K-mean Algorithm (1)

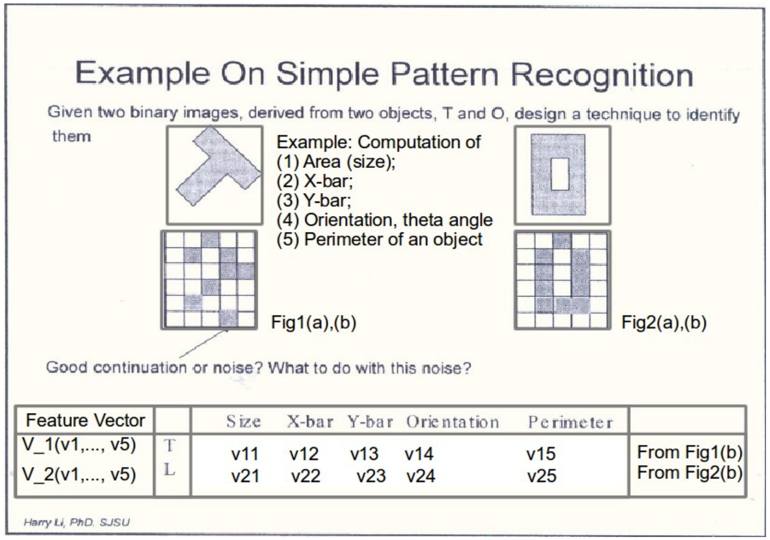
https://en.wikipedia.org/wiki/K-means_clustering

Given a set of observations ($\mathbf{x}_1$, $\mathbf{x}_2$, ..., $\mathbf{x}_n$), where each observation is a d-dimensional real vector, k-means clustering aims to partition the n observations into k ($\leq n$) sets $\mathbf{S} = \{S_1, S_2, ..., S_k\}$ so as to minimize the within-cluster sum of squares (WCSS) (i.e. variance). Formally, the objective is to find:

$$\underset{\mathbf{S}}{\arg\min} \sum_{i=1}^{k} \sum_{\mathbf{x} \in S_i} \left\| \mathbf{x} - \boldsymbol{\mu}_i \right\|^2 = \underset{\mathbf{S}}{\arg\min} \sum_{i=1}^{k} |S_i| \operatorname{Var} S_i$$

Example:

Form feature vectors


Example On Simple Pattern Recognition

Given two binary images, derived from two objects, T and O, design a technique to identify them

Example: Computation of
(1) Area (size);
(2) X-bar;
(3) Y-bar;
(4) Orientation, theta angle
(5) Perimeter of an object

Fig1(a),(b)

Fig2(a),(b)

Good continuation or noise? What to do with this noise?

Feature Vector		Size	X-bar	Y-bar	Orientation	Perimeter	
V_1(v1,..., v5)	T	v11	v12	v13	v14	v15	From Fig1(b)
V_2(v1,..., v5)	L	v21	v22	v23	v24	v25	From Fig2(b)

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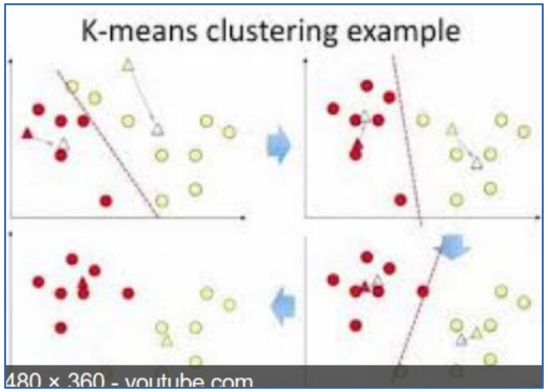


Cluster Seekingg

K-mean Algorithm (2)

https://en.wikipedia.org/wiki/K-means_clustering

Assignment step: Assign each observation to the cluster whose mean has the least squared Euclidean distance, this is intuitively the "nearest" mean.[7] (Mathematically, this means partitioning the observations according to the Voronoi diagram generated by the means).

