Développements limités

Formule de Taylor-Young

Si f est de classe $\mathcal{C}^n$ au voisinage de a, alors f admet le $\mathrm{DL}_n(a)$ suivant :

$$\begin{aligned} f(x) &\underset{x\to a}{=} f(a)+f'(a)(x-a)+\frac{f''(a)}{2}(x-a)^2+\cdots+\frac{f^{(n)}(a)}{n!}(x-a)^n+o((x-a)^n) \\ &\underset{x\to a}{=} \sum_{k=0}^{n}\frac{f^{(k)}(a)}{k!}(x-a)^k+o((x-a)^n) \end{aligned}$$

Puissance, logarithme, exponentielle

$$\frac{1}{1-x} \underset{x\to 0}{=} \sum_{k=0}^{n} x^k+o(x^n) \underset{x\to 0}{=} 1+x+x^2+\cdots+x^n+o(x^n)$$

$$\frac{1}{1+x} \underset{x\to 0}{=} \sum_{k=0}^{n} (-1)^k x^k+o(x^n) \underset{x\to 0}{=} 1-x+x^2-x^3+x^4\cdots+(-1)^n x^n+o(x^n)$$

$$\ln(1+x) \underset{x\to 0}{=} \sum_{k=1}^{n} (-1)^{k-1}\frac{x^k}{k}+o(x^n) \underset{x\to 0}{=} x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots+(-1)^{n-1}\frac{x^n}{n}+o(x^n)$$

$$e^x \underset{x\to 0}{=} \sum_{k=0}^{n} \frac{x^k}{k!}+o(x^n) \underset{x\to 0}{=} 1+x+\frac{x^2}{2}+\frac{x^3}{6}+\cdots+\frac{x^n}{n!}+o(x^n)$$

$$\begin{aligned} (1+x)^\alpha &\underset{x\to 0}{=} 1+\alpha x+\frac{\alpha(\alpha-1)x^2}{2}+\frac{\alpha(\alpha-1)(\alpha-2)x^3}{6}+\cdots+\frac{\alpha(\alpha-1)(\alpha-2)\ldots(\alpha-n+1)x^n}{n!}+o(x^n) \\ &\underset{x\to 0}{=} \sum_{k=0}^{n}\frac{\prod_{j=0}^{k-1}(\alpha-j)}{k!}x^k \end{aligned}$$

Fonctions circulaires

$$\sin x \underset{x\to 0}{=} \sum_{k=0}^{n}(-1)^k\frac{x^{2k+1}}{(2k+1)!}+o(x^{2n+1}) \underset{x\to 0}{=} x-\frac{x^3}{6}+\frac{x^5}{120}-\cdots+(-1)^n\frac{x^{2n+1}}{(2n+1)!}+o(x^{2n+1})$$

$$\cos x \underset{x\to 0}{=} \sum_{k=0}^{n}(-1)^k\frac{x^{2k}}{(2k)!}+o(x^{2n}) \underset{x\to 0}{=} 1-\frac{x^2}{2}+\frac{x^4}{24}-\cdots+(-1)^n\frac{x^{2n}}{(2n)!}+o(x^{2n})$$

$$\arctan x \underset{x\to 0}{=} \sum_{k=0}^{n}(-1)^k\frac{x^{2k+1}}{2k+1}+o(x^{2n+1}) \underset{x\to 0}{=} x-\frac{x^3}{3}+\frac{x^5}{5}-\cdots+(-1)^n\frac{x^{2n+1}}{2n+1}+o(x^{2n+1})$$

Fonctions hyperboliques

$$\operatorname{sh} x \underset{x\to 0}{=} \sum_{k=0}^{n}\frac{x^{2k+1}}{(2k+1)!}+o(x^{2n+1}) \underset{x\to 0}{=} x+\frac{x^3}{6}+\frac{x^5}{120}+\cdots+\frac{x^{2n+1}}{(2n+1)!}+o(x^{2n+1})$$

$$\operatorname{ch} x \underset{x\to 0}{=} \sum_{k=0}^{n}\frac{x^{2k}}{(2k)!}+o(x^{2n}) \underset{x\to 0}{=} 1+\frac{x^2}{2}+\frac{x^4}{24}+\cdots+\frac{x^{2n}}{(2n)!}+o(x^{2n})$$