Forward kinematics of one finger in our design

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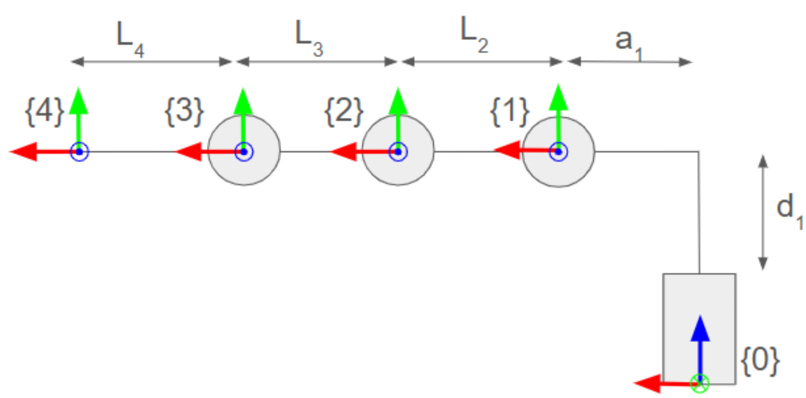
1 DH table and Link Frames

DH table:

i	θ	d	a	α
1	θ_1	d_1	a_1	90°
2	θ_2	0	L_2	0
3	θ_3	0	L_3	0
4	θ_4	0	L_4	0



2 Forward Kinematics

Using the general formula for each transformation matrix:

$$
{}^{i-1}T_i(\theta_i) = \begin{bmatrix} cos\theta_i & -cos\alpha_i sin\theta_i & sin\alpha_i sin\theta_i & a_i cos\theta_i \\ sin\theta_i & cos\alpha_i cos\theta_i & -sin\alpha_i cos\theta_i & a_i sin\theta_i \\ 0 & sin\alpha_i & cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}
$$

$$
{}^0T_1 = \begin{bmatrix} c_1 & 0 & s_1 & a_1c_1 \\ s_1 & 0 & -c_1 & a_1s_1 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}
$$

$$
{}^1T_2 = \begin{bmatrix} c_2 & -s_2 & 0 & L_2c_2 \\ s_2 & c_2 & 0 & L_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}
$$

$$
{}^2T_3 = \begin{bmatrix} c_3 & -s_3 & 0 & L_3c_3 \\ s_3 & c_3 & 0 & L_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}
$$

$$
{}^3T_4 = \begin{bmatrix} c_4 & -s_4 & 0 & L_4c_4 \\ s_4 & c_4 & 0 & L_4s_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}
$$

$$
{}^0T_1 \cdot {}^1T_2 = \begin{bmatrix} c_1c_2 & -s_2c_1 & s_1 & L_2c_1c_2 + a_1c_1 \\ s_1c_2 & -s_1s_2 & -c_1 & L_2s_1c_2 + a_1s_1 \\ s_2 & c_2 & 0 & L_2s_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}
$$

$$
{}^0T_1 \cdot {}^1T_2 \cdot {}^2T_3 = \begin{bmatrix} c_1c_2c_3 - c_1s_2s_3 & -c_1c_2s_3 - c_1s_2c_3 & s_1 & L_3c_1c_2c_3 - L_3c_1s_2s_3 + L_2c_1c_2 + a_1c_1 \\ s_1c_2c_3 - s_1s_2s_3 & -s_1c_2s_3 - s_1s_2c_3 & -c_1 & L_3s_1c_2c_3 - L_3s_1s_2s_3 + L_2s_1c_2 + a_1s_1 \\ s_2c_3 + c_2s_3 & -s_2s_3 + c_2c_3 & 0 & L_3s_2c_3 + L_3c_2s_3 + L_2s_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}
$$

$$
{}^0T_1 \cdot {}^1T_2 \cdot {}^2T_3 \cdot {}^3T_4 = {}^0T_4 = \tag{8}
$$

Column 1:

$$
\begin{bmatrix} (c_1c_2c_3 - c_1s_2s_3)c_4 + (-c_1c_2s_3 - c_1s_2c_3)s_4 \\ (s_1c_2c_3 - s_1s_2s_3)c_4 + (-s_1c_2s_3 - s_1s_2c_3)s_4 \\ (s_2c_3 + c_2s_3)c_4 + (-s_2s_3 + c_2c_3)s_4 \\ 0 \end{bmatrix} \tag{9}
$$

