MODULE TITLE	**Integral Equations**	**CREDIT VALUE**	**15**
MODULE CODE	**MTH3042**	**MODULE CONVENER**	**Dr Layal Hakim (Coordinator)**

DURATION: TERM	**1**	**2**	**3**
DURATION: WEEKS	**0**	**11**	**0**

Number of Students Taking Module (anticipated)	**150**

DESCRIPTION - summary of the module content

Similarly to differential equations, integral equations provide an effective way to model real life situations, particularly those that arise in physics and engineering. Using certain techniques, many initial and boundary value problems can be converted to integral equations where the unknown function lies in the integrand. This module will introduce students to the mathematics of integral equations, techniques of analysing such equations, and methods of solving them, analytically or numerically.
The module MTH2003 is prerequisite for this. MTH2001 or MTH2008 are highly recommended.

AIMS - intentions of the module

Following the introduction of what integral equations, students will be introduced to a large class of integral equation. Volterra integral equations and Fredholm integral equations will be explained and methods on how to solve these equations will be described for cases of having integral equations of the first kind and the second kind, as well as looking at homogeneous and nonhomogeneous equations. Examples, that model real life problems, will be given and solutions will be interpreted.

INTENDED LEARNING OUTCOMES (ILOs) (see assessment section below for how ILOs will be assessed)

On successful completion of this module ***you should be able to***:
Module Specific Skills and Knowledge:
1. Classify integral equations;

2. Define the Laplace transform and implement its use to solve integral equations;

3. Explicitly solve several classes of integral equations, both analytically and numerically;

4. Deploy the analysis of integral equations;

5. Illustrate the use of integral equations to model real life problems.

Discipline Specific Skills and Knowledge:

6. Analyse qualitative information about the solution;

7. Develop further the ability of problem structuring, problem solving, and logical thinking;

8. Assemble the necessary parts of a proof that form a final result by a chain of reasoning.

Personal and Key Transferable / Employment Skills and Knowledge:

9. Describe real life applications using integral equations through examples and exercises;

10. Build the ability to identify which techniques are suitable for which problems;

11. Communicate ideas effectively by learning the analysis of integral equations.

SYLLABUS PLAN - summary of the structure and academic content of the module

Classification of integral equations:
- Linear/nonlinear, Fredholm/Volterra, homogeneous/inhomogeneous, first/second kind.
Structure of kernels:
- convolution/non-convolution type, separable kernels, finite rank kernels, and weakly singular kernels.
Laplace transforms:
- An introduction to Laplace transforms with a focus on integral equations with a convolution type kernel;
- When to use, and how to obtain, the inverse Laplace transform.
iii. Linear integral equations:
- Conditions under which the solutions to first and second kind integral equations exist;
- Linear operators and The Fredholm Alternative;
- Methods of obtain the exact and numerical solutions to Fredholm and Volterra integral equations;
- Bounded linear integral operators and how to find the bound using norms of integral operators;
- Iterative techniques, particularly the Neumann iteration method, for second kind equations;
- Approximation techniques using Taylor series;
- Analysis of integral equations: criteria for convergence; existence and uniqueness of solutions; continuity of integral operators;
- The Fredholm Theorem and its proof, the Herbert-Shmidt Theorem for self-adjoint kernels and its proof;
- Error estimates in the numerical solution;
- Nonlinear integral equations: Methods of obtaining solutions to simple cases of the Fredholm integral equation of the Hammerstein type;
- Applications to where integral equations are used to model the behaviour of real life problems.

LEARNING AND TEACHING

LEARNING ACTIVITIES AND TEACHING METHODS (given in hours of study time)

Scheduled Learning & Teaching Activities	33.00	**Guided Independent Study**	117.00	**Placement / Study Abroad**	

DETAILS OF LEARNING ACTIVITIES AND TEACHING METHODS

Category	Hours of study time	Description
Lectures	28	Definitions are stated and explained, theorems are stated and proved, examples are thoroughly carried out interactively, and results and techniques are discussed.
Tutorials	5	Students can practice exercises, examples and ask questions in a supportive environment.
Independent Study	117	Independent reading and problem solving.

ASSESSMENT

FORMATIVE ASSESSMENT - for feedback and development purposes; does not count towards module grade

Form of Assessment	Size of Assessment (e.g. duration/length)	ILOs Assessed	Feedback Method
Exercise sheets	5 x 10 hours	All	The lecturer will discuss problems during tutorials. Solutions to the sheets will be uploaded onto the VLE at some point after the tutorial.

SUMMATIVE ASSESSMENT (% of credit)

Coursework	20	**Written Exams**	80	**Practical Exams**	0

DETAILS OF SUMMATIVE ASSESSMENT

Form of Assessment	% of Credit	Size of Assessment (e.g. duration/length)	ILOs Assessed	Feedback Method
Coursework 1- based on questions submitted for assessment	10	15 hours	All	Annotated script and written/verbal feedback
Coursework 2- based on questions submitted for assessment	10	15 hours	All	
Written Exam – closed book	80	2 hours (Summer)	All	Written/verbal on request, SRS

DETAILS OF RE-ASSESSMENT (where required by referral or deferral)

Original Form of Assessment	Form of Re-assessment	ILOs Re-assessed	Time Scale for Re-assessment
Written Exam *	Written Exam (2 hours)	All	During the August Ref/Def Period
Coursework 1 *	Coursework 1	All	August Ref/Def Period
Coursework 2 *	Coursework 2	All	August Ref/Def Period

*Please refer to reassessment notes for details on deferral vs. Referral reassessment

RE-ASSESSMENT NOTES

Deferrals: Reassessment will be by coursework and/or written exam in the deferred element only. For deferred candidates, the module mark will be uncapped.
Referrals: Reassessment will be by a single written exam worth 100% of the module only. As it is a referral, the mark will be capped at 40%.

RESOURCES

INDICATIVE LEARNING RESOURCES - The following list is offered as an indication of the type & level of information that you are expected to consult. Further guidance will be provided by the Module Convener

Basic reading:
ELE – http://vle.exeter.ac.uk/

Web based and electronic resources:

Other resources:

Reading list for this module:

Type	Author	Title	Edition	Publisher	Year	ISBN	Search
Set	RP Kanwal	Linear Integral Equations	2nd ed.	Birkhauser, Boston	1997		[Library]
Set	AC Pipkin	A Course on Integral Equations		Springer-Verlag, New York	1991		[Library]
Set	BJ Moiseiwitsch	Integral Equations		Longman, London	1977		[Library]
Set	D Porter	Integral Equations. A practical treatment, from Spectral Theory to Applications		DSG, Stirling, CUP	1990		[Library]

CREDIT VALUE	15	**ECTS VALUE**	7.5
PRE-REQUISITE MODULES	MTH2003		
CO-REQUISITE MODULES			
NQF LEVEL (FHEQ)	6	**AVAILABLE AS DISTANCE LEARNING**	No
ORIGIN DATE	Wednesday 03 April 2019	**LAST REVISION DATE**	Thursday 26 January 2023
KEY WORDS SEARCH	Integral equations; Volterra integral equations; Fredholm integral equations; Laplace transforms.		