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# PSQT

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## Home Assignment - 1

① i) compute Probability that individual likes both vehicle 1 & vehicle 2. The probability he like both vehicle is :

$$P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2)$$
$$= 0.55 + 0.65 - 0.80$$
$$= 0.40$$

ii) Determine and interpret $P(A_2/A_3)$.

$$P(A_2/A_3) = \frac{P(A_2 \cap A_3)}{P(A_3)} = \frac{0.40}{0.70} = 0.571$$

iii) Are $A_2$ & $A_3$ independent events..?

Two events A & B are independent if

$$P(A \cap B) = P(A)P(B)$$
$$P(A/B) = P(A), \quad P(B/A) = P(B)$$

Given, $P(A_2 \cap A_3) = 0.40$

The value of $P(A_2) * P(A_3) = 0.65 \times 0.70$
$= 0.455$

$\because P(A_2 \cap A_3) \neq P(A_2) \cdot P(A_3)$ (or)

$\because P(A_2/A_3) = 0.571 \neq P(A_2) = 0.65$

$\because P(A_2/A_3) \neq P(A_2)$

The events $A_2, A_3$ are independent.

5 2 t

(2) i) Person is male, he has secondary education.

Total male = 88, out of which they have SE = 28

Total secondary educated adults = 28 + 50 = 78

Let M be male, S be secondary educated

$$P(M/S) = \frac{P(M \cap S)}{P(S)} = \frac{28}{78} = 0.358$$

ii) Person don't have college degree, person is female.

Total no. of females = 40 + 50 + 17 = 112

out of which 45 + 50 = 95 don't have college degree

Let C don't have college degree, F = female student

$$P(C/F) = \frac{P(C \cap F)}{P(F)} = \frac{95}{112} = 0.848$$

(3) i) The next purchases request atleast one of 3 options.

p(requesting atleast one) = P(either A (or) B (or) C)

$P(A \cup B \cup C) = 85\% = 0.85$

ii) The next purchaser select none of 3 options

P(None of select) = 1 - P(A (or) B (or) C)

$= 1 - P(A \cup B \cup C)$

$= 1 - 0.85 = 0.15$

iii) Next purchaser last only an automatic transmission & not either of other 2 options.

$$P(C) = P(C) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$
$$= 0.4 + 0.55 - 0.63 = \underline{0.32}$$

$$P(A \cap C) = P(A) + P(C) - P(A \cup C)$$
$$= 0.4 + 0.70 - 0.77 = \underline{\underline{0.33}}$$

$$P(B \cap C) = P(B) + P(C) - P(B \cup C)$$
$$= 0.55 + 0.70 - 0.80 = \underline{0.45}$$

$$P(A \text{ and } B \text{ and } C) = P(A \cap B \cap C)$$
$$= P(A \cup B \cup C) - P(A) - P(B) - P(C) + P(A \cap B) + P(B \cap C) + P(A \cap C)$$

$$= 0.85 - 0.40 - 0.55 - 0.70 + 0.32 + 0.33 + 0.45$$
$$= 0.30$$

$$\therefore P(C) = P(C) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$
$$= 0.70 - 0.33 - 0.45 + 0.30$$
$$= 0.22$$

(4) (a) $$P(A \cap B \cap C) = P\left(C / A \cap B\right) \times P(A \cap B)$$
$$= P\left(C / A \cap B\right) \cdot P(B/A) \cdot P(A)$$
$$= (0.2)(0.72)(0.32)$$
$$= 0.046$$

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(b) $P(B' \cap C) = P(A \cap B' \cap C) + P(A' \cap B' \cap C)$

$= P(C/A \cap B') \cdot P(A \cap B') + P(C/A' \cap B') + P(A' \cap B')$

$= P(C/A \cap B') \cdot P(B'/A) \cdot P(A) + P(C/A' \cap B') - P(B'/A') P(A')$

$= P(C/A \cap B') [1 - P(B/A)] \cdot P(A) + P(C/A' \cap B') = [1 - P(B/A')]$

$[1 - P(A)]$

$= (0.80)(1-0.72)(0.32) + (0.91)(1-0.20)(1-0.32)$

$= 0.0716 + 0.495$

$= 0.5667$

(5) we have to show that

$P(B_1/A)$

Let A be initial repair that was incomplete

$B_1$ = initial repair made by Janet

$B_2$ by tom, $B_3$ by georgea, $B_4$ by peter.

$$P(B_1/A) = \frac{(0.20)(0.05)}{(0.20)(0.05) + (0.60)(0.10) + (0.15)(0.10) + (0.05)(0.05)}$$

$$= \frac{0.01}{0.0875} = 0.1142$$

(6) i) verify validation of density functions

To verify validation of density functions.

→ To domain of density function is set to all possible values that random variable x can take in [2,5]

→ The density function must be non-negative, $f(x) \geq 0 \ \forall x$ in domain. we see it is true because the funel $2(1+x)/27$ is positive $\forall x$.

→ The integral of density function over domain is 1.

$$\int f(x)dx = \int \frac{2(1+x)}{27} dx$$

$$= \frac{2}{27} \int (1+x)dx \Rightarrow \frac{2}{27}\left[x + \frac{x^2}{2}\right]$$

$$= \frac{2}{27}\left[5 + \frac{25}{2}\right] + c$$

$$= 1 + c \left[\because \text{Integrated is } 1\right]$$

∴ Density function is valid.

ii) Find cumulative distribution function.