$$S_i^{(t)} = \big\{ x_p : \big\| x_p - m_i^{(t)} \big\|^2 \le \big\| x_p - m_j^{(t)} \big\|^2 \ \forall j, 1 \le j \le k \big\},$$

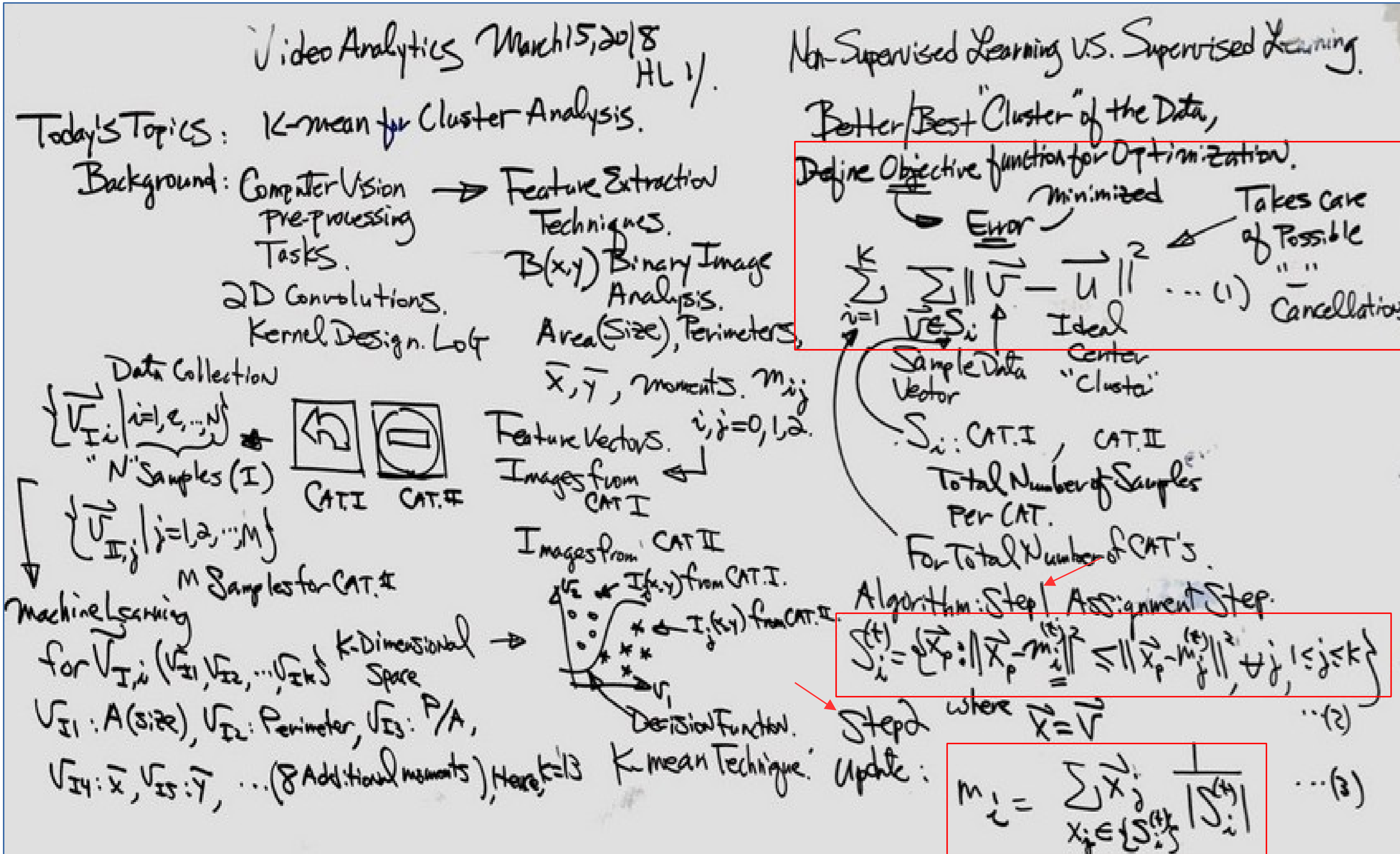
Update step: Calculate the new means to be the centroids

$$m_i^{(t+1)} = \frac{1}{|S_i^{(t)}|} \sum_{x_j \in S_i^{(t)}} x_j$$

Algorithm:

