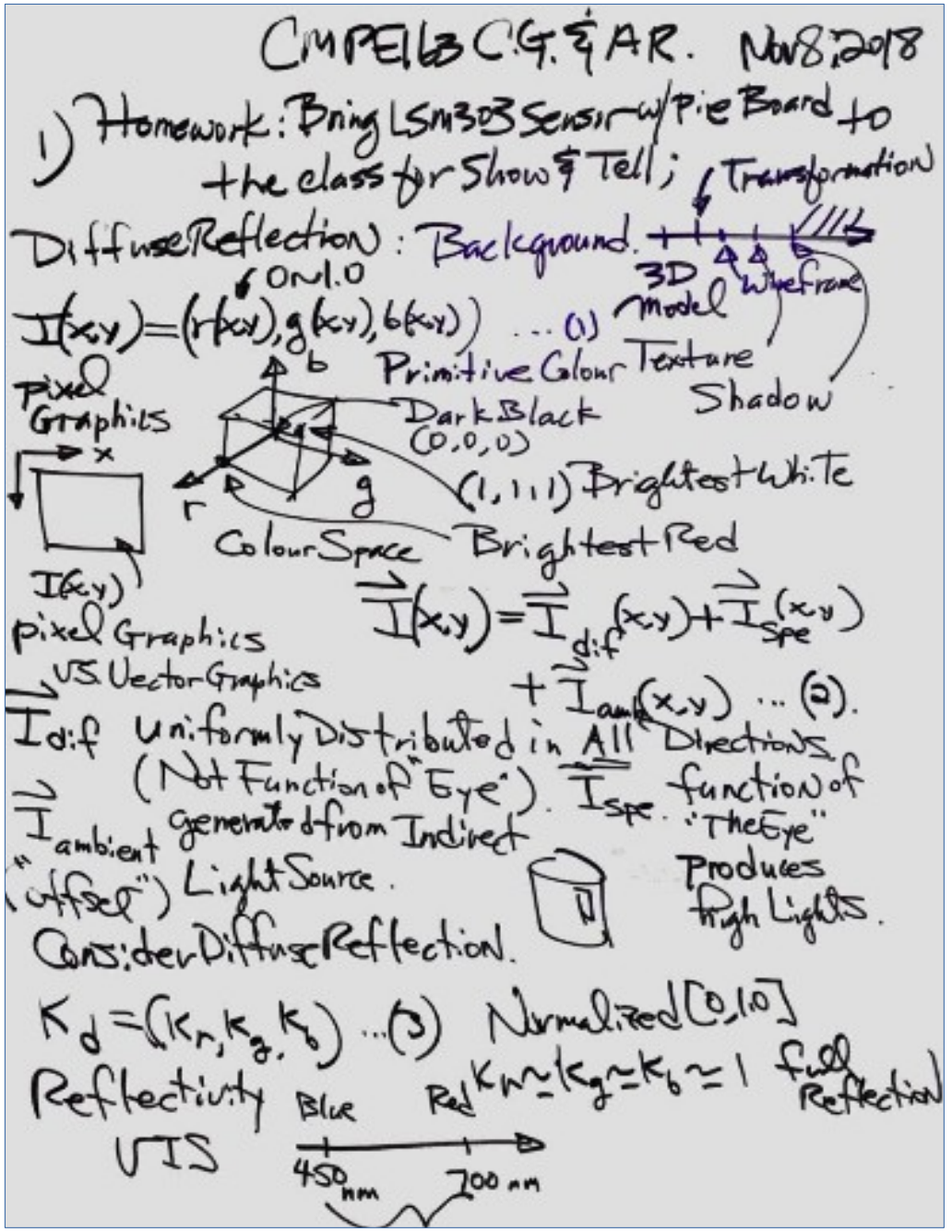
$$\|\vec{r}\|_2^2 = x_r^2 + y_r^2 + z_r^2$$

