Inverse Kinematics of 2 and 3 link finger - only done 2 so far... need to check it too not sure if correct

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1 Inverse Kinematics of 1 finger

As 2 link manipulator
For end effector position $[a,b,c]^T$

$$a = L_1c_1 + L_2c_{12}$$

$$b = L_1s_1 + L_2s_{12}$$

$$c = 0$$

1. Finding θ_2

To find θ_2 first the geometry of the right angled triangle with sides a and b and a hypotenuse of r_1:

$$a^2 + b^2 = r_1^2$$

Only need one solution (elbow up) because the elbow of the finger doesn't bend backwards

$$r_1^2 = L_1^2 + L_2^2 + 2L_1L_2(c_{12} + s_{12})$$

since $c_{12} + s_{12} = c_2$

$$c_2 = \frac{r_1^2 - L_1^2 - L_2^2}{2L_1L_2}$$

It therefore follows that

$$\theta_2 = cos^{-1}(\frac{r_1^2 - L_1^2 - L_2^2}{2L_1L_2})$$

Case 1

$$\frac{r_1^2 - L_1^2 - L_2^2}{2L_1L_2} > 1,\ 0\ solutions\ for\ \theta_2$$

Case 2

$$\frac{r_1^2 - L_1^2 - L_2^2}{2L_1L_2} = 0,\ cos(\theta_2) = 1$$

Therefore

$$a = (L_1 + L_2)c_1,\ b = (L_1 + L_2)s_1$$

Rearranging:

$$c_1 = \frac{a}{L_1 + L_2} s_1 = \frac{b}{L_1 + L_2}$$

$$tan(\theta_1) = \frac{s_1}{c_1}$$

$$\theta_1 = Atan2(b, a)$$

Case 3
When

$$-1 < \frac{r_1^2 - L_1^2 - L_2^2}{2L_1L_2} < 1,$$

$$\theta_2 = cos^{-1}(\frac{r_1^2 - L_1^2 - L_2^2}{2L_1L_2}) \text{ and } \theta_2 = 2\pi - cos^{-1}(\frac{r_1^2 - L_1^2 - L_2^2}{2L_1L_2})$$

Case 4
When

$$\frac{r_1^2 - L_1^2 - L_2^2}{2L_1L_2} = -1$$

So

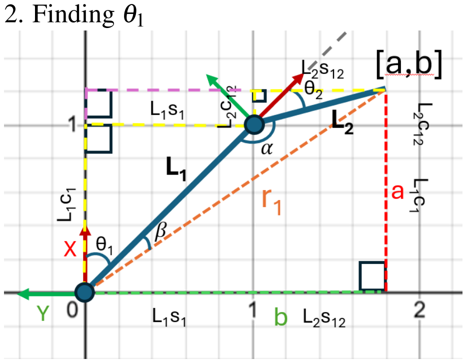
$$\theta_2 = \pi$$

Case 5
When

$$\frac{r_1^2 - L_1^2 - L_2^2}{2L_1L_2} < -1$$

So 0 solutions. The solution we require is when elbow is up only.

2. Finding θ_1



The line r_1 makes an angle $\theta_1 + \beta$ with the x-axis. This makes a right angled triangle with sides a and b, which gives:

$$tan(\theta_1 + \beta) = \frac{b}{a}, \text{ where } b = L_1s_1 + L_2s_{12} \text{ and } a = L_1c_1 + L_2c_{12}.$$

$$\theta_1 = tan^{-1}(\frac{b}{a}) - \beta \tag{1}$$

And using the law of cosines

$$cos(\beta) = \frac{r_1^2 + L_1^2 - L_2^2}{2r_1L_2} \tag{2}$$

$$\beta = cos^{-1}(\frac{r_1^2 + L_1^2 - L_2^2}{2r_1L_2}) \tag{3}$$

Meaning:

$$\theta_1 = tan^{-1}(\frac{b}{a}) - cos^{-1}(\frac{r_1^2 + L_1^2 - L_2^2}{2r_1L_2}) \tag{4}$$

As 3 link manipulator
For end effector position $[a, b, c]^T$

$$a = L_1c_1 + L_2c_{12} + L_3c_{123} \tag{5}$$

$$b = L_1s_1 + L_2s_{12} + L_3s_{123} \tag{6}$$

$$c = 0 \tag{7}$$