Dynamics considerations

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1 Calculating the Jacobian matrix and consideration of joint velocities

Considering the forward kinematics of the manipulator: Forward Kinematics:

$$^{0}T_3 = \begin{bmatrix} c_{123} & -s_{123} & 0 & L_1c_1 + L_2c_{12} + L_3c_{123} \\ s_{123} & c_{123} & 0 & L_1s_1 + L_2s_{12} + L_3s_{123} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

The position and orientation of the end effector [a, b, ϕ]^T is given by:

$$a = L_1c_1 + L_2c_{12} + L_3c_{123} \tag{2}$$

$$b = L_1s_1 + L_2s_{12} + L_3s_{123} \tag{3}$$

$$\phi = \theta_1 + \theta_2 + \theta_3 \tag{4}$$

In order to find the manipulator's dynamics, the following relationship between Cartesian and joint space coordinates can be considered:

$$\dot{\mathbf{x}} = \mathbf{J}\dot{\mathbf{q}} \tag{5}$$

Where $\dot{\mathbf{x}}$ represents the vector of the Cartesian (end effector) velocities, $\mathbf{J} \in \mathbb{R}^{mxn}$(where m is the number of Cartesian coordinates and n is the degrees of freedom, or number of joints, of the manipulator, so in this case m=3 and n=3) is the Jacobian matrix, and $\dot{\mathbf{q}}$ is the vector of joint-space velocities.
To find the Cartesian space velocities the time derivatives of a, b, and ϕ are first taken:

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{a} \\ \dot{b} \\ \dot{\phi} \end{bmatrix} \tag{6}$$

$$\dot{a} = -\dot{\theta}_1L_1s_1 - (\dot{\theta}_1 + \dot{\theta}_2)L_2s_{12} - (\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)L_3s_{123} \tag{7}$$

$$\dot{b} = \dot{\theta}_1L_1c_1 + (\dot{\theta}_1 + \dot{\theta}_2)L_2c_{12} + (\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)L_3c_{123} \tag{8}$$

$$\dot{\phi} = \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3 \tag{9}$$

$\dot{\mathbf{q}}$, The Jacobian matrix, given by the following formula:

$$\mathbf{J} = \begin{bmatrix} \frac{\partial x_1}{\partial \theta_1} & \frac{\partial x_1}{\partial \theta_2} & \frac{\partial x_1}{\partial \theta_3} \\ \frac{\partial x_2}{\partial \theta_1} & \frac{\partial x_2}{\partial \theta_2} & \frac{\partial x_2}{\partial \theta_3} \\ \frac{\partial \phi}{\partial \theta_1} & \frac{\partial \phi}{\partial \theta_2} & \frac{\partial \phi}{\partial \theta_3} \end{bmatrix} \tag{10}$$

So dividing the values for

References

[1] "Introduction to Robot Geometry and Kinematics." Accessed: Jan. 10, 2025. [Online]. Available: `http://fricke.co.uk/Teaching/CS591_Swarm_Robotics_2017fall/Readings/IntroRobotKinematics5.pdf`