1) Assum Number of Classes $= K$.
   Pick $\vec{m}_i(t)\big|_{t=1}$, $i = 1, 2, \cdots, K$.
   Arbitrary.
   e.g. $\vec{m}_i(t) = \vec{X}_i$
2) $\|\vec{X}_\ell - \vec{m}_i(t)\| \le \|\vec{X}_\ell - \vec{m}_j(t)\|$ for $j \ne i$, $1 \le j \le K$
   Regroup $\vec{X}_\ell$ such that $\vec{X}_\ell \in \omega_i(S_i(t))$
3) Update $\vec{m}_i(t+1)$ for $i = 1, 2, \dots, K$, Check
   Termination $\vec{m}_i(t+1) = \vec{m}_i(t)$ Yes, Stop
   No, Goto 2

K-mean Example



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Hand Calculation

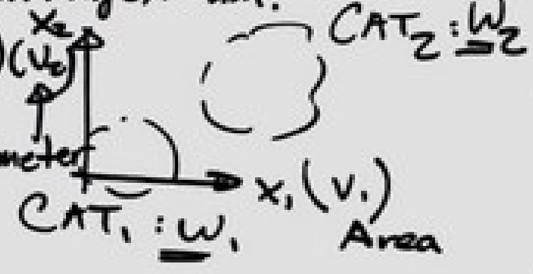
March 15, 2018 HL. 21.

Example: Given 2 CATs (classes), find the means, m_j, for $j=1,2$ Based ON K-mean Algorithm.

(See Handout pp1.) In Class.

$\vec{X} : \vec{V}$

Notation



Step 1. Arbitrary 2 pts as 2 classes & means $K=2$

$$m_i(t) \begin{cases} m_1(1) = \vec{X}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ m_2(1) = \vec{X}_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = (1,0)^t \end{cases}$$

mean.

Step 2. Compare Each pt with the mean from Each Class, to make sure Eqn (2) Satisfied.

$$\|\vec{X}_1 - \vec{m}_1\| \leq \|\vec{X}_1 - \vec{m}_2\| \text{ holds good}$$

Since,

$$\left\| \begin{pmatrix} x_{11} \\ x_{12} \end{pmatrix} - \begin{pmatrix} m_{11} \\ m_{12} \end{pmatrix} \right\| = (x_{11}-m_{11})^2 + (x_{12}-m_{12})^2 = 0^2+0^2 = 0$$

Similarly

$$\|\vec{X}_1 - \vec{m}_2\| = (x_{11}-m_{21})^2 + (x_{12}-m_{22})^2 = (0-1)^2+(0-0)^2 = 1 \rightarrow \vec{X}_1 \in \omega_1 \left(S_1(t=1) \text{ or, }\right)$$

And $\vec{X}_2$, we have

$$\|\vec{X}_2 - \vec{m}_2\| \leq \|\vec{X}_2 - \vec{m}_1\|$$

or.

$$\vec{X}_2 \in \omega_2 \left(S_2(t=1)\right)$$

for $\vec{X}_3$,

$$\underbrace{\|\vec{X}_3 - \vec{m}_1\|}_{=1} \leq \underbrace{\|\vec{X}_3 - \vec{m}_2\|}_{=2}$$

$$\vec{X}_3 \in \omega_1 \left(S_1(1)\right)$$

or.