CDF $f(x) = P(X \leq x)$

we can find CDF by calculating integral of density function from lower bound of domain to $x$.

$$F(x) = \int f(t)dt$$

$$= \int \frac{2(1+t)}{27} dt$$

$$= \frac{2}{27} \int (1+t)\, dt$$

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$= \frac{2}{27}\left[t + \frac{t^2}{2}\right] + C$

CDF is valid if :

1.) $F(x)$ is non decreasing

2.) $F(x)$ is continuous

$F(x)$ has range of $[0,1]$

we see CDF satisfies all above properties

iii) $P(x<4) = \int_{-\infty}^{4} F(x)\,dx$

$= \int_{-\infty}^{2} F(x) + \int_{2}^{4} F(x)\,dx$

$= \int_{-\infty}^{2} 0\,dx + \int_{2}^{4} \frac{2(1+x)}{27}\,dx$

$= \frac{2}{27}\left[x + \frac{x^2}{2}\right]_2^4 \Rightarrow \frac{2}{27}\left[(4+8) - (2+2)\right]$

$= \frac{2}{27}(8) = \underline{\underline{\frac{16}{27}}}$

iv) $P(3 \le x \le 4) = \int_{3}^{4} F(x)\,dx = \int_{3}^{4} \frac{2(1+x)}{27}\,dx$

$= \frac{2}{27}\left[x + \frac{x^2}{2}\right]_3^4 = \frac{2}{27}\left[(4+8) - \left(3 + \frac{9}{2}\right)\right]$

$= \frac{2}{27}(4.5)$

$= \frac{9}{27} = \underline{0.33}$

(7) (a) obtain probability student scores distinction atleast 75% correct

$$= \frac{75}{100} \times 8 = 6 \qquad \left[ p(x) = {}^nC_x\, p^x q^{n-x} \right]$$

he must get atleast 6 crct answers.

$$p(x \geq 6) = p(X=6) + p(X=7) + p(X=8)$$

$p$ = success = $1/3$, $q$ = failure = $1 - 1/3 = 2/3$ $n = 8$

$$p(x=6) + p(x=7) + p(x=8)$$

$$= {}^8C_6 (1/3)^6 (2/3)^2 + {}^8C_7 (1/3)^7 (2/3)^1 + {}^8C_8 (1/3)^8 (2/3)^0$$

$$= (1/3)^6 \left[ {}^8C_6 (2/3)^0 + {}^8C_7 (1/3)(2/3)^1 + {}^8C_8 (1/3)^2 \right]$$

$$= \frac{1}{729} \left[ 28 \times 4/9 + 8 \times 2/9 + 1/9 \right]$$

$$= \underline{0.0196}$$

b) exactly 3 correct answers

$$p(X = 3) = {}^nC_x\, p^x q^{n-x}$$

$$= {}^8C_3 (1/3)^3 (2/3)^5$$

$$= 56 \times (1/27)\left(\frac{32}{243}\right)$$

$$= \frac{1792}{6561} = 0.273$$

(8) i) how many of 15 trucks have blowouts

The expected no.of trucks have blowout

$$\mu = 15 \times 0.25 = 3.75$$

$$\boxed{\mu = 3.75}$$

ii) Find variance of no. of blowouts

$$\text{variance} = E[(x - E[x])^2]$$
$$= E[x^2] - (E[x])^2$$
$$= (15 * (0.25)^2] - [15 * (0.75)^2] - (3.75)^2$$
$$= \underline{2.8125}$$

(5)

9 (a) Fewer that 7 terms possessing bubbles.

here $n = 8000$, $p = 0.001$ $\left(\because \frac{1}{1000}\right)$

Let s use poisson distribution $p(x) = \frac{e^{-\lambda t} (\lambda t)^x}{x!}$

$\lambda = np = 8000 \times 0.001 = 8$ ; $t = 1$

$$p(x<7) = p(x \le 6) = \sum_{x=0}^{6} e^{-8.1} \cdot \frac{(8.1)^x}{x!}$$

(b) None will posses a bubble.

probability that : → ∵ Item has atleast one bubble in 1000 $= \frac{1}{1000}$

$= 0.001$

∵ It has no bubble is $\frac{999}{1000}$

∵ It has no bubble in 8000 glass sample is

$$= \left(\frac{999}{1000}\right)^{8000} = (0.999)^{8000}$$

(10) a) 6 particles enter in given ms:

$$P(x,\lambda t) = \frac{e^{-\lambda t}(\lambda t)^x}{x!}$$

$$P(6,4) = \frac{e^{-4}(4)^6}{6!}$$

$$= \frac{e^{-4} \times 4096}{720}$$

b) None will enter the counter

$$P(0,4) = \frac{e^{-4}(4)^0}{0!} = \frac{1}{e^4}$$

$$= 0.0183$$

c) Atleast 2 will enter the counter.

$$P(2) = \frac{e^{-4}(4)^2}{2!} = \frac{8}{e^4} = 0.0902$$

d) Atmost 2 will enter the counter in given m.sec

$$P(1) + P(0)$$

$$= \frac{e^{-4}\cdot(4)^1}{1!} + \frac{e^{-4}(4)^0}{0!}$$

$$= \frac{4}{e^4} + \frac{1}{e^4}$$

$$= 5/e^4 = \underline{\underline{0.08}}$$