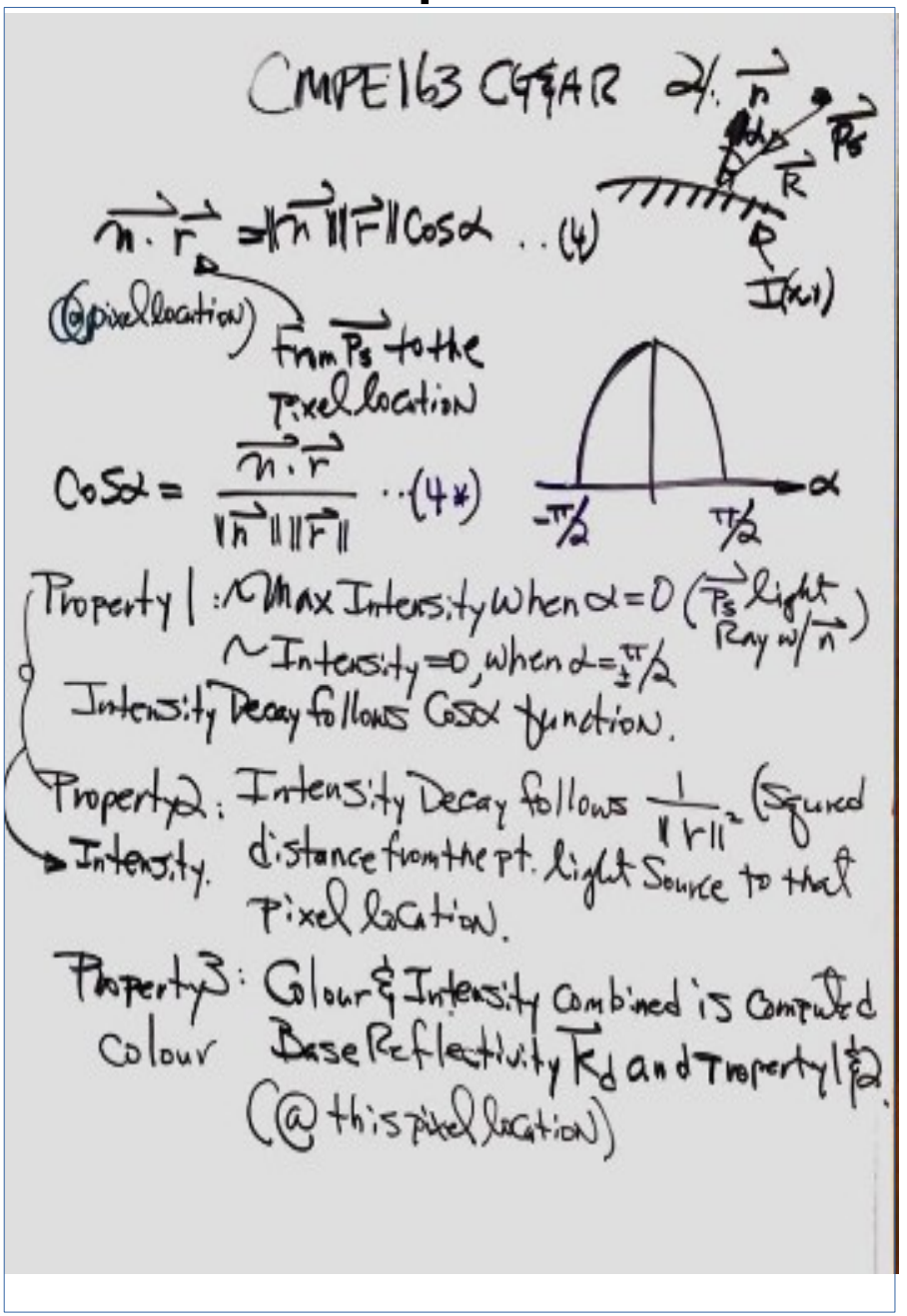
$$I_g = K_{dg} \frac{\vec{n} \cdot \vec{r}}{\|\vec{n}\| \|\vec{r}\|} \cdot \frac{1}{\|\vec{r}\|_2^2} \quad \ldots (1.2)$$

$$I_b = K_{db} \frac{\vec{n} \cdot \vec{r}}{\|\vec{n}\| \|\vec{r}\|} \cdot \frac{1}{\|\vec{r}\|_2^2} \quad \ldots (1.3)$$

Reference: Computer Graphics, C. K. Pokorny, C. F. Gerald, pp. 514

11-8-2018 Diffuse Reflection Equation





11-8-2018 Diffuse Reflection Calculation

CMPE163 CG&AR. Nov8 2018 3/

From Eqn (1.1) (on github) Example: PP4.

$$I_r = K_{dr}\frac{\vec{n}\cdot\vec{r}}{\|\vec{n}\|\|\vec{r}\|}\frac{1}{\|\vec{r}\|^2} \quad \cdots(1.1)$$

$\vec{T_3}(x_3,y_3,z_3)=(10,200,500)$, $\vec{P_i}(x_i,y_i,z_i)$, $i=2$.

$\vec{n}(0,0,1)$, $\vec{r}=\vec{P_s}+\lambda(\vec{P_s}-\vec{P_i})=$

$(x_s,y_s,z_s)+\lambda(x_s-x_i,y_s-y_i,z_s-z_i)\Big|_{\lambda=-1}$ $\vec{P_i}$

Assume $K_{dr}=0.85$, therefore $[0,255]$

$$I_r = 0.85\cdot\frac{(0,0,1)(x_i,y_i,z_i)}{\|(0,0,1)\|\,\|(x_i,y_i,z_i)\|}\cdot\frac{1}{[(x_s-x_i)^2+(y_s-y_i)^2+(z_s-z_i)^2]}$$

$$=0.85\cdot\frac{z_i}{\sqrt{x_i^2+y_i^2+z_i^2}}\cdot\frac{1}{\sqrt{(x_s-x_i)^2+(y_s-y_i)^2+(z_s-z_i)^2}}$$ Sqrt

for Red color

C/C++ `#define K_d_red 0.85`

```
I_red = K_d_red * z[i];
float tmp = Sqrt(x[i]*x[i]+y[i]*y[i]+z[i]*z[i])
float tmp2 = Sqrt((xs-x[i])*(xs-x[i]) +
        (ys-y[i])*(ys-y[i])+(zs-z[i])*(zs-z[i]));
I_red = I_red/(tmp*tmp2);
```

CMPE163 CG&AR 4/

Intensity 220 ... (x_2,y_2) ~50 Linear Interpolation.

