

# 现代通信系统与网络仿真技术 大作业

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## Problem1

The energy spectral density of an MSK (minimum shift keyed) signal is defined by where  $T_b$  is the bit time.

$$G_{MSK}(f) = \frac{16T_b \cos^2(2\pi T_b f)}{\pi^2 [1 - (4T_b f)^2]^2}$$

(1) Develop a MATLAB program to plot the signal to aliasing noise ratio (SNR)<sub>a</sub> as the number of samples per symbol varies from 4 to 20.

(2) Compare the result with that of the rectangular pulse shape by plotting both on the same set of axes. Explain the results.

After sampling, we have the energy spectral density

$$G_{MSKsampling}(f) = f_s^2 \sum_{n=-\infty}^{\infty} G_{MSK}(f - n f_s).$$

Hence we get the energy of signal and aliasing noise

$$E_{signal} = \int_{-\frac{f_s}{2}}^{\frac{f_s}{2}} f_s^2 G_{MSK}(f) df,$$

$$E_{noise} = \int_{-\frac{f_s}{2}}^{\frac{f_s}{2}} f_s^2 \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} G_{MSK}(f - n f_s) df,$$

separately. By observing the representation of  $E_{noise}$  and the propriety of  $G_{MSK}(f)$ , we derive

$$E_{noise} = \int_{\frac{f_s}{2}}^{\infty} 2f_s^2 G_{MSK}(f) df,$$

Then the resultant  $(SNR)_a = \frac{E_{signal}}{E_{noise}}$ . However, the integral is hard to solve, so we derive an approximation of  $(SNR)_a$  by numerical integration:

$$(SNR)_a \approx \frac{\sum_{j=0}^{\frac{k f_s}{2}} G_{MSK}\left(\frac{j}{k}\right)}{\sum_{j=-\frac{k f_s}{2}}^{\infty} G_{MSK}\left(\frac{j}{k}\right)}.$$

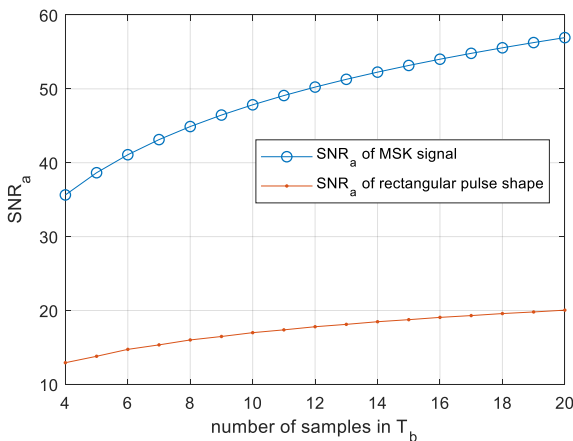


Figure P1.a

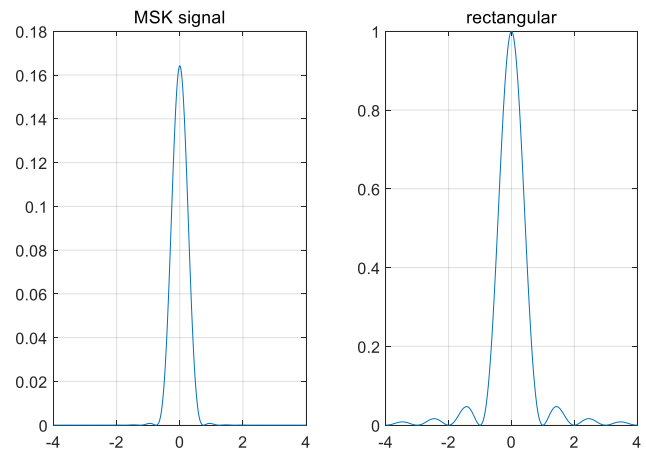


Figure P1.b

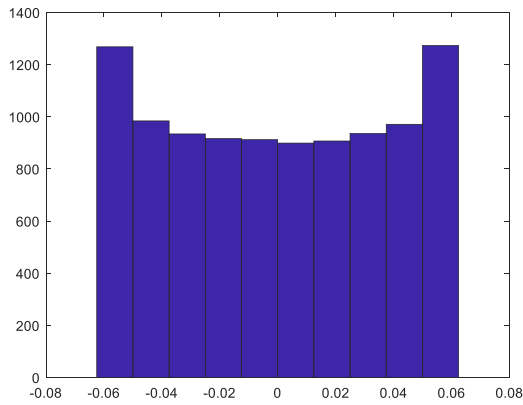


Figure P2.a

## Problem 2

In evaluating the effect of a fixed-point quantizing process, the assumption was made that the error induced by the quantizing process can be represented by a uniformly distributed random value. In this problem we investigate the validity of this assumption.

