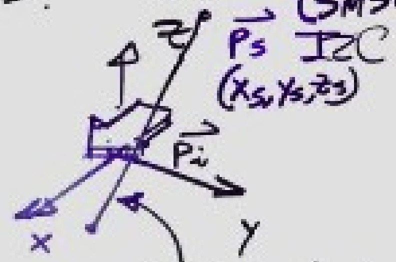
10-11-2018 Design of 3D Cursor

CMPE163 Introduction To CG & AR

Midterm Exam: 25th, 2 weeks. Oct. 11, 2018
(SM303

Example: $\overrightarrow{P_s}(x_s, y_s, z_s)$
$\{P_i(x_i, y_i, z_i) \mid i = 0, 1, \cdots 7\}$



Ray Equation.

$$\begin{cases} \vec{P} = \vec{P_s} + \lambda(\vec{P_s} - \vec{P_i}) \\ \vec{n} \cdot (\vec{a} - \vec{v}) \Big|_{\vec{v} = \vec{P}} = 0 \end{cases}$$

$$\lambda = \frac{\vec{n} \cdot (\vec{a} - \vec{P_s})}{\vec{n} \cdot (\vec{P_s} - \vec{P_i})} = \frac{(n_x, n_y, n_z)(x_a - x_s, y_a - y_s, z_a - z_s)}{(n_x, n_y, n_z)(x_s - x_i, y_s - y_i, z_s - z_i)}$$

$$= \frac{n_x(x_a - x_s) + n_y(y_a - y_s) + n_z \cdot (z_a - z_s)}{n_x(x_s - x_i) + n_y \cdot (y_s - y_i) + n_z \cdot (z_s - z_i)} \quad \cdots (1)$$

```
float
tmp = nx*(xa-xs )+ny*(ya-ys )+nz*(za-zs);
lamda = tmp/(nx*(xs-x[i])+ny*(ys-y[i])
        +nz*(zs-z[i]));
```

Exampl: $\vec{P_s}(50, 50, 200)$, $\vec{P_i}(15, 15, 50)$;
plane π: x_w-y_w plane, Find Intersection pt.
Step 1: From Eq(1), find λ first, $\vec{n}(n_x, n_y, n_z) = (0, 0, 1)$
Hence, $\lambda = \frac{0 + 0 + (z_a - 200)}{0 + 0 + (200 - 50)} = -\frac{200}{150} = -4/3$

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Therefore, Intersection Pt is given by the Ray Equation

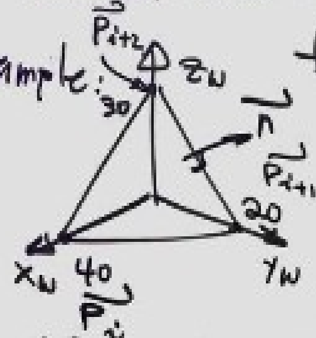
$$\vec{P} = \vec{P_s} + \lambda(\vec{P_s} - \vec{P_i}) \Big|_{\lambda = -\frac{4}{3}} = \vec{P_s}\left(1 - \tfrac{4}{3}\right) + \frac{4\vec{P_i}}{3}$$

$$= -\frac{1}{3}\vec{P_s} + \frac{4\vec{P_i}}{3} = -\frac{1}{3}(50, 50, 200) + \frac{4}{3}(15, 15, 50)$$

$$= \frac{1}{3}(60 - 50, 60 - 50, 0) = \frac{1}{3}(10, 10, 0)$$

$\vec{n}$ Normal Vector Computation;

Example:



for x_w-y_w plane: $\vec{n}_{x\text{-}y} = (0, 0, 1)$
y_w-z_w plane; $\vec{n}_{y\text{-}z} = (1, 0, 0)$
$z_w x_w$ plane; $\vec{n}_{z\text{-}x} = (0, 1, 0)$

$$\vec{n} = (\vec{P_{i+1}} - \vec{P_i}) \times (\vec{P_{i+2}} - \vec{P_i})$$

$$= \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_{i+1} - x_i & y_{i+1} - y_i & z_{i+1} - z_i \\ x_{i+2} - x_i & y_{i+2} - y_i & z_{i+2} - z_i \end{bmatrix} = \quad \vec{i} \quad \vec{j} \quad \vec{k}$$

From the given condition.