Step 3. Update means.

$$\vec{m}(2) = \frac{1}{|S(1)|} \sum_{x_j \in \{S(1)\}} \vec{X}_j = \frac{1}{2}\left(\vec{X}_1 + \vec{X}_3\right)$$

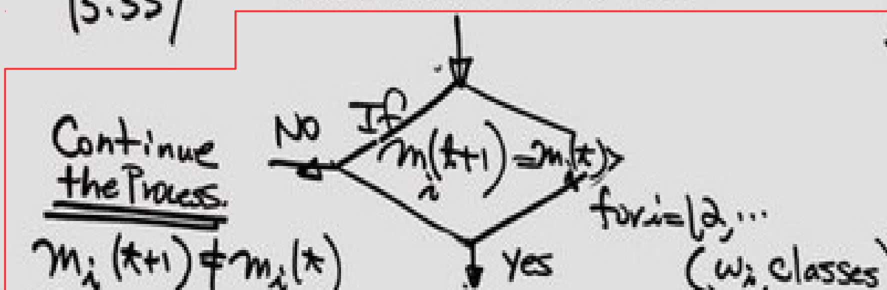
$$= \frac{1}{2}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix}$$

K-mean

CMPE297 March 5, 2018, Harry Li 3/.

$$\vec{m}_2(2) = \frac{1}{|S_2(1)|}\sum_{j\in\{S_2(1)\}} \vec{X}_j = \frac{1}{18}\left(\vec{X}_2+\vec{X}_4+\cdots+\vec{X}_{20}\right) = \begin{pmatrix}5.67\\5.33\end{pmatrix}$$

Terminate the Process?



Stop

Step 4

$$\|\vec{X}_l-\vec{m}_1(2)\| \le \|\vec{X}_l-\vec{m}_2(2)\|, \text{ for } l=1,2,\cdots,8$$

and

$$\|\vec{X}_p-\vec{m}_2(2)\| \le \|\vec{X}_p-\vec{m}_1(2)\|, \text{ for } p=9,10,\cdots,20$$

Hence, $S_1(2)=\{\vec{X}_1,\vec{X}_2,\cdots,\vec{X}_8\}$, $S_2(2)=\{\vec{X}_9,\vec{X}_{10},\cdots,\vec{X}_{20}\}$

update mean,

$$\vec{m}_1(3)=\frac{1}{10}\left(\vec{X}_1+\vec{X}_2+\cdots+\vec{X}_8\right)=\begin{pmatrix}1.25\\1.53\end{pmatrix}$$

$$\vec{m}_2(3)=\frac{1}{10}\left(\vec{X}_9+\vec{X}_{10}+\cdots+\vec{X}_{20}\right)=\begin{pmatrix}7.67\\7.33\end{pmatrix}$$

Check if $\vec{m}_i(t+1)\overset{?}{=}\vec{m}_i(t)$, $i=1,2$.

Since, not, continue,

Step 5. Check

$$\|\vec{X}_l-\vec{m}_1(3)\| \overset{?}{\le} \|\vec{X}_l-\vec{m}_2(3)\|$$

for $l=1,2,\cdots,8$.

