MODULE TITLE	**Groups, Rings and Fields**	**CREDIT VALUE**	**15**
MODULE CODE	**MTH2010**	**MODULE CONVENER**	**Prof Mohamed Saidi (Coordinator)**

DURATION: TERM	**1**	**2**	**3**
DURATION: WEEKS	**11**	**0**	**0**

Number of Students Taking Module (anticipated)	**120**

DESCRIPTION - summary of the module content

In this module, you will explore some of the key techniques of modern algebra, including groups, rings, and fields. These topics have their roots in the desire to solve certain equations that arise from arithmetic and geometry.

The most familiar example of a ring is the set of all integers Z=...,-3,-2,-1,0,1,2,3... equipped with the usual operations of addition and multiplication. The familiar properties of these operations serve as a model for the axioms for rings. We can consider whether certain equations have solutions in rings such as the integers. For example, Fermat's Last Theorem famously asserts that if n is a fixed integer that is at least 3, then the equation x^n + y^n = z^n has no solutions for which x, y and z are non-zero integers. Though this problem is easy to state, its solution is extremely difficult: it was first stated in 1637 but the first complete and correct proof was given in 1994. Ring theory is essential for the fourth year module MTHM028 Algebraic Number Theory, which in turn lays the foundations for solving problems such as Fermat's Last Theorem.

Fields are special types of ring in which every non-zero element has a multiplicative inverse. Examples include the rational numbers Q, the real numbers R and the complex numbers C.

Group theory was introduced in the first year and will be developed further in this module. Not only does group theory underpin ring theory, but is also interesting and useful in its own right. For example, we shall see that group actions can be used to solve certain counting problems.

The material in this module is essential for the study of many of our pure mathematics modules at levels 3 and M, including MTH3004 Number Theory, MTH3026 Cryptography, MTH3038 Galois Theory, MTHM010 Representation Theory of Finite Groups, MTHM028 Algebraic Number Theory and MTHM029 Algebraic Curves.

Prerequisite module: MTH1001 (or equivalent).

AIMS - intentions of the module

This module aims to develop the theories and techniques of modern abstract algebra, particularly in relation to groups, rings, and fields. The main emphasis will be on rigorous definitions and proofs, but there will also be some applications such as how to solve certain counting problems.

INTENDED LEARNING OUTCOMES (ILOs) (see assessment section below for how ILOs will be assessed)

On successful completion of this module, **you should be able to**:

Module Specific Skills and Knowledge:
1 recall key definitions concerning abstract algebraic structures of groups, rings and fields;
2 understand examples of each of these, significant results on their structure and the relation between them.

Discipline Specific Skills and Knowledge:
3 tackle problems in many branches of mathematics that require the use of groups, rings and fields;
4 reveal sufficient knowledge of the fundamental algebraic concepts needed for advanced studies in pure mathematics.

Personal and Key Transferable / Employment Skills and Knowledge:
5 appreciate that concrete problems often require abstract theories for their solution;
6 show the ability to monitor your own progress, to manage time, and to formulate and solve complex problems.

SYLLABUS PLAN - summary of the structure and academic content of the module

- review of group axioms and basic examples: cyclic, symmetric and dihedral groups;
- group homomorphisms, kernel, image, isomorphisms;
- left and right cosets, normal subgroups;
- quotient groups, the first isomorphism theorem;
- group actions and permutation representations;
- group acting on itself by left multiplication;
- Orbit-Stabiliser Theorem, Orbit Counting Lemma;
- group acting on itself by conjugation, conjugacy classes, centre of a group, conjugacy in Sn, simple groups, A5 is simple;
- Sylow's theorems;
- axioms for rings, examples: integers, integers modulo n, matrix ring, polynomial ring (over C, R, and Q);
- definition and examples of fields including Q, R and C;
- units, zero divisors, integral domains, fields, field of fractions of an integral domain;
- ring homomorphisms, kernel, image;
- quotient rings, the first isomorphism theorem;
- existence of greatest common divisors in Z and K[X];
- extended Euclidean Algorithm;
- polynomial rings over a field, and over an integral domain;
- ideals: principal, prime, and maximal ideals;
- prime and irreducible elements;
- principal ideal domain, unique factorisation domain.

