

Tutorial-1

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Pre-Tutorial:-

① Define DFA and NFA normally?

Ans:- DFA:- For each input symbol, one can determine the state to which the machine will move. As it has a finite no. of states, the machine is called Deterministic Finite Machine / Deterministic Finite Automaton.

A DFA can be represented by a 5-tuple.

$(Q, \Sigma, \delta, q_0, F)$ where

Q - finite set of states

Σ - finite set of symbols called alphabet

δ - transition function where $\delta: Q \times \Sigma \rightarrow Q$

q_0 - Initial state ($q_0 \in Q$)

F - set of final state / states of Q ($F \subseteq Q$)

NFA:-

The finite automata are called NFA. when there exist many paths for specific input from the current state to next state. Each NFA can be translated into DFA but every NFA is Non DFA.

(Here $\delta: Q \times \Sigma \rightarrow 2^Q$)

The two exceptions are

* It contains multiple next states.

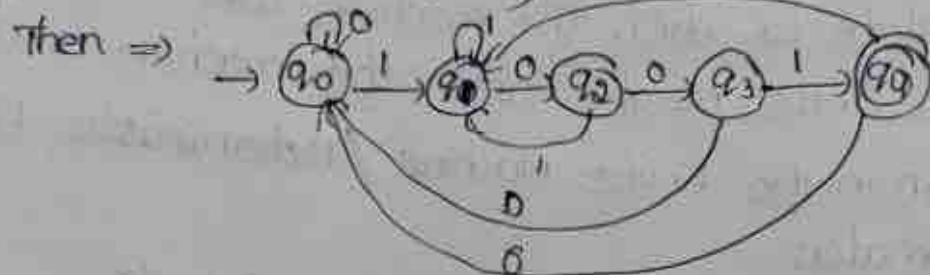
* It contains ϵ transitions.

- 2) construct a DFA that accepts the language
 $L = \{w \in \{0,1\}^+ \mid w \text{ contains } 1001 \text{ or } 0110\}$

Sol: $\Sigma = \{0,1\}$

$L = \{1001, 001001, 101001, \dots\}$

DFA: $M = (Q, \Sigma, \delta, q_0, F)$



	0	1
$\rightarrow q_0$	q_0	q_1
q_1	q_2	q_1
q_2	q_3	q_1
q_3	q_0	q_4
(q_4)	q_0	q_1

$$\delta(q_0, 0) = q_0$$

$$\delta(q_0, 1) = q_1$$

$$\delta(q_1, 0) = q_2$$

$$\delta(q_1, 1) = q_1$$

$$\delta(q_2, 0) = q_3$$

$$\delta(q_2, 1) = q_1$$

$$\delta(q_3, 1) = q_4$$

$$\delta(q_3, 0) = q_0$$

$$\delta(q_4, 0) = q_0$$

$$\delta(q_4, 1) = q_1$$

3) write the steps for converting ϵ -NFA to DFA and vice-versa with an example for each?

ans: steps for converting NFA to DFA:

step 1: Initially $Q' = \emptyset$

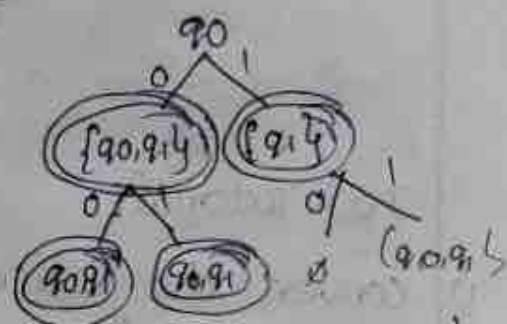
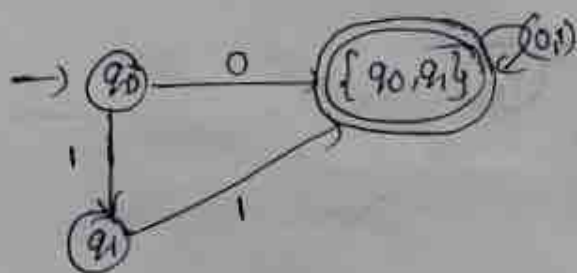
2) Add q_0 to NFA to Q' . Then find the transitions from this start state.

3) In Q' , find possible set of states for each input symbol, if this set of states is not in Q' , then add it to Q' .

4) In DFA, the final state will be all states which contain (Final state of NFA)



	0	1
q_0	$\{q_0, q_1\}$	$\{q_1\}$
q_1	\emptyset	$\{q_0, q_1\}$



$$\begin{aligned}\delta(\{q_0, q_1\}, 0) &\Rightarrow \delta\{q_0, 0\} \cup \delta\{q_1, 0\} \\ &= \{q_0, q_1\} \cup \emptyset \\ &= \{q_0, q_1\}\end{aligned}$$

$$\begin{aligned}\delta\{q_1, 0\} \cup \delta\{q_1, 1\} \\ \emptyset \cup \{q_0, q_1\}\end{aligned}$$

$$\begin{aligned}\delta(\{q_0, q_1\}, 1) &= \delta\{q_0, 1\} \cup \delta\{q_1, 1\} \\ &= \{q_1\} \cup \{q_0\} \\ &= \{q_0, q_1\}\end{aligned}$$

steps for converting DFA to NFA:

① Let's assume DFA has state set.

$$Q = \{q_0, q_1, \dots, q_n\}$$

2) Now, we build NFA N' as follows

1) Start with DFA D

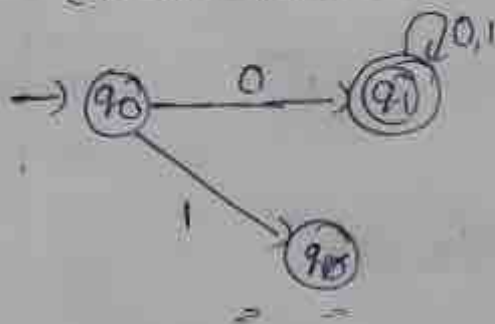
2) Add an additional accepting state for NFA N' such that N' will have $n+1$ total no. of states.

