

| MODULE TITLE | Topology and Metric Spaces | CREDIT VALUE | 15 |
|------------------------|----------------------------|-----------------|-----------------------------|
| MODULE CODE | мтн3040 | MODULE CONVENER | Dr Demi Allen (Coordinator) |
| DURATION: TERM | 1 | 2 | 3 |
| DURATION: WEEKS | o | 11 weeks | 0 |
| Number of Students Ta | king Module (anticipated) | 24 | |

DESCRIPTION - summary of the module content

Topology and metric spaces provide a set of powerful tools that are used in many other branches of mathematics (from Algebraic Topology and Numerical Analysis to Dynamical Systems and Ergodic Theory). Fundamental to these topics is the idea of generalising the idea of "closeness" of two objects in a set to a very general setting. These techniques are fundamental to the understanding of more advanced topics in mathematics such as Measure Theory, Functional Analysis, Algebraic Topology and Algebraic Geometry.

This course aims to give an introduction to topology and metric spaces as well as applications to basic concepts of measure theory. In every section covered in this course we will start by studying the definitions and then we will present examples and some basic properties. Some important theorems will be stated and proved. With this module you will have the opportunity to further refine your skills in problem-solving, axiomatic reasoning and the formulation of mathematical proofs.

Pre-requisite - MTH2001 or MTH2008

AIMS - intentions of the module

The objective of this module is to provide you an introduction to Topology and Metric Spaces. Our main objective will be to define the basic concepts clearly and to provide proofs of useful theorems.

INTENDED LEARNING OUTCOMES (ILOs) (see assessment section below for how ILOs will be assessed)

On successful completion of this module you should be able to:

Module Specific Skills and Knowledge

- 1. Recall and apply key definitions in Analysis;
- 2. State, prove and apply core theorems in Topology and metric spaces.

Discipline Specific Skills and Knowledge

- 3. Extract abstract problems from a diverse range of problems;
- 4. Use abstract reasoning to solve a range of problems.

Personal and Key Transferable / Employment Skills and Knowledge

- ${\bf 5.}\ {\bf Think\ analytically\ and\ use\ logical\ argument\ and\ deduction;}$
- 6. Communicate results in a clear, correct and coherent manner.

SYLLABUS PLAN - summary of the structure and academic content of the module

- Review of some real analysis: Real numbers, real sequences, limits of functions, continuity, intervals, set theory. (3 lectures)
- Metric spaces: Definition and examples, open and closed sets in metric spaces, equivalent metrics, examples. (4 lectures)
- Topological spaces: Bases, sub-bases and weak topologies, topologies of subspaces and products, homeomorphisms. (4 lectures)
- The Hausdorff condition: separation axioms, Hausdorff space, regular topological space. (3 lectures)
- Compact spaces: Definition, Compactness of [a,b], properties of compact spaces, continuous maps on compact spaces. An inverse function theorem. (3 lectures)
- Connected spaces: Connectedness, components, path-connectedness. (3 lectures)
- Complete metric spaces: Definition and examples, Fixed point theorems, the contraction mapping theorem. (4 lectures)
- Introduction to measure theory: Measure of plane sets. Outer and inner measure of a set. Measurable set (in the sense of Lebesgue). Some fundamental properties of Lebesgue measure and measurable sets. Definition and fundamental properties of measurable functions. (3 lectures)
- If time allows, a selection from the following: σ -algebras. Positive Borel measures. The Riesz representation theorem. Lp-spaces. Elementary Hilbert space theory. Banach spaces. Baire's theorem. (3 lectures)
- Revision (3 lectures)

| LEARNING AND TEACHING | | | | | | | | | |
|---|---------------------|--|--------------------------|--------|--------------------------|------|--|--|--|
| LEARNING ACTIVITIES AND TEACHING METHODS (given in hours of study time) | | | | | | | | | |
| Scheduled Learning & Teaching Activities | | | Guided Independent Study | 127.00 | Placement / Study Abroad | 0.00 | | | |
| DETAILS OF LEARNING ACTIVITIES AND TEACHING METHODS | | | | | | | | | |
| Category | Hours of study time | Description | | | | | | | |
| Scheduled learning and teaching activities | 33 | Lectures | | | | | | | |
| Example classes Guided Independent Study | 127 | Example classes Studying additional recordings complementing lectures, and reading material, examples sheets and revision | | | | | | | |

