l1p2

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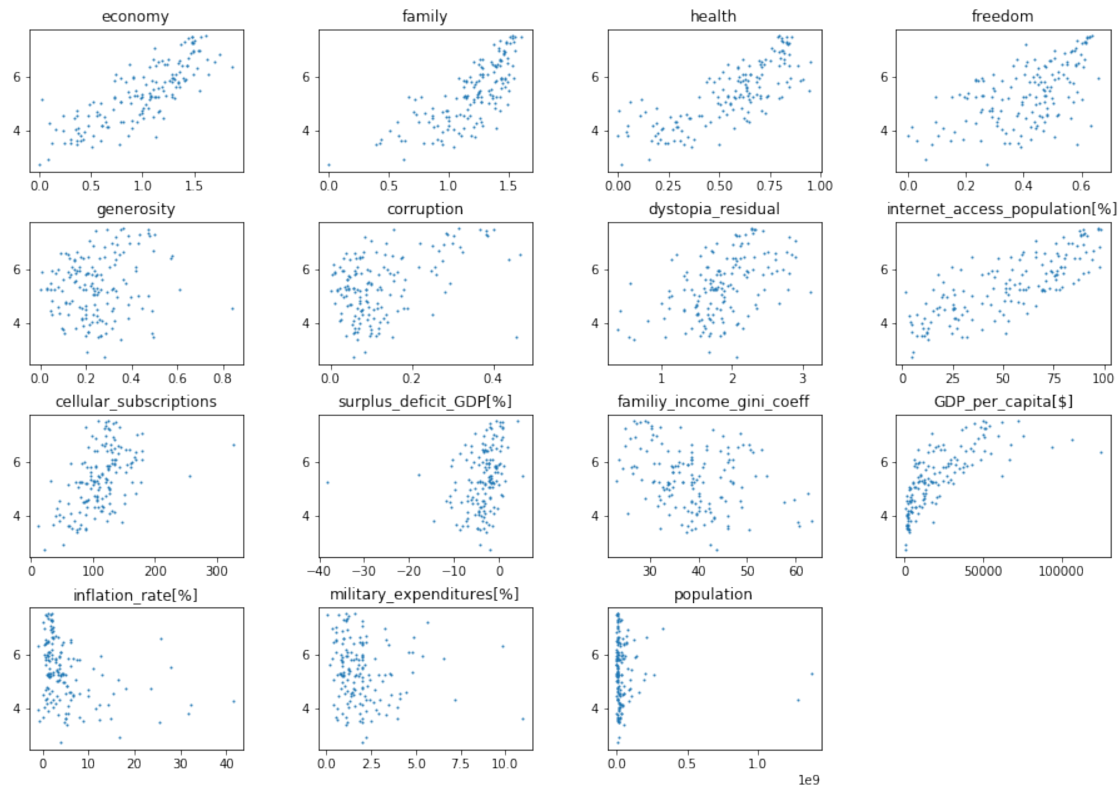
Robert Ernstbrunner, MatNr.: 01403753