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2/

Therefore, Intersection Pt is given by the Ray Equation

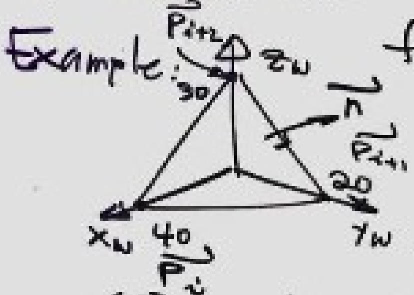
$$\vec{P} = \vec{P_s} + \lambda(\vec{P_s}-\vec{P_i})\Big|_{\lambda=-\frac{4}{3}} = \vec{P_s}\left(1-\frac{4}{3}\right) + \frac{4\vec{P_i}}{3}$$

$$= -\frac{1}{3}\vec{P_s} + 4\frac{\vec{P_i}}{3} = -\frac{1}{3}(50,50,200) + \frac{4}{3}(15,15,50)$$

$$= \frac{1}{3}(60-50, 60-50, 0) = \frac{1}{3}(10,10,0)$$

$\vec{n}$ Normal Vector Computation;

Example:



for x_w-y_w plane: $\vec{n}_{x-y} = (0,0,1)$

y_w-z_w plane; $\vec{n}_{y-z} = (1,0,0)$

$z_w x_w$ plane; $\vec{n}_{z-x} = (0,1,0)$

$$\vec{n} = (\vec{P}_{i+1}-\vec{P_i}) \times (\vec{P}_{i+2}-\vec{P_i})$$

$$= \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_{i+1}-x_i & y_{i+1}-y_i & z_{i+1}-z_i \\ x_{i+2}-x_i & y_{i+2}-y_i & z_{i+2}-z_i \end{bmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0-40 & 20-0 & 0-0 \\ 0-40 & 0-0 & 30-0 \end{vmatrix}$$

From the given Condition

$$= \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ -40 & 20 & 0 \\ -40 & 0 & 30 \end{bmatrix} = \vec{i}\,20\times30 + \vec{j}\,40\times30 + \vec{k}(20\times(-40))$$

$$= \vec{i}\,600 + \vec{j}\,1200 + \vec{k}\,800$$

$$= \vec{i}\,6 + \vec{j}\,12 + \vec{k}\,8$$

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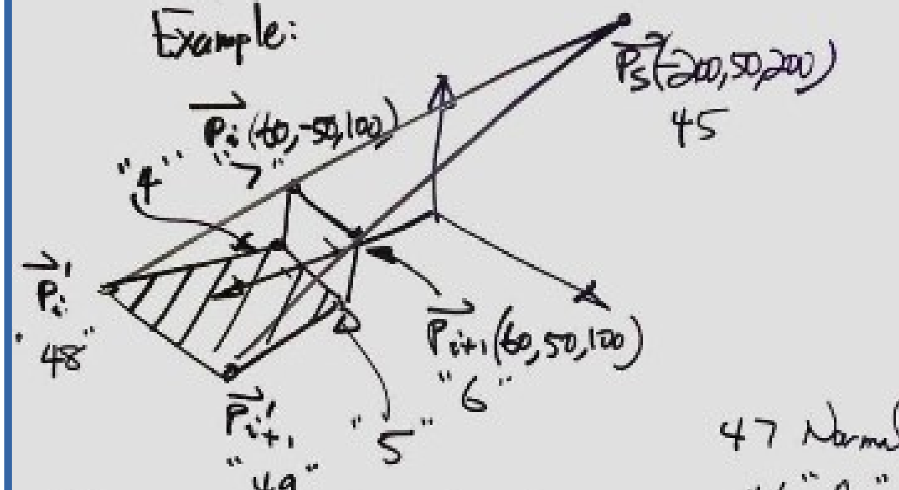
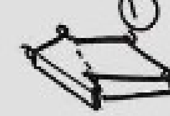
Example:



