# Epistemic Action Logic

Yudai Kubono

JAIST

February 11, 2022

# Epistemic Action Logic

- Epistemic Action Logic (EAL) is a kind of dynamic epistemic logic to describe updating agents' epistemic states.
- An action are described by an action model, which is a structure of the action.
- An action model makes it possible to treat more complex communicative actions than PAL.

# Example: Misleading Private Announcement

**Example 1**

*Agent $a$ is secretly informed that $p$ is true, but the other agents misunderstand that $a$ secretly received the information that $p$ is false.*

# Syntax

- Let $\mathcal{P}$ be a set of atomic propositions, and $\mathcal{G}$ be a set of agents. The language $\mathcal{L}$ of $\mathrm{EAL}$ is the set of formulas generated by the following grammar:

  $$\mathcal{L} \ni \varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid K_i\varphi \mid [A, e]\varphi,$$

  where $p \in \mathcal{P}$, $i \in \mathcal{G}$, and $(A, e)$ is a pointed action model with
  (1) a finite domain $E$, and
  (2) for all $e \in E$, $\mathrm{pre}(e)$ is in $\mathcal{L}$ that is already constructed in the previous stage of the inductively defined hierarchy.
  Other connectives $\vee$, $\rightarrow$, and $\leftrightarrow$ are defined in the usual manner.

# Semantics

- A Kripke model and an action model give semantics.
- A Kripke model is a tuple $\langle W, \{R_i\}_{i\in\mathcal{G}}, V\rangle$ where a non-empty set of possible worlds $W$, an accessibility relation $R$ on $W$, and a valuation $V$.

# Action Model

**Definition 1**

*An action model is a tuple* $A = \langle E, \{R_i\}_{\mathcal{G}}, \mathrm{pre}\rangle$, *where:*

- $E$ *is a non-empty finite set of possible communicative events;*
- $R_i$ *is a binary possibility relation on* $E$*;*
- $\mathrm{pre} : E \to \mathcal{L}$ *is a function that assign a precondition to each event.*

- A pair $(A, e)$ is called a pointed action model. It refers to an action itself.
- $\{E, \{R_i\}_{i\in\mathcal{G}}, \mathrm{pre}\}$ in $A$ is denoted by $\{E^A, \{R_i^A\}_{i\in\mathcal{G}}, \mathrm{pre}^A\}$.

# Satisfaction Relation

**Definition 2**

*For any Kripke model $M$ and possible worlds $w \in W$, the satisfaction relation $\vDash$ is given as follows:*

$$
\begin{aligned}
M, w \vDash p &:\Leftrightarrow w \in V(p);\\
M, w \vDash \neg\varphi &:\Leftrightarrow M, w \nvDash \varphi;\\
M, w \vDash \varphi \wedge \psi &:\Leftrightarrow M, w \vDash \varphi \text{ and } M, w \vDash \psi;\\
M, w \vDash K_i\varphi &:\Leftrightarrow \text{ for all } v \text{ such that } (w, v) \in R_i, M, v \vDash \varphi;\\
M, w \vDash [A, e]\varphi &:\Leftrightarrow (M, w \vDash \mathrm{pre}^A(e) \Rightarrow M[A], (w, e) \vDash \varphi).
\end{aligned}
$$

# Product Update

**Definition 3**

*The Kripke model* $M[A] = \langle W', \{R'_i\}_{i \in \mathcal{P}}, V' \rangle$ *via the product update operation is defined as follows:*

- $W' := \{(w, e) \in W \times E \mid M, w \vDash \mathrm{pre}^A(e)\}$;
- $((w, e), (w', e')) \in R'_i :\Leftrightarrow (w, w') \in R_i$ *and* $((e, e')) \in R^A_i$;
- $V'(p) := V(p) \times E$.

# Representation of the Example

**Example 1**

*Agent $a$ is secretly informed that $p$ is true, but the other agents misunderstand that $a$ secretly received the information that $p$ is false.*

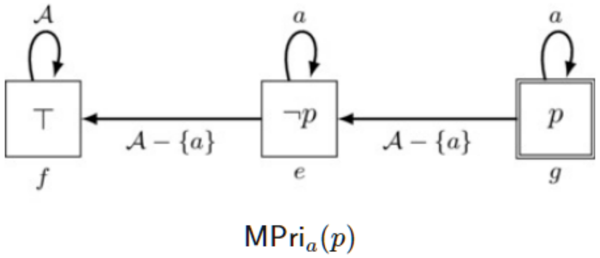


Figure: The pointed action model $(\mathrm{MPria}(p), g)$ (Baltag and Renne, 2016).

# Proof System **EAL**

Table: Axiom schemas and inference rules of **EAL**.

| Axiom schemas | |
|---|---|
| TAUT | The set of propositional tautologies |
| K | $K_i(\varphi \to \psi) \to (K_i\varphi \to K_i\psi)$ |
| T | $K_i\varphi \to \varphi$ |
| 5 | $\neg K_i\varphi \to K_i\neg K_i\varphi$ |
| AP | $[A,e]p \leftrightarrow (\mathrm{pre}(e) \to p)$ |
| AN | $[A,e]\neg\varphi \leftrightarrow (\mathrm{pre}(e) \to \neg[A,e]\varphi)$ |
| ACN | $[A,e](\varphi \wedge \psi) \leftrightarrow ([A,e]\varphi \wedge [A,e]\psi)$ |
| AK | $[A,e]K_i\varphi \leftrightarrow (\mathrm{pre}(e)$ $\to \bigwedge_{(e,f)\in R_i^A} K_i[A,f]\varphi)$ |
| ACM | $[A,e][B,f]\varphi \leftrightarrow [(A,e);(B,f)]\varphi$ |
| **Inference rules** | |
| MP | If $\vdash \varphi$ and $\vdash \varphi \to \psi$, then $\vdash \psi$ |
| KG | If $\vdash \varphi$ then $\vdash K_i\varphi$ |
| AG | If $\vdash \varphi$ then $\vdash [A,e]\varphi$ |

[1] A. Baltag and B. Renne. (2016). Dynamic epistemic logic. https://plato.stanford.edu/entries/dynamic-epistemic.

[2] Hans Van Ditmarsch, Wiebe van Der Hoek, and Barteld Kooi. (2007). *Dynamic epistemic logic*. Springer Science & Business Media.