

MODULE TITLE	Galois Theory	CREDIT VALUE	15
MODULE CODE	MTH3038	MODULE CONVENER	Prof Mohamed Saidi (Coordinator)
DURATION: TERM	1	2	3
DURATION: WEEKS	11	0	0
Number of Students Taking Module (anticipated)	30		

DESCRIPTION - summary of the module content

Drawing on key ideas in the theory of groups and fields, you will learn core elements of the theory of field extensions. You are already familiar with the idea that the real numbers can be extended to the complex numbers by introducing a new number as the square root of -1; Galois theory formalises such constructions and explores the intriguing relationship between groups and field extensions. As an important application of Galois Theory, you will understand why there can be no algebraic solution to the general quintic polynomial with rational coefficients.

Prerequisite module: MTH2002 or both MTH2010 (Groups, Rings, and Fields) and MTH2011 (Linear Algebra), or equivalent.

AIMS - intentions of the module

The aim of this module is to motivate and develop Galois Theory both as an abstract theory and through the study of important applications.

INTENDED LEARNING OUTCOMES (ILOs) (see assessment section below for how ILOs will be assessed)

On successful completion of this module, **you should be able to:**

Module Specific Skills and Knowledge:

1. State and apply key definitions in Galois theory;

2. State, prove and apply core theorems in Galois theory.

Discipline Specific Skills and Knowledge:

3. Perform computations accurately;

4. Use abstract reasoning to solve a range of problems.

Personal and Key Transferable / Employment Skills and Knowledge

5. Communicate your findings effectively in writing;

6. Work independently and manage your time and resources effectively.

SYLLABUS PLAN - summary of the structure and academic content of the module

- Review of the field axioms, the characteristic of a field, examples. Field extensions, degree, finite and algebraic extensions, extensions obtained by adjoining a root of an irreducible polynomial, degree in a tower of extensions; irreducibility criteria for Polynomials: Gauss' Lemma and Eisenstein's criterion.

- Splitting fields and algebraic closure. Separable and inseparable extensions. Cyclotomic polynomials and extensions. Automorphisms of a field. The group of automorphisms, the fixed field of a subgroup of automorphisms, the Galois correspondence. The fundamental theorem of Galois theory. Finite fields. Finite extensions of finite fields. The Galois theory of finite fields. Composite extensions and simple extensions. The primitive element theorem;

- Cyclotomic extensions and abelian extensions over \mathbb{Q} . Abelian groups as Galois groups over \mathbb{Q} . Cyclic extensions and Kummer theory. Galois groups of polynomials. Solvable and radical extensions: solution of cubic and quartic equations by radicals, insolubility of the quintic. Computation of Galois groups over \mathbb{Q} . Hilbert's irreducibility theorem. Polynomials with Galois groups S_n and A_n .

LEARNING AND TEACHING

LEARNING ACTIVITIES AND TEACHING METHODS (given in hours of study time)

Scheduled Learning & Teaching Activities	33.00	Guided Independent Study	117.00	Placement / Study Abroad	0.00
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DETAILS OF LEARNING ACTIVITIES AND TEACHING METHODS

Category	Hours of study time	Description
Scheduled learning and teaching activities	33	Lectures including example classes
Guided independent study	117	Lecture and assessment preparation; wider reading

ASSESSMENT

FORMATIVE ASSESSMENT - for feedback and development purposes; does not count towards module grade

Form of Assessment	Size of Assessment (e.g. duration/length)	ILOs Assessed	Feedback Method
Exercises	One sheet fortnightly (or equivalent)	All	Verbal and generic feedback in example classes. Annotated script and written feedback

SUMMATIVE ASSESSMENT (% of credit)

Coursework	20	Written Exams	80	Practical Exams	0
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DETAILS OF SUMMATIVE ASSESSMENT

Form of Assessment	% of Credit	Size of Assessment (e.g. duration/length)	ILOs Assessed	Feedback Method
Coursework 1 - based on questions submitted for assessment	10	15 hours	All	Annotated script and written/verbal feedback
Coursework 2 - based on questions submitted for assessment	10	15 hours	All	Annotated script and written/verbal feedback
Written Exam - closed book	80	2 hours (summer)	All	Written/verbal on request, SRS

DETAILS OF RE-ASSESSMENT (where required by referral or deferral)

Original Form of Assessment	Form of Re-assessment	ILOs Re-assessed	Time Scale for Re-assessment
Written Exam*	Written Exam (2 hours)	All	August Ref/Def Period
Coursework 1 *	Coursework 1	All	August Ref/Def Period
Coursework 2 *	Coursework 2	All	August Ref/Def Period

*Please refer to reassessment notes for details on deferral vs. Referral reassessment

RE-ASSESSMENT NOTES

Deferrals: Reassessment will be by coursework and/or written exam in the deferred element only. For deferred candidates, the module mark will be uncapped.

Referrals: Reassessment will be by a single written exam worth 100% of the module only. As it is a referral, the mark will be capped at 40%

RESOURCES

INDICATIVE LEARNING RESOURCES - The following list is offered as an indication of the type & level of information that you are expected to consult. Further guidance will be provided by the Module Convener

ELE: <http://vle.exeter.ac.uk/>

Reading list for this module:

Type	Author	Title	Edition	Publisher	Year	ISBN	Search
Set	Stewart, I.	Galois Theory		Chapman and Hall	2004		[Library]
Set	Rotman, J.	Galois Theory		Springer	1998		[Library]

CREDIT VALUE	15	ECTS VALUE	7.5
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PRE-REQUISITE MODULES	MTH2002, MTH2010, MTH2011
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CO-REQUISITE MODULES	
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NQF LEVEL (FHEQ)	6	AVAILABLE AS DISTANCE LEARNING	No
ORIGIN DATE	Tuesday 10 July 2018	LAST REVISION DATE	Thursday 26 January 2023
KEY WORDS SEARCH	Galois; field; extension; group; polynomial.		