Let call new accepting state q_{n+1}

3) Now add an exception state of epsilon ϵ transition for all accepting states to new accepting state q_{n+1} and make all the original accepting states just normal states.

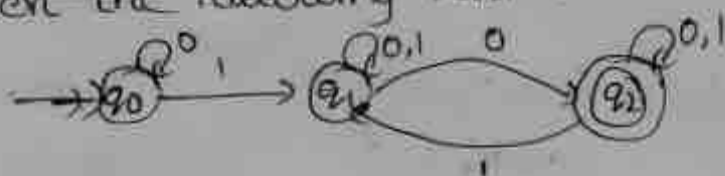
ex- $\epsilon = \{0,1\}$ starts with '0'

$L = \{0, 00, 01, 000, 001, 010, 0010, 0101, \dots\}$

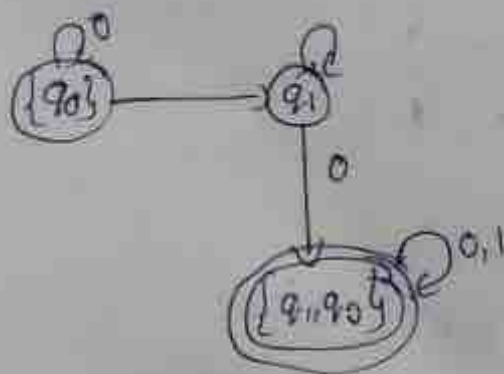


In - Tutorial:

1) convert the following NFA to DFA.



Ans:



	0	1
q_0	q_0	q_1
q_1	$\{q_1, q_2\}$	$\{q_1\}$
q_2	$\{q_1, q_2\}$	$\{q_1, q_2\}$

$$\delta(\{q_1, q_2\}, 0) = \delta(q_1, 0) \cup \delta(q_2, 0)$$

$$= \{q_1, q_2\} \cup \{q_1, q_2\}$$

$$\delta(\{q_1, q_2\}, 1) = \delta(q_1, 1) \cup \delta(q_2, 1) = \{q_1, q_2\}$$

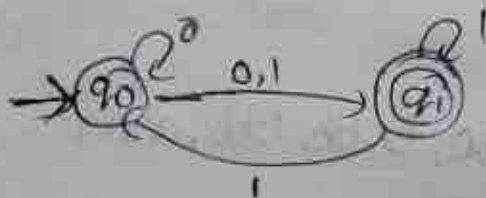
$$= \{q_1, q_2\}$$

Ans:

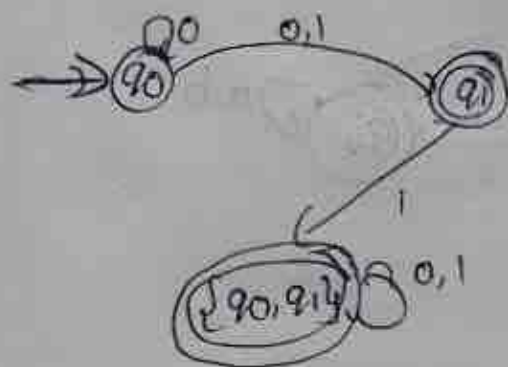
- ② write the algorithm that converts NFA to DFA.
explain your algorithm works using the below NFA?

A) Algorithm:-

- steps:-
- ① Initially $Q' = \emptyset$
 - ② Add q_0 of NFA to Q' of DFA
 - ③ In Q' , find possible states for each input symbol. If this set of states is not in Q' , then add it to Q' .
 - ④ In DFA, the final state will be all states which contain F (final state of NFA)



	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_1\}$
q_1	\emptyset	$\{q_0, q_1\}$



$$\begin{aligned}
 &\delta(\{q_0, q_1\}, 0) \\
 &= \delta(q_0, 0) \cup \delta(q_1, 0) \\
 &= \{q_0, q_1\} \cup \{\emptyset\} \\
 &= \{q_0, q_1\} \\
 &\delta(\{q_0, q_1\}, 1) \\
 &= \delta(q_0, 1) \cup \delta(q_1, 1) \\
 &= \{q_1\} \cup \{q_0, q_1\} \\
 &= \{q_0, q_1\}
 \end{aligned}$$

Post-tutorial:-

- ① Differentiate NFA and DFA

NFA	DFA
$\textcircled{1} (Q, E, \delta, q_0, F)$ $\delta = Q \times E \rightarrow 2^Q$	$\textcircled{1} (Q, E, \delta, q_0, F)$ $\delta = Q \times E \rightarrow Q$

② $Q = \{q_0, q_1\}$, $\Sigma = \{0, 1\}$
 $q_0 \rightarrow 0$, $q_1 \rightarrow 1$

③ Transition may leads to multiple states.

④ Back track is not required

⑤ practical implementation of DFA is feasible

$Q = \{q_0, q_1\}$, $\Sigma = \{0, 1\}$
 $q_0 \xrightarrow{0} q_0$, $q_1 \xrightarrow{0} q_1$

③ Transition leads to unique state.