46 "A" $\vec{a}$

Homework: Compute Shade of the floating Arrow. Due Next Week (18th, Thursday) — CANVAS



① $\{\vec{P_i} \mid i=0,1,\dots,6\}$

Use Top 7 Vertexes Only

② Find Optimal $\vec{E}(400,400,400)$

$\vec{P_s}$ (longer Shadow preferred, with Negative x_s)

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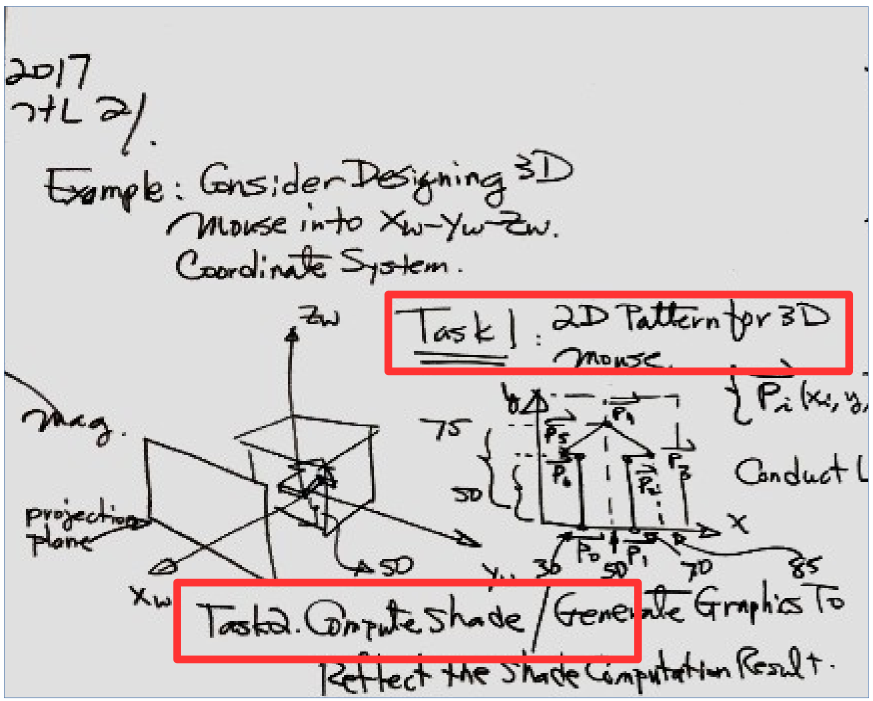
Design of 3D Cursor



