タブロー法による恒真性の判定

Yudai Kubono

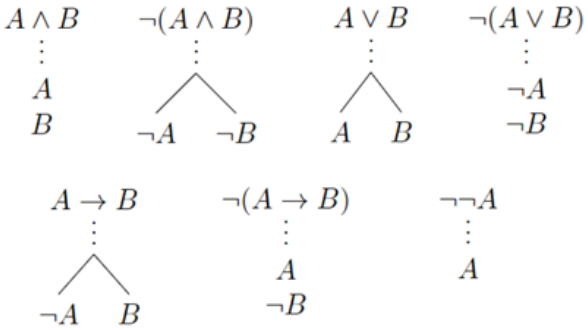
JAIST

July 16, 2022

Tableau System / Method

- Tableau method is a type of proof procedure that defines proof.
- We can genarate a tree (a directed acyclic graph satisfying the three conditions) as a proof by tableau system.
- The first step is creating a tree with the negation of the formula that is verified whether it is provable or not as its root. Next step is applying extention rules to a tree. It ends when any rules can not be applied to any unapplied nodes.

Branch Extention Rules



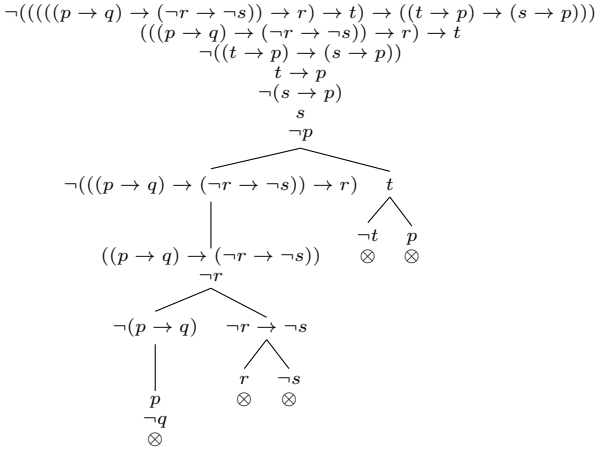
Closed Tableau and Open Tableau

Definition 1

- A path is a finite sequence of nodes $(v_1, \cdots, v_n)$ (i.e. formulas) where there are edges $\langle v_i, v_{i+1} \rangle$ for $i = 1, \cdots, n-1$.
- A branch (maximal path) is a path such that there are only literals as nodes that the rules are not applied to.
- A branch is closed if it contains A and $\neg A$.
- A tableau is closed if all branches are closed, and A tableau is open if it retains an open branch.
- A closed tableau for $\neg A$ is a proof of A.

Example

Meredith single axiom : $((((p \to q) \to (\neg r \to \neg s)) \to r) \to t) \to ((t \to p) \to (s \to p))$



Completeness Theorem

Theorem 1

There is a closed tableau for $\neg A$ ($\vdash A$) iff A is tautology ($\vDash A$).

Proof Strategy

Soundness ($\vdash A \Rightarrow \vDash A$)

1. $\nvDash A \Rightarrow \nvdash A$.
2. There is at least one branch such that all formulas in it are true, since $\neg A$ is satisfiable, and the fact that all rules preserve truth value from a upper formula to lower formulas.
3. It is an open branch.
4. There is an open tableau for $\neg A$.

Completeness ($\vDash A \Rightarrow \vdash A$)

1. $\nvdash A \Rightarrow \nvDash A$.
2. There is an open tableau for $\neg A$.
3. We can consider a valuation such that p is true when p appear in that open branch, and p is false when $\neg p$ appear in it for all literals in it.
4. Since all rules preserve truth value from lower formulas to a upper formula, all formulas in the branch are true on that valuation.
5. $\neg A$ is satisfiable.

Prefixed Tableau for Modal Logic

- 様相論理のタブロー法として、論理式に可能世界のラベル付けを行った冠頭論理式を扱う冠頭タブロー法がある。
- K4 に対応するタブローはその長さが無限に伸びてしまう可能性があり、kriple model における filtration に相当する操作をタブロー上で行い、有限の長さのタブローへと変換する必要がある（次回予定）。

[1] M. Fitting and R.L. Mendelsohn. *First-order modal logic*. Kluwer Academic Publishers, 1998

[2] 戸田山和久. 『論理学をつくる』. 名古屋大学出版会, 2000.