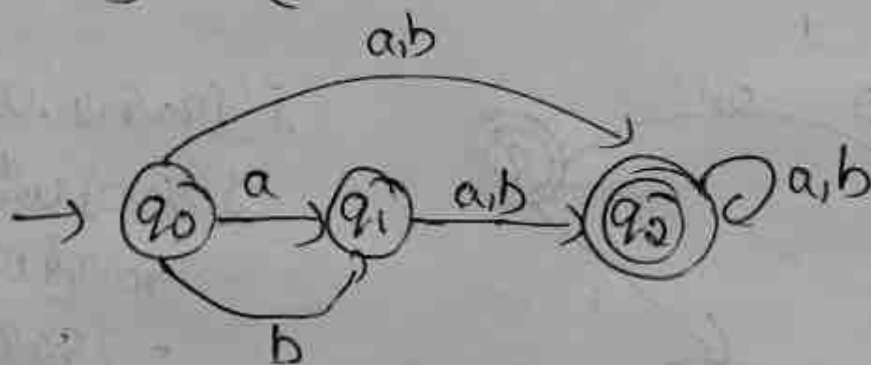
④ Back tracking is required.

⑤ Not feasible, convert NFA to DFA.

② construct NFA for language $L = \{w \in \{0, 1\}^* \mid w \text{ contains an even number of } 0\text{'s and an odd number of } 1\text{'s}\}$

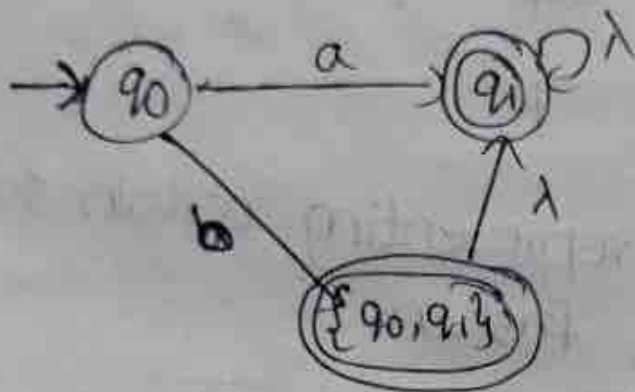
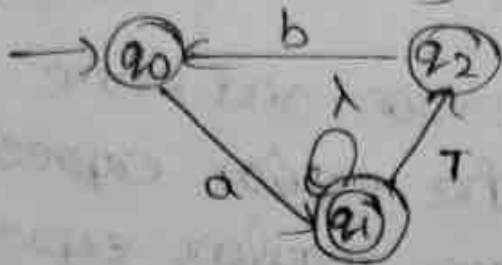
Sol:

language = $\{a, aa, b, bb, aaaa, \dots\}$



3)

convert the following E-NFA to DFA - 8100032325



	a	b	λ
q0	q1	∅	∅
q1	∅	∅	{q1, q2}
q2	∅	q0	∅

$$\delta(\{q_0, a\} = \{q_1\}$$

$$\delta\{q_0, b\} = \{\emptyset\}$$

$$\delta\{q_1, a\} = \{\emptyset\}$$

$$\delta\{q_1, b\} = \{\emptyset\}$$

$$\delta(\{q_1, q_2\}, b)$$

$$= \delta(q_1, b) \cup \delta(q_2, b)$$

$$\emptyset \cup q_0 = q_0$$

$$\delta(\{q_1, q_2\}, \lambda)$$

$$= \delta(q_1, \lambda) \cup \delta(q_2, \lambda)$$

$$= \{q_1, q_2\} \cup \{\emptyset\}$$

$$= \{q_1, q_2\}$$

$$\delta(\{q_1, q_2\}, a)$$

$$= \delta(q_1, a) \cup \delta(q_2, a)$$

$$= \emptyset \cup \emptyset = \emptyset$$

$$\delta(\{q_1, q_2\}, b)$$

Tutorial-2

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Pre-Tutorial:-

- ① Explain regular expression and name some of identity rules for the regular expression? Assume a, b and c are regular expressions in the identity rules.

A) Regular expression:-

It is used for representing certain sets of strings in algebraic fashion.

Identity rules:-

$$\emptyset + \gamma = \gamma$$

$$\emptyset \cdot \gamma = \gamma \cdot \emptyset = \emptyset$$

$$\epsilon \cdot \gamma = \gamma \cdot \epsilon = \gamma$$

$$\gamma + \gamma = \gamma$$

$$\gamma^* \cdot \gamma^* = \gamma^*$$

$$\gamma^* \cdot \gamma = \gamma \gamma^* = \gamma^+$$

$$(\gamma^*)^* = \gamma^*$$

$$R R^* = R^* R = R^+$$

$$\epsilon + \gamma^* \gamma = \epsilon + \gamma^+ = \gamma^+$$

$$(ab)^* a = a(ba)^*$$

$$(a+b)^* = (a^* b^*)^*$$

$$= (a^* + b^*)^*$$

$$(a+b)c = ac+bc$$

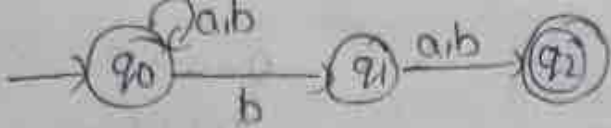
$$\epsilon^* = \epsilon$$

$$\emptyset^* = \epsilon$$

- ② Consider Language L given by regular expression $(a+b)^* b (a+b)$ over the alphabet $\{a, b\}$. Design a DFA that accepts L .