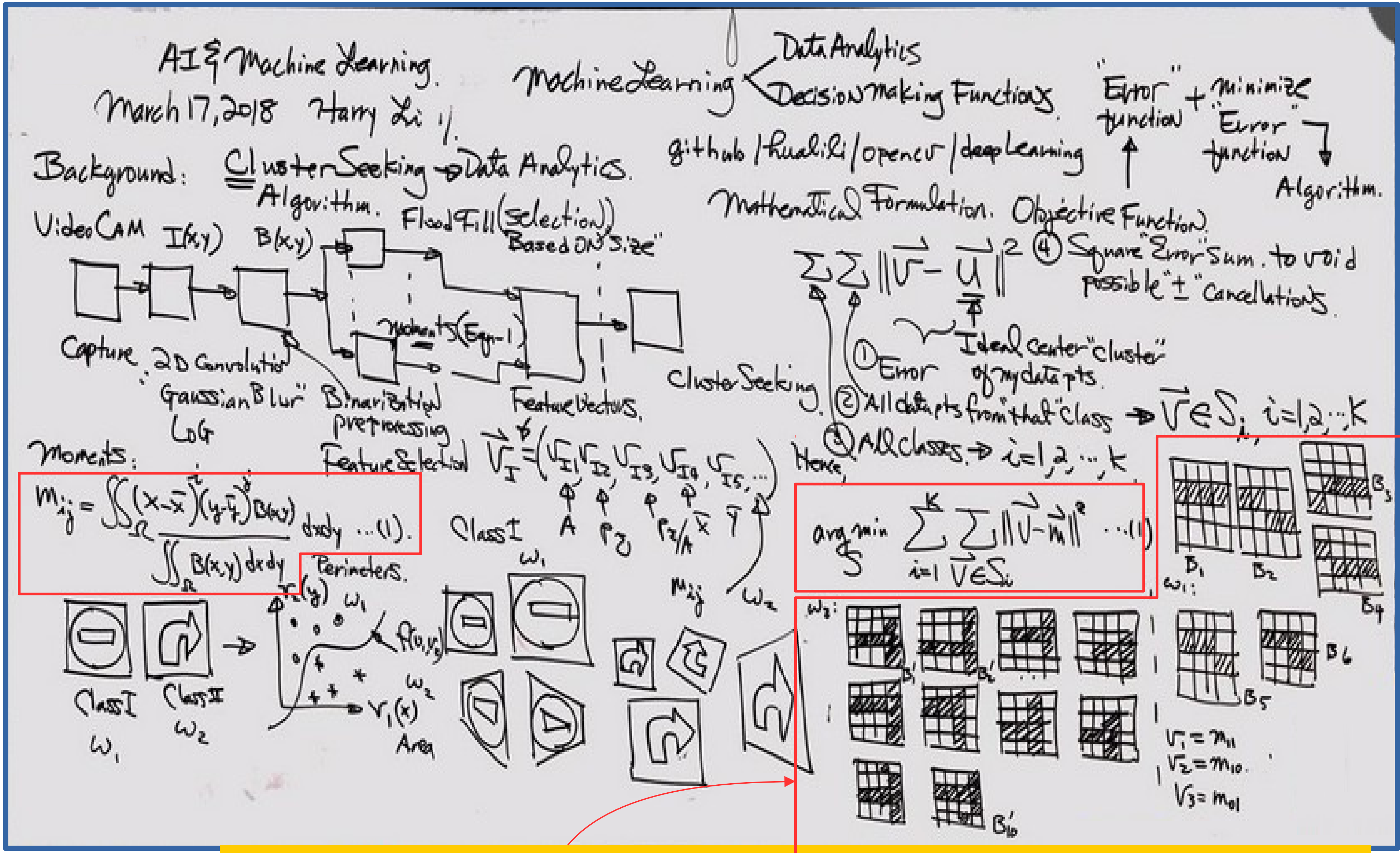
Similarly, $\|\vec{X}_p-\vec{m}_2(3)\| \overset{?}{\le} \|\vec{X}_p-\vec{m}_1(3)\|$

Both is Satisfied, No Regrouping, therefore,

$\vec{m}_i(t+1)\overset{?}{=}\vec{m}_i(t)$, for $i=1,2$.

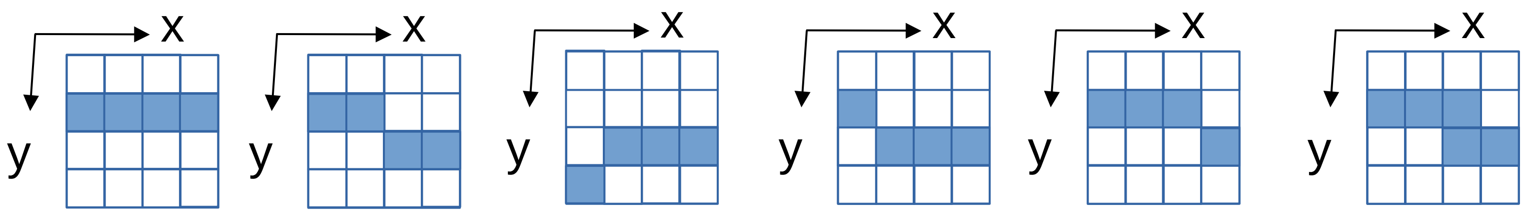
Yes! Converged, Stop.