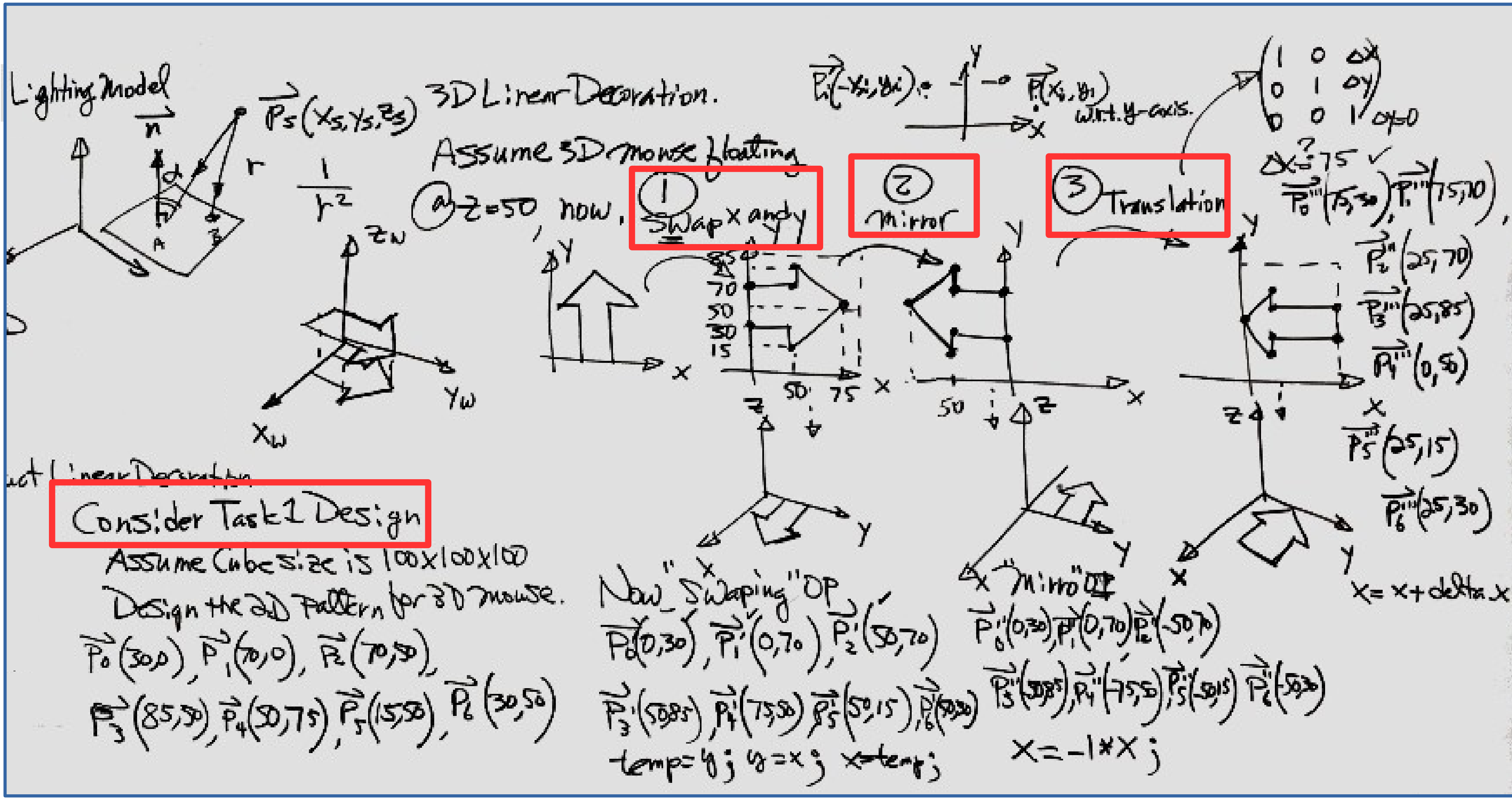
Three tasks:

1. 2D pattern for 3D mouse and perform 3D linear decoration algorithm

2. Compute shade

3. Hidden Line/Surface Removal

Design 2D Cursor Pattern then 3D Decoration



3D Decoration

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Now, with **Linear Decoration**, we can change $\{\vec{P}_i'''(x_i''', y_i''') \mid i=0,1,\cdots,6\}$ to 3D mouse, by adding z-dimension, such that $z_i'''=50$, Hence, we have

$$\{\vec{P}_i(x_i, y_i, z_i) \mid i=0,1,2,\cdots,6;\ z_i=50\}$$

$\vec{P}_0(75,30,50)$

$\vec{P}_1(75,70,50)$, $\vec{P}_2(25,70,50)$, $\vec{P}_3=(25,85,50)$

$\vec{P}_4(0,50,50)$, $\vec{P}_5(25,15,50)$, $\vec{P}_6(25,30,50)$

Make the pattern with Thickness=5.

$S1: \{\vec{P}_i(x_i, y_i, z_i) \mid i=0,1,2,\cdots,6\}$

Then, (Layer Beneath S_1)

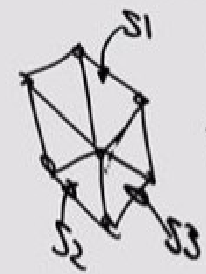
$S2: \{\vec{P}_i'(x_i, y_i, z_i-5) \mid i=0,1,2,\cdots,6\}$

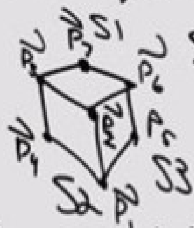
"Wireframe" → Solid Object

Hidden Line/Surface Removal:
- Simple ~ Based ON Vector Cross Product
- Z-Buffer Algorithm.