(x_1,y_1) dist $\times\ \frac{y_2-y}{y_2-y_1}=\frac{x_2-x}{x_2-x_1}$ ...(6)

Min $\vec{P_1}$(Near) $\vec{P_2}$(far) Max

$$\frac{y_2-y}{x_2-x}=\frac{y_2-y_1}{x_2-x_1},\quad y_2-y=\frac{y_2-y_1}{x_2-x_1}(x_2-x)$$

$$-y=-y_2+\frac{y_2-y_1}{x_2-x_1}(x_2-x)=-y_2-\frac{y_2-y_1}{x_2-x_1}x+\frac{y_2-y_1}{x_2-x_1}x_2$$

$$y=y_2+\frac{y_2-y_1}{x_2-x_1}x-x_2\frac{y_2-y_1}{x_2-x_1}$$

$$y=\frac{y_2-y_1}{x_2-x_1}x+\left(y_2-x_2\frac{y_2-y_1}{x_2-x_1}\right)\quad\cdots(6^*)$$

Suppose: $\dfrac{1}{\sqrt{x_i^2+y_i^2+z_i^2}\cdot\sqrt{(x_s-x_i)^2+(y_s-y_i)^2+(z_s-z_i)}}=\Delta_i$ ...(7)

find $\min_{i\in[0,1,\cdots,6]}\{\Delta\}$, $\max_{i\in[0,1,\cdots,6]}\{\Delta\}$ ...(7*)

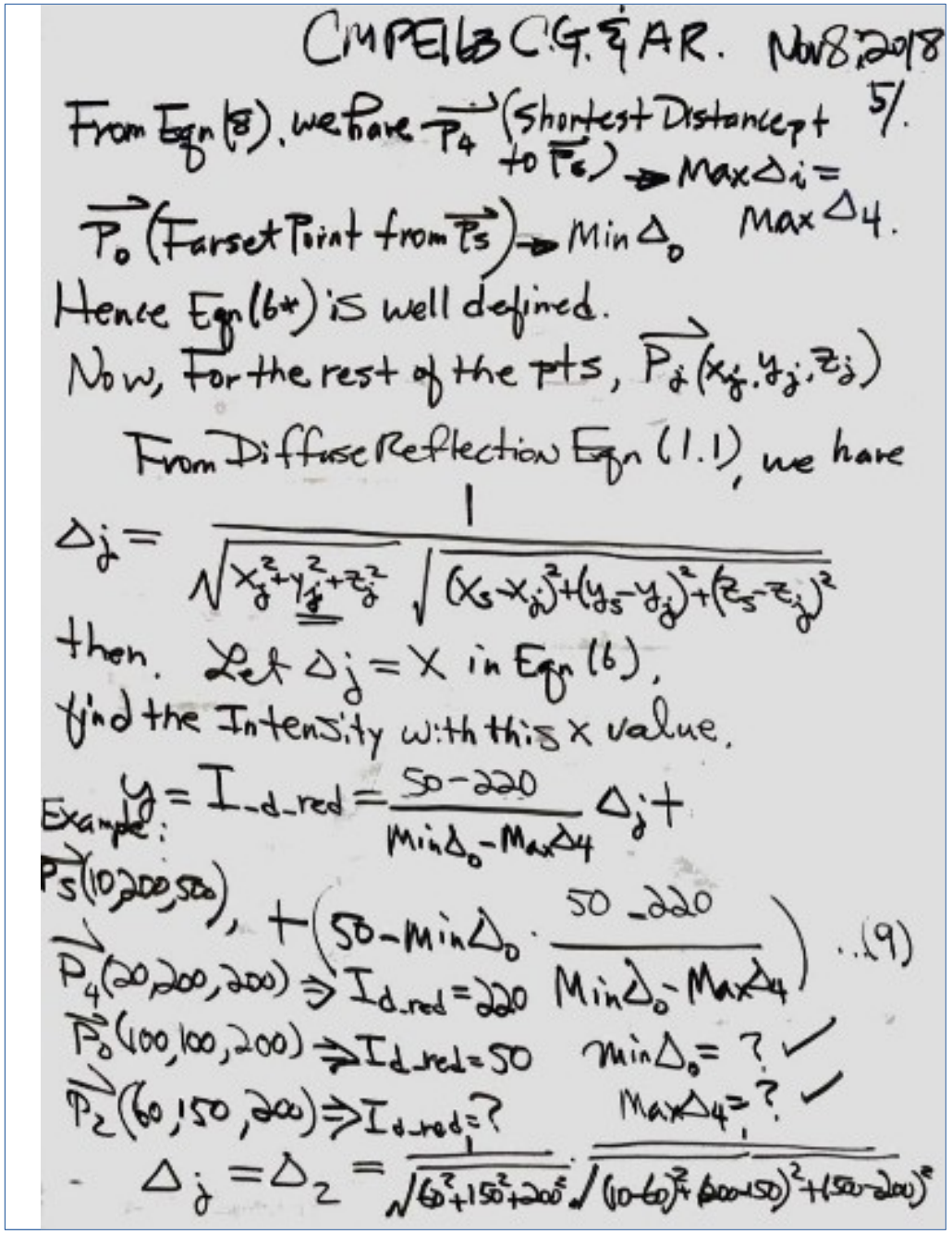
Hence, 2 pts can be found, in Eqn (6*)

Indep. Variable is Δ_i, function (Y, Intensity) is intensity.

$(x_2,y_2)=(\min\Delta_i, 50)$

$(x_1,y_1)=(\max\Delta_i, 220)$ ... (8)

11-8-2018 Diffuse Reflection Calculation (2)



CMPE163 C.G. & AR. Nov 8, 2018 5/.

