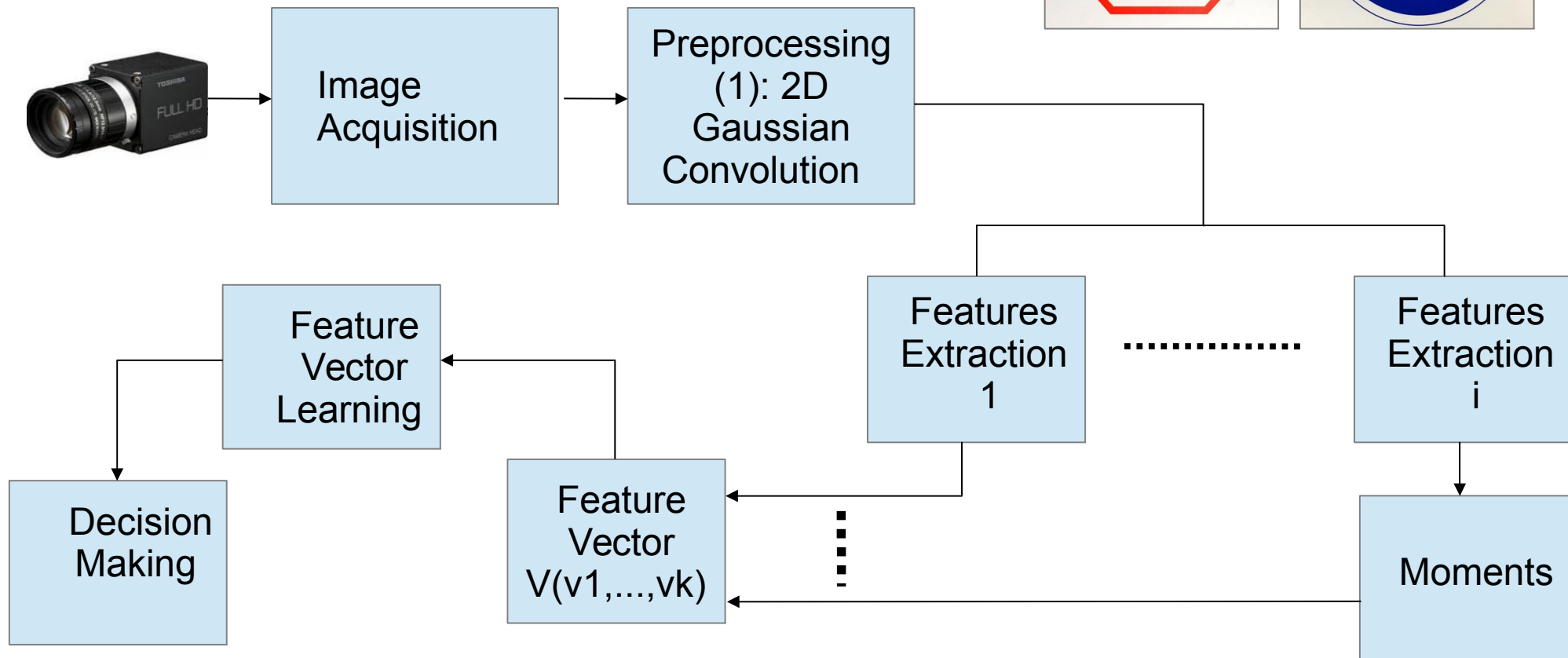
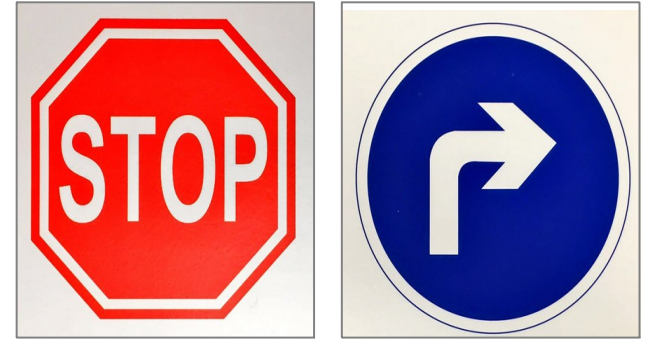