1 Q2.1

1. Use `matplotlib` to show scatterplots of each variable

```
In [1]: from matplotlib import pyplot as plt
        import numpy as np
        import pandas as pd

        # added
        from sklearn.preprocessing import StandardScaler

        data = pd.read_csv("happiness.csv")

        # fill impossible data with NaN
        data.loc[data['inflation_rate[%]'] > 100, 'inflation_rate[%]'] = np.nan

        # drop useless columns
        data.drop(columns=['country', 'happiness_rank', 'map_reference', \
                           'biggest_official_language'], inplace=True)

        # replace NaNs with column-mean
        data.fillna(data.mean(), inplace=True);

        # set target
        y = data.loc[:,'happiness_score']

        s = 1
        %matplotlib inline
        plt.subplot(4, 4, 1)
        plt.scatter(data.loc[:,'economy'], y, s)
        plt.title('economy')
        plt.subplot(4, 4, 2)
        plt.scatter(data.loc[:,'family'], y, s)
        plt.title('family')
        plt.subplot(4, 4, 3)
```

```
plt.scatter(data.loc[:,'health'], y, s)
plt.title('health')
plt.subplot(4, 4, 4)
plt.scatter(data.loc[:,'freedom'], y, s)
plt.title('freedom')
plt.subplot(4, 4, 5)
plt.scatter(data.loc[:,'generosity'], y, s)
plt.title('generosity')
plt.subplot(4, 4, 6)
plt.scatter(data.loc[:,'corruption'], y, s)
plt.title('corruption')
plt.subplot(4, 4, 7)
plt.scatter(data.loc[:,'dystopia_residual'], y, s)
plt.title('dystopia_residual')
plt.subplot(4, 4, 8)
plt.scatter(data.loc[:,'internet_access_population[%]'], y, s)
plt.title('internet_access_population[%]')
plt.subplot(4, 4, 9)
plt.scatter(data.loc[:,'cellular_subscriptions'], y, s)
plt.title('cellular_subscriptions')
plt.subplot(4, 4, 10)
plt.scatter(data.loc[:,'surplus_deficit_GDP[%]'], y, s)
plt.title('surplus_deficit_GDP[%]')
plt.subplot(4, 4, 11)
plt.scatter(data.loc[:,'familiy_income_gini_coeff'], y, s)
plt.title('familiy_income_gini_coeff')
plt.subplot(4, 4, 12)
plt.scatter(data.loc[:,'GDP_per_capita[$]'], y, s)
plt.title('GDP_per_capita[$]')
plt.subplot(4, 4, 13)
plt.scatter(data.loc[:,'inflation_rate[%]'], y, s)
plt.title('inflation_rate[%]')
plt.subplot(4, 4, 14)
plt.scatter(data.loc[:,'military_expenditures[%]'], y, s)
plt.title('military_expenditures[%]')
plt.subplot(4, 4, 15)
plt.scatter(data.loc[:,'population'], y, s)
plt.title('population')

plt.subplots_adjust(top=2, bottom=0, left=0, right=2, hspace=0.35, wspace=0.35)
```



After looking at the plots, I now present these out of the gut conclusions:

1. economy: positive linear correlation
2. family: positive linear/quadratic correlation
3. health: positive linear correlation
4. freedom: linear / quadratic / cubic correlation
5. generosity: weak quadratic correlation / no correlation
6. corruption: quadratic correlation / no correlation
7. dystopia residual: no correlation
8. internet acces population [%]: positive linear correlation / cubic correlation
9. cellular subscriptions: quadratic correlation
10. surplus deficit GDP [%]: no correlation
11. family income gini coeff: no correlation / weak negative linear correlation
12. GDP per capital [$]: quadratic correlation
13. inflation rate [%]: no correlation
14. military expenditures [%]: no correlation
15. population: no correlation

What features are important? I'd say all features that highly correlate with the happines score are relevant, e.g., economy, family, health and internet access population. However the data is heavily spread out in most of the cases and therefore, I expect all models to perform rather poorly.

2 Q2.2

Load data and set up packages

In [2]:
```
from sklearn.kernel_ridge import KernelRidge
# not needed from the template
# from sklearn.model_selection import train_test_split
# from sklearn.model_selection import RepeatedKFold

# added
from sklearn.model_selection import cross_val_score
```

Check the dataset for missing values and, if any are found, address them programmatically

In [3]:
```
# Unwanted columns were dropped, extreme outliers
# were replaced with NaNs and NaNs were eventually
# replaced by their column means before plotting.
```