Background

1° Define Vertices of 3D Object(s) in Counter Clockwise Direction. (When Viewing the Object from the outside)



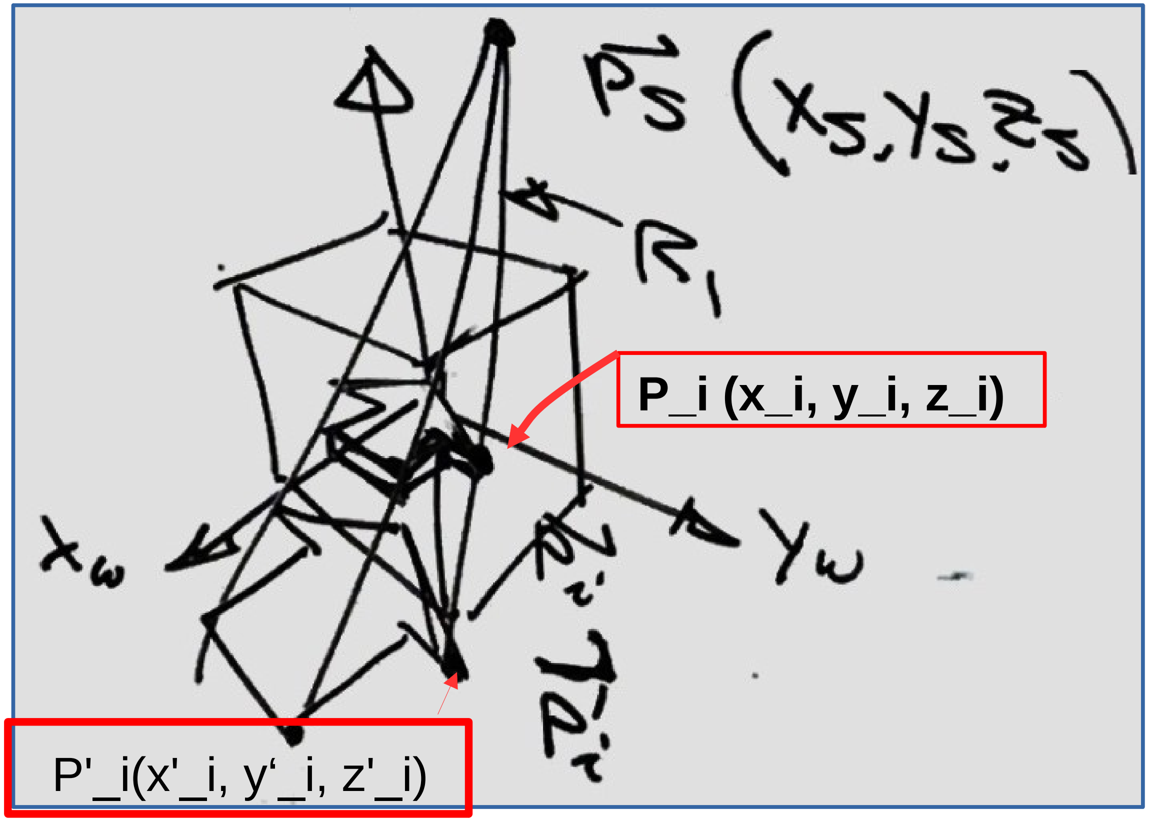


S_1: $\vec{P}_2-\vec{P}_6-\vec{P}_7-\vec{P}_3$ C.C.