*) convert RE into NFA and then find out DFA from NFA.

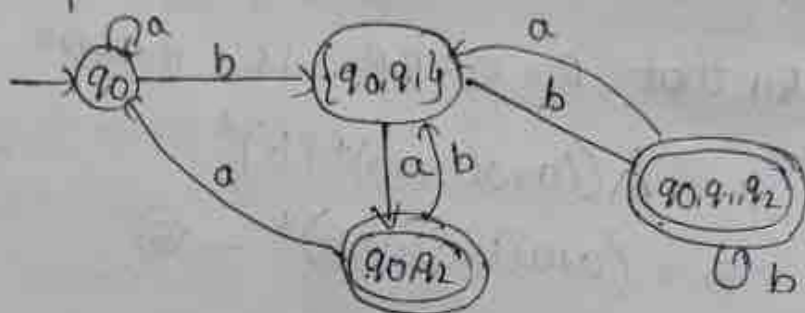
$$RE = (a+b)^* b (a+b)$$



	a	b
→ q0	q0	{q0, q1}
q1	q2	q2
q2	∅	∅

DFA table From NFA.

	a	b
→ q0	q0	{q0, q1}
{q0, q1}	{q0, q1}	{q0, q1, q2}
* {q0, q1}	q0	{q0, q1}
* {q0, q1, q2}	{q0, q1}	{q0, q1, q2}



In tutorial:-

- i) provide algorithm to convert NFA into RE create a RE for following NFA.



$$q_1 = q_1 a + q_2 b + \lambda \rightarrow (1)$$

$$q_2 = q_1 a + q_2 b + q_3 b \rightarrow (2)$$

$$q_3 = q_2 a \rightarrow (3)$$

Now, $q_1 = q_1$ substitute q_2 in (2) in (3)

$$q_1 = q_1 a + q_2 b$$

$$q_3 = (q_1 a + q_2 b) a$$

$$q_3 = q_1 a a + q_2 a b + q_3 a b \rightarrow (4)$$

subst q_1 in q_2 from eq's (2)

$$q_2 = q_1 a + q_2 b + q_3 b$$

$$q_2 = q_1 a + q_2 b + (q_2 a) b$$

$$q_2 = q_1 a + q_2 b + q_2 a b$$

[sub's value q_3 in eq]

$$q_2 = q_1 a + q_2 (b + ab)$$

$$R = Q + R P$$

$$R = Q + R P$$

$$R = Q P^*$$

$$q_2 = (q_1 a)(b + ab)^* \rightarrow (5)$$

From eq (1)

$$q_1 = q_1 a + q_2 b + \lambda \quad [\text{sub eq 5 in eq (1)}]$$

$$q_1 = q_1 a + ((q_1 a)(b + ab)^*) b + \lambda$$

$$q_1 = \lambda + q_1 (a + a(b + ab)^* b)$$

$$\text{By that } R = Q + R P \quad \text{i.e. } R = Q P^*$$

$$q_1 = \lambda ((a + a(b + ab)^* b))^*$$

$$q_1 = (a + a(b + ab)^* b)^* \rightarrow (6)$$

Final state (23)

$$q_3 = q_2 a \quad [\text{sub (5) in (3)}]$$

$$q_3 = q_1 a (b + ab)^* a$$

subst eq (6) :

$$q_3 = (a + a(b + ab)^* b)^* a (b + ab)^* a$$

The required regular expression.

Algorithm

- ① create a new state for the ^{start} state of the NFA.
- ② Create a new state for the accept state of the NFA.
- ③ For each state of transition in the NFA, create a new transition in the new automaton, using rules to determine regular expression for the transition.
- ④ Add an epsilon transition for the new start state to the old start state.
- ⑤ This algorithm will convert any NFA into regular expression.

② Explain use of

Ans:- Arden's
If P a contain
 $R = Q$
steps:-

- ① For eq into st
- ② Add
- ③ calc
- ④ Res solution

$$R =$$

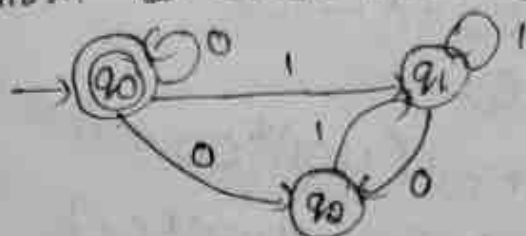
from

$$q_1 =$$

$$q_1$$

$$R$$

- ② Explain Arden's theorem to convert FA to RE.
use algorithm to convert following DFA to RE.



Ans: Arden's Theorem:-

If P and Q are 2 RE over Σ and if P doesn't contain ϵ then the following eq is R by

$R = Q + RP$ has a unique solution i.e., $R = QP^+$

steps:-

- ① For each state q_1, q_2, q_3, \dots all exists that comes into state written in eq format.
- ② Add epsilon to initial state.
- ③ calculate all equations.
- ④ Result is value of final state.

solution for example:-

$$q_0 = q_0 0 + \lambda \rightarrow \textcircled{1}$$

$$q_1 = q_0 1 + q_1 1 \rightarrow \textcircled{2}$$

$$q_2 = q_0 0 + q_1 0 \rightarrow \textcircled{3}$$

$$R = QP \rightarrow \textcircled{1} \Rightarrow Q + QP^+P$$

$$\Rightarrow Q [\epsilon + P^+P] \Rightarrow Q[\epsilon + P^+P]$$

$$\underline{R = QP^+} \rightarrow \text{Proved.}$$

from ② & ③

$$q_1 = q_0 1 + q_1 1 + q_1 0 1$$

$$q_1 = q_0 1 + q_1 (1 + 0 1)$$

$R = Q + RP$ by Arden's theorem.

