CMPE163 August 20 (Fri)
Organizational Meeting

1) Harry Li E-mail:
hua.li@sjsu.edu
(650) 400-1116 Text
Office: M.W. 3:40-4:40 pm.
Zoom ID + Passcode
is the Same as
what you have today.
Lecture Zoom Link sent to
the class today.
Note: Homework, Projects
Announcements will be made
in Class, posted on Line as
github
CANVAS, Submission of homwork
projects will be
on CANVAS.

Text Books + References (optional)
a. Unity Tutorial, 3D Graphics
Game Dev. Engine
b. Other Optional Text Books —
Reference Only.

Programming Languages + Software IDE
1. Unity, Student or Personal
Edition. → Karting Game

2. Python for Graphics
Video, Version 3.6 or
higher.
Anaconda; Tool for
Python Programming →
3. C/C++ for 2D & 3D
Graphics, Videos.
4. C# for Interface to
Unity IDE.
→ 5. Open CV. Homework:
Installation of Open CV.
In 2 Weeks Sept. 2nd (Th)
→ 6. OpenGL Installation
of OpenGL. Homework:
Installation, and have it
ready By Next week
Aug. 26 (Th) Before
five pm..
→ 7. O.S. Ubuntu 18.04

Installation of Unity
By Aug. 26 (Th).
Before 4:00 pm.

Grading Policy:
30% Projects, Homework etc.
30% midterm (ONE)
40% Final (Comprehensive)

Conduct of the Class
1° Lecture 2° Show+Tell
3° Form A team, 2-3 Person Team.
All homework, coding have to be individual, however teamwork is encouraged, and be required

Projects, Homework: Assigned Projects. (3 projects)
plus A-Semester-Long project (Team Project)
a 2-3 person team;
b Proposal of A-Semester_Long Project;
c Progress Report & Presentation During Class show+tell
d Final Presentation (P.P.T. Demo)

3 projects.
Project to Build 3D Animated Graphics.
Virtual Camera + Video

"Game"-Like Environment
{ Robotics
Self-Driving.

August. 26 (Th)
Topics 1° Software Development Tool
2° Vector Graphics
3D Vector Graphics.
Reference Link: github/flyalibi
Software Tool: First, Unity Up By Friday
OpenGL Installation on your Machine
Example: Running Unity.
"Karting" Game
Start the Unity.
Step 1. On the Right hand UI. Interactive Tutorial Panel (Window)
Select/Go through 2 Tutorial
First Tutorial — play the Karting Game
Step 2. UI Editor
a Scene View Window
b Hierarchy Window

3D Graphics + Video

Hierarchy: "Everything" Defined in this Window,

a PAN ;
b Zoom In/Out, c Orbit Movment (Virtual Camera)

Use this platform to modify the "Karting" Game. Removal of Some/all
- 3D Objects
- Re-Building 3D Scene. (3D World coordinate)

Introduction to 2D Vector Graphics.

Dimensional Description ⇑ Vertices (Vertex) To Define Graphics Pattern(s) ⇑ $\{\vec{P_i} \mid i=1,2,\dots,4\}$