S_2: $\vec{P}_1-\vec{P}_2-\vec{P}_3-\vec{P}_4$ C.C

S_3: $\vec{P}_1-\vec{P}_5-\vec{P}_6-\vec{P}_2$ C.C

Single Point Light Source



Give a single point light source P_s, and the 3D cursor as

{P_i | i = 0, 1, …, N-1}

Find the intersection points on Xw-Yw plane,

{P'_i | i = 0, 1, …, N-1}

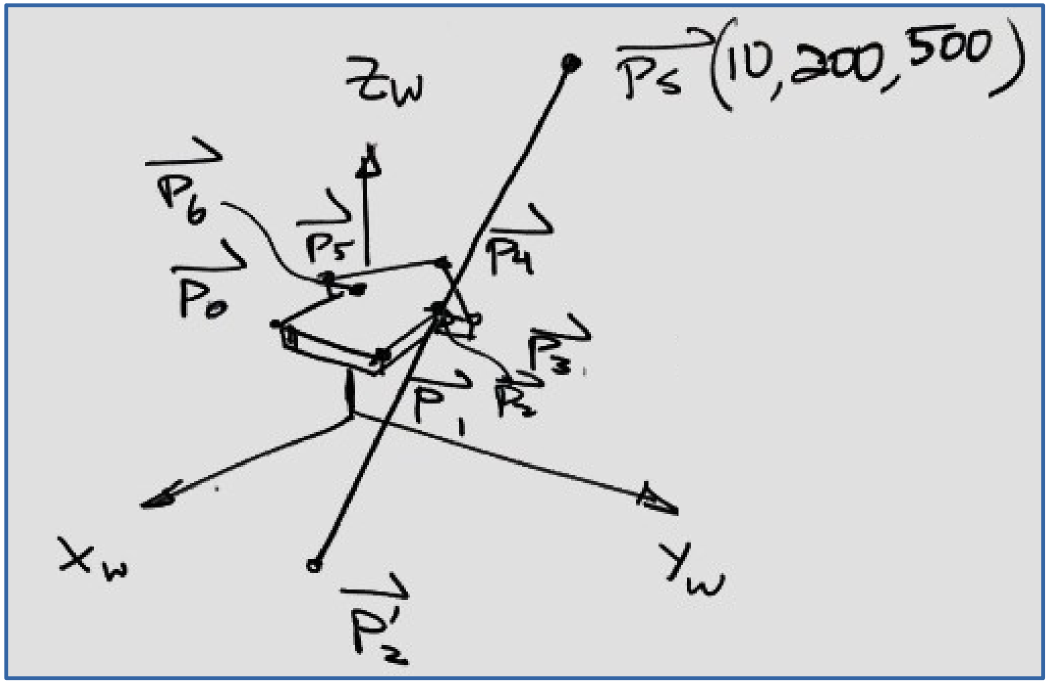
e.g.,

P_i (x_i, y_i, z_i) from the 3D cursor, linked to single point light source

P_s(x_s, y_s, z_s) and formed intersection point

P'_i(x'_i, y'_i, z'_i)

Computing Shade From A Single Point Light Source (1)



Ray Equation (Line~)

$$\vec{R_2} = \vec{P_s} + \lambda(\vec{P_s} - \vec{P_2}) \quad (1)$$

Plane equation below:

$$\vec{n} \cdot (\vec{r} - \vec{a}) = 0 \quad (2)$$

Where the normal vector of the Xw-Yw plane is

$$\vec{n} = (0,0,1)$$

And the known point vertex a is:

$$\vec{a} = (0,0,0)$$

for $i=0,1,2,\cdots,6$, we have generalized

$$\vec{R_i}(x_i,y_i,z_i) = \vec{P_s}(x_s,y_s,z_s) + \lambda(x_s-x_i,\ y_s-y_i,\ z_s-z_i)$$

7 Ray equations for each vertex of the 3D cursor (1.1)

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Computing Shade From A Single Point Light Source (2)

Substitute the known condition into the plane equation, we have

$$(0,0,1)*(\vec{v}-\vec{a})\Big|_{\vec{a}=0}=0$$

(3)

Note vector V is the common shared point (intersection) of the ray vector, so we have

$$\vec{n}*\vec{v}\Big|_{\vec{v}=\vec{R}_i}=0$$

(4)

e.g.

$$\vec{n}*\vec{R}_i\Big|_{\vec{R}_i=\vec{P}_S+\lambda(\vec{P}_S-\vec{P}_i)}=0$$

Hence,

$$\vec{n}*\left(\vec{P}_S+\lambda(\vec{P}_S-\vec{P}_i)\right)=0$$

(5)

Or

$$\vec{n}*\vec{P}_S+\lambda\vec{n}*(\vec{P}_S-\vec{P}_i)=0$$

(5.1)

Solve for lamda,

$$\lambda=-\frac{\vec{n}*\vec{P}_S}{\vec{n}*(\vec{P}_S-\vec{P}_i)}$$

(6)

e.g.

