

MODULE TITLE	Mathematical Structures	CREDIT VALUE	30
MODULE CODE	MTH1001	MODULE CONVENER	Prof Nigel Byott (Coordinator)
DURATION: TERM	1	2	3
DURATION: WEEKS	11	11	0
Number of Students Taking Module (anticipated)		200	

DESCRIPTION - summary of the module content

A key aspect of mathematics is its ability to unify and generalise disparate situations exhibiting similar properties by developing the concepts and language to describe the common features abstractly and reason about them rigorously. In this module, you will be introduced to the language of logic, sets, and functions which underpins of all modern pure mathematics, and will learn how to use it to construct clear and logically correct mathematical proofs. The content goes beyond mathematics taught at A-level: you will learn and use methods to prove rigorous general results about the convergence of sequences and series, justifying the techniques developed in MTH1002 and laying the foundations for a deeper study of Analysis in MTH2008. You will also learn the definitions and properties of abstract algebraic structures such as groups and vector spaces. These ideas are developed further in MTH2010 and MTH2011. The material in this module is fundamental to many other modules in the mathematics degree programmes. It underpins the topics you will see in more advanced modules in fundamental mathematics and enables a deeper understanding and rigorous justification of the mathematical tools you will meet in more applied mathematics modules and which are widely used in physics, economics, and many other disciplines.

AIMS - intentions of the module

The purpose of this module is to provide you with an introduction to axiomatic reasoning in mathematics, particularly in relation to the perspective adopted by modern algebra and analysis. The building blocks of mathematics will be developed, from logic, sets and functions through to proving key properties of the standard number systems. We will introduce and explore the abstract definition of a group, and rigorously prove standard results in the theory of groups, before progressing to consider vector spaces, both in the abstract and with a specific focus on finite-dimensional vector spaces over the real numbers. The ideas and techniques of this module are essential to the further development of these themes in the two second-year streams Analysis and Algebra, and subsequent fundamental mathematics modules in years 3 and 4.

INTENDED LEARNING OUTCOMES (ILOs) (see assessment section below for how ILOs will be assessed)

On successful completion of this module, **you should be able to:**

Module Specific Skills and Knowledge:

- 1 read, write and evaluate expressions in formal logic relating to a wide variety of mathematical contexts;
- 2 use accurately the abstract language of sets, relations, functions and their mathematical properties;
- 3 identify and use common methods of proof and understand their foundations in the logical and axiomatic basis of modern mathematics;
- 4 state and apply properties of familiar number systems (\mathbb{N} , \mathbb{Z} , $\mathbb{Z}/n\mathbb{Z}$, \mathbb{Q} , \mathbb{R} , \mathbb{C}) and the logical relationships between these properties;
- 5 recall key definitions, theorems and proofs in the theory of groups and vector spaces;
- 6 read and write formal proofs using interactive theorem prover

Discipline Specific Skills and Knowledge:

- 7 evaluate the importance of abstract algebraic structures in unifying and generalising disparate situations exhibiting similar mathematical properties;
- 8 explore open-ended problems independently and clearly state their findings with appropriate justification;

Personal and Key Transferable/ Employment Skills and Knowledge:

- 9 formulate and express precise and rigorous arguments, based on explicitly stated assumptions;
- 10 reason using abstract ideas and communicate reasoning effectively in writing;
- 11 use learning resources appropriately;
- 12 exhibit self-management and time management skills.

SYLLABUS PLAN - summary of the structure and academic content of the module

- Writing proofs using the Lean interactive theorem prover
- Sets; relations; functions; countability; logic; proof;
- Primes; elementary number theory;
- Limits of sequences; convergence of series
- Groups; examples; basic proofs; homomorphisms & isomorphisms;
- Vector spaces; linear independence; spanning; bases; linear maps; isomorphisms; n -dimensional spaces over \mathbb{R} are isomorphic to \mathbb{R}^n .

LEARNING AND TEACHING

LEARNING ACTIVITIES AND TEACHING METHODS (given in hours of study time)

Scheduled Learning & Teaching Activities	76.00	Guided Independent Study	224.00	Placement / Study Abroad
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DETAILS OF LEARNING ACTIVITIES AND TEACHING METHODS

Category	Hours of study time	Description
Scheduled Learning and Teaching Activities	66	Lectures
Scheduled Learning and Teaching Activities	10	Tutorials
Guided Independent Study	224	Studying additional recordings complementing lectures, and reading material, example sheets and revision.