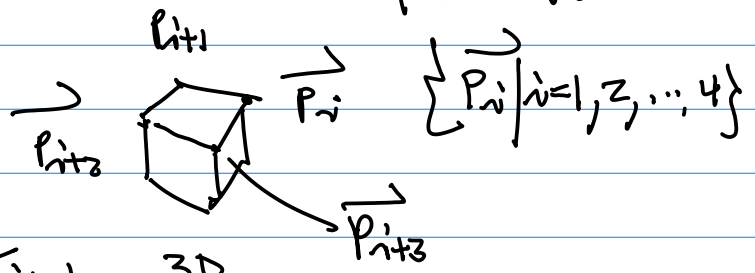


Fig. 1. 3D

Vector ⇒ Vertex ⇒ Point
⇐ ⇐

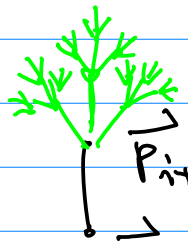


Fig. 2 2D Vector Graphics

2D Vector Definition of a Line Segment

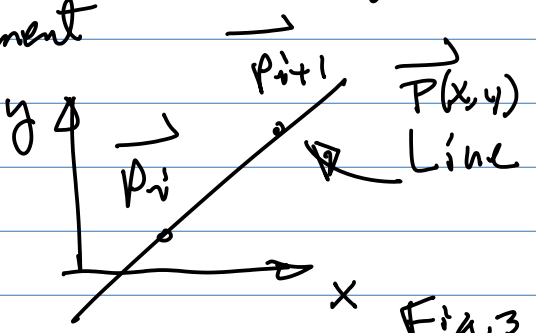


Fig. 3

x-y Coordinate System
"Virtual" Display Coordinate System

Primitive Graphics
2 pts to uniquely define a line
$\vec{P_i}$, $\vec{P_{i+1}}$

Notation
$\vec{P_i}$ Short Hand Notation
$\vec{P_i}(x_i, y_i)$, x_i -, y_i - Comp.
$\vec{P_i}(x_i, y_i) = (x_i, y_i)$ for Coding in C/C++, Python, ...

To Define A Line
(1) Direction of the Line

$$\vec{d} = \vec{P_{i+1}} - \vec{P_i} \quad \dots (1)$$

Ending pt. Starting pt

Eqn(1), C can be written as follows

$$\vec{d}(x_d, y_d) = \vec{P}_{i+1}(x_{i+1}, y_{i+1}) - \vec{P}_i(x_i, y_i) \quad \cdots (1-a)$$

For Coding purpose,

$$\begin{cases} x_d = x_{i+1} - x_i & (1-b) \\ y_d = y_{i+1} - y_i & (1-c) \end{cases}$$

Write C-Code for the directional vector in Eqn(1-b), (1-c)

Question: How to find the Ending pt from Eqn (2a) ?

if $\lambda = 1$

$$\vec{P}(x,y) = \vec{P}_i(x_i, y_i) + 1 \cdot \left(\vec{P}_{i+1}(x_{i+1}, y_{i+1}) - \vec{P}_i(x_i, y_i)\right)$$

$$= \cancel{\vec{P}_i(x_i, y_i)} + \vec{P}_{i+1}(x_{i+1}, y_{i+1}) - \cancel{\vec{P}_i(x_i, y_i)}$$

$$= \vec{P}_{i+1}(x_{i+1}, y_{i+1}) \text{ Ending pt.}$$

```
x_d[i] = x[i+1] - x[i] ; // for x-Comp of the directional vector
y_d[i] = y[i+1] - y[i] ; // for y-Comp. of the directional Vector.
```

$\cdots (1-d), (1-e)$

(2) Need A pt to make an Unique Line

$$\vec{P}(x,y) = \vec{P}_i(x_i, y_i) + \lambda \vec{d}(x,y) \quad \cdots (2)$$

where λ is scalar

Physical meaning: $\vec{P}(x,y)$ Any pt. on the Line

$\vec{P}_i(x_i, y_i)$ A given pt (Known) on this Line

$d(x,y)$, A directional vector of the Line

Let $\lambda = 0$, $\vec{P}(x,y) = \vec{P}_i(x_i, y_i)$ starting pt.

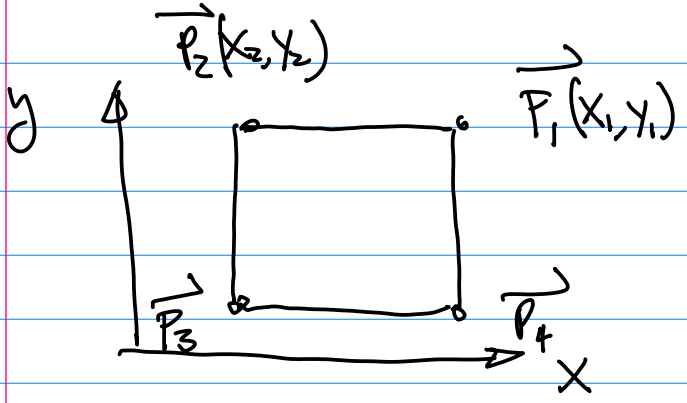
From Eqn(2), $\vec{P}(x,y) = \vec{P}_i(x_i, y_i) + \lambda \left(\vec{P}_{i+1}(x_{i+1}, y_{i+1}) - \vec{P}_i(x_i, y_i)\right) \cdots (2a)$

Screen Saver a collection of 2D Rotating Patterns. (Squares)

Example: Using Eqn (2a) to Create 2D Rotating Squares as a Screen Saver.

