様相論理で弱模倣性を定義したい

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動機や設定した課題

1. 動機
   - **ブラックボックステストの知識表現**：外部から見えないプロセスを含むようなシステムを解析するための形式理論を考えたい.
2. Issues
   - **関係**：到達可能関係は存在するかしないかの 2 通りだけで、エージェントの知識状態を扱う. しかし、ブラックボックスのような知識状態が分からないエージェントに対する表現には、**到達可能関係が存在するか否か分からない**という意味の付値も考えられる.
   - **テストとしての適切さ**：ブラックボックステストとして適切であるかをどのように示すと良いのだろうか.(何か基準が欲しい)

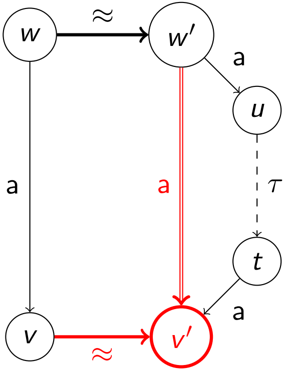
Undefined Relation

1. Motivation
   - 未定義な関係を導入したい. There are three value E, N and U.
   - $R^a_b(w, v) = E$: Agent a knows agent b's relation from w to v exists.
   - $R^a_b(w, v) = N$: Agent a knows agent b's relation from w to v doesn't exist.
   - $R^a_b(w, v) = U$: Agent a doesn't know whether agent b's relation from w to v exists or not. Actually, $R^a_b(w, v)$ may exist or may not exist.
2. Issue
   - If $U \equiv E \vee N$, then $E \to U$ and $N \to U$. But E and N isn't U. Is there a model satisfied these properties?
   - **Composition of relation** : If $R^a_b(w, v) = T$ and $R^a_b(v, u) = U$, then $R^a_b(w, u) = \ldots$?
3. Proposal
   - **Modal Logic** : I deal with Knowledge about valuation of relation by modal logic.

Representation of Testing

- Issue
  - What is how to show the correctness as black box testing ?
- Proposal
  - **Dynamic Epistemic Logic** : To deal with Knowledge and information change according to testing. I would like to infer others' knowledge by observing their action.
  - **Weak Simulation** を適切さの基準にしよう.

Weak Simulation(1/3)



- **Behavioural Equivalence** : Two labeled transition system have the same behavior.
- **Observational Equivalence** : For a given system, equality based purely on behavior observable from outside the system
- グラフの構造が同じならば、知識状態も同じになるだろうから、観測等価性を基準とするの良いかも.

Weak simulation(2/3)

Definition

- for each label $\alpha \in \Sigma$, there is an associated binary relation $\xrightarrow{\alpha}$ on S.
- $\tau \in \Sigma$ is the silent step.
- $\Rightarrow$ is the reflaxive and transitive closires of $\xrightarrow{\tau}$.
- $\stackrel{\alpha}{\Rightarrow} \quad := \quad \xrightarrow{\alpha} \circ \Rightarrow \circ \xrightarrow{\alpha}$
- $\stackrel{(\alpha)}{\Rightarrow} \quad := \quad \begin{cases} \Rightarrow & if\, \alpha = \tau \\ \stackrel{\alpha}{\Rightarrow} & otherwise. \end{cases}$

Weak Simulation(3/3)

Definition

Let $M = (S_1, \Sigma, \rightarrow_1)$ and $N = (S_2, \Sigma, \rightarrow_2)$ be two labelled state transition systems, with $\tau \in \Sigma$ the silent step.
A relation $\approx \subseteq S_1 \times S_2$ is called **weak simulation** if whenever $p \approx q$ and any labelled transition $p \xrightarrow{\alpha}_1 p'$, there is a state $q' \in S_2$ such that $p' \approx q'$ and $p' \overset{(\alpha)}{\Rightarrow}_2 q$.

Weak Simulation for the proposal

R^i_j : j's relation from the perspective of i.

Definition

Let $M = (W_1, \mathcal{A}, \{R^i_j \mid i, j \in \mathcal{A}\}, V_1)$ and $M = (W_2, \mathcal{A}, \{R^i_j \mid i, j \in \mathcal{A}\}, V_2)$ be Epistemic models.

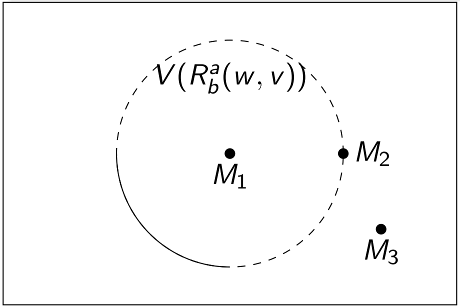
A relation $\approx \subset W_1 \times W_2$ defined by the following is called **weak simulation(仮)**:

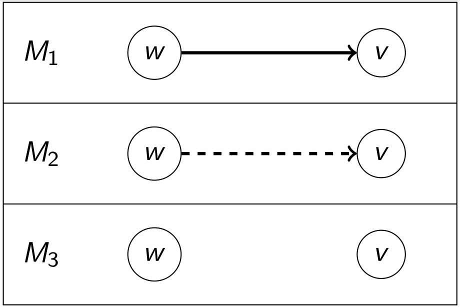
- if whenever $w \approx v$ and any relation $R^i_j(w, w') = U$, there is a world $v' \in W_2$ such that $v \approx v'$ and $R^i_j(v, v') = U$.
- if whenever $w \approx v$ and any relation $R^i_j(w, w') = E$, there is a world $v' \in W_2$ such that $v \approx v'$ and $R^i_j(v, v') \neq N$.
- if whenever $w \approx v$ and any relation $R^i_j(w, w') = N$, there is a world $v' \in W_2$ such that $v \approx v'$ and $R^i_j(v, v') \neq E$.