ASSESSMENT

FORMATIVE ASSESSMENT - for feedback and development purposes; does not count towards module grade

Form of Assessment	Size of Assessment (e.g. duration/length)	ILOs Assessed	Feedback Method
Exercise Sheets	10 x 10 hours	All	Tutorial; model answers provided on ELE and discussed in class
Mid-Term Tests	2 x 1 hour	All	Feedback on marked sheets, class feedback

SUMMATIVE ASSESSMENT (% of credit)

Coursework	10	Written Exams	90	Practical Exams	0
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DETAILS OF SUMMATIVE ASSESSMENT

Form of Assessment	% of Credit	Size of Assessment (e.g. duration/length)	ILOs Assessed	Feedback Method
Written Exam A - Closed Book (Jan)	45	2 hours	All	Via SRS
Written Exam B - Closed Book (May)	45	2 hours	All	Via SRS
Computer-based proof A	5	5 x 1 Hour	All	Online tools
Computers-based proof B	5	5 x 1 hour	All	Online tools

DETAILS OF RE-ASSESSMENT (where required by referral or deferral)

Original Form of Assessment	Form of Re-assessment	ILOs Re-assessed	Time Scale for Re-reassessment
Written Exam A - Closed Book*	Written Exam A (45%, 2hr)	All	Ref/Def Period
Written Exam B - Closed Book*	Written Exam B (45%, 2hr)	All	Ref/Def Period
Computer-based proof A*	Computer-based proof A (5%)	All	tRef/Def Period
Computer-based proof B*	Computer-based proof B (5%)	All	Ref/Def Period

RE-ASSESSMENT NOTES

Deferrals: Reassessment will be by coursework and/or written exam in the deferred element only. For deferred candidates, the module mark will be uncapped.

Referrals: Reassessment will be by a written exam (worth 50% of the module mark) for each term where the mark for the term is less than 40%. As it is a referral, the module mark will be capped at 40%.

RESOURCES

INDICATIVE LEARNING RESOURCES - The following list is offered as an indication of the type & level of information that you are expected to consult. Further guidance will be provided by the Module Convener

ELE - <http://vle.exeter.ac.uk>

Reading list for this module:

Type	Author	Title	Edition	Publisher	Year	ISBN	Search
Set	Liebeck, M.	A Concise Introduction to Pure Mathematics	3rd	Chapman & Hall/CRC Press	2010	978-1439835982	[Library]
Set	Allenby, R.B.J.T.	Numbers and Proofs		Arnold	1997	000-0-340-67653-1	[Library]
Set	Stewart, J.	Calculus	5th	Brooks/Cole	2003	000-0-534-27408-0	[Library]
Set	McGregor, C., Nimmo, J. & Stothers, W.	Fundamentals of University Mathematics	2nd	Horwood, Chichester	2000	000-1-898-56310-1	[Library]
Set	Allenby, R.B.	Linear Algebra, Modular Mathematics		Arnold	1995	000-0-340-61044-1	[Library]
Set	Hamilton, A.G.	Linear Algebra: An Introduction with Concurrent Examples		Cambridge University Press	1989	000-0-521-32517-X	[Library]
Set	Jordan, C. and Jordan, D. A.	Groups		Arnold	1994	0-340-61045-X	[Library]
Set	Houston, K.	How to Think Like a Mathematician: A Companion to Undergraduate Mathematics	1st	Cambridge University Press	2009	978-0521719780	[Library]
Set	Thomas, G., Weir, M., Hass, J.	Thomas' Calculus	12th	Pearson	2010	978-0321643636	[Library]
Set	Lipschutz, S., Lipson, M.	Schaum's Outlines: Linear Algebra	4th	McGraw-Hill	2008	978-0071543521	[Library]

CREDIT VALUE	30	ECTS VALUE	15
PRE-REQUISITE MODULES	None		
CO-REQUISITE MODULES	None		
NQF LEVEL (FHEQ)	4	AVAILABLE AS DISTANCE LEARNING	No
ORIGIN DATE	Tuesday 10 July 2018	LAST REVISION DATE	Tuesday 11 July 2023
KEY WORDS SEARCH	Proof; Logic; Number Systems; Symmetries; Groups; Vectors; Matrices; Geometry; Linear Algebra		