Intro to Moments for Objects Recognition

Objectives: Develop a technical to detect different shaped, different color, different size objects



Pattern Recognition For Binary Images

The tool box for pattern recognition for binary images

1. Size
2. Moments

\bar{x}

\bar{y}

\bar{x}^k

\bar{y}^k etc.

3. Perimeter
4. Orientation
5. Compositions of the above

Perimeter and moments: vector

6. Invariant operators
 - size invariant
 - orientation invariant
 - illumination invariant

Note: Starting from binary images, extended to color images

Biologically inspired techniques

- Rule 1. Proximity
- Rule 2. Similarity
- Rule 3. Closure
- Rule 4. Good continuation
- Rule 5. Symmetry
- Rule 6. Simplicity

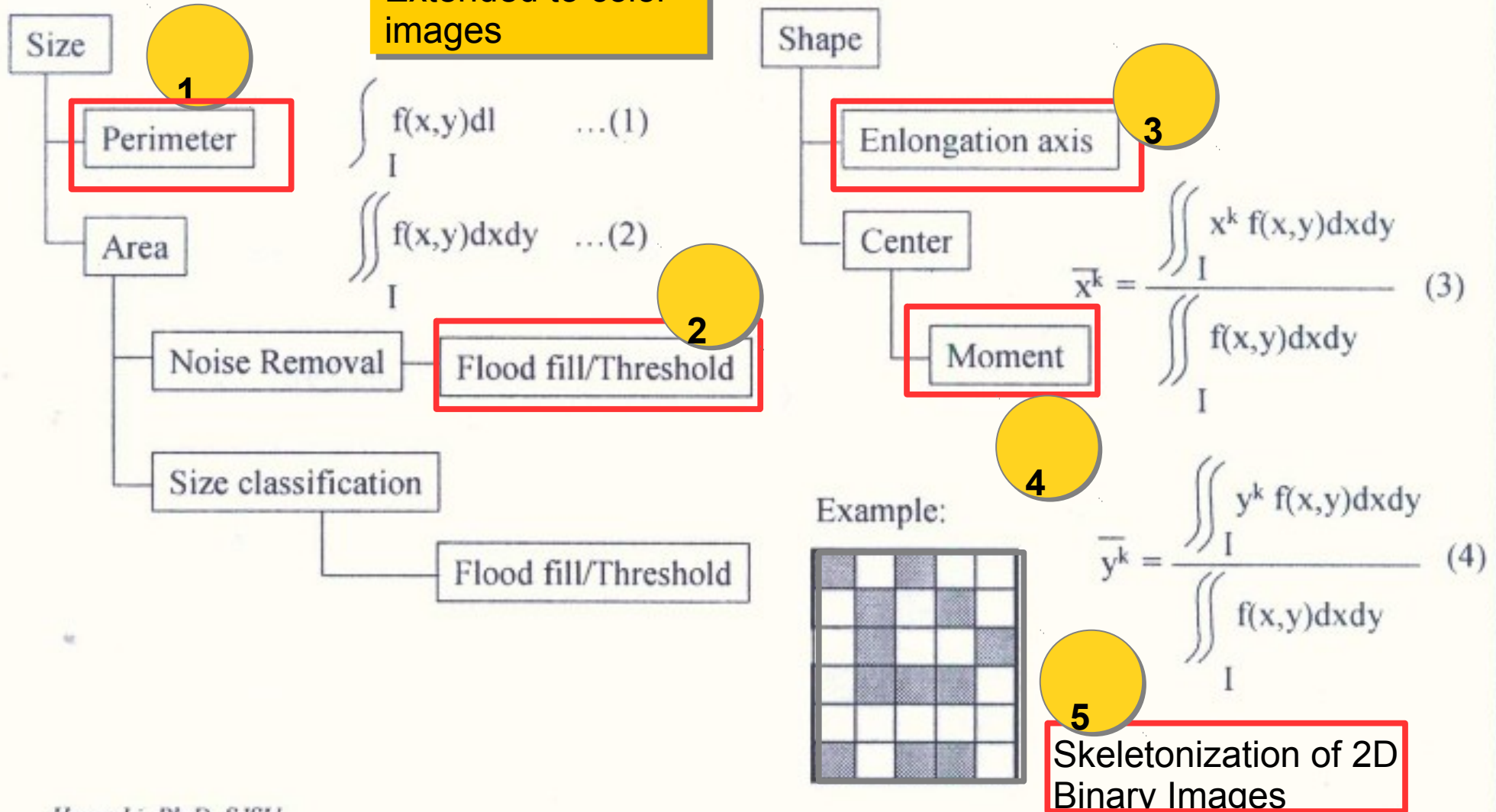
Note: 'Proximity' usage for clean up binary image and remove noise, as well as growing boundary points per 'good continuation' rule to form a better edge map.