(1) Use  $\sin(6t)$  as a signal. Using a sampling frequency of 20 Hz, generate, using MATLAB, a vector of 10,000 samples of this waveform. Note that the signal frequency and the sampling frequency are not harmonically related. Why was this done?

(2) Develop a MATLAB model for a fixed-point quantizer that contains 16 quantizing levels ( $b = 4$ ). Using this model quantize the sample values generated in (1). Generate a vector representing the 10,000 values of quantizing error.

(3) Compute the values of  $E\{e[k]\}$  and  $E\{e^2[k]\}$ . Compare with the theoretical values and explain the results.

(4) Using the MATLAB function hist, generate a histogram of the quantizing errors. What do you conclude?

(1) To avoid the vector being periodic, which will reduce the valid data.

(3) Quantizing level  $\Delta = \frac{2}{2^4} = 0.125$ . Theoretical values:

$$E\{e[k]\} = 0.$$

$$E\{e^2[k]\} = \frac{\Delta^2}{12} = 0.0013.$$

Experimental values:

$$\hat{E}\{e[k]\} = -5.5155 \times 10^{-5} \approx 0$$

$$\hat{E}\{e^2[k]\} = 0.0015.$$

The theoretical values and experimental values are close.

(4) Histogram is shown as Figure P2.a. We conclude that it's a proper model to view quantizing error variant  $e[k] = x_q[k] - x[k]$  as a uniformly distributed variant. But in this case of sin function, the probability of large error is greater than that of small error.

The comparison of SNR is shown in Figure P1.a, which shows the signal to aliasing noise ratio of MSK signal is much higher than that of rectangular shape pulse signal, under same sampling frequency.

By plotting the energy spectral density in Figure P1.b, we can easily explain the gap: Energy spectral density of the rectangular shape pulse has a relative much higher sidelobe compared with that of MSK signal, whose sidelobe is very close to zero.

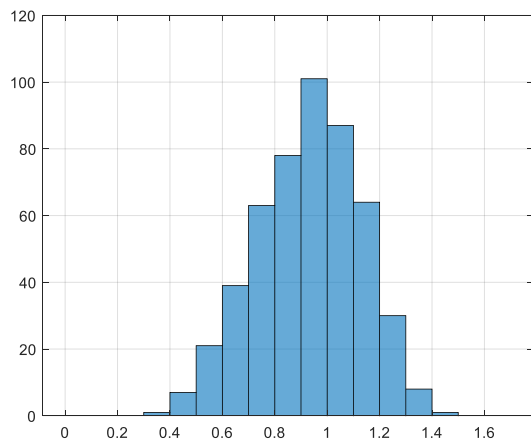


Figure P3.a

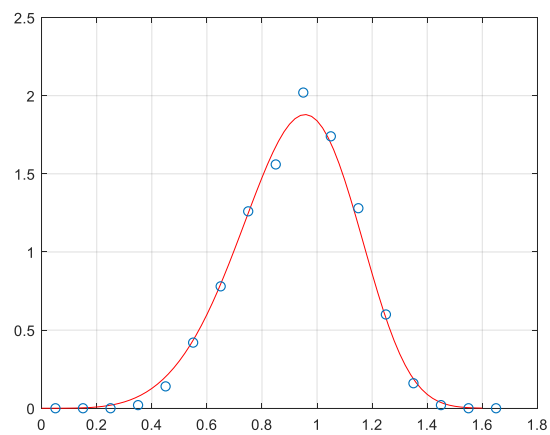


Figure P3.b

### Problem 3

A Weibull random variable is defined by the pdf

$$f_X(x) = ax^{a-1}e^{-x^a}u(x)$$

where  $a$  is a parameter and denotes the unit step. Using the inverse transform method, develop an algorithm for generating a sequence of random numbers having a Weibull distribution. Using the algorithm just generated with  $a = 5$ , generate 500 samples of a Weibull random variable. Plot the histogram and compare with the pdf.

Solution:

$$\begin{aligned} f_X(x) &= ax^{a-1}e^{-x^a}u(x) \\ F_X(x) &= \int f_X(x)dx \\ &= \int_0^\infty ax^{a-1}e^{-x^a}dx \\ &= 1 - e^{-x^a} (x > 0) \end{aligned}$$

Let  $U = F_X(X)$ , then  $X = F_X^{-1}(U)$

$$X = \sqrt[a]{-\ln(1-U)}$$

where  $U$  is uniformly distributed in  $(0,1)$ .