$$\text{i.e., } R = QP^+$$

$$\underline{q_1} = q_0 1 (1 + 0 1)^+ \rightarrow \textcircled{4}$$

From ① & ③

$$q_0 = q_{00} + q_{100} + \lambda \rightarrow ⑤$$

From ④ & ⑤

$$q_0 = q_{00} + q_{01}(1+01)^*00 + \lambda$$

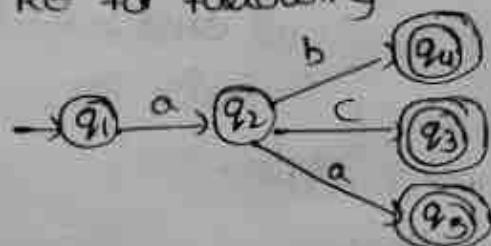
$$q_0 = \lambda + q_0(0+1(1+01)^*00)$$

$$R = Q + RP \Rightarrow R = QP^*$$

$$q_0 = \lambda(0+1(1+01)^*00) \Rightarrow q_0 = (0+1(1+01)^*00)$$

post Tutorial:-

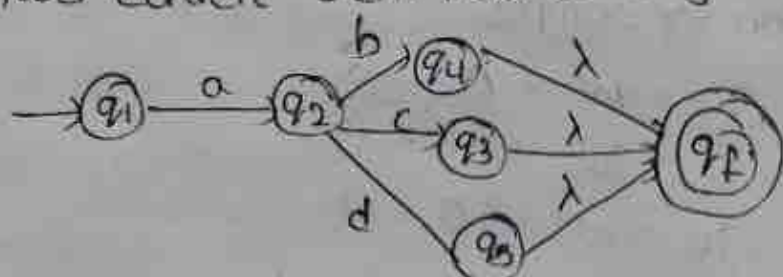
① Find RE for following DFA



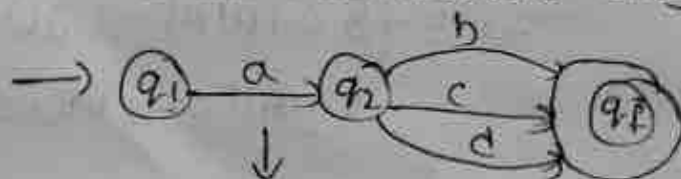
Ans:-

there exist multiple final states.

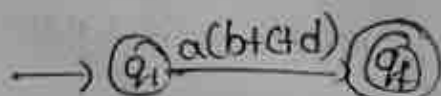
so, we convert them into a single final state



eliminate all intermediate stage q_4, q_3, q_5



let's eliminate q_2



Regular Expression = $a(b+cd)$

② For $\Sigma = \{a, b\}$ Let us consider the regular language $L = \{x/x = a^{2+3k} \text{ or } x = b^{10+2k}, k \geq 0\}$ what could be the minimum pumping length the constant guaranteed by the pumping lemma for L ?

Ans: $L = \{x/x = a^{2+3k} \text{ (or) } x = b^{10+2k}, k \geq 0\}$
 $L = \{a^2, a^5, a^8, a^{11}, \dots \cup b^{10}, b^{12}, b^{14}, \dots\}$

pumping lemma:

Let L be an infinite RL. Then there exists some positive integer m such that any $w \in L$ with $|w| \geq m$ can be decomposed as

$$w = xyz$$

with $|xy| \leq m$ such that $w_i = xy^i z$

is also L for all $i = 0, 1, 2, \dots$

\therefore Minimum pumping length should be 11, because string with length 10 ($w = b^{10}$) does not repeat anything, but string with length 11 (ie; $w = b^{11}$) will repeat states length of pumping lemma is

Pre Tutorial

- ① what is the context free grammar. Explain with an example?

sol: Context free grammar is a formal grammar which is used to generate all possible strings in a given formal language.

Context free grammar G can be defined by four types as:

$$G = (V, T, P, S)$$

where G : describes the grammar.

V : describes set of non-terminal symbols

T : Finite set of Terminal symbols

P : production rules

S : start symbol.

ex: $L = \{wcw^R \mid w \in (a,b)^*\}$

production rules:- $S \rightarrow aSa$

$S \rightarrow bSb$

$S \rightarrow \epsilon$

Now, check the string $abbcbba$ derived from given.

$S \Rightarrow aSa \Rightarrow abSba \Rightarrow abbsbba \Rightarrow abbcbbba$

By applying the productions $S \rightarrow aSa$, $S \rightarrow bSb$, $S \rightarrow \epsilon$

we get string $abbcbba$.

In-Tutorial:-

① construct a CFG for a language

$$L = \{ w c w^R / w \in (a,b)^* \}$$

Sol:-

$$L = \{ aa, bb, abba, aabbba, abaaba, \dots \}$$

production rules:-

$$S \rightarrow asa$$

$$S \rightarrow bsb$$

$$S \rightarrow c$$

Now, check the strings ~~are~~^{are not} reverse (or) not
for example:- abbcbbba.

$$S \Rightarrow asa \Rightarrow absba \Rightarrow abbsbba \Rightarrow \underline{\underline{abbcbbba}}$$

It convert and get a string.

② derive the string "aabbabba" for leftmost derivation and right most derivation using a CFG.

