TRIGONOMÉTRIE HYPERBOLIQUE

Définition

$$\forall x \in \mathbb{R}$$

$$\sin x = \frac{e^x - e^{-x}}{2}$$

$$\operatorname{ch} x = \frac{e^x + e^{-x}}{2}$$

$$th x = \frac{sh x}{ch x}$$

Formule fondamentale : $ch^2 - sh^2 = 1$.

Formules d'addition et de soustraction (hors programme)

$$\operatorname{ch}(a+b) = \operatorname{ch} a \operatorname{ch} b + \operatorname{sh} a \operatorname{sh} b$$

$$\operatorname{sh}(a+b) = \operatorname{sh} a \operatorname{ch} b + \operatorname{ch} a \operatorname{sh} b$$

$$\operatorname{th}(\mathfrak{a}+\mathfrak{b}) = \frac{\operatorname{th}\mathfrak{a} + \operatorname{th}\mathfrak{b}}{1 + \operatorname{th}\mathfrak{a}\operatorname{th}\mathfrak{b}}$$

$$\operatorname{ch}(\mathfrak{a}-\mathfrak{b})=\operatorname{ch}\mathfrak{a}\operatorname{ch}\mathfrak{b}-\operatorname{sh}\mathfrak{a}\operatorname{sh}\mathfrak{b}$$

$$\operatorname{sh}(\mathfrak{a}-\mathfrak{b})=\operatorname{sh}\mathfrak{a}\operatorname{ch}\mathfrak{b}-\operatorname{ch}\mathfrak{a}\operatorname{sh}\mathfrak{b}$$

$$\operatorname{th}(a-b) = \frac{\operatorname{th} a - \operatorname{th} b}{1 - \operatorname{th} a \operatorname{th} b}$$

Formules de duplication (hors programme) -

$$ch 2a = ch^2 a + sh^2 a = 2 ch^2 a - 1 = 2 sh^2 a + 1$$

$$\operatorname{sh} 2\mathfrak{a} = 2\operatorname{sh} \mathfrak{a}\operatorname{ch} \mathfrak{a}$$

$${\rm th}\, 2\alpha = \frac{2\,{\rm th}\,\alpha}{1+{\rm th}^2\,\alpha}$$

Parité

Les fonctions sh et th sont impaires. La fonction ch est paire.

Dérivation

$$\mathrm{sh}^{\,\prime}=\mathrm{ch}$$

$$\operatorname{ch}' = \operatorname{sh}$$

$$th' = 1 - th^2 = \frac{1}{ch^2}$$

Limites -

$$\lim_{x\to +\infty} \operatorname{sh} x = +\infty$$

$$\lim_{x \to -\infty} \sin x = -\infty$$

$$\lim_{x\to +\infty} \operatorname{ch} x = +\infty$$

$$\lim_{x \to -\infty} \operatorname{ch} x = +\infty$$

$$\lim_{x \to +\infty} \operatorname{th} x = 1$$

$$\lim_{x \to -\infty} \operatorname{th} x = -1$$

