Logic of Action: STIT Logic

Yudai Kubono

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Shizuoka University, Graduate School of Science and Technology

Logic of Action

- PDL (Propositional Dynamic Logic)

  $[p]\varphi$: 'For every possible execution of program p, φ holds afterwards.'
- PAL (Public Announcement Logic)

  $[\varphi!]\psi$: 'ψ is true after the truthful public announcement φ.'
- STIT logic

STIT Logic

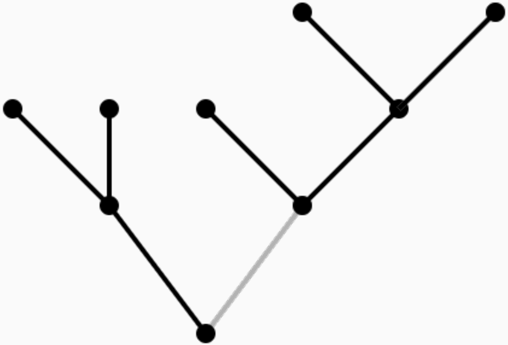
- STIT stands for ‘See To It That.’
- $\mathrm{stit}\varphi$: ‘an agent see to it that φ is true.’
- STIT logic does not directly describe actions.

Fundamental Notion

To formalize the notion of action, begin with two general observations:

i . *usually an agent is not able to select one possible future to become the unique actual future, but*

ii . *by his action he can make sure that certain futures, which before his action are possible, are no longer possible after his action. (Segerberg et.al., 2013)*



Syntax

Let $\mathcal{P}$ be a countable set of atomic propositions, $\mathcal{G}$ be a countable set of agents. The language $\mathcal{L}$ is the set of formulas generated by the following grammar:

$$\mathcal{L} \ni \varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \mathrm{stit}_i\varphi,$$

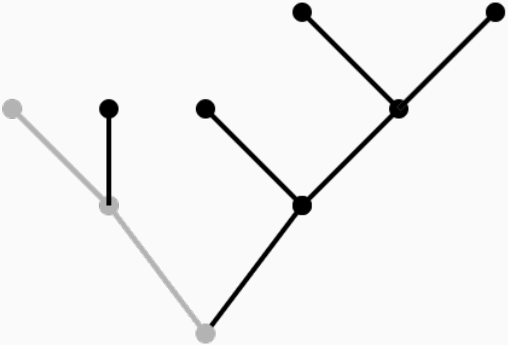
where $p \in \mathcal{P}$ and $i \in \mathcal{G}$. Other logical connectives $\vee$, $\to$, and $\leftrightarrow$ are defined in the usual manner.

Semantics

A stit model M is a tuple $\langle T, \leq, \{C_i^m\}_{m \in T, i \in \mathcal{G}}, V\rangle$, where:

- T is a set of moments;
- $\leq$ is a partial order of T, such that, for any m_1, m_2, m_3, if $m_1 \leq m_3$ and $m_2 \leq m_3$, then $m_1 \leq m_2$ or $m_2 \leq m_1$;
- $C_i^m : H_m \to 2^{(H_m)}$ is a fuction, where H_m is a collection of maximal sets of linearly ordered moments that contain m.
- $V : \mathcal{P} \times H \times T \to \{1, 0\}$, where H is a collection of maximal sets of linearly ordered moments;

- We call $h \in H$ a history.



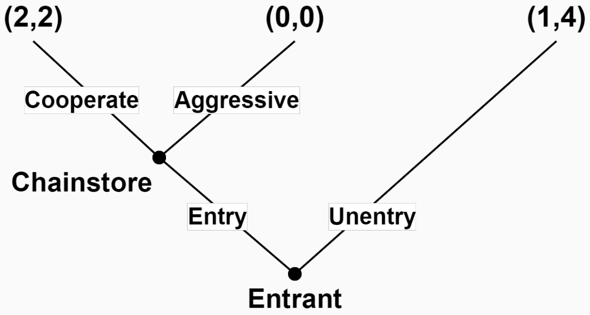
For a history $h \in H$ and a moment $m \in M$, a satisfaction relation $\vDash$ is given as follows:

$$
\begin{aligned}
(h, m) \vDash p &\textit{ iff } V(p, h, m) = 1; \\
(h, m) \vDash \neg\varphi &\textit{ iff } (h, m) \nvDash \varphi; \\
(h, m) \vDash \varphi \wedge \psi &\textit{ iff } (h, m) \vDash \varphi, \text{ and } (h, m) \vDash \psi; \\
(h, m) \vDash \mathrm{stit}_i\varphi &\textit{ iff } C_i^m(h) \subseteq \|\varphi\|_m,
\end{aligned}
$$

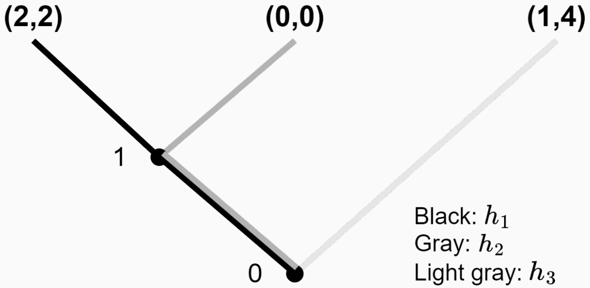
where $\|\varphi\| \coloneqq \{h \in H_m \mid (h, m) \vDash \varphi\}$.

- For each m, i, h, $C_i^m(h)$ refers to the possible histories after all actions open to i at m in h.
- The definition of stit operator is called that of Chellas stit.

Example: a Chainstore Game



- $\mathcal{G} = \{e, c\}$
- $p^i_{\{0,1,2,4\}}$: 'i obtains payoff $\{0, 1, 2, 4\}$'



- $(h_1, 1) \vDash \mathrm{stit}_e \neg p_4^c$: '$e$ see it to that c is not able to obtain payoff 4 at 1 in h_1.'
- $C_e^m(h) \subseteq \{h' \mid (h', m) \vDash \neg p_4^c\}$

Conclusion

- STIT logic expresses futures that agents achieve by their actions, whatever the opponents do.

- Analysis of strategic action.
- Applying the idea to other logic, such as awareness logic.
- 足立さん誕生日おめでとうございます。

Reference

[1] Segerberg, K., Meyer, J.-J., and Kracht, M.. (2013). The Logic of Action. https://plato.stanford.edu/entries/logic-action.

[2] Horty, J. F. and Belnap, N.. (1995). The Deliberative Stit: A Study of Action, Omission, Ability, and Obligation. *Journal of Philosophical Logic*, 24(6):583-644.