Sol:-

$$S \rightarrow AB/BA$$

$$A \rightarrow a/as/bAA$$

$$B \rightarrow b/bb/aBB$$

aabbabba

left most derivation:-

$$S \Rightarrow AB \Rightarrow aABB \Rightarrow aabbB \Rightarrow aabbbs \Rightarrow \downarrow$$

$$\in aabbabbaA \in aabbabbs \in aabbabB \\ \hookrightarrow \underline{\underline{aabbabba}}$$

Right most derivation:-

$$S \Rightarrow AB \Rightarrow aABB \Rightarrow aABbs \Rightarrow aABbba \Rightarrow$$

$$\underline{\underline{aabbabba}} \in aabbAbba \in aabbsbba \in aABbba$$

post Tutorial:-

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① Generate CFG for the language

$$L = \{ 0^i 1^j 0^k / j > i+k \}$$

sol:-

$$\text{Given } L = \{ 0^i 1^j 0^k / j > i+k / i, k \geq 1 \}$$

$$\Rightarrow 0 \ 1 \cdot 1 \ 10$$

$$\text{let } i=1, k=1$$

$$j=3$$

$$L = \{ 0^1 1^3 0^1, \dots \}$$

$$S \Rightarrow XYZ$$

$$X \Rightarrow 0X1 / 01$$

$$Y \Rightarrow 1Y / 1$$

$$Z \Rightarrow 1Z0 / 10$$

$$S \Rightarrow XYZ \Rightarrow 0X1Yz \Rightarrow 0011Yz \Rightarrow 00111Yz$$

$$\Rightarrow 001111z \Rightarrow 00111110$$

$$i=2 \ j=5, k=1$$

$$5 > 3$$

$$\underline{j > i+k}$$

PARSE TREE, AMBIGUITY INCFG:-

Pre-Tutorial:-

① differentiate ambiguous and unambiguous grammar.

Ambiguous	unambiguous
* The leftmost and Right most derivations are not same.	* The left most and rightmost derivations are same.
* Amount of non-terminal in ambiguous grammar is less than unambiguous grammar.	* Amount of non-terminals In unambiguous grammar is greater than in ambiguous grammar. Length of parse tree is large.
* Length of parse tree is short.	* It generates only one parse tree.
* It generate more than one parse tree.	* It does not contain any ambiguity.
* It contains ambiguity.	

In-Tutorial:-

① consider the following grammar

$$S \rightarrow AB/C$$

$$A \rightarrow \epsilon / aA$$

$$B \rightarrow \epsilon / bB$$

Derive the string acb using left-most and rightmost derivation. show the parse trees in your derivation.

Ans! Left most derivation

S

ASB $S \rightarrow ASB$

aASB $A \rightarrow aA$

aESB $A \rightarrow E$

aECB $S \rightarrow C$

aECbB $B \rightarrow bB$

aEcbe $B \rightarrow E$

abc

Right most derivation

S

ASB $S \rightarrow ASB$

ASbB $B \rightarrow bB$

ASbE $B \rightarrow E$

AcbE $S \rightarrow C$

aAcbe $A \rightarrow aA$

aEcbe $A \rightarrow E$

abc

② consider the following grammar :

$$S \rightarrow aS / \epsilon$$

The language generated by this grammar :

$$L = \{ a^n, n \geq 0 \} \text{ or } a^*$$

- i) Find the Leftmost Derivation and Rightmost Derivation.
- ii) Also, prove all the strings generated from this grammar have their leftmost derivation and right most derivation exactly same draw the parse of the same.

Sol:

i) Let us consider string $w = aaaa$

Leftmost derivation

$S \rightarrow as|e$

S
 $as \quad (S \rightarrow as)$
 $aaS \quad (S \rightarrow as)$
 $aaas \quad (S \rightarrow as)$
 $aaaE \quad (S \rightarrow E)$
aaa

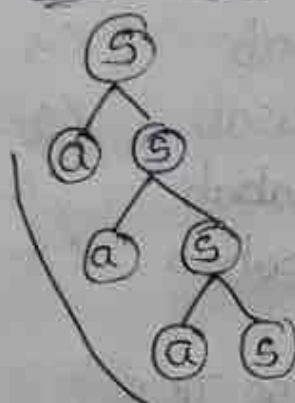
Rightmost derivation

$S \rightarrow as|e$

$as \quad (S \rightarrow as)$
 $aaS \quad (S \rightarrow as)$
 $aaas \quad (S \rightarrow as)$
 $aaaE \quad (S \rightarrow E)$
aaa

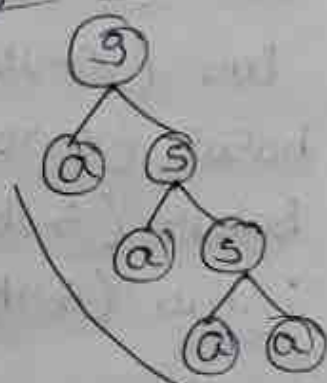
ii) parse tree for the given

Leftmost Tree



aaa

Rightmost Tree



aaa

clearly, the leftmost derivation parse tree = the Rightmost derivation parse tree.

POST-Tutorial:-

① consider the following grammar.

$S \rightarrow sas|b$

It is an ambiguous grammar. Generate the string babab from this grammar to prove your point.

Soln The given grammar

$S \rightarrow Sas|b$

$w = babab$

Leftmost derivation

S
 $\underline{S}aS \quad (S \rightarrow SaS)$
 $SaS aS \quad (S \rightarrow SaS)$
 $baSaS \quad (S \rightarrow b)$
 $babas \quad (S \rightarrow b)$
 $babab$

Rightmost derivation

S
 $SaS \quad (S \rightarrow SaS)$
 $SaS aS \quad (S \rightarrow SaS)$
 $SaS ab \quad (S \rightarrow b)$
 $Sabab \quad (S \rightarrow b)$
 $babab$

(con.)