From Eqn (8), we have $\vec{P_4}$ (Shortest Distance pt to $\vec{P_s}$) → Max Δ_i = $\vec{P_0}$ (Farset Point from $\vec{P_s}$) → Min Δ_0 Max Δ_4.

Hence Eqn (6*) is well defined.

Now, For the rest of the pts, $\vec{P_j}(x_j, y_j, z_j)$

From Diffuse Reflection Eqn (1.1), we have

$$\Delta_j = \frac{1}{\sqrt{x_j^2+y_j^2+z_j^2}\ \sqrt{(x_s-x_j)^2+(y_s-y_j)^2+(z_s-z_j)^2}}$$

then. Let $\Delta_j = X$ in Eqn (6),

find the Intensity with this X value.

Example: $y = I_{d\_red} = \frac{50-220}{\text{Min}\Delta_0 - \text{Max}\Delta_4}\Delta_j +$

$$+\left(50 - \text{Min}\Delta_0 \cdot \frac{50-220}{\text{Min}\Delta_0 - \text{Max}\Delta_4}\right) \quad ..(9)$$

$P_s(10,200,50)$,

$P_4(20,200,200) \Rightarrow I_{d\_red} = 220$

$P_0(400,100,200) \Rightarrow I_{d\_red} = 50$

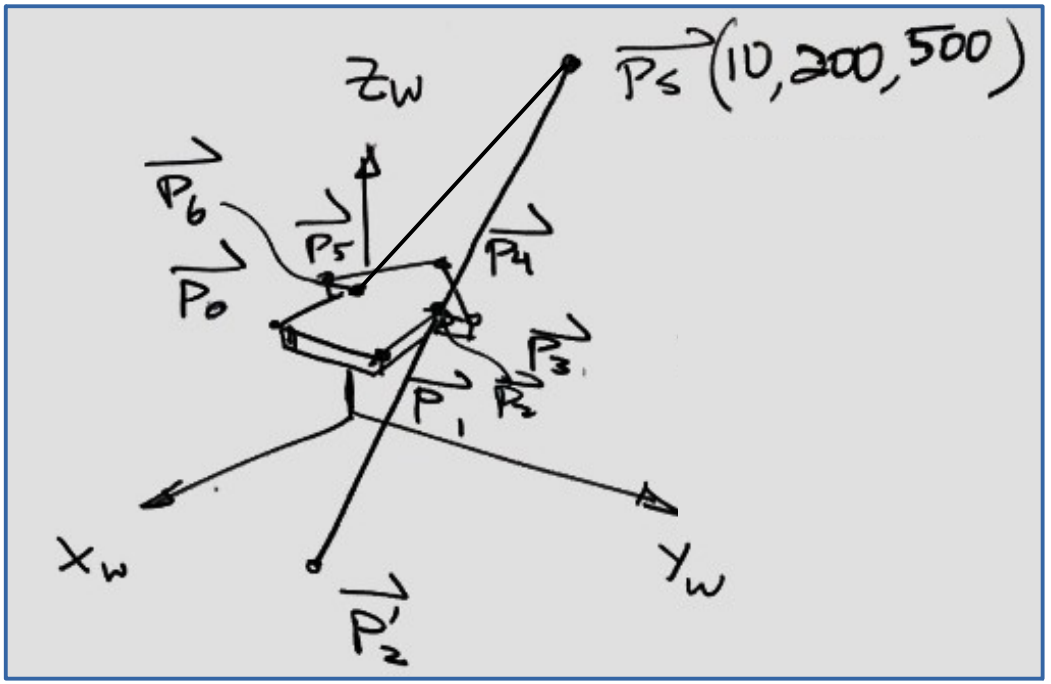
$P_2(60,150,200) \Rightarrow I_{d\_red} = ?$

MinΔ_0 = ? ✓

MaxΔ_4 = ? ✓

$$\Delta_j = \Delta_2 = \frac{1}{\sqrt{60^2+150^2+200^2}\ \sqrt{(10-60)^2+(200-150)^2+(50-200)^2}}$$

Diffuse Reflection Example



Example: Suppose we have a single light source P_s(10,200, 500), now define its (r, g, b) color, so we have single color light source as I_s(r_s, g_s, b_s) = (1.0, 0.0, 0.0), Find the diffuse reflection on the 3D floating arrow by first find color intensity on each of the marked vertex, and then find the color of each pixel of the cursor.
Assume reflection coefficient Kd=(1.0, 0.0, 0.0)

From equation (1.1),

$$I_r = K_{dr} \frac{\vec{n} \cdot \vec{r}}{\|\vec{n}\| \|\vec{r}\|} \frac{1}{\|\vec{r}\|_2^2}$$

… (1.1)