LEARNING AND TEACHING

LEARNING ACTIVITIES AND TEACHING METHODS (given in hours of study time)

Scheduled Learning & Teaching Activities	38.00	**Guided Independent Study**	112.00	**Placement / Study Abroad**	

DETAILS OF LEARNING ACTIVITIES AND TEACHING METHODS

Category	Hours of study time	Description
Scheduled learning and teaching activities	33	Lectures including example classes
Scheduled learning and teaching activities	5	Tutorials

Guided independent study	112	Lecture and assessment preparation; wider reading

ASSESSMENT

FORMATIVE ASSESSMENT - for feedback and development purposes; does not count towards module grade

Form of Assessment	Size of Assessment (e.g. duration/length)	ILOs Assessed	Feedback Method
Exercise sheets	5 x 10 hours	All	Exercises discussed in tutorials: tutor feedback.

SUMMATIVE ASSESSMENT (% of credit)

Coursework	10	Written Exams	90	Practical Exams	0

DETAILS OF SUMMATIVE ASSESSMENT

Form of Assessment	% of Credit	Size of Assessment (e.g. duration/length)	ILOs Assessed	Feedback Method
Written Exam – closed book	90%	2 hours (January)	All	Written/verbal on request, SRS
Coursework exercises 1	5%	15 hours	All	Annotated script and written/verbal feedback
Coursework exercises 2	5%	15 hours	All	Annotated script and written/verbal feedback

DETAILS OF RE-ASSESSMENT (where required by referral or deferral)

Original Form of Assessment	Form of Re-assessment	ILOs Re-assessed	Time Scale for Re-assessment
Written Exam*	Written Exam (2 hours) (90%)	All	August Ref/Def period
Coursework exercises 1 *	Coursework exercises 1 (5%)	All	August Ref/Def period
Coursework exercises 2 *	Coursework exercises 2 (5%)	All	August Ref/Def period

*Please refer to reassessment notes for details on deferral vs. Referral reassessment

RE-ASSESSMENT NOTES

Deferrals: Reassessment will be by coursework and/or exam in the deferred element only. For deferred candidates, the module mark will be uncapped.

Referrals: Reassessment will be by a single written exam worth 100% of the module only. As it is a referral, the mark will be capped at 40%.

RESOURCES

INDICATIVE LEARNING RESOURCES - The following list is offered as an indication of the type & level of information that you are expected to consult. Further guidance will be provided by the Module Convener

ELE: http://vle.exeter.ac.uk

Reading list for this module:

Type	Author	Title	Edition	Publisher	Year	ISBN	Search
Set	Wallace D.A.R.	Groups Rings and Fields		Springer	2001	000-3-540-76177-2	[Library]
Set	Durbin, J.	Modern Algebra: An Introduction	Sixth	John Wiley & Sons	2009	978-0-470-53035-1	[Library]
Set	Cameron, P.J.	Fields Introduction to Algebra	Second	Oxford Science Publications	2008	978-0-19-852793-0	[Library]
Set	Dummit, D S and Foote, R M	Abstract Algebra	Third	Prentice Hall	2003	978-0471433347	[Library]

CREDIT VALUE	15	ECTS VALUE	7.5
PRE-REQUISITE MODULES	MTH1001		
CO-REQUISITE MODULES			
NQF LEVEL (FHEQ)	5	AVAILABLE AS DISTANCE LEARNING	No
ORIGIN DATE	Wednesday 26 February 2020	LAST REVISION DATE	Thursday 26 January 2023
KEY WORDS SEARCH	Rings; groups; fields; isomorphism; group actions; ideals		