$\rightarrow S$
 $SaS \quad (S \rightarrow SaS)$
 $baS \quad (S \rightarrow b)$
 $baSaS \quad (S \rightarrow SaS)$
 $babas \quad (S \rightarrow b)$
 $babab$

$\rightarrow S$
 $SaS \quad (S \rightarrow SaS)$
 $Sab \quad (S \rightarrow b)$
 $Sasab \quad (S \rightarrow SaS)$
 $Sabab \quad (S \rightarrow b)$
 $babab$

\rightarrow More than one left most parse Tree

\rightarrow More than one right most parse Tree

\rightarrow More than one parse Tree

\therefore The given grammar is ambiguous.

① Explain the ^{simplification} grammar? Mention its use? Elaborate the steps that are followed in simplification process?

A) Simplification of grammar means reduction of grammar by removing useless symbols.

Its use includes facilitating language learning making test more accessible to individuals with language difficulties, and improving reading comprehensions for language learning.

The steps followed by in automated simplification productions:

- ① Inputting the original text into the software
- ② Running algorithms to identify complex sentence (or) vocabulary and structures.
- ③ Modifying the text according to predefined rules or guidelines for grammar simplification.
- ④ Evaluating the output for accuracy and clarity.
- ⑤ Making further adjustments as necessary to ensure that the simplified text conveys the same meaning as the original.

② Find a reduced grammar equivalent to the grammar G , having production rules.

$$S \rightarrow AC/B$$

$$A \rightarrow a$$

$$C \rightarrow C/BC$$

$$E \rightarrow aA/e$$

sol

Phase 1:- $T_{\epsilon} = \{a, c, e\}$

$$W_1 = \{A, C, E\}$$

$$W_2 = \{A, C, E, S\}$$

$$W_3 = \{A, C, E, S\}$$

$$G' = \{(A, C, E, S), \{a, c, e\}, P, (S)\}$$

$$P: S \rightarrow AC$$

$$A \rightarrow a$$

$$C \rightarrow C$$

$$E \rightarrow aA/e$$

Phase 2:- $Y_1 = \{S\}$

$$Y_2 = \{S, A, C\}$$

$$Y_3 = \{S, A, C, a, c\}$$

$$Y_4 = \{S, A, C, a, c\}$$

$$G'' = \{(A, C, S), (a, c), P(S)\}$$

$$P: S \rightarrow AC, A \rightarrow a, C \rightarrow X$$

In Tutorial:-

① Remove unit productions from the following grammar.

$$S \rightarrow AC, A \rightarrow a, C \rightarrow x/b, X \rightarrow Y, Y \rightarrow z, z \rightarrow a$$

sol procedure for Removal:-

step 1:- To remove $A \rightarrow B$, add production.

$A \rightarrow X$ to the grammar rule whenever
 $B \rightarrow X$ occurs in the grammar.

step 2:- Delete $A \rightarrow B$ from the grammar.

step 3:- Repeat from step 1 until all unit productions are removed.

The given grammar is

$P: S \rightarrow AC, A \rightarrow a.C, C \rightarrow X/b, X \rightarrow Y, Y \rightarrow z, z \rightarrow a$

productions:-

$C \rightarrow X, X \rightarrow Y, Y \rightarrow z$

$Y \rightarrow z, z \rightarrow a$ from step 1 add $Y \rightarrow a$

1) $P: S \rightarrow AC, A \rightarrow a, C \rightarrow X/b, X \rightarrow Y, Y \rightarrow a, z \rightarrow a$

2) Since $Y \rightarrow a$, we add $z \rightarrow a$

$P: S \rightarrow AC, A \rightarrow a, C \rightarrow X/b, z \rightarrow a, Y \rightarrow a, z \rightarrow a$

3) Since $X \rightarrow a$, we add $C \rightarrow a$

$P: S \rightarrow AC, A \rightarrow a, C \rightarrow a/b, z \rightarrow a, Y \rightarrow a, z \rightarrow a$

Remove the unreachable symbols

$\therefore P \Rightarrow S \rightarrow AC, A \rightarrow a, C \rightarrow a/b$

② A grammar G is defined with rules $S \rightarrow XA/BB, B \rightarrow b/SB, X \rightarrow b, A \rightarrow a$. Write the productions obtained after normalized GNF of G .

sol steps to convert a given CFG to GNF:-

step 1, check if the given CFG has any unit

productions (or) null productions and remove
of these are any.

step 2:- check whether the CFG is already in
Chomsky Normal Form (CNF) and convert it to
CNF if it is not.

step 3:- change the names of the Non-Terminal
symbols into the same A_i in ascending order of i .