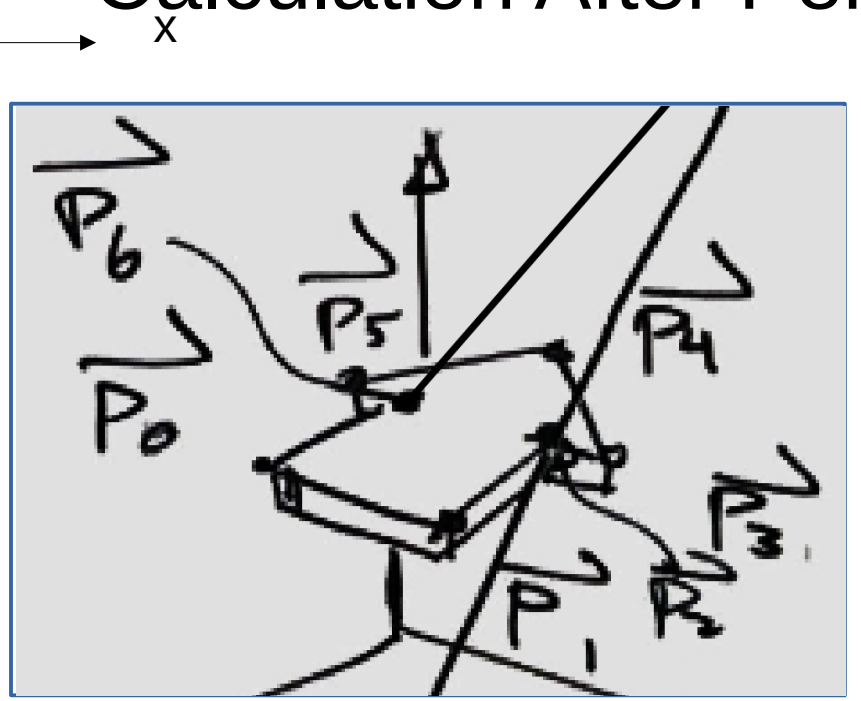
First, find ray equation to, say, one of the vertex, P2(25, 70, 50).

Then find the distance from light source to P2.

Then use the given condition, find the color internsity at P2 location.

Repeat this process to find color intensity for all the vertex from P0 to P6.

Calculation After Perspective Projection



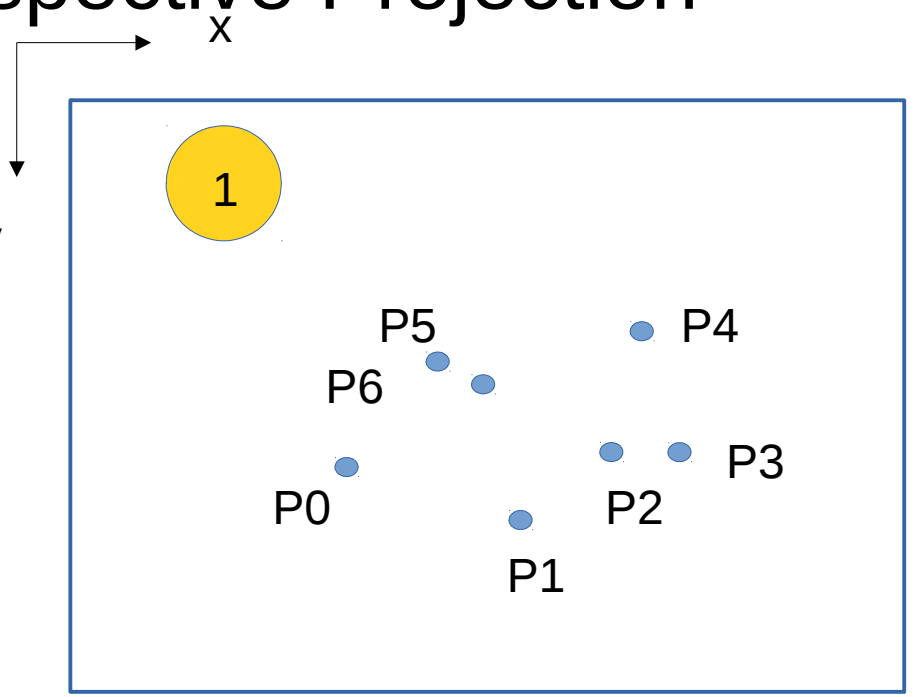
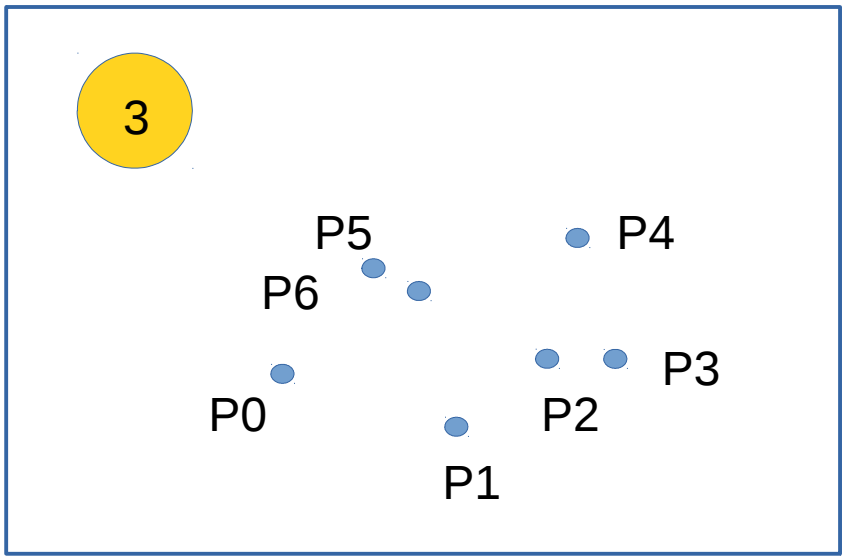
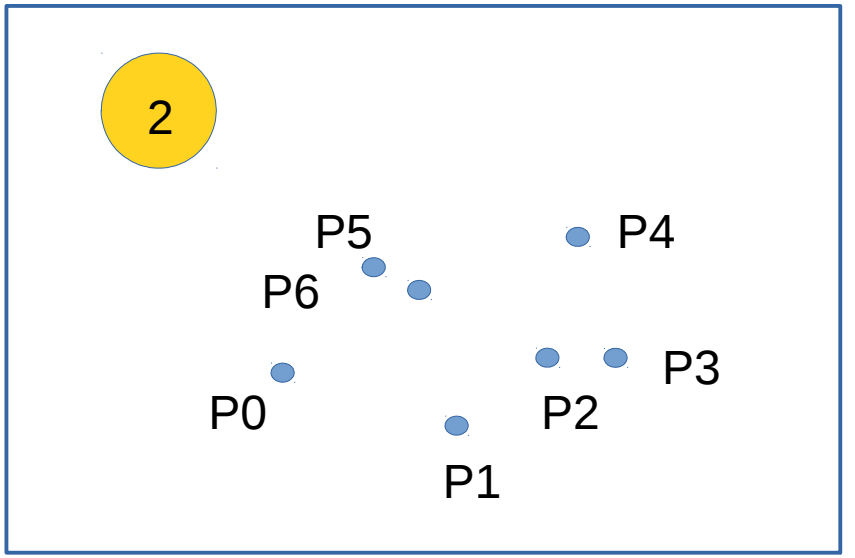
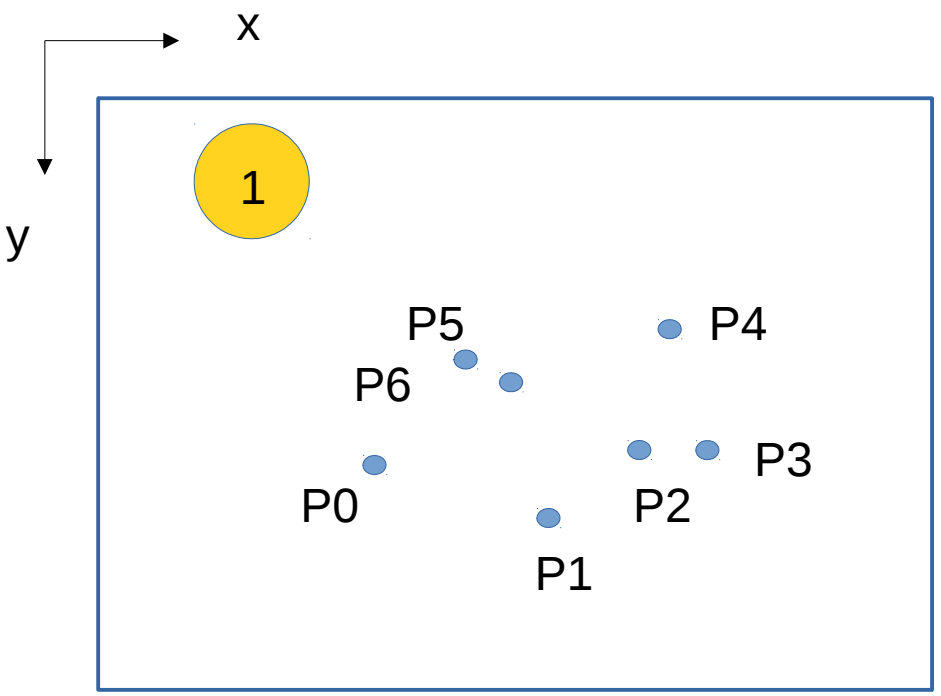


