

# TRIGONOMÉTRIE

## Formules d'addition et de soustraction

$$\begin{aligned}\cos(a+b) &= \cos a \cos b - \sin a \sin b & \sin(a+b) &= \sin a \cos b + \cos a \sin b & \tan(a+b) &= \frac{\tan a + \tan b}{1 - \tan a \tan b} \\ \cos(a-b) &= \cos a \cos b + \sin a \sin b & \sin(a-b) &= \sin a \cos b - \cos a \sin b & \tan(a-b) &= \frac{\tan a - \tan b}{1 + \tan a \tan b}\end{aligned}$$

## Cas particuliers

$$\begin{aligned}\cos\left(x + \frac{\pi}{2}\right) &= -\sin x & \sin\left(x + \frac{\pi}{2}\right) &= \cos x & \tan\left(x + \frac{\pi}{2}\right) &= -\frac{1}{\tan x} \\ \cos\left(x - \frac{\pi}{2}\right) &= \sin x & \sin\left(x - \frac{\pi}{2}\right) &= -\cos x & \tan\left(x - \frac{\pi}{2}\right) &= -\frac{1}{\tan x} \\ \cos\left(\frac{\pi}{2} - x\right) &= \sin x & \sin\left(\frac{\pi}{2} - x\right) &= \cos x & \tan\left(\frac{\pi}{2} - x\right) &= \frac{1}{\tan x} \\ \cos(x + n\pi) &= (-1)^n \cos x & \sin(x + n\pi) &= (-1)^n \sin x & & \end{aligned}$$

## Formules de duplication

$$\begin{aligned}\cos 2a &= \cos^2 a - \sin^2 a & \sin 2a &= 2 \sin a \cos a & \tan 2a &= \frac{2 \tan a}{1 - \tan^2 a} \\ &= 2 \cos^2 a - 1 = 1 - 2 \sin^2 a & & & & \\ \cos^2 a &= \frac{1 + \cos 2a}{2} & \sin^2 a &= \frac{1 - \cos 2a}{2} & & \end{aligned}$$

## Formules de factorisation

$$\begin{aligned}\cos a + \cos b &= 2 \cos \frac{a+b}{2} \cos \frac{a-b}{2} & \sin a + \sin b &= 2 \sin \frac{a+b}{2} \cos \frac{a-b}{2} \\ \cos a - \cos b &= -2 \sin \frac{a+b}{2} \sin \frac{a-b}{2} & \sin a - \sin b &= 2 \cos \frac{a+b}{2} \sin \frac{a-b}{2}\end{aligned}$$

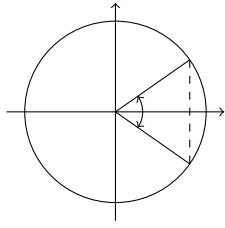
## Formules de linéarisation

$$\begin{aligned}\cos a \cos b &= \frac{1}{2} (\cos(a+b) + \cos(a-b)) & \sin a \sin b &= \frac{1}{2} (\cos(a-b) - \cos(a+b)) \\ \sin a \cos b &= \frac{1}{2} (\sin(a+b) + \sin(a-b)) & \cos a \sin b &= \frac{1}{2} (\sin(a+b) - \sin(a-b))\end{aligned}$$

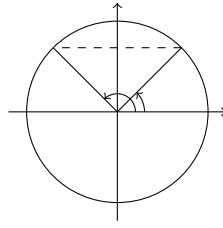
## Paramétrage rationnel du cercle trigonométrique

$$\cos \theta = \frac{1-t^2}{1+t^2} \quad \sin \theta = \frac{2t}{1+t^2} \quad \tan \theta = \frac{2t}{1-t^2} \quad \left( \text{avec } t = \tan \frac{\theta}{2} \right)$$

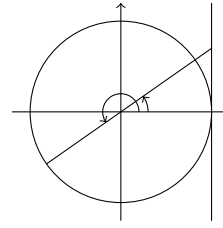
## Equations trigonométriques



$$\cos a = \cos b \Leftrightarrow \begin{cases} a \equiv b[2\pi] \\ \text{ou} \\ a \equiv -b[2\pi] \end{cases}$$



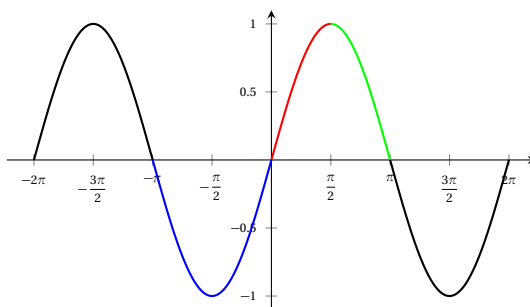
$$\sin a = \sin b \Leftrightarrow \begin{cases} a \equiv b[2\pi] \\ \text{ou} \\ a \equiv \pi - b[2\pi] \end{cases}$$



$$\tan a = \tan b \Leftrightarrow a \equiv b[\pi]$$

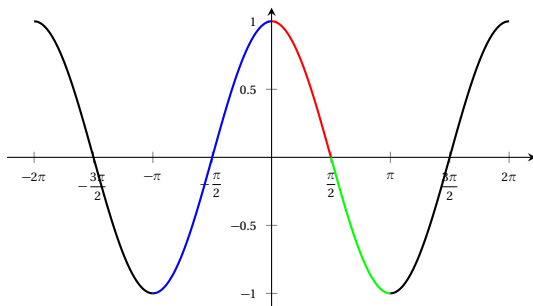
## Graphes

Graphe de sin



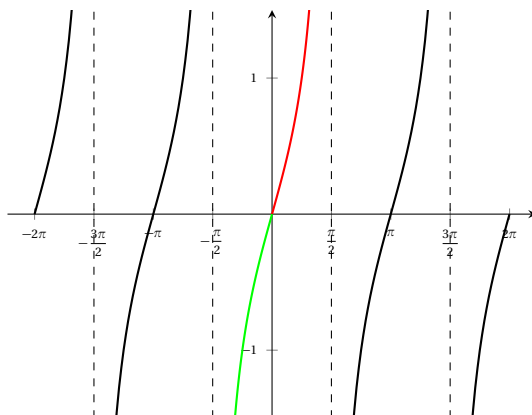
- Etude sur  $\left[0, \frac{\pi}{2}\right]$
- Prolongement par symétrie d'axe  $x = \frac{\pi}{2}$
- Prolongement par parité
- Prolongement par  $2\pi$ -périodicité

Graphe de cos



- Etude sur  $\left[0, \frac{\pi}{2}\right]$
- Prolongement par symétrie de centre  $\left(\frac{\pi}{2}, 0\right)$
- Prolongement par parité
- Prolongement par  $2\pi$ -périodicité

Graphe de tan



- Etude sur  $\left[0, \frac{\pi}{2}\right]$
- Prolongement par parité
- Prolongement par  $\pi$ -périodicité