Note: Similarity defines a interesting question, how to describe one object is similar, or somewhat similar to others, neural network and fuzzy logic may help.

Ground rule: signature of a image, tools including 3 invariant characteristics

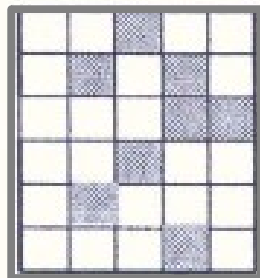
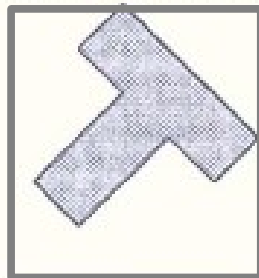
Binary Image Processing

Extended to color images



Example On Simple Pattern Recognition

Given two binary images, derived from two objects, T and O, design a technique to identify them



Example: Computation of
 (1) Area (size);
 (2) X-bar;
 (3) Y-bar;
 (4) Orientation, theta angle
 (5) Perimeter of an object

Fig1(a),(b)

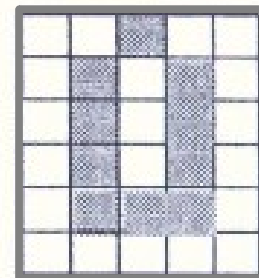
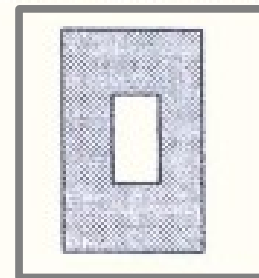


Fig2(a),(b)

Good continuation or noise? What to do with this noise?

Feature Vector		Size	X-bar	Y-bar	Orientation	Perimeter	
V_1(v1,..., v5)	T	v11	v12	v13	v14	v15	From Fig1(b)
V_2(v1,..., v5)	L	v21	v22	v23	v24	v25	From Fig2(b)

Intro Feature Characterization

Example: Fill out this table based on the characteristics of each feature

	Perimeter v1	Area v2	x_bar v3	y_bar v4	Theta v5	Moments v6-vi	Hu-Moments v(i+1)-vk
Illumination invariant							
Scale invariant			Y	Y	Y		Y
Orientation Invariant							Y

Perimeter:

$$P = \int_{\Omega} f(x,y) dl$$

Or,

$$P = \sum_{k_1=1}^N \sum_{k_2=1}^M B'(x,y)$$

Where $B'(x,y)$ from object whose neighboring pixels belong to background

x_bar:

$$\bar{x} = \frac{\iint_{\Omega} x B(x,y) dx dy}{\iint_{\Omega} B(x,y) dx dy}$$

y_bar can be defined similarly

Moments:

$$M_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^p y^q f(x,y) dx dy$$

Central moments:

