

TRIGONOMETRIE HYPERBOLIQUE

Définition

$$\forall x \in \mathbb{R} \quad \operatorname{sh} x = \frac{e^x - e^{-x}}{2} \quad \operatorname{ch} x = \frac{e^x + e^{-x}}{2} \quad \operatorname{th} x = \frac{\operatorname{sh} x}{\operatorname{ch} x}$$

Formule fondamentale : $\operatorname{ch}^2 - \operatorname{sh}^2 = 1$.

Formules d'addition et de soustraction (hors programme)

$$\begin{aligned} \operatorname{ch}(a+b) &= \operatorname{ch} a \operatorname{ch} b + \operatorname{sh} a \operatorname{sh} b & \operatorname{ch}(a-b) &= \operatorname{ch} a \operatorname{ch} b - \operatorname{sh} a \operatorname{sh} b \\ \operatorname{sh}(a+b) &= \operatorname{sh} a \operatorname{ch} b + \operatorname{ch} a \operatorname{sh} b & \operatorname{sh}(a-b) &= \operatorname{sh} a \operatorname{ch} b - \operatorname{ch} a \operatorname{sh} b \\ \operatorname{th}(a+b) &= \frac{\operatorname{th} a + \operatorname{th} b}{1 + \operatorname{th} a \operatorname{th} b} & \operatorname{th}(a-b) &= \frac{\operatorname{th} a - \operatorname{th} b}{1 - \operatorname{th} a \operatorname{th} b} \end{aligned}$$

Formules de duplication (hors programme)

$$\begin{aligned} \operatorname{ch} 2a &= \operatorname{ch}^2 a + \operatorname{sh}^2 a = 2 \operatorname{ch}^2 a - 1 = 2 \operatorname{sh}^2 a + 1 \\ \operatorname{sh} 2a &= 2 \operatorname{sh} a \operatorname{ch} a \\ \operatorname{th} 2a &= \frac{2 \operatorname{th} a}{1 + \operatorname{th}^2 a} \end{aligned}$$

Parité

Les fonctions sh et th sont impaires. La fonction ch est paire.

Dérivation

$$\operatorname{sh}' = \operatorname{ch} \quad \operatorname{ch}' = \operatorname{sh} \quad \operatorname{th}' = 1 - \operatorname{th}^2 = \frac{1}{\operatorname{ch}^2}$$

Limites

$$\begin{aligned} \lim_{x \rightarrow +\infty} \operatorname{sh} x &= +\infty & \lim_{x \rightarrow +\infty} \operatorname{ch} x &= +\infty & \lim_{x \rightarrow +\infty} \operatorname{th} x &= 1 \\ \lim_{x \rightarrow -\infty} \operatorname{sh} x &= -\infty & \lim_{x \rightarrow -\infty} \operatorname{ch} x &= +\infty & \lim_{x \rightarrow -\infty} \operatorname{th} x &= -1 \end{aligned}$$

Graphes