Define 2 Vectors (pts)

Step 1. $\vec{P}_i(x_i, y_i)$, and $\vec{P}_{i+1}(x_{i+1}, y_{i+1})$



$\vec{P}_1(x_1, y_1) = (60, 60)$, $\vec{P}_2(x_2, y_2) = (10, 60)$

And to Define A Line in Parallel with $\vec{P}_1$ & $\vec{P}_2$

$\vec{P}_3(x_3, y_3) = (10, 10)$, $\vec{P}_4(x_4, y_4) = (60, 10)$

Connect $\vec{P}_2$ to $\vec{P}_3$, Similarly $\vec{P}_1$ to $\vec{P}_4$

Therefore, we have formed A square

Line Equation for Line (Top Line)

$$\vec{P}(x,y) = \vec{P}_1(x_1, y_1) + \lambda\left(\vec{P}_2(x_2, y_2) - \vec{P}_1(x_1, y_1)\right) \quad \ldots(3a)$$

Line for $\vec{P}_2(x_2, y_2)$ and $\vec{P}_3(x_3, y_3)$

$$\vec{P}(x,y) = \vec{P}_2(x_2, y_2) + \lambda_2\left(\vec{P}_3(x_3, y_3) - \vec{P}_2(x_2, y_2)\right) \quad \ldots(3b)$$

And for the other 2 Lines

$$\vec{P}(x,y) = \vec{P}_3(x_3, y_3) + \lambda_3\left(\vec{P}_4(x_4, y_4) - \vec{P}_3(x_3, y_3)\right) \quad \ldots(3c)$$

And

$$\vec{P}(x,y) = \vec{P}_4(x_4, y_4) + \lambda_4\left(\vec{P}_1(x_1, y_1) - \vec{P}_4(x_4, y_4)\right) \quad \ldots(3d)$$

These 4 equations define the Boundary of the Square.

From Coding Aspect:

From Sample/Example (1-d), & (1-e)

Eqn (3a) becomes

$$\begin{cases} x = x_1 + \lambda(x_2 - x_1) & \ldots(4a) \\ y = y_1 + \lambda(y_2 - y_1) & \ldots(4b) \end{cases}$$

Define A buffer for x, And a buffer for y.

Each x_1, y_1, x_2, y_2 are also

Therefore C/C++ Coding Implementation for(i=a), (i=b) can be done accordingly.

Homework: Install openGL on your machine, By Next Lecture, So we will use it for Rotating Squares implementation.

Sept 2nd (Th)
Topics: 1° 2D Screen Saver Implementation;
Ref: github/tinwuli/openGL/
Homework: (To Be Submitted in 1 week) Submission 4:00pm. Sept. 9th (Th) (1pt)

Visit Homework Assignment on OpenGL, Source code .cpp has been posted.
Example: OpenGL CPP code
1° Create A program header template, Start your Program with this unified template
a. Program Name
b. Coded by
c. Date, d. Version
e. Status (Debugging, Release). f. Compilation and Built; g. Ref. (URL)
b. glBegin( );
⋮
glEnd( );
glClear( );
GL_POLYGON Keyword.
Vertex (pt)

Homework: GL_LINES
Modify the Sample code to draw a line with
$\overrightarrow{P_i(x_i, y_i)} = \overrightarrow{P_1(x_1, y_1)} = (50, 50)$
$\overrightarrow{P_{i+1}(x_{i+1}, y_{i+1})} = \overrightarrow{P_2(x_2, x_2)} = (60, 100)$

Compile your program, run it.
E-mail your Screen Capture, or 5 seconds Video Clips.

Submission in e-mail. Before Sept 9 4:30pm. (No point)

Create Rotating Squares for a Screen Saver.

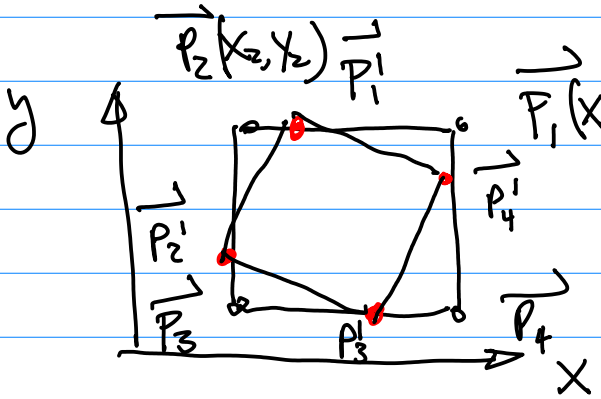


Fig 1. Note: $\vec{P}_1, \vec{P}_2, \cdots, \vec{P}_4$ are defined in A Counter Clockwise direction (Later in 3D Graphics we will do Hidden Line/Surface Removal)