And

$$\mu_{pq} = \frac{\iint_{\Omega} (x-\bar{x})^p (y-\bar{y})^q B(x,y) dx dy}{A}$$

Orientation Computation

$$\tan 2\phi \triangleq \frac{b}{a-c}$$

$$a = \iint_{\Omega} (x - \bar{x})^2 B(x, y) dx dy \quad \dots (2)$$

$$b = \iint_{\Omega} 2(x - \bar{x})(y - \bar{y}) B(x, y) dx dy \quad \dots (3)$$

$$c = \iint_{\Omega} (y - \bar{y})^2 B(x, y) dx dy \quad \dots (4)$$

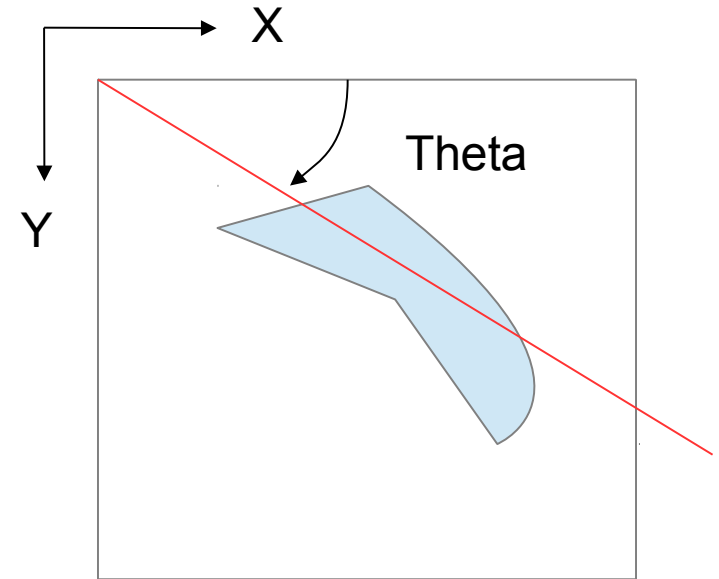
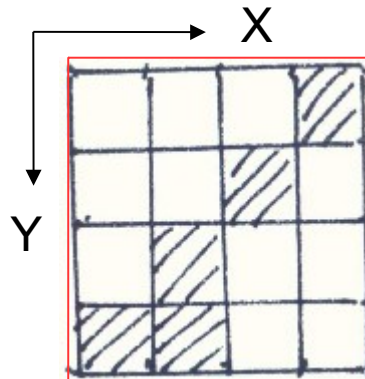
Example: See my handout

$$a = 7$$

$$b = -8$$

$$c = 6$$

$$\text{Theta} = -41.4375$$



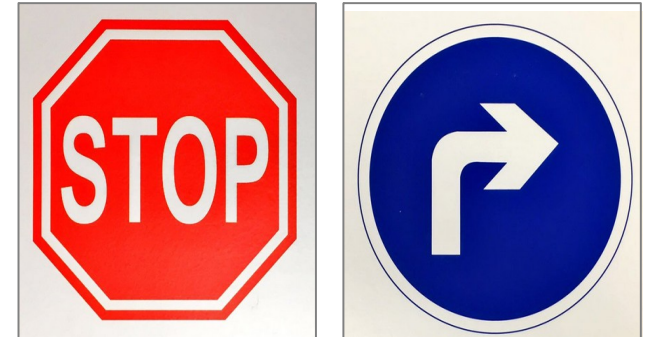
Reference: Robot Vision, by
BPK, Horn, Chapter 3, pp. 46-
64

Note: my hand calculation use
integer, when have access to
computer, use Float!
($\bar{x} = 2.8$ changed to 3, and
 $\bar{y} = 2.4$ changed to 2)

Raw Moments

The "raw moment" of order $(p + q)$ for image $f(x,y)$ is defined as:

$$M_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^p y^q f(x, y) dx dy \quad (1)$$



For the discrete function, we have:

$$M_{ij} = \sum_x \sum_y x^i y^j I(x, y) \quad (2)$$

Note: image $I(x,y)$ can be binary image or gray scale images. But we start the discussion from the binary images first.