The given grammar with rules.

$$S \rightarrow XA / BB$$

$$B \rightarrow b / SB$$

$$X \rightarrow b$$

$$A \rightarrow a$$

Replace: S with A_1

X with A_2

A with A_3

B with A_4

we get

$$A_1 \rightarrow A_2 A_3 / A_4 A_4$$

$$A_4 \rightarrow b / A_1 A_4$$

$$A_2 \rightarrow b$$

$$A_3 \rightarrow a$$

step 4:- After the rules so that the Non-Terminals
are in ascending order, such that,

If the production is of the form $A_i \rightarrow A_j X$,
then $i < j$ and should never be $i \geq j$

$$A_4 \rightarrow b / A_1 A_4$$

$$A_4 \rightarrow b / A_2 A_3 A_4 / A_4 A_4 A_4$$

$$A_4 \rightarrow b / A_3 A_4 / A_4 A_4 A_4 \rightarrow \text{Left Recursion.}$$

⑤ Remove Left recursion.

$$Z \rightarrow A_4 A_4 Z / A_4 A_4$$

$$A_4 \rightarrow bA_3A_4 / b\bar{z} / bA_3A_4\bar{z}$$

Now the grammar is:

$$A_1 \rightarrow A_2A_3 / A_4A_4$$

$$A_4 \rightarrow b / bA_3A_4 / b\bar{z} / bA_3A_4\bar{z}$$

$$\bar{z} \rightarrow A_4A_4 / A_4A_4\bar{z}$$

$$A_2 \rightarrow b$$

$$A_3 \rightarrow a$$

In GNF we are not allow to have variable in the beginning we need to modify A_1 ,

$$A_1 \rightarrow bA_3 / bA_4 / bA_3A_4A_4 / b\bar{z}A_4 / bA_3A_4\bar{z}A_4$$

$$A_4 \rightarrow b / bA_3A_4A_4 / b\bar{z}A_4 / bA_3A_4\bar{z}A_4 / bA_4\bar{z} / bA_3A_4A_4\bar{z} / b\bar{z}A_4\bar{z} / bA_3A_4\bar{z}A_4\bar{z}$$

$$A_2 \rightarrow b$$

$$A_3 \rightarrow a$$

Post-Tutorial:

① convert the following

a) CFG into CNF

$$S \rightarrow ASA / aB, A \rightarrow B/S, B \rightarrow b/\epsilon.$$

sol: step 1 - Since S appears in RHS, we add a new state, S and $S' \rightarrow S$ is added to the production.

$$P: S' \rightarrow S, S \rightarrow ASA / aB, A \rightarrow B/S, B \rightarrow b/\epsilon$$

② Remove the Null productions:

$$B \rightarrow \epsilon \text{ and } A \rightarrow \epsilon;$$

$$\text{After removing } B \rightarrow \epsilon : P: S' \rightarrow S, S \rightarrow ASA / aB / a,$$

$$A \rightarrow B/S/\epsilon, B \rightarrow b$$

$$\text{After removing } A \rightarrow \epsilon : P: S' \rightarrow S, S \rightarrow ASA / aB / a / AS / SA / S, A \rightarrow B/S, B \rightarrow b$$

③ Remove the unit productions

$S \rightarrow S$, $S' \rightarrow S$, $A \rightarrow B$ and $A \rightarrow S$.

After removing $S \rightarrow S$

P: $S' \rightarrow S$, $S \rightarrow ASA/aB/a/AS/SA$,
 $A \rightarrow B/S$, $B \rightarrow b$

After removing $S' \rightarrow S$

P: $S' \rightarrow ASA/aB/a/AS/SA$,
 $S \rightarrow ASA/aB/a/AS/BA$,
 $A \rightarrow B/S$, $B \rightarrow b$

After removing $A \rightarrow B$

P: $S' \rightarrow ASA/aB/a/AS/SA$,
 $S \rightarrow ASA/aB/a/AS/SA$,
 $A \rightarrow b/S$, $B \rightarrow b$

After removing $A \rightarrow S$

P: $S' \rightarrow ASA/aB/a/AS/SA$,
 $S \rightarrow ASA/aB/a/AS/SA$,
 $A \rightarrow b/ASA/aB/a/AS/SA$,
 $B \rightarrow b$

④ Now change the productions

$S' \rightarrow aB$, $S \rightarrow aB$, and $A \rightarrow aB$

Finally we get

P: $S' \rightarrow AX/YB/a/AS/SA$,
 $S \rightarrow AX/YB/a/AS/SA$,
 $A \rightarrow b/AX/YB/a/AS/SA$,
 $B \rightarrow b$
 $X \rightarrow SA$
 $Y \rightarrow a$

which is required chomsky Normal form for the given CFG.

- ② write the steps for removing null productions and unreachable symbols? Explain with an ex of your own?

Sol:- ① Remove null productions:-

A production is considered null if its right hand side is empty.

For ex, consider a CFG with the following production: $A \rightarrow \epsilon$, This production can be removed.

② Remove unreachable symbols:-

*) A symbol is considered unreachable if it can never appear in any string generated by the CFG.

. To identify unreachable symbols, start from the start symbol and make all symbols reachable.

Then remove all the symbols that were not marked as reachable.

Ex:- consider the following CFG:

$$G = (N, T, P, S)$$

$$\text{when } N = \{S, A, B\}$$

$$T = \{a, b\}$$

$$P = \{S \rightarrow \epsilon, S \rightarrow A, A \rightarrow B, B \rightarrow a\}$$

$$S = S$$

① Remove the null production:-

The null production: $(S \rightarrow \epsilon)$ can be removed

New grammar:

$$G = (N, T, P, S)$$

$$\text{where } N = \{S, A, B\}$$

$$T = \{a, b\}$$

$$P = \{S \rightarrow A, A \rightarrow B, B \rightarrow a\}$$

$$S = S$$

② Remove unreachable symbols.

starting from the start symbols "S", we can reach "A", "B" and "a". so, "A", "B", and "a" are reachable symbols.

The unreachable symbols can be removed.

New grammar:

$$G = (N, T, P, S)$$

$$\text{where } N = \{S, B\}$$

$$T = \{a, b\}$$

$$P = \{S \rightarrow A, B, a\}$$

$$S = S.$$