# Développements limités

## Formule de Taylor-Young

Si f est de classe $\mathcal{C}^n$ au voisinage de $a$, alors f admet le $\mathrm{DL}_n(a)$ suivant :

$$\begin{aligned}
f(x) &\underset{x\to a}{=} f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + o((x-a)^n) \\
&\underset{x\to a}{=} \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k + o((x-a)^n)
\end{aligned}$$

## Puissance, logarithme, exponentielle

$$\begin{aligned}
\frac{1}{1-x} &\underset{x\to 0}{=} \sum_{k=0}^{n} x^k + o(x^n) &&\underset{x\to 0}{=} 1 + x + x^2 + \cdots + x^n + o(x^n) \\
\frac{1}{1+x} &\underset{x\to 0}{=} \sum_{k=0}^{n} (-1)^k x^k + o(x^n) &&\underset{x\to 0}{=} 1 - x + x^2 - x^3 + x^4 \cdots + (-1)^n x^n + o(x^n) \\
\ln(1+x) &\underset{x\to 0}{=} \sum_{k=1}^{n} (-1)^{k-1}\frac{x^k}{k} + o(x^n) &&\underset{x\to 0}{=} x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n-1}\frac{x^n}{n} + o(x^n) \\
e^x &\underset{x\to 0}{=} \sum_{k=0}^{n} \frac{x^k}{k!} + o(x^n) &&\underset{x\to 0}{=} 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots + \frac{x^n}{n!} + o(x^n)
\end{aligned}$$

$$\begin{aligned}
(1+x)^\alpha &\underset{x\to 0}{=} 1 + \alpha x + \frac{\alpha(\alpha-1)x^2}{2} + \frac{\alpha(\alpha-1)(\alpha-2)x^3}{6} + \cdots + \frac{\alpha(\alpha-1)(\alpha-2)\dots(\alpha-n+1)x^n}{n!} + o(x^n) \\
&\underset{x\to 0}{=} \sum_{k=0}^{n} \frac{\prod_{j=0}^{k-1}(\alpha-j)}{k!} x^k
\end{aligned}$$

## Fonctions circulaires

$$\begin{aligned}
\sin x &\underset{x\to 0}{=} \sum_{k=0}^{n} (-1)^k \frac{x^{2k+1}}{(2k+1)!} + o(x^{2n+1}) &&\underset{x\to 0}{=} x - \frac{x^3}{6} + \frac{x^5}{120} - \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+1}) \\
\cos x &\underset{x\to 0}{=} \sum_{k=0}^{n} (-1)^k \frac{x^{2k}}{(2k)!} + o(x^{2n}) &&\underset{x\to 0}{=} 1 - \frac{x^2}{2} + \frac{x^4}{24} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n}) \\
\arctan x &\underset{x\to 0}{=} \sum_{k=0}^{n} (-1)^k \frac{x^{2k+1}}{2k+1} + o(x^{2n+1}) &&\underset{x\to 0}{=} x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + o(x^{2n+1}) \\
\tan x &= \underset{x\to 0}{=} x + \frac{x^3}{3} + o(x^3)
\end{aligned}$$

## Fonctions hyperboliques

$$\begin{aligned}
\operatorname{sh} x &\underset{x\to 0}{=} \sum_{k=0}^{n} \frac{x^{2k+1}}{(2k+1)!} + o(x^{2n+1}) &&\underset{x\to 0}{=} x + \frac{x^3}{6} + \frac{x^5}{120} + \cdots + \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+1}) \\
\operatorname{ch} x &\underset{x\to 0}{=} \sum_{k=0}^{n} \frac{x^{2k}}{(2k)!} + o(x^{2n}) &&\underset{x\to 0}{=} 1 + \frac{x^2}{2} + \frac{x^4}{24} + \cdots + \frac{x^{2n}}{(2n)!} + o(x^{2n})
\end{aligned}$$