2.1 Linear model

In [4]:
```
X = data.iloc[:,1:].copy()

# compute corrcoef
print("\n%-13sCorrcoef:"%(""))
for x in range(0,len(X.columns)):
    cc = np.ma.corrcoef(X.iloc[:,x], y)[1][0]
    print("%-30s: %.3f"% (X.columns[x], cc), end=" "),
    if (np.abs(cc) < 0.3):
        print("<- no correlation, drop candidate")
    else:
        print("")

# carefully drop possibly uncorrelated columns
X.drop(columns=['surplus_deficit_GDP[%]', 'military_expenditures[%]', \
                'population'], inplace=True);

# function to compute the VIF
def VIF_scores(K):
    cc = np.corrcoef(K, rowvar=False)
    VIF = np.linalg.inv(cc)
    print("\n%-16sVIF:"%(""))
    for x in range(0,len(K.columns)):
        print("%-30s: %.3f"%(K.columns[x], VIF.diagonal()[x]), end=" ")
        if (VIF.diagonal()[x] > 5):
            print("<- VIF too high")
        else:
            print("")

# compute VIF
```

```python
VIF_scores(X)

# drop features with VIF too high
X.drop(columns=['internet_access_population[%]'], inplace=True)
X.drop(columns=['GDP_per_capita[$]'], inplace=True)

# compute updated VIF
VIF_scores(X)

#define model
kr = KernelRidge(alpha=1e-6, kernel='linear', gamma=None, coef0=1, kernel_params=None)

# compute R^2
print("\nR_sqrd:", kr.fit(X,y).score(X,y))

# cross-validation
scores = cross_val_score(kr, X, y, cv=5)
print("\n5-fold cross validation (R_sqrd):\n", scores)
```

```
             Corrcoef:
economy                       : 0.812
family                        : 0.753
health                        : 0.782
freedom                       : 0.570
generosity                    : 0.155 <- no correlation, drop candidate
corruption                    : 0.429
dystopia_residual             : 0.475
internet_access_population[%] : 0.791
cellular_subscriptions        : 0.508
surplus_deficit_GDP[%]        : 0.282 <- no correlation, drop candidate
familiy_income_gini_coeff     : -0.303
GDP_per_capita[$]             : 0.719
inflation_rate[%]             : -0.329
military_expenditures[%]      : -0.128 <- no correlation, drop candidate
population                    : -0.032 <- no correlation, drop candidate

                VIF:
economy                       : 11.015 <- VIF too high
family                        : 2.275
health                        : 4.629
freedom                       : 1.934
generosity                    : 1.311
corruption                    : 2.138
dystopia_residual             : 1.077
internet_access_population[%] : 6.254 <- VIF too high
cellular_subscriptions        : 1.879
familiy_income_gini_coeff     : 1.374
```

```
GDP_per_capita[$]              : 5.723 <- VIF too high
inflation_rate[%]              : 1.351

                VIF:
economy                        : 5.318 <- VIF too high
family                         : 2.197
health                         : 4.158
freedom                        : 1.931
generosity                     : 1.262
corruption                     : 1.528
dystopia_residual              : 1.063
cellular_subscriptions         : 1.869
familiy_income_gini_coeff      : 1.329
inflation_rate[%]              : 1.317

R_sqrd: 0.9999999407343686

5-fold cross validation (R_sqrd):
 [0.99999958 0.99999715 0.99999662 0.99999816 0.99999938]
```

Summary Plotting wasn't enough insight for me. Therefore, the correlation coefficients for each variable were calculated and factors that do not correlate were considered candidates to be tossed. Just as a note: one has to be really brave to fit a line through some of the plots, but at this point I had no other strategy to build my model. E.g., I tried to find transformations for a better fit of the linear model, but couldn't find any.

Selecting factors before the cross-validation (CV) process is not a good idea and should be ideally done during the CV process [1]. Eventually, I carefully removed the drop candidates one by one and concluded that all could be tossed except for the 'generosity' factor.
Next, the variance inflation factor (VIF) was calculated to check for multicolinearity (i.e., to check if independent variables are correlated among each other). A VIF of 1 is good, a VIF between 5 and 10 indicates high correlation and might be a problem. From the first VIF computations (see the first VIF table above) 3 variables indicate too much correlation.
Dropping the 'economy' factor with a VIF over 10 seems reasonable at first but is a bad idea, because it leads to siginficant drops in the CV eventually. This might be because the correlation for 'economy' is the strongest among all independent variables to the dependent variable. Dropping the 'internet access population' and 'GDP per capita' variables with bad VIFs leads to an even better model (although, the 'economy' factor is still too high in the second VIF table above). Like the 'economy' variable, the 'internet access population' and 'GDP per capita' variables highly correlate with the happiness score, but do not impair the final results when being removed after the VIF check.

Best model parameters: $\alpha = 10^{-6}$, γ has no effect on the result.

References:

[1] T. Hastie, The Elements of Statisitcal Learning, Chapter 7.10.2 The wrong and right way to do Cross-validation

Strategies for optimizing the linear model:
https://www.youtube.com/watch?v=dQNpSa-bq4M&list=PLIeGtxpvyG-IqjoU8IiF0Yu1WtxNq_4z-&index=1

cross_val_score code:
https://scikit-learn.org/stable/modules/generated/sklearn.model_selection.cross_val_score.html
VIF code:
https://stackoverflow.com/questions/42658379/variance-inflation-factor-in-python

