MODULE TITLE	**Nonlinear Systems and Control**	**CREDIT VALUE**	**15**
MODULE CODE	**MTH3011**	**MODULE CONVENER**	**Prof Prathyush P Menon (Coordinator)**

DURATION: TERM	**1**	**2**	**3**
DURATION: WEEKS	**0**	**11 weeks**	**0**

Number of Students Taking Module (anticipated)	**11**

DESCRIPTION - summary of the module content

Lyapunov theory is a landmark in the stability of dynamical systems and differential equations which has profoundly influenced both significant mathematical results and important applications. This theory is built around a study of energy-like Lyapunov functions of a system. Such functions are first motivated via simple examples and then developed into a powerful tool for analysis and control of nonlinear systems. We study stability types of equilibria and basins of attraction. Rate of change of energy is manipulated via feedback control to force equilibria to have the desired qualitative properties. Applications to mechanical, bio-chemical and economic systems will be developed. Feedback design techniques such as recursive back-stepping and adaptation are studied.

Energy-like functions play a key role in the qualitative study of the dynamical behaviour of nonlinear systems, replacing algebraic tools like eigenvalues so important for linear systems. Mechanical systems and electrical circuits have naturally defined energy. Energy can be manipulated via external control and especially feedback control. The module will develop a conceptual framework interwoven with several case studies. The emphasis for the mathematics is the application of the theory, not so much in the development of the theory. Case studies will include examples such as inverted pendula, rotating bodies, bio-reactors, etc.

On this module, there is ample opportunity for use of computer software (e.g. Maple and similar packages). You will find out how the need to choose suitable Lyapunov functions or stabilising feedbacks lends itself for developing creative mathematical processes and intuition.

Pre-requisite: MTH2003 Differential Equations, MTH2005 Modelling: Theory & Practice or equivalent

AIMS - intentions of the module

The aims of the modules include helping you to understand the nonlinear models and nonlinear phenomena, and the qualitative behavior of second order linear systems and near equilibrium points. On this module, you will learn how to locally linearise a nonlinear system and study the qualitative behavior and discover Lyapunov methods for studying stability of nonlinear systems. Furthermore, you will study the stability of the perturbed systems, the small gain theorem, controllability condition, the principles of Lyapunov based feedback design techniques and a set of mechanical, biological examples. Finally, the course will give you a good understanding of stability and analysis techniques for nonlinear systems, and prepare you to carry out basic feedback design techniques based on Lyapunov theory such as recursive back-stepping.

INTENDED LEARNING OUTCOMES (ILOs) (see assessment section below for how ILOs will be assessed)

On successful completion of this module, ***you should be able to***:
Module Specific Skills and Knowledge:

1 use the Lyapunov stability method to study stability of equilibria of simple differential equations;

2 understand the subtleties of stability in its various forms;

3 utilise the Lyapunov method as a framework for feedback stabilisation.

Discipline Specific Skills and Knowledge:

4 appreciate that Lyapunov theory is both qualitative and quantitative and the mathematical arguments and methods built around it have broader implications in applied mathematics, to some extent in pure mathematics and even in economics and engineering.

Personal and Key Transferable/ Employment Skills and Knowledge:

5 demonstrate further logical reasoning.

SYLLABUS PLAN - summary of the structure and academic content of the module

- motivating examples: linear systems, mechanical systems, bio-reactors, economic systems, free systems and forced (controlled) systems, models and preliminaries, case studies and applications;

- stability method of Lyapunov: Lyapunov functions and stability criteria, stability of equilibria of conservative and gradient systems, Lyapunov functions for linear systems and linearisation;

- further concepts to include: invariance principles, Barbalat's Lemma, converse Lyapunov results, basins of attraction;

- case study applications of Lyapunov methods: simple pendulum, rotating bodies, recurrent neural networks, Walrasian equilibria in economic models, bio-reactors;

- feedback design: Lyapunov design, adaptive control, recursive back-stepping, feedback control of coupled systems, case studies for recursive control design.

LEARNING AND TEACHING

LEARNING ACTIVITIES AND TEACHING METHODS (given in hours of study time)

Scheduled Learning & Teaching Activities	33.00	**Guided Independent Study**	117.00	**Placement / Study Abroad**	0.00

DETAILS OF LEARNING ACTIVITIES AND TEACHING METHODS

Category	Hours of study time	Description
Scheduled learning and teaching activities	28	Lectures
Scheduled learning and teaching activities	5	Example classes/group discussion
Guided independent study	117	Lecture and assessment preparation; private study

ASSESSMENT

FORMATIVE ASSESSMENT - for feedback and development purposes; does not count towards module grade

Form of Assessment	Size of Assessment (e.g. duration/length)	ILOs Assessed	Feedback Method

Form of Assessment	Size of Assessment (e.g. duration/length)	ILOs Assessed	Feedback Method
Coursework: problem sheets	10 hours, 4-6 questions per problem sheet (2 sheets)	All	Written

SUMMATIVE ASSESSMENT (% of credit)

Coursework	20	Written Exams	80	Practical Exams	0

DETAILS OF SUMMATIVE ASSESSMENT

Form of Assessment	% of Credit	Size of Assessment (e.g. duration/length)	ILOs Assessed	Feedback Method
Coursework – based on questions submitted for assessment	20	1 assignment, 24 hours total	All	Annotated script and written/verbal feedback
Written Exam – closed book	80	2 hours (Summer)	All	Written/verbal on request, SRS

DETAILS OF RE-ASSESSMENT (where required by referral or deferral)

Original Form of Assessment	Form of Re-assessment	ILOs Re-assessed	Time Scale for Re-reassessment
Written Exam *	Written Exam (2 hours)	All	August Ref/Def period
Coursework *	Coursework	All	August Ref/Def period

*Please refer to reassessment notes for details on deferral vs. Referral reassessment

RE-ASSESSMENT NOTES

Deferrals: Reassessment will be by coursework and/or written exam in the deferred element only. For deferred candidates, the module mark will be uncapped.
Referrals: Reassessment will be by a single written exam worth 100% of the module only. As it is a referral, the mark will be capped at 40%.

RESOURCES

INDICATIVE LEARNING RESOURCES - The following list is offered as an indication of the type & level of information that you are expected to consult. Further guidance will be provided by the Module Convener

ELE: http://vle.exeter.ac.uk
Reading list for this module:

Type	Author	Title	Edition	Publisher	Year	ISBN	Search
Set	Khalil, H.K.	Nonlinear Systems		Prentice-Hall	2000	000-0-132-28024-8	[Library]

CREDIT VALUE	15	ECTS VALUE	7.5
PRE-REQUISITE MODULES	MTH2003, MTH2005		
CO-REQUISITE MODULES			
NQF LEVEL (FHEQ)	6	AVAILABLE AS DISTANCE LEARNING	No
ORIGIN DATE	Tuesday 10 July 2018	LAST REVISION DATE	Thursday 26 January 2023
KEY WORDS SEARCH	None Defined		