ASSESSMENT

FORMATIVE ASSESSMENT - for feedback and development purposes; does not count towards module grade

| Form of Assessment Size of Assessment (e.g. duration/length) | | ILOs Assessed | Feedback Method | |
|--|----------|---------------|-----------------------------|--|
| Coursework problem sheets | 10 hours | All | Written comments on scripts | |

SUMMATIVE ASSESSMENT (% of credit)

| Coursework | 20 | Written Exams | 80 | Practical Exams | 0 |
|------------|----|---------------|----|-----------------|---|

DETAILS OF SUMMATIVE ASSESSMENT

| Form of Assessment | % of Credit | Size of Assessment (e.g. duration/length) | ILOs Assessed | Feedback Method |
|---|----------------|---|------------------|--|
| Coursework 1- based on questions submitted for assessment | 10 | 15 hours | All | Annotated script and written/verbal feedback |
| Coursework 2- based on questions submitted for assessment | 10 | 15 hours | All | Annotated script and written/verbal feedback |
| Written Exam- closed book | 80 | 2 hours (Summer) | All | Written/verbal on request, SRS |

DETAILS OF RE-ASSESSMENT (where required by referral or deferral)

| Original Form of Assessment | m of Assessment Form of Re-assessment | | Time Scale for Re-assessment | | | | |
|---|---------------------------------------|-----|------------------------------|--|--|--|--|
| Written Exam * | Written Examination (2 hours) | All | August Ref/Def Period | | | | |
| Coursework 1 * | Coursework 1 | All | August Ref/Def Period | | | | |
| Coursework 2 * | Coursework 2 | All | August Ref/Def Period | | | | |
| *Please refer to reassessment notes for details on deferral vs. Referral reassessment | | | | | | | |

RE-ASSESSMENT NOTES

Deferrals: Reassessment will be by coursework and/or written exam in the deferred element only. For deferred candidates, the module mark will be uncapped. Referrals: Reassessment will be by a single written exam worth 100% of the module only. As it is a referral, the mark will be capped at 40%.

RESOURCES

INDICATIVE LEARNING RESOURCES - The following list is offered as an indication of the type & level of information that you are expected to consult. Further guidance will be provided by the Module Convener

ELE: http://vle.exeter.ac.uk/

Web based and Electronic Resources:

Reading list for this module:

| Тур | Author | Title | | Edition | Publisher | | Year | ISBN | Search |
|------------|-----------------------------------|-------------|--|------------|---|-----------------|--------|----------------|------------------------|
| Set Set | Falconer, K. Sutherland, W.A. | Fractal Geo | ometry n to Metric and Topological Spaces | 2nd editio | n Wiley Oxford Science Publications | | 2003 | 978-0470848623 | [Library] [Library] |
| Set Set | Rudin, W. Charles Chapman Pugh | | omplex Analysis matical Analysis | Third | McGraw Hill Undergraduate Texts in Mathema | atics, Springer | | 978-0070619876 | [Library] [Library] |
| Set | James R Munkres | Topology | , | | Prentice Hall | . , 3 | | | [Library] |
| CRE | DIT VALUE | | 15 | ECTS | VALUE | 7.5 | | | |
| PRE | REQUISITE MODULES | | MTH2001, MTH2008 | | | | | | |
| CO-I | REQUISITE MODULES | | | | | | | | |
| NQF | LEVEL (FHEQ) | | 6 | AVAI | LABLE AS DISTANCE LEARNING | No | | | |
| ORIG | GIN DATE | | Tuesday 10 July 2018 | LAST | REVISION DATE | Thursday 26 | Januar | y 2023 | |
| KEY | WORDS SEARCH | | None Defined | | | | | | |