From Eqn (2-a),

$$\vec{P}(x,y) = \vec{P}_1(x_1,y_1) + \lambda\left(\vec{P}_2(x_2,y_2) - \vec{P}_1(x_1,y_1)\right)$$

Let $\lambda = 0.8$ $\quad$ ~~0.5~~

$$\vec{P}(x,y) = \vec{P}_1(x_1,y_1) + \lambda\left(\vec{P}_2(x_2,y_2) - \vec{P}_1(x_1,y_1)\right) \quad \cdots (3a)$$

If $\lambda = 0.8$, $\vec{P}$ is 80% pt on the line formed by $\vec{P}_1$ & $\vec{P}_2$;

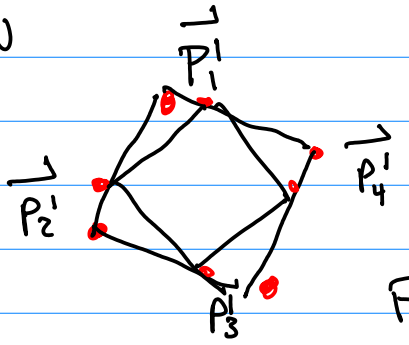


Fig. 2

Repeat the same process, however with New set of Points, $\vec{P}_1'$, $\vec{P}_2'$, $\vec{P}_3'$, $\vec{P}_4'$

Continue this process, we have:

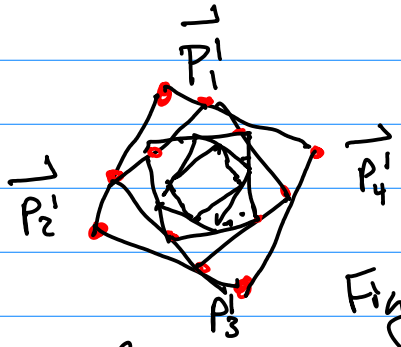


Fig. 3

To generalize this process, we introduce a superscript j as follows,

From Eqn (2a)

$$\vec{P}(x,y) = \vec{P_i}(x_i,y_i) + \lambda\left(\vec{P}_{i+1}(x_{i+1},y_{i+1}) - \vec{P_i}(x_i,y_i)\right) \quad \cdots (2a)$$

becomes

$$\vec{P_i}^{j+1}(x_i^{j+1}, y_i^{j+1}) = \vec{P_i}^{j}(x_i^j, y_i^j) + \lambda\left(\vec{P}_{i+1}^{j}(x_{i+1}^j, y_{i+1}^j) - P_i^j(x_i^j, y_i^j)\right) \quad \cdots (1)$$

The Above Equation can be written in Explicit form (x-Comp, y-Comp)

For x-Comp.

$$\begin{cases} x_i^{j+1} = x_i^j + \lambda(x_{i+1}^j - x_i^j) & \cdots (2a) \\ y_i^{j+1} = y_i^j + \lambda(y_{i+1}^j - y_i^j) & \cdots (2b) \end{cases}$$

C/C++ Code

```
x_buf[i][j+1] = x[i][j] - lamda*(x[i+1][j]-x[i][j]);
y_buf[i][j+1] = y[i][j] - lamda*(y[i+1][j]-y[i][j]);
```

Sample code Example: github/hualili/openCV ... /1_line.cpp.

https://github.com/hualili/opencv/blob/master/ComputerGraphics_AR/F2018/1_line.c

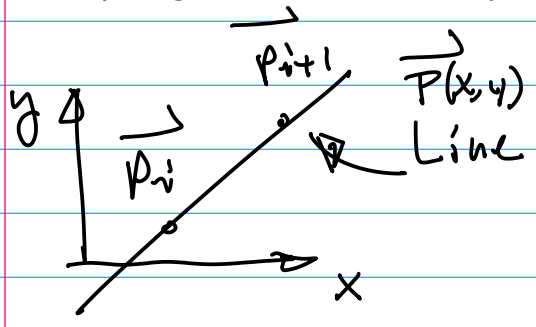


Fig.4

① Header

```
1  /*****************************************************
2   * Program: line.c   Coded by: Harry Li              *
3   * Version: x1.0;    status: tested;                 *
4   * Compile and build:                                *
5   * gcc main.cpp -o main.o -lGL -lGLU -lglut          *
6   * Date: Jun 5, 2014                                 *
7   * Purpose: Graphics Demo.                           *
8   *****************************************************/
9  #include<GL/glut.h>
10 #include<stdio.h>
```