Image (graphics) plane after perspective projection.





Color On The Line Segments (DDA)



Given P_i and P_(i+1), find line equation first;

Then, rasterization of the line by using DDA (Digital Differential Algorithm);

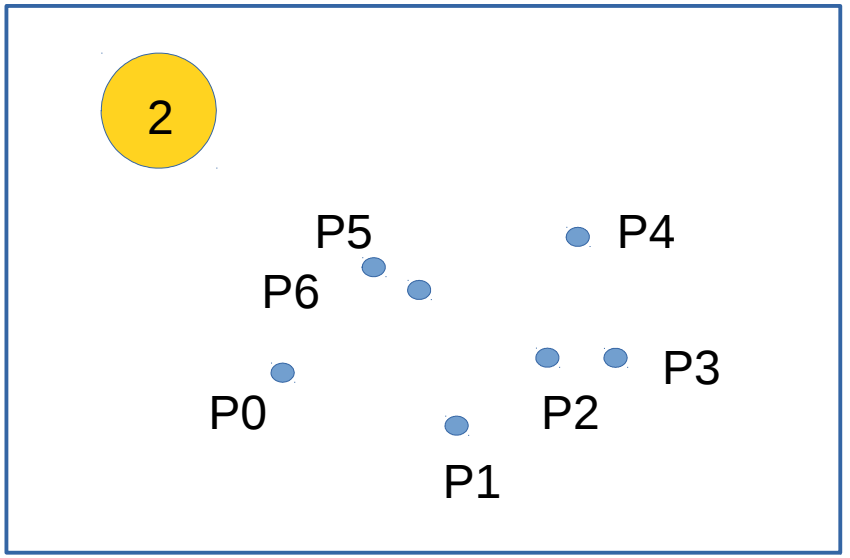
Then use bilinear interpolation to find the color on each point on the line. As a result you will have figure labeled as figure 2.