We can treat image intensity as its probability density function

$$\sum_x \sum_y I(x, y) \quad (3)$$

Reference: Robot Vision, by BPK, Horn, Chapter 3, pp. 46-64

Python Example For Moments

First, let's find contours, by openCV.org definition, “Contours can be explained simply as a curve joining all the continuous points (along the boundary), having same color or intensity. The contours are a useful tool for shape analysis and object detection and recognition.”

Note: In OpenCV, object to be found should be white and background should be black when applying contour finding function.

```
cv2.findContours(thresh,cv2.RETR_TREE,cv2.CHAIN_APPROX_SIMPLE)
```

The arguments: the 1st is source image, 2nd is contour retrieval mode, 3rd is contour approximation method. And it outputs the contours and hierarchy. contours is a Python list of all the contours in the image. Each individual contour is a Numpy array of (x,y) coordinates of boundary points of the object.

```
im = cv2.imread('test.jpg')
imgray = cv2.cvtColor(im,cv2.COLOR_BGR2GRAY)
ret,thresh = cv2.threshold(imgray,127,255,0)
im2, contours, hierarchy = cv2.findContours(thresh,cv2.RETR_TREE,cv2.CHAIN_APPROX_SIMPLE)
```


Compute Contours Features

https://docs.opencv.org/3.1.0/dd/d49/tutorial_py_contour_features.html

1. Moments

```
1 import cv2
2 import numpy as np
3
4 img = cv2.imread('star.jpg',0)
5 ret,thresh = cv2.threshold(img,127,255,0)
6 contours,hierarchy = cv2.findContours(thresh, 1, 2)
7
8 cnt = contours[0]
9 M = cv2.moments(cnt)
10 print M
```

2. Contour Area

```
area = cv2.contourArea(cnt)
```

3. Contour Perimeter

```
perimeter = cv2.arcLength(cnt,True)
```

4. Contour Approximation

```
1 epsilon = 0.1*cv2.arcLength(cnt,True)
2 approx = cv2.approxPolyDP(cnt,epsilon,True)
```

5. Convex Hull

 Convexity defects

checks a curve for convexity defects and corrects it

```
hull = cv2.convexHull(cnt)
```

6. Checking Convexity

```
k = cv2.isContourConvex(cnt)
```

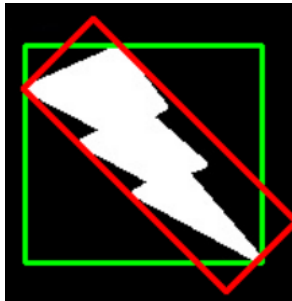


7.a. Straight Bounding Rectangle

```
1 x,y,w,h = cv2.boundingRect(cnt)
2 cv2.rectangle(img,(x,y),(x+w,y+h),(0,255,0),2)
```

7.b. Rotated Rectangle

```
1 rect = cv2.minAreaRect(cnt)
2 box = cv2.boxPoints(rect)
3 box = np.int0(box)
4 cv2.drawContours(img,[box],0,(0,0,255),2)
```



Compute Contours Features

https://docs.opencv.org/3.1.0/dd/d49/tutorial_py_contour_features.html

8. Minimum Enclosing Circle

```
1 (x,y),radius = cv2.minEnclosingCircle(cnt)
2 center = (int(x),int(y))
3 radius = int(radius)
4 cv2.circle(img,center,radius,(0,255,0),2)
```



