Introduction to Game Theory

Yudai Kubono

JAIST

November 20, 2022

Outline

- Non-cooperative static game
- Non-cooperative dynamic game with perfect information
- Mixed strategy
- Advanced concepts of game

Example 1: Prisoner's Dilemma

Example 1

Two prisoners, A and B, supposed to have committed a common crime, are being interrogated (取り調べ) in separate rooms. In order to get them to confess, the prosecutor (検事) offers them the following plea bargain (司法取引).

If they both confess, they will both be imprisoned for five years.

If only one of them confesses, the one who confesses will be released, but the one who keeps silent will be imprisoned for ten years.

If they both remain silent, their sentences (判決) will be reduced to two years in prison for lack of evidence.

A and B have no means of communicating with each other.

	B remains silent	B confesses
A remains silent	(-2,-2)	(-10,0)
A confesses	(0,-10)	(-5,-5)

Figure: A payoff matrix of prisoner's dilemma.

Non-cooperative static games with perfect information 1/2

Definition 1

A strategic form game with ordinal payoffs is a tuple $\langle \mathrm{Ag}, \{S_i\}_{i \in \mathrm{Ag}}, \{\pi_i\}_{i \in \mathrm{Ag}} \rangle$*, where:*

Ag *is a set of agents;*

S_i *is a set of* i*'s strategies;*

$\pi_i : S \to \mathbb{R}$ *is a* i*'s payoff function, where* $S = \prod_{i \in \mathrm{Ag}} S_i$*.*

- We denote a sequence of the players' strategies other than i as s_{-i}.

Non-cooperative static games with perfect information 2/2

- Typical solution concepts: Nash equilibrium

Definition 2

(s_i, s_{-i}) is Nash equilibrium if for all $i \in \mathrm{Ag}$ and $s_i' \in S_i$, $\pi_i(s_i, s_{-i}) \geq \pi_i(s_i', s_{-i})$.

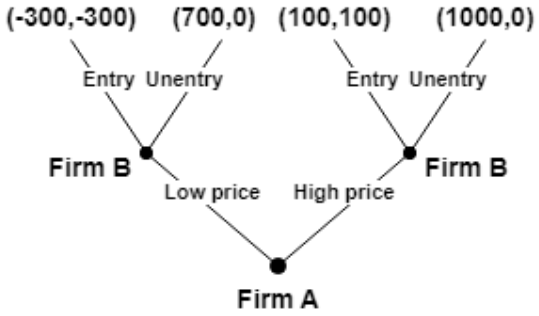
- Iterated Deletion of Strictly Dominated Strategies (IDSDS)

  We repeatedly exclude i's strictly dominated strategies from S_i for each i. Then, the tuple of strategies that remain at last is Nash equilibrium.

Example 2: Entry Deterrence Game

Example 2

Firm B is considering the entry into the monopoly market by firm A. A can prevent the entry by lowering the price, but excessive price reductions can cause severe damage to its own firm.



Non-cooperative dynamic games with perfect information 1/2

Definition 3

A extensive form game with ordinal payoffs is a tuple $\langle \mathrm{Ag}, T, \mathcal{X}, U, \pi \rangle$ *, where:*

Ag *is a set of agents;*

T *is a tree that is represented as* $(X, \bigcup_{i \in \mathrm{Ag}} A_i)$*, where* A_i *is a set of* i*'s alternatives;*

$\mathcal{X} = \{\mathcal{X}_i\}_{i \in \mathrm{Ag}}$ *is a partition of* $X \setminus L$*, where* L *is a set of leaves in* T*;*

U *is a refinement of* $\mathcal{X}$ *such that* $u \in U$ *is a singleton;*

$\pi : L \to \mathbb{R}^{|\mathrm{Ag}|}$ *is a payoff function.*

- Nodes other that leaves in a tree $(X \setminus L)$ is called moves.
- $\mathcal{X}_i$ represents a set of i's moves.
- $U_i = \{u \mid u \subseteq \mathcal{X}_i\}$ is called i's information set.
- A i's strategy in dynamic games is a function $s_i : U_i \to A_i$.

Non-cooperative dynamic games with perfect information 2/2

- Typical solution concepts: subgame perfect Nash equilibrium

Definition 4

Given $G = \langle \mathrm{Ag}, T, \mathcal{X}, U, \pi \rangle$, $G' = \langle \mathrm{Ag}, T', \mathcal{X}, U, \pi \rangle$ is a subgame of G, where T' is a subtree that is a tree such as a part of the original tree.

Definition 5

(s_i, s_{-i}) is subgame perfect Nash equilibrium if (s_i, s_{-i}) is Nash equilibrium in all the subgames.

- Backward induction

  Starting from the closest nodes to leaves downward, we sequentially exclude the leaf that is not optimal. Then, the leaves that remain are subgame perfect Nash equilibrium.

Mixed strategy

- Each player selects one probability distribution on pure strategies instead of one pure strategy.
- Suppose that $\{a, b, c\}$ is a set of pure strategies, an example of i's mixed strategy q_i can be expressed as $(q_i(a) = 1/3, q_i(b) = 1/3, q_i(c) = 1/3)$.
- A pure strategy can be interpreted as a special case of a mixed strategy.

Advanced Analysis: Game with imperfect information

- Game with imperfect information is a game where some players at the move do not know the past of other players' alternatives.
- These games are expressed by extensive form game removing the condition for U: $u \in U$ is a singleton and adding belief that is a function assigning probabilities to an information set.

Advanced Analysis: Game with incomplete information and Cooperative Game

- A game with incomplete information is a game where some of the rules are not common knowledge, such as players, alternatives, or payoff functions.
- This game is expressed by adding a concept of type representing information that gives an effect to a payoff function.
- Cooperative game is a game where players have a method of external enforcement of cooperative behavior.

[1] 岡田章. (2011) ゲーム理論　新版. 有斐閣.

[2] 梶井厚志・松井彰彦. (2000) ミクロ経済学　戦略的アプローチ. 日本評論社.