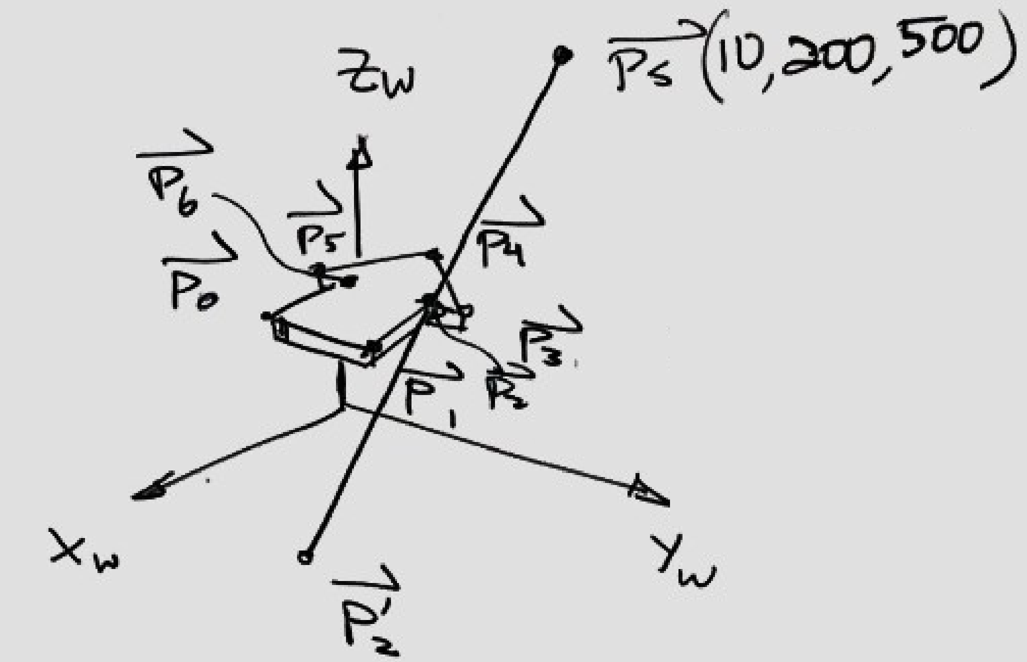
$$\lambda=\frac{-(n_x,n_y,n_z)*(x_S,y_S,z_S)}{(n_x,n_y,n_z)*(x_S-x_i,y_S-y_i,z_S-z_i)}$$

(6.1)

Computing Shade From A Single Point Light Source (3)

Example: Compute the Shade point $\vec{P_2'}$

Given $\vec{P_2}(25, 70, 50)$, $\vec{P_s}(10, 200, 500)$



From equation (6), compute lamda

$$\lambda = -\frac{\vec{n} * \vec{P_s}}{\vec{n} * (\vec{P_s} - \vec{P_2})} = -\frac{n_x \cdot x_s + n_y y_s + n_z z_s}{n_x(x_s - x_2) + n_y(y_s - y_2) + n_z(z_s - z_2)}$$

$$= -\frac{0 + 0 + 1 \cdot z_s}{0 + 0 + 1 \cdot (z_s - z_2)} = -\frac{z_s}{z_s - z_2} = -\frac{500}{500 - 50} = -\frac{500}{450} = -\frac{10}{9}$$

Then substitute the lamda back to the ray equation to find the intersection point as follows

$$\vec{P_2'} = \vec{P_s} + \lambda(\vec{P_s} - \vec{P_2})$$

$$= (10, 200, 500) - \frac{10}{9}(10 - 25, 200 - 70, 500 - 50)$$

$$= \left(10 + \frac{150}{9}, 200 - \frac{1300}{9}, \underbrace{500 - \frac{4500}{9}}_{0}\right)$$

The rest of the points can be computed similarly.

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