② Libraries

③ $\vec{P_1}(x_1,y_1)$, $\vec{P_2}(x_2,y_2)$, $\vec{P_1}(x_1,y_1)=(1,1)$, $\vec{P_2}=(-1,-1)$

```
11 void mydisplay()
12 {
13 float p1x=1.0f,p1y=1.0f;    //the window coordinates (-1.0, 1.0)
14 float p2x=-1.0f,p2y=-1.0f;
```

```
15  glClear(GL_COLOR_BUFFER_BIT);
16  glLoadIdentity();
```

(4) Note: House Keeping for 2D Graphics

(5) { glBegin( );
    { glEnd( );

```
17  glBegin(GL_LINES);
18  glVertex2f(p1x,p1y);
19  glVertex2f(p2x,p2y);
20  glEnd();
```

GL_LINES

glVertex2f(x,y) → $\vec{P_i}(x_i, y_i) = (x_i, y_i)$

Note: In your homework, please 2D Sample code,

Sept. 9 (Th)

1. Show-And-Tell
2. Today's Topics: 2D Vector Graphics for Screen Saver Application, Squares (Rotating Pattern), Trees

Show+Tell Ken, Bill. Patrick, Anh

Homework: Implementation of Rotating Squares Based on Eqn (2.6), on pp 8.

Due A week from Today.

Show+Tell

Optional Homework: Write a script CS (C#) to Control your Design.

~ 2 weeks

Example: Creating a Tree By 2D Vector Graphics.

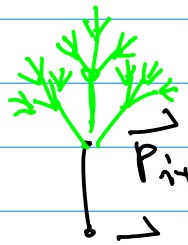


Fig. 1 2D Vector Graphics

Note: 1° The Levels of Iteration should be at Least 7 or Higher;

2° Random Function Generator to Allow Each tree to be placed at Random Locations. rand( );

3° Python Version Openbal for the Implementation is encouraged.

Description of the Algorithm:

1. You Design the Length of A Tree Trunck. By 2D Vectors. $\vec{P_i}$, $\vec{P_{i+1}}$

$\vec{P_i}$: Starting pt; $\vec{P_{i+1}}$ Ending pt.

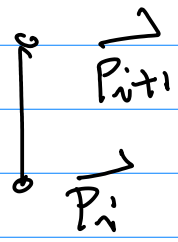


Fig 2

$\vec{P}_i(0,-10)$

$\vec{P}_{i+1}(0,0)$ ... (*)

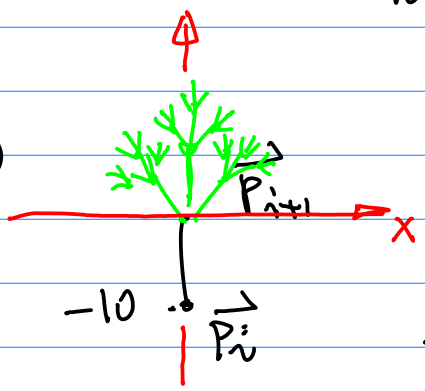
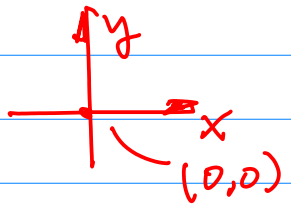


Fig 4

2. Generate Another (Next Level) Tree Branch → Main Branch (Same Direction) as its previous Level;

{ a. Same Direction

b. Reduction By 20%



From Eqn (1a), (1b), with the Given Condition (*), we have

$$\vec{P}'_{i+1} = \vec{P}_{i+1} + \lambda(\vec{P}_{i+1} - \vec{P}_i)$$

$$= (0,0) + \lambda((0,0) - (0,-10))$$

$$= (0,0) + \lambda(0,10) \quad \ldots (2)$$

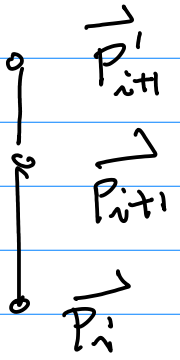


Fig 3

To find New Ending pt $\vec{P}'_{i+1}$, Let $\lambda = 0.8$, Substitute λ into the above equation

$$\vec{P}'_{i+1}(x'_{i+1}, y'_{i+1}) = \vec{P}_{i+1} + \lambda(\vec{P}_{i+1} - \vec{P}_i) \quad \ldots (1)$$