9. Fitting an Ellipse

```
1 ellipse = cv2.fitEllipse(cnt)
2 cv2.ellipse(img,ellipse,(0,255,0),2)
```



10. Fitting a Line

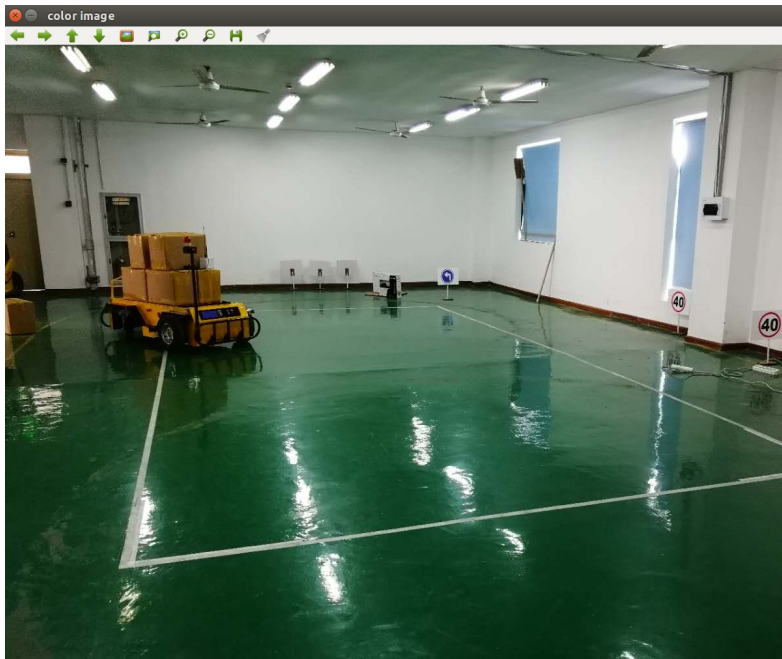
```
1 rows,cols = img.shape[:2]
2 [vx,vy,x,y] = cv2.fitLine(cnt, cv2.DIST_L2,0,0.01,0.01)
3 lefty = int((-x*vy/vx) + y)
4 righty = int(((cols-x)*vy/vx)+y)
5 cv2.line(img,(cols-1,righty),(0,lefty),(0,255,0),2)
```



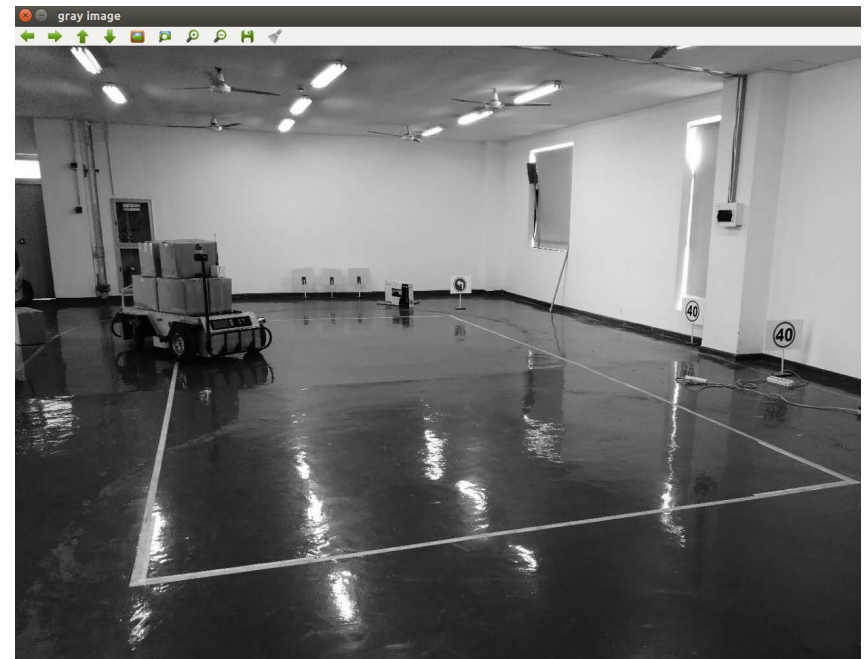
Separation of Floor Track Example With Practical Challenge