2.2 Quadratic model

In [5]:
```
X = data.iloc[:,1:].copy();

############################
## Spearman corrcoef code ##
############################
# from scipy.stats import spearmanr

#print("\n%-13sCorrcoef:"%(""))
#for x in range(1,len(data.columns)):
#    cc = spearmanr(data.iloc[:,x], y)
#    print("%-30s: corr.: %.3f, p-value: %.2e"% (data.columns[x],cc[0],cc[1]),end=" ")
#    if (np.abs(cc[0]) < 0.3):
#        print("<- no correlation, drop candidate")
#    else:
#        print("")

# carefully drop possibly uncorrelated features
X.drop(columns=['familiy_income_gini_coeff', \
                'military_expenditures[%]', 'population'], inplace=True)
# additional careful drops
X.drop(columns=['internet_access_population[%]'], inplace=True)
X.drop(columns=['GDP_per_capita[$]'], inplace=True)

# model definition
kr = KernelRidge(alpha=1e-8, kernel='poly', gamma=1e-5, coef0=1, degree=2)

# compute R^2
print("\nR_sqrd:", kr.fit(X,y).score(X,y))

# cross-validation
scores = cross_val_score(kr, X, y, cv=5)
print("\n5-fold cross validation (R_sqrd):\n", scores)
```

```
R_sqrd: 0.9999999209795366

5-fold cross validation (R_sqrd):
 [0.99999834 0.99999628 0.99998579 0.99999333 0.99999149]
```

Summary Like in the linear model, correlations between factors were also considered. However, this time around, the Spearman correlation coefficient for nonlinear correlations was used. The results are similar to the Pearsons method in the linear model. Since the Spearman method is not included in numpy, scipy had to be used and since incorporating other packages is not allowed, I only present the results in the table below.

feature	corrcoef	p-value	result
economy	0.825	9.36e-40	
family	0.774	4.00e-32	
health	0.788	5.37e-34	
freedom	0.556	5.81e-14	
generosity	0.136	9.05e-02	no correlation, drop candidate
corruption	0.301	1.42e-04	
dystopia_residual	0.504	2.41e-11	
internet_access_population[%]	0.792	1.26e-34	
cellular_subscriptions	0.553	8.50e-14	
surplus_deficit_GDP[%]	0.408	1.33e-07	
familiy_income_gini_coeff	-0.285	3.30e-04	no correlation, drop candidate
GDP_per_capita[$]	0.827	3.45e-40	
inflation_rate[%]	-0.372	1.85e-06	
military_expenditures[%]	-0.196	1.43e-02	no correlation, drop candidate
population	-0.108	1.81e-01	no correlation, drop candidate

From this insight removing all drop candidates except for 'generosity' did not affect the quality of the model. Addiditonally, removing 'internet access population' and 'GDP per capita', like in the linear model, did not impair the model and made scaling unnecessary.

Best model parameters: $\alpha = 10^{-8}, \gamma = 10^{-5}$

2.3 Gaussian model

```
In [6]: X = data.iloc[:,1:].copy();

        # model definition
        kr = KernelRidge(alpha=1e-8, kernel='rbf', gamma=1e-6, kernel_params=None)

        # compute R^2
        print("\nR_sqrd:", kr.fit(X,y).score(X,y))

        # cross-validation
        scores = cross_val_score(kr, X, y, cv=5)
```

```
print("\n5-fold cross validation (R_sqrd):\n", scores)

print("\n\n## With feature scaling:")

# scale features
scaler = StandardScaler().fit(X)
temp = scaler.transform(X)
X = pd.DataFrame(data=temp, index=X.index, columns=X.columns)

# recompute R^2
print("\nR_sqrd:", kr.fit(X,y).score(X,y))

# redo cross-validation
scores = cross_val_score(kr, X, y, cv=5)
print("\n5-fold cross validation (R_sqrd):\n", scores)
```

```
R_sqrd: 0.999995457664676

5-fold cross validation (R_sqrd):
 [ -344.01071139  -849.2932515  -1105.29309945  -399.93350013
  -102.85021507]


## With feature scaling:

R_sqrd: 0.9999999430168296

5-fold cross validation (R_sqrd):
 [0.99999911 0.99999702 0.99999503 0.99999781 0.99999923]
```

Summary For the Gaussian model I did not remove any features and did not get any 'matrix is near singular' warnings, due to the nature of the model. I had to scale the features eventually in order to get a usable model. I used up all my research power in the linear model. Therefore, I didn't investigate as much here and do not have too much to say.

Best model parameters: $\alpha = 10^{-8}$, $\gamma = 10^{-6}$

2.4 Comparison

All models worked surprisingly well eventually. So well, that it might be reasonable to become suspicious (keyword: overfitting). Since all models work well, I would say the linear model is the best since it is the simplest model among the three.