Column 2:

$$
\begin{bmatrix} (c_1c_2c_3 - c_1s_2s_3)(-s_4) + (-c_1c_2s_3 - c_1s_2c_3)c_4 \\ (s_1c_2c_3 - s_1s_2s_3)(-s_4) + (-s_1c_2s_3 - s_1s_2c_3)c_4 \\ (s_2c_3 + c_2s_3)(-s_4) + (-s_2s_3 + c_2c_3)c_4 \\ 0 \end{bmatrix} \tag{10}
$$

Column 3:

$$
\begin{bmatrix} s_1 \\ -c_1 \\ 0 \\ 0 \end{bmatrix} \tag{11}
$$

Column 4:

$$\begin{bmatrix} (c_1c_2c_3 - c_1s_2s_3)L_4c_4 + (-c_1c_2s_3 - c_1s_2c_3)L_4s_4 + (L_3c_1c_2c_3 - L_3s_1s_2s_3 + L_2c_1c_2 + a_1c_1) \\ (s_1c_2c_3 - s_1s_2s_3)L_4c_4 + (-s_1c_2s_3 - s_1s_2c_3)L_4s_4 + (L_3s_1c_2c_3 - L_3s_1s_2s_3 + L_2s_1c_2 + a_1s_1) \\ (s_2c_3 + c_2s_3)L_4c_4 + (-s_2s_3 + c_2c_3)L_4s_4 + L_3s_2c_3 + L_3c_2s_3 + L_2s_2 + d_1 \\ 1 \end{bmatrix} \tag{12}$$

Now, using the trigonometric identities:

$$sinAcosB \pm cosAsinB = sin(A \pm B) \tag{13}$$

$$cosAcosB \mp sinAsinB = cos(A \pm B) \tag{14}$$

each column can be simplified to the following:
Column 1:

$$\begin{bmatrix} c_1(c_4c_{23} - s_4s_{23}) \\ s_1(c_4c_{23} - s_4s_{23}) \\ s_{23}c_4 + c_{23}s_4 \\ 0 \end{bmatrix} \tag{15}$$

Column 2:

$$\begin{bmatrix} -c_1(s_4c_{23} + c_4s_{23}) \\ -s_1(s_4c_{23} + c_4s_{23}) \\ -s_4s_{23} + c_4c_{23} \\ 0 \end{bmatrix} \tag{16}$$

Column 3 (same as above):

$$\begin{bmatrix} s_1 \\ -c_1 \\ 0 \\ 0 \end{bmatrix} \tag{17}$$

Column 4:

$$\begin{bmatrix} c_1(L_4(c_{23}c_4 - s_{23}s_4) + L_3c_{23} + L_2c_2 + a_1) \\ s_1(L_4(c_{23}c_4 - s_{23}s_4) + L_3c_{23} + L_2c_2 + a_1) \\ L_4(s_{23}c_4 + c_{23}s_4) + L_3s_{23} + L_2s_2 + d_1 \\ 1 \end{bmatrix} \tag{18}$$

Simplfying further using:

$$sin(a+b+c) = sin(a+b)cosc + cos(a+b)sin(c) \tag{19}$$

and

$$cos(a+b+c) = cos(a+b)cos(c) - sin(a+b)sin(c) \tag{20}$$

Column 1:

$$\begin{bmatrix} c_1c_{234} \\ s_1c_{234} \\ s_{234} \\ 0 \end{bmatrix} \tag{21}$$

Column 2:

$$\begin{bmatrix} -c_1 s_{234} \\ -s_1 s_{234} \\ c_{234} \\ 0 \end{bmatrix} \tag{22}$$

Column 3 (same as previously):

$$\begin{bmatrix} s_1 \\ -c_1 \\ 0 \\ 0 \end{bmatrix} \tag{23}$$

Column 4:

$$\begin{bmatrix} c_1(L_4 c_{234} + L_3 c_{23} + L_2 c_2 + a_1) \\ s_1(L_4 c_{234} + L_3 c_{23} + L_2 c_2 + a_1) \\ L_4 s_{234} + L_3 s_{23} + L_2 s_2 + d_1 \\ 1 \end{bmatrix} \tag{24}$$

So full forward kinematics is given by:

$$\begin{bmatrix} c_1 c_{234} & -c_1 s_{234} & s_1 & c_1(L_4 c_{234} + L_3 c_{23} + L_2 c_2 + a_1) \\ s_1 c_{234} & -s_1 s_{234} & -c_1 & s_1(L_4 c_{234} + L_3 c_{23} + L_2 c_2 + a_1) \\ s_{234} & c_{234} & 0 & L_4 s_{234} + L_3 s_{23} + L_2 s_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{25}$$