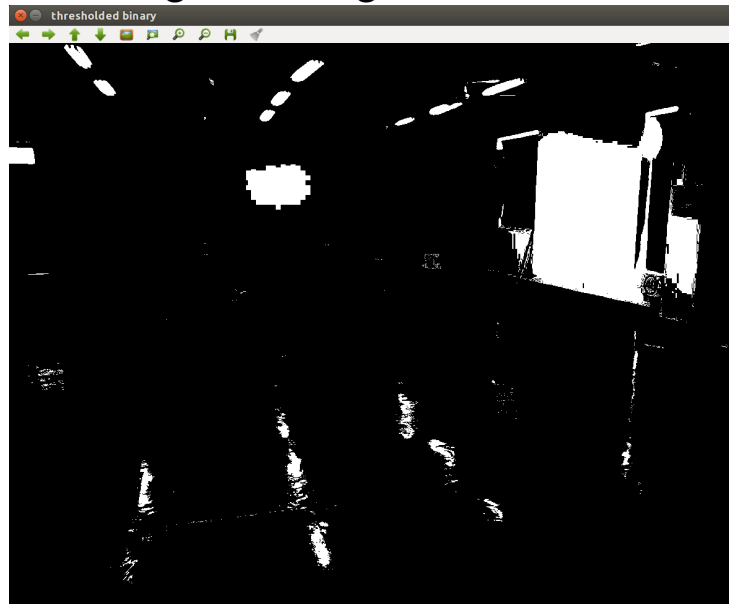
Team Homework Separation of Floor Track



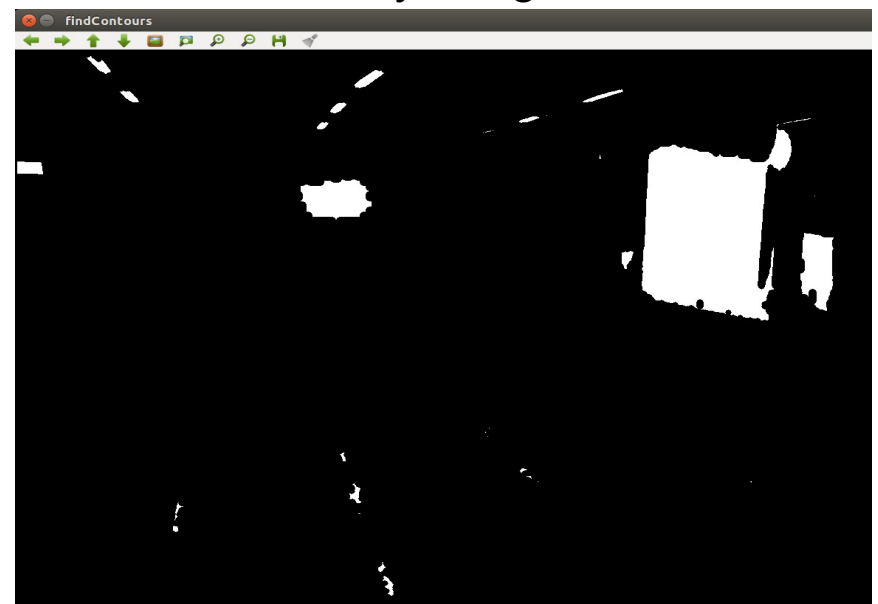
Original image



Gray-image



thresholdbinary



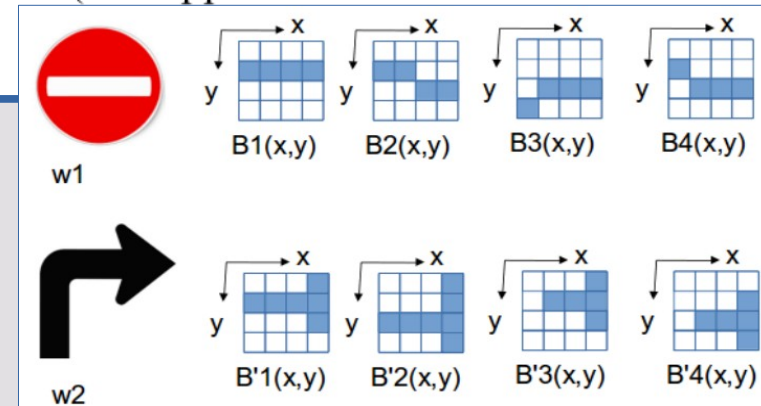
findcontour

Difference =
Original – high
intensity
thresholded
image
>> image with
removal of high
intensity region

Computation of Moments

QUESTION 3 (15 Points) Given two traffic signs and their binarized images taken from different conditions as shown in the following figure, design a machine learning technique by answering the following questions:

5.1 (5 pts) Based on given 2 classes of image, find moments m_{01} , m_{10} for each of the image, and form feature vector space with your computation result (see Appendix for m_{pq} definition if needed).



	w_{11}	w_{12}	w_{13}	w_{14}	w_{21}	w_{22}	w_{23}	w_{24}
u_y/m_{01}	2/5	2/5	3/5	2/5	2/10	(194)/10	(8/5)/(2/5)	3/10
u_x/m_{10}	2.5/5	2.5/5	2.5/5	2.5/5	(18/6)/10	(22/7)/(1/7)	(17/5)/10	(17/5)/10

$$m_{10,23} = (4 - \frac{17}{5}) + (2 - \frac{17}{5}) + (3 - \frac{17}{5}) + (4 - \frac{17}{5})$$