Starting Pt. ↑ Direction Vector of the previous Level

$$\vec{P}'_{i+1} = (0,0) + 0.8(0,10)$$

$$= (0 + 0.8 \times 0,\ 0 + 0.8 \times 10)$$

$$= (0,8)$$

$$\begin{cases} x'_{i+1} = x_{i+1} + \lambda(x_{i+1} - x_i) & \ldots (1a) \\ y'_{i+1} = y_{i+1} + \lambda(y_{i+1} - y_i) & \ldots (1b) \end{cases}$$

3. (Create the other 2 Side Branches (Left Branch, Right Branch)

Define A pt $\vec{P}_i(x_i, y_i)$
Rotate this pt. $\vec{P}_i(x_i, y_i)$ By α

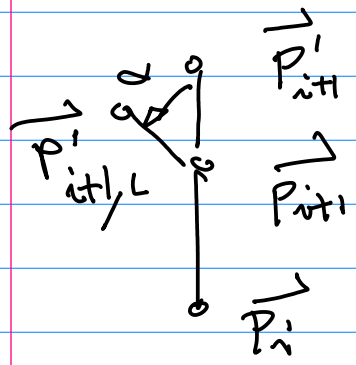


Fig 5.

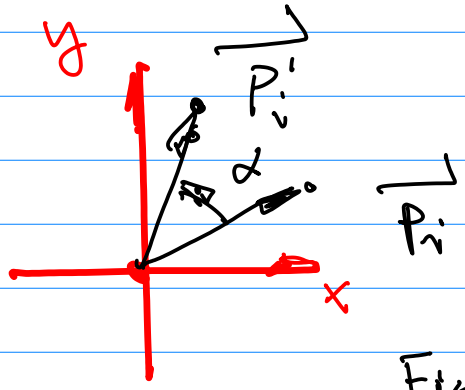


Fig 6.

Left Branch: Rotating the main Branch (at the Same Level) Counter Clockwise By α (Angle) Denote the New Branch as $\vec{P}'_{i+1,L}$

Note: Need a Newer Math. Formulation for this Rotation.

$$R_{3\times3} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \ldots(3)$$

$\vec{P}_i(x_i, y_i) = (x_i, y_i)$,

$\vec{P}_{i+1}(x_{i+1}, y_{i+1}) = (x_{i+1}, y_{i+1})$

$\Downarrow$ Col. Vector

$$\vec{P}'_i(x_i, y_i) = (x_i, y_i)^T = \begin{pmatrix} x_i \\ y_i \end{pmatrix}$$

$$\vec{P}'_{i+1} = \begin{pmatrix} x_{i+1} \\ y_{i+1} \end{pmatrix}$$

For Simplicity,

$$\vec{P}_i(x_i, y_i) = \begin{pmatrix} X_i \\ Y_i \end{pmatrix},$$

$$\vec{P}_{i+1}(x_{i+1}, y_{i+1}) = \begin{pmatrix} X_{i+1} \\ Y_{i+1} \end{pmatrix}$$

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A Pt $\overrightarrow{P_i} = \begin{pmatrix} x_i \\ y_i \end{pmatrix}$, Before the Rotation,

Pt. $\overrightarrow{P_i'} = \begin{pmatrix} x_i' \\ y_i' \end{pmatrix}$, After the Rotation

After the Rotation $\overset{?}{=}$ Before the Rotation,

Hence, we need Rotation matrix

Eqn (3),

Now, for the Branch to the Right,

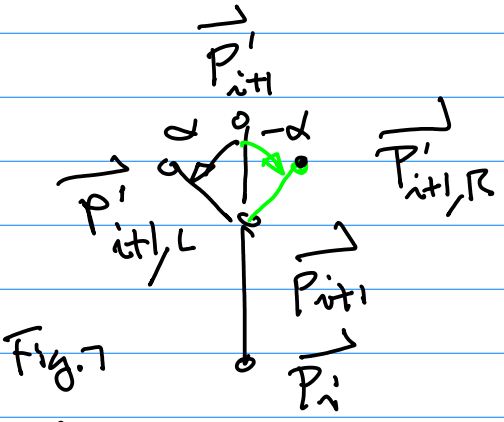

Fig. 7

After

$$\begin{pmatrix} x_i' \\ y_i' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix} \quad \text{Before} \quad \cdots (3b)$$

Rotation

Therefore, $-\sin(-\alpha) = \sin\alpha$

$$\begin{pmatrix} x'_{i+1,R} \\ y'_{i+1,R} \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x'_{i+1} \\ y'_{i+1} \\ 1 \end{pmatrix} \quad \cdots (5)$$