DDA Algorithm Example

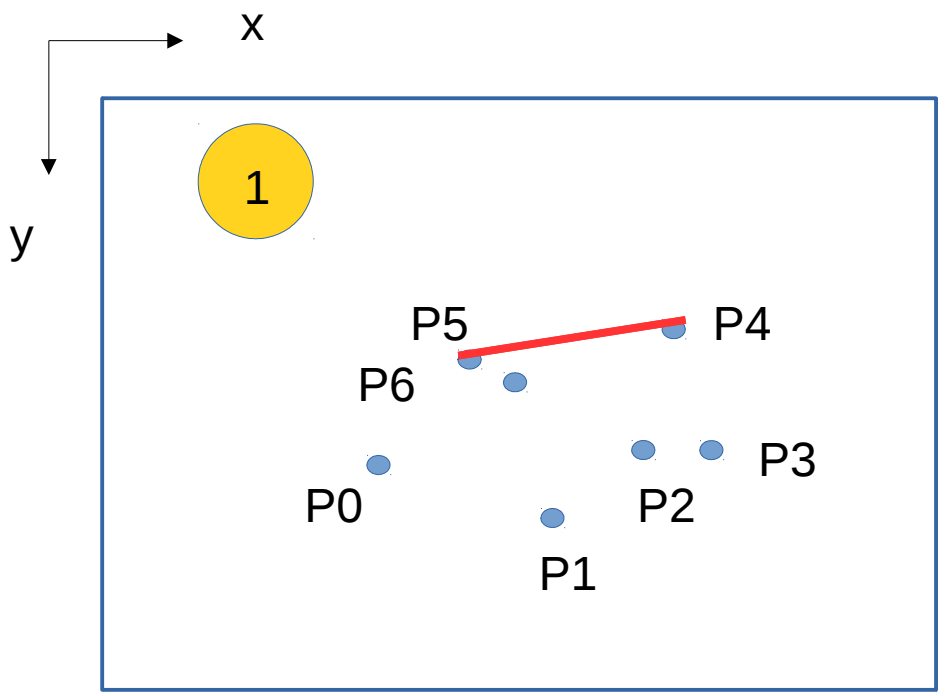
The key :
Slop |a| should be less than or equal to 1,
Otherwise will have to swap x and y;

$y_{(k+1)} = a * x_{(k+1)} + b$ … (1)

$x_{(k+1)} = x_k;$ … (2)



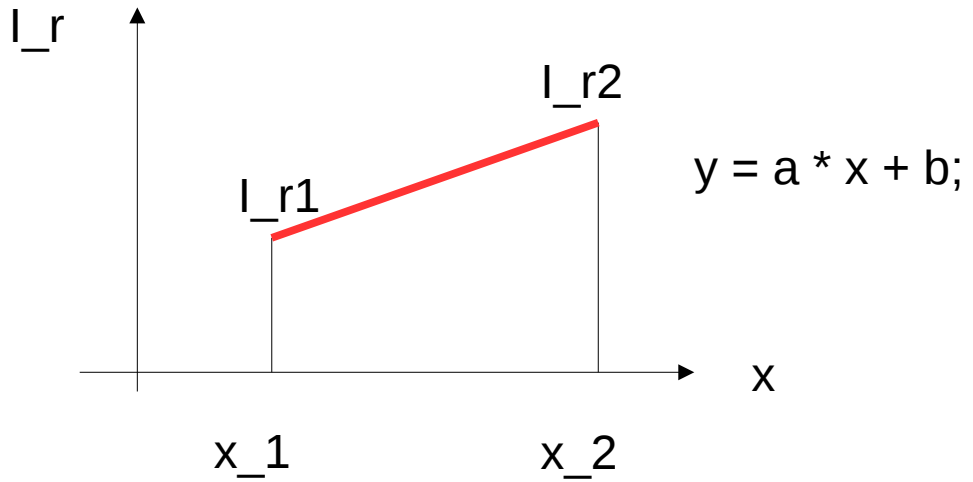
Color On The Line Segments (Interpolation)



Example: when rasterization is done, then find color for each point on the line by interpolation.

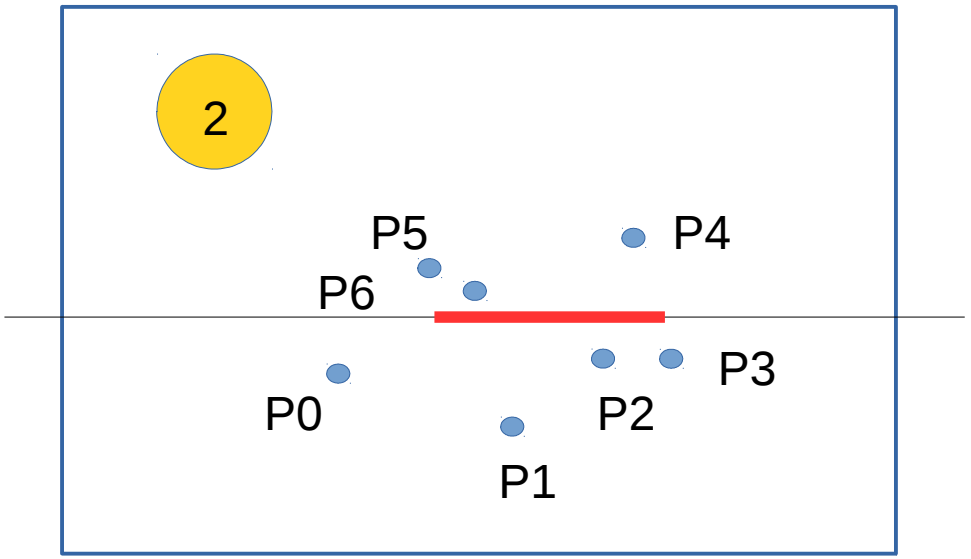
Example (Interpolation)