$$= \frac{6}{5} + (-\frac{7}{5}) + (-\frac{2}{5}) + \frac{3}{5} = 0$$

$$m_{10,24} = (4 - \frac{17}{5}) \times 2 + (2 - \frac{17}{5}) + (3 - \frac{17}{5}) + (4 - \frac{17}{5}) = 0$$

(2) PART II. K-means Cluster Algorithm.
 Use (u_x, u_y) to form feature vectors.
 Then, apply OpenCV. K-mean

For Class 2, $w_{aj}, j=1,2,3,4$

$$u_{x21} = [4 + (1+2+3+4) + 4] \frac{1}{A} = 18/6$$

$$u_{x22} = [4 \times 3 + (1+2+3+4)] \frac{1}{A} = 22/7$$

$$u_{x23} = [4 \times 2 + (2+3+4)] \frac{1}{A} = 17/5$$

$$u_{x24} = [4 \times 2 + (2+3+4)] \frac{1}{A} = 17/5$$

$$m_{10,21} = [(4 - \frac{18}{6}) + (1 - \frac{18}{6}) + (2 - \frac{18}{6}) + (3 - \frac{18}{6}) + (4 - \frac{18}{6})] \frac{1}{A}$$

$$= [\frac{6}{6} - \frac{12}{6} - \frac{6}{6} + 0 + \frac{6}{6} + \frac{6}{6}] \frac{1}{6} = 0$$

$$m_{10,22} = [(4 - \frac{22}{7}) \times 3 + (1 - \frac{22}{7}) + (2 - \frac{22}{7}) + (3 - \frac{22}{7}) + (4 - \frac{22}{7})] \frac{1}{A}$$

$$= (-\frac{45}{7} - \frac{15}{7} - \frac{8}{7} - \frac{1}{7} + \frac{6}{7}) \frac{1}{A} = (-\frac{63}{7}) \frac{1}{A} = (-9) \frac{1}{A}$$