Use kmean.py To Compute Example



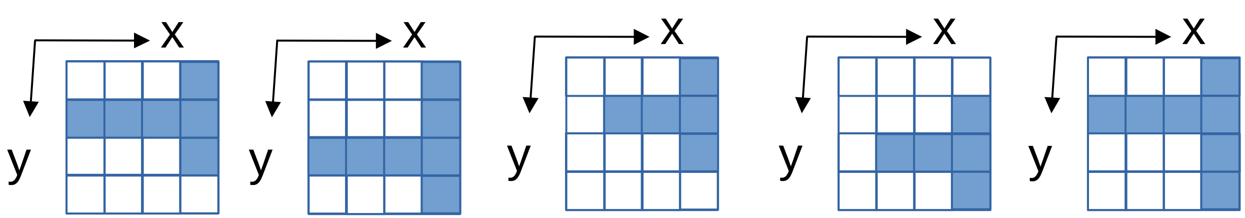
Example: w1: B1(x,y), …, B6(x,y), w2: B'1(x,y), …, B'10(x,y), V = (v1, v2, v3), where v1 = m11, v2=m10, v3=m01, find m1 and m2 by using kmean.py

kmean.py Example



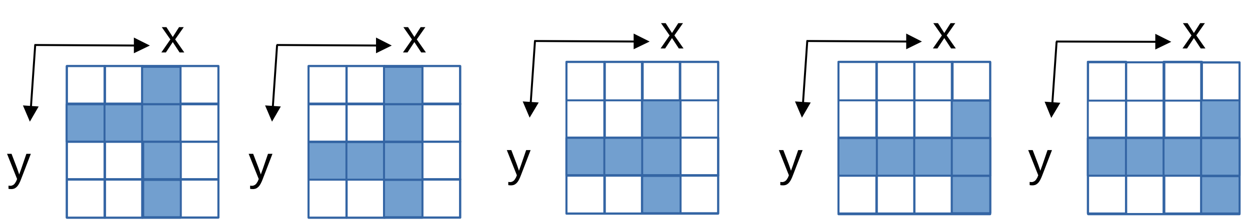
B1(x,y) B2(x,y) B3(x,y) B4(x,y) B5(x,y) B6(x,y)

w1



B'1(x,y) B'2(x,y) B'3(x,y) B'4(x,y) B'5(x,y)

w2



B'6(x,y) B'7(x,y) B'8(x,y) B'9(x,y) B'10(x,y)

Midterm Review

Midterm Review. Questions. Basic Concepts.

Colour Image Plane, Coordinate System Setup, Convolution (Shift + mult + Addition)

Kernels. (math Formulation to a mask)

- Forward Difference
- Backward ~
- Central Difference

LoG

$\nabla^2 G(x,y)$ ←

Computer Vision Filter OPS. ←

$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

$G(x,y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-x_0)^2+(y-y_0)^2}{2\sigma^2}}$ Non-Skewed

$(\sigma_x = \sigma_y)$

↓

Binarization $B(x,y)$

→ Flood Fill Algorithm Implementation → Feature Extraction: Area/Perimeter, moments.

$\vec{V_I} = (v_{I1}, v_{I2}, \ldots v_{IN})$

→ Cluster Analysis. K-mean Algorithm.

Computation:

Question: On Kernel Design, 2D Convolution, Feature Extraction. → Cluster Analysis.

Programming Language. Python

↓

Q/A. 14~20