Suppose $\alpha = 30°$

$$\begin{cases} x_i' = x_i \cos\alpha - y_i \sin\alpha & \cdots (4a) \\ y_i' = x_i \sin\alpha + y_i \cos\alpha & \cdots (4b) \end{cases}$$

$$\begin{cases} x'_{i+1,R} = \cos\alpha \cdot x'_{i+1} + \sin\alpha \cdot y'_{i+1} & \cdots (6a) \\ y'_{i+1,R} = -\sin\alpha \cdot x'_{i+1} + \cos\alpha \cdot y'_{i+1} & \cdots (6b) \end{cases}$$

Note: 1° Positive α: α in Counter Clockwise direction;
Negative α: Clockwise

2° The Rotation in (3b) defines the Rotation w.r.t the origin of the x-y Coordinate System.

Team 1
- Christiana 1702,
- Anh 3807,
- Charles 0436
- Ken 3381

Team 2
- Christoffer 7230
- Patric 8001
- Anmol 9654

Team3 { Yong 4236
Zhonglin 6764

Semester Long - Project Proposal.

A Paragraph, 50 words, proposal.

Tech. Elements:
1. 3D Graphics
2. Interative
3. Presentation mode — Script features
4. Integrate Real/Live Video from File(s) And/or from a Camera

Guidelines for Topics Selection
Enhance/Improve Productive
Promote/Present Better Communication

By Sept. 23rd Submission to Email; fina.li@sjsu.edu

Show + Tell

Note: For more general cases,

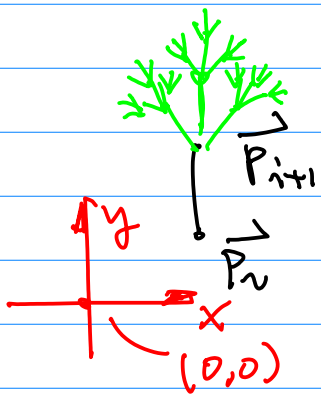

Fig.8

Sept 16 (Th)
Topics: 1° 2D Transforms
To project Implementation
Trees/Forest
Note: Homework (Show + Tell)
Homework (Un-Official)
Rotating Squares OpenGL
Reference: On git
github/fnwalli/OpenGL/
Computer~/F2018/~
2021F-b-... Homework.

(2 pts.)

Sept. 2nd
Homework: Implementation of Rotating Squares Based on Eqn(2a)-(2b), on pp 8.

Example: ON Window.

Note:
1. Always start your program with a Header;
a. Program Name:
Coded by:
Date:
Version:
Status:
Note: (Environment, O.S Compilation & Build)

2. Implementation.
   Vector Formulation w/o Rotation Matrix.

Note: SinX or CosX
Taylor Expansion (Taylor Series)

$$f(x)=f(x_0)+\frac{f'(x)}{1!}(x-x_0)+\frac{f''(x)}{2!}(x-x_0)^2+\cdots+R_n(x)$$

Look-up table.

3. Change Polygons to Lines
   2D Vertex is better for this implementation;
4. $\lambda = 0.8$ to Begin with, then when finish the entire program execution, modify λ, $\lambda = 0.95$, Observe the difference patterns.
   try $\lambda = 0.05$
5. Add Delay.
   Frame Rate (Refrash Rate for Display).
   30 FPS (Frames per Second)
   $f = 30Hz$;
   $T = 1/f = 33.3\ mS$.
6. Keep one color at time for a set of all rotating squares
7. Create multiple sets of Rotating Squares, By Defining "Anchor point" for Each set. Change
   2D Vertex (x_i, y_i)
   to
   2D Vertex (x_i+x_0, y_i+y_0)
   where my anchor point is (x_0, y_0)

Later, you can randomize the anchor point.

8. Modify the Line width

Note for the Homework, 1.3

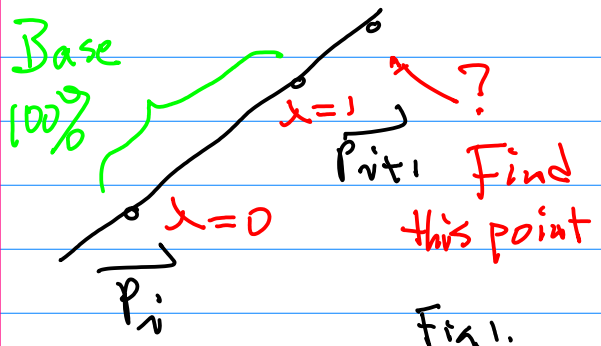

Fig1.