Definition of Undefined Relation(1/2)

Kripke Model

- $\mathfrak{M}$ is a set of kripke models ($|\mathfrak{M}| = 3^{|R|}$).
- $\mathcal{N} : M \to \mathcal{P}(\mathcal{P}(M))$ is a relation on $\mathfrak{M}$.
- $V(R_j^i(w,v))$ is a neighborhood of world M such that $M \models R_j^i(w,v)$.





Definition of Undefined Relation(2/2)

Neighborhood relation

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\begin{aligned}
\mathcal{N}(M) = \{M' \mid\ & \forall R^i_j(w,v)(M' \models R^i_j(w,v) = U \rightarrow M \models R^i_j(w,v) = U), \\
& \forall R^i_j(w,v)(M' \models R^i_j(w,v) = T \leftrightarrow M \not\models R^i_j(w,v) = F), \\
& \forall R^i_j(w,v)(M' \models R^i_j(w,v) = F \leftrightarrow M \not\models R^i_j(w,v) = T), \}
\end{aligned}
$$

Topological semantics

- $[\![\varphi]\!]_{\mathfrak{M}} = \{M \mid \mathfrak{M}, M \models \varphi\}$,
- $\mathfrak{M}, M \models \neg R^a_b(w,v) :\iff M \in \mathfrak{M} - ([\![R^a_b(w,v)]\!]_{\mathfrak{M}})$,
- $\mathfrak{M}, M \models \Box R^a_b(w,v) :\iff M \in Int([\![R^a_b(w,v)]\!]_{\mathfrak{M}})$,
- $\mathfrak{M}, M \models \Diamond R^a_b(w,v) :\iff M \in Cl([\![R^a_b(w,v)]\!]_{\mathfrak{M}})$.

Definition

- $\mathfrak{M}, M \models R^a_b(w,v) = E :\iff \mathfrak{M}, M \models \Box R^a_b(w,v) \iff M \in Int([\![R^a_b(w,v)]\!]_{\mathfrak{M}})$,
- $\mathfrak{M}, M \models R^a_b(w,v) = N :\iff M \in Int(\mathfrak{M} - [\![R^a_b(w,v)]\!]_{\mathfrak{M}})$
  $\iff \mathfrak{M}, M \models \Box\neg R^a_b(w,v)$,
- $\mathfrak{M}, M \models R^a_b(w,v) = U :\iff M \in Cl([\![R^a_b(w,v)]\!]_{\mathfrak{M}}) \cap Cl(\mathfrak{M} - [\![R^a_b(w,v)]\!]_{\mathfrak{M}})$,
  $\iff \mathfrak{M}, M \models \Diamond R^a_b(w,v) \wedge \Diamond\neg R^a_b(w,v)$,

Proof

- Proposition
  - **(i)** If $R^i_j(w,v) = E$ and $R^i_j(v,u) = N$, then $R^i_j(w,u) = N$.
  - **(ii)** If $R^i_j(w,v) = E$ and $R^i_j(v,u) = U$, then $R^i_j(w,u) = U$.
- Proof
  - **(i)** : Assume that $R^i_j(w,u) = E$ for each M. Since R^i_j is symmetric relation, $R^i_j(u,w) = E$. Since R^i_j is transitive relation, $R^i_j(u,v) = E$. Since R^i_j is symmetric relation, $R^i_j(v,u) = E$. Contradiction.
    Thus, There is no model M such that $M \models \Box R^i_j(w,u)$. Therefore, $R^i_j(w,u) = N$.
  - **(ii)** : Since $R^i_j(v,u) = U$, there are M_1, M_2 such that $M_1, M_2 \in \mathcal{N}(M)$, $M_1 \models R^i_j(v,u) = E$ and $M_2 \models R^i_j(v,u) = N$. By Transitivity, $M_1 \models R^i_j(w,u) = E$. By (i), $M_2 \models R^i_j(w,u) = N$. Thus, $M \models R^i_j(w,u) = U$.

Further studies

- **Awareness (Bi)simulation** : I would like to read the paper: "*Knowledge, Awareness, and Bisimulation*" [1]. Can we deal with the awareness bisimulation in Kubono's awareness logic with partition?
- **Relation may not satisfy symmetry** : In Epistemic Logic, S5 may not be natural. (For example, S4.2. S4.2 = S4 + directed)
  - Reflective and transitive.(S4)
  - directed : $(\forall x, y, z(xRy \land xRz) \rightarrow \exists u(yRu \land zRu))$.
    (intuition : the existence of u guarantees that there is no contradiction between the two poeces of information.)

Reference I

[1] Hans van Ditmarsch, Tim French, Fernando R Velázquez-Quesada, and Yì N Wáng.
Knowledge, awareness, and bisimulation.
arXiv preprint arXiv:1310.6410, 2013.