Step 1. Interpolation along x-direction;
Step 2. Interpolation along y-direction;
Where at each direction, we just use one dimensional interpolation.



$y = a * x + b;$

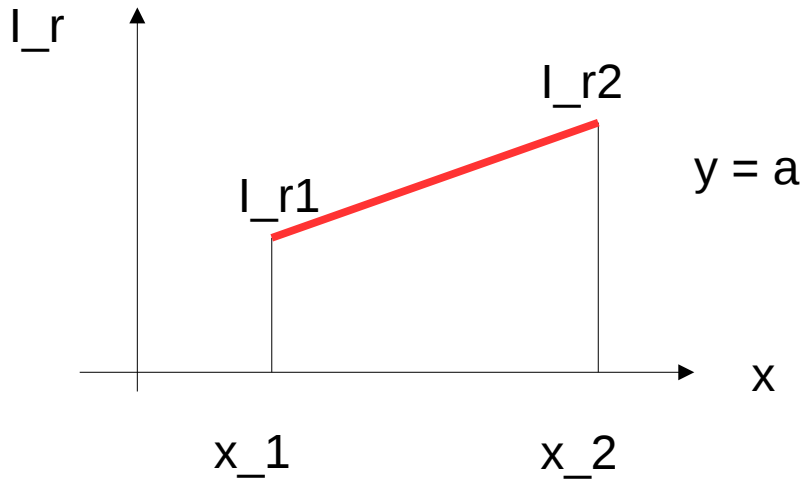
Color Inside the Line Boundary (Interpolation)



Example: when rasterization is done, then find color for each point on the line inside the boundary lines by interpolation.

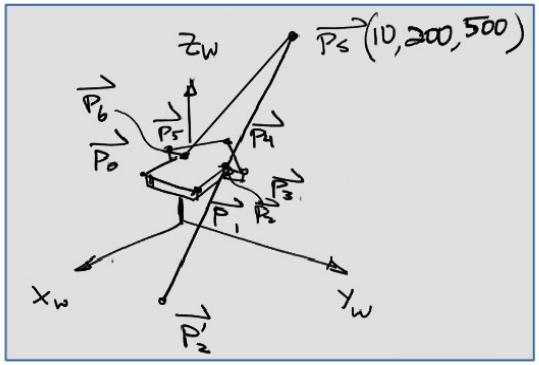
Example (Interpolation)

Raster scan the image, use linear interpolation find the color between boundaries.



$y = a * x + b;$

Diffuse Reflection Example



Example: Suppose we have a single light source P_s(10,200, 500), now define its (r, g, b) color, so we have single color light source as I_s(r_s, g_s, b_s) = (1.0, 0.0, 0.0), Find the diffuse reflection on the 3D floating arrow by first find color intensity on each of the marked vertex, and then find the color of each pixel of the cursor.
Assume reflection coefficient Kd=(1.0, 0.0, 0.0)

Harry Li, Ph.D

From equation (1.1),

$$I_r = K_{dr} \frac{\vec{n} \cdot \vec{r}}{\|\vec{n}\| \|\vec{r}\|} \frac{1}{r^2}$$

... (1.1)

First, find ray equation to, say, one of the vertex, P2(25, 70, 50).

Then find the distance from light source to P2.

Then use the given condition, find the color internsity at P2 location.

Repeat this process to find color intensity for all the vertex from P0 to P6.

Calculation After Perspective Projection

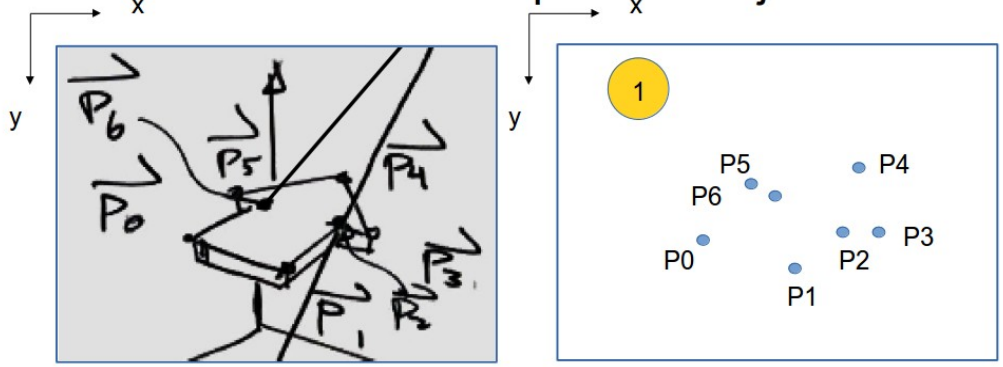


Image (graphics) plane after perspective projection.