Makarenkov Artem

CT 2

№1

1. Nonlinear optimization problem is given in the following form

$$\min\{f(x) = -3\,x_1{}^4 + 6\,x_1{}^2x_2{}^2 - 4\,x_1{}^2x_3{}^2 + 10\,x_2{}^2x_3{}^2 + 2\,x_3{}^4\},$$

subject to

$$h_1(x) = 8\,x_1{}^2 + 10\,x_1x_2 + 6\,x_1x_3 - 48\,x_1 + 8\,x_2{}^2 + 8\,x_2x_3 - 14\,x_2 + 3\,x_3{}^2 + 36\,x_3 - 120 = 0,$$

$$g_1(x) = -8\,x_1 - 3\,x_2 + 4\,x_3 - 2 \leqslant 0,$$

$$g_2(x) = -4\,x_1 + 104.5\,x_2 + 97\,x_3 - 406 \leqslant 0.$$

Check whether $\bar{x} = (0, 2, 2)^\top$, $\tilde{x} = (0.989103, 1.906098, 2.172878)^\top$, $\hat{x} = (3, 4, 0)^\top$ are stationary points. (1.0)

$$f'_{x_1} = -12x_1^3 + 12x_1x_2^2 - 8x_1x_3^2$$

$$f'_{x_2} = 12x_2x_1^2 + 20x_2x_3^2$$

$$f_{x_3} = -8x_3x_1^2 + 20x_3x_2^2 + 8x_3^3$$

$$h'_{x_1} = 16x_1 + 10x_2 + 6x_3 - 48$$

$$h'_{x_2} = 10x_1 + 16x_2 + 8x_3 - 14$$

$$h'_{x_3} = 6x_1 + 8x_2 + 6x_3 + 36$$

Value of h, g_1, g_2 for all points

```
1:h1(x_bar) = 0
1:g1(x_bar) = 0
1:g2(x_bar) = -3.0
2:h1(x_bar) = -1.627774801704618e-05
2:g1(x_bar) = -6.939606000000001
2:g2(x_bar) = -4.9999999873762135e-06
3:h1(x_bar) = 0
3:g1(x_bar) = -38
3:g2(x_bar) = 0.0
```

Value of derivatives for each point

```
f1_x1(x_bar) = 0
f1_x2(x_bar) = 160
f1_x3(x_bar) = 224
h1_x1(x_bar) = -16
h1_x2(x_bar) = 34
h1_x3(x_bar) = 64
----------------------------
f1_x1(x_bar) = -5.848143030439637
f1_x2(x_bar) = 202.36637037618843
f1_x3(x_bar) = 222.9563697477327
h1_x1(x_bar) = -0.07610400000000084
h1_x2(x_bar) = 43.771622
h1_x3(x_bar) = 70.22067
----------------------------
f1_x1(x_bar) = 252
f1_x2(x_bar) = 432
f1_x3(x_bar) = 0
h1_x1(x_bar) = 40
h1_x2(x_bar) = 80
h1_x3(x_bar) = 86
```

compute points for each derivative:

Checking point 1

Stationarity condition

$$\begin{cases} 0 + \lambda \cdot -16 - 8\mu_1 - 4\overset{=0}{\mu_2} = 0 \\ 160 + \lambda \cdot 34 - 3\mu_1 + 104{,}5\,\overset{=0}{\mu_2} = 0 \\ 224 + \lambda \cdot 64 + 4\mu_1 + 97\,\overset{=0}{\mu_2} = 0 \end{cases}$$

solving system, we got

$$\overset{\text{exist}}{\lambda = -4} \; ; \; \begin{matrix} \mu_1 = 8 \geq 0 \\ \mu_2 = 0 \geq 0 \end{matrix} \Bigg| \Rightarrow \bar{x} \text{ is stationary point.}$$

checking point 2:

$f_{x_1} = -5{,}84814$; $f_{x_2} = 202{,}36637$; $f_{x_3} = 222{,}9564$

$h_{x_1} = -0{,}0761$; $h_{x_2} = 43{,}7716$; $h_{x_3} = 70{,}22067$

stationary condition

$$\begin{cases} f_{x_1} + \lambda h_{x_1} = 0 \\ f_{x_2} + \lambda h_{x_2} = 0 \\ f_{x_3} + \lambda h_{x_3} = 0 \end{cases}$$

Check solution: there is no solution $\Rightarrow$

$\Rightarrow \tilde{x}$ is not stationary point

3) Check point 3:

Stationary condition

$$\begin{cases} 252 + \lambda 40 - \mu_2 4 \\ 432 + \lambda 80 + \mu_2 104{,}5 \\ 0 + \lambda 86 + \mu_2 \cdot 97 \end{cases}$$

Checking solution: No solution exist $\Rightarrow$

$\Rightarrow \hat{x}$ is not stationary point

Answer: $\bar{x}$ - stationary

$\tilde{x}$ - not stationary

$\hat{x}$ - not stationary

N 3

You can find code in

Makarenkov_task_3.py

Results:

Final point: [-5.00001514e-01 -5.00001514e-01
-5.00001514e-01 -5.00001514e-01
-5.00001514e-01 -5.00001514e-01 -5.00001514e-01
-5.00001514e-01
-5.00001514e-01 -1.43849011e-06]
Num of interation: 7
Value of the function: 30.625000000213618

N 2

You can find code in

Makarenkov_task_2.py

Results:

3.98105386e-01 3.95477026e-01 4.01001309e-01
3.81302155e-01
4.51697028e-01 3.23586330e-05]
Num of interation: 53
Value of the function: 15.77250962951612