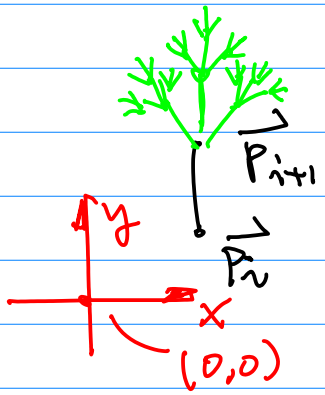

Fig3.

Now, Consider Question 2

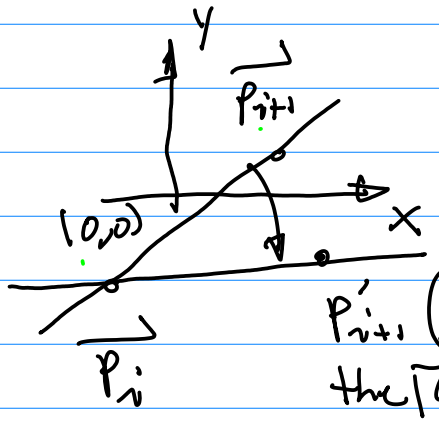

Fig2

$\vec{P}'_{i+1}$ (After the Rotation)

To Rotate the Tree Branches, we want to make sure the rotation is performed wrt the origin.

Translate $\vec{P}_{i+1}$ to the origin,

Translation matrix T to move $\vec{P}_{i+1}$ to the origin.

Pre-processing: to move $\vec{P}_i$ to the origin (0,0)

then, Rotation;

Post-processing.

$$x'_{i+1} = x_{i+1} + \Delta x \quad \text{(After)} \quad \text{(Before)} \quad \ldots (1)$$

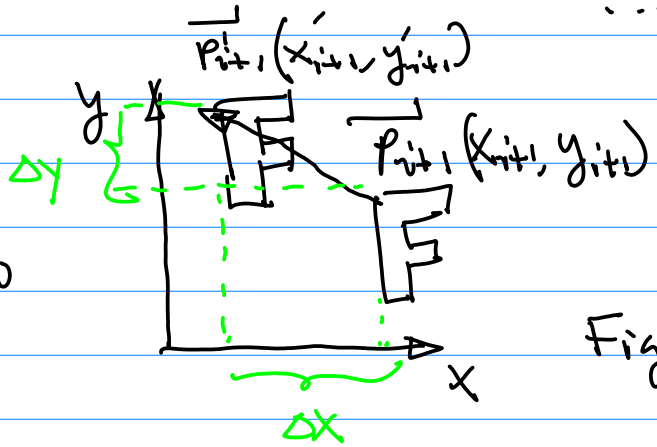

Fig4.

$$y'_{i+1} = y_{i+1} + \Delta y \quad \ldots (1b)$$

Sept. 23rd

Topic: Trees & 2D Transformation

Homework (One A week)

Composition.

Composition of 2D Transforms

$$\begin{pmatrix} x'_{i+1} \\ y'_{i+1} \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & \Delta x \\ 0 & 1 & \Delta y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{i+1} \\ y_{i+1} \\ 1 \end{pmatrix} \quad \cdots (2)$$

After Before

$\Delta x = -10$

$\Delta y = -10$

$\vec{P_1}$

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -10 \\ 0 & 1 & -10 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 10 \\ 10 \\ 1 \end{pmatrix}$$ verified.

Then, do this for $\vec{P_2}(10,20)$, find $\vec{P_2'}$

then, Rotation

$$R = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\alpha = 30$

$$\begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -10 \\ 0 & 1 & -10 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 10 \\ 20 \\ 1 \end{pmatrix} \quad \cdots (3)$$

Then post Processing.

Post Processing to undo the Pre-processing.

$$T^{-1} = \begin{pmatrix} 1 & 0 & -\Delta x \\ 0 & 1 & -\Delta y \\ 0 & 0 & 1 \end{pmatrix} \quad \cdots (4)$$

Substitute the given condition, we have

$$T^{-1} = \begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{pmatrix}$$

Integrate this into the Composition matrices, we have

$$T^{-1} R T =$$

$$\begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha & \\ \sin\alpha & \cos\alpha & \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -10 \\ 0 & 1 & -10 \\ 0 & 0 & 1 \end{pmatrix} \quad \cdots (5)$$