Histogram is shown in Figure P3.a and the PDF is shown in P3.b. To compare them, we use points in Figure P3.b to represent the relative value of histogram bins. Generating only 500 samples makes the gap between PDF and experimental values significant, but it still shows the correspondence between them, indicating that the algorithm is correct.

### Problem 4

#### 题目内容略

总服务时间见图P4.a, 平均吞吐量见P4.b, 平均接入时延见P4.c

可以看出在总服务时间随MTCD数量增加浮动较大, 但总体趋势为MTCD数量越多, 总服务时间越长。

在设备数量并不很多时, 随着设备数量增多, 平均吞吐量和平均接入时延都较快增加, 但当设备数量

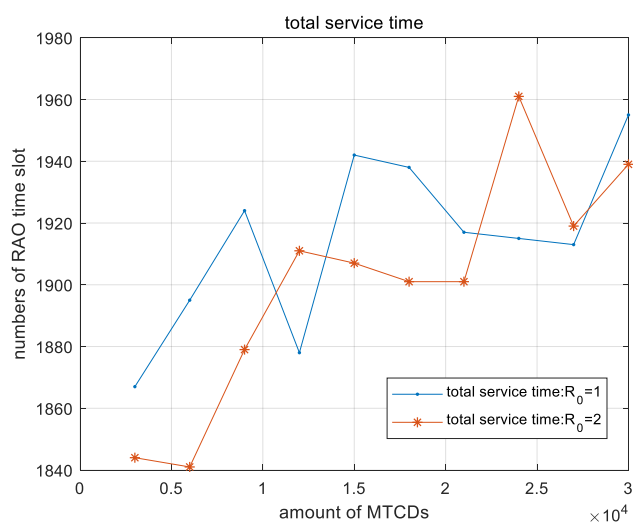


Figure P4.a

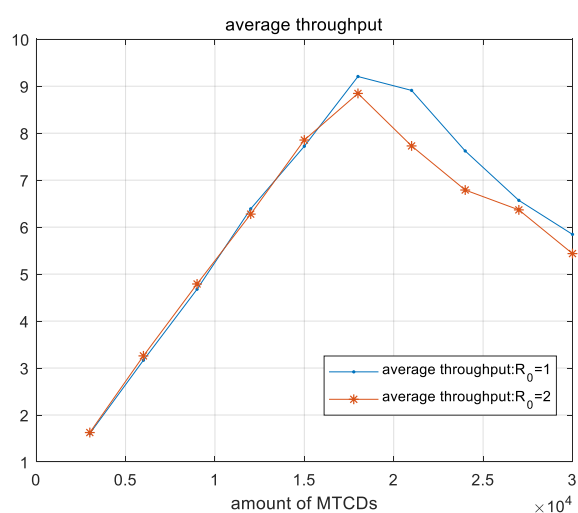


Figure P4.b

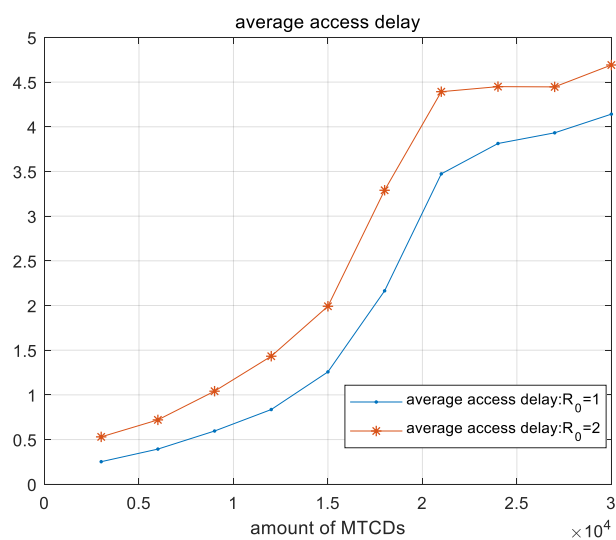


Figure P4.c

进一步增加时，平均吞吐量反而下降，而平均接入时延增速放缓。

对比不同最小传输速率约束 $R_0$ 下的系统性能，发现其对总服务时间的影响规律性不强；而对于平均吞吐量，在设备数量较小时， $R_0$ 对平均吞吐量基本没有影响，而在设备数量较大时， $R_0 = 2$ 相较 $R_0 = 1$ 时的吞吐量更小； $R_0$ 对于平均接入时延的影响的规律性最强， $R_0 = 2$ 时的接入时延显著高于 $R_0 = 1$ 时的平均接入时延。

