

Inverse Kinematics of 2 and 3 link finger - only done 2 so far... need to check it too not sure if correct

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1 Inverse Kinematics of 1 finger

As 2 link manipulator

For end effector position $[a, b, c]^T$

$$a = L_1 c_1 + L_2 c_{12}$$

$$b = L_1 s_1 + L_2 s_{12}$$

$$c = 0$$

1. Finding θ_2

To find θ_2 first the geometry of the right angled triangle with sides a and b and a hypotenuse of r_1 :

$$a^2 + b^2 = r_1^2$$

$$r_1^2 = L_1^2 + L_2^2 + 2L_1 L_2 (c_{12} + s_{12})$$

since $c_{12} + s_{12} = c_2$

$$c_2 = \frac{r_1^2 - L_1^2 - L_2^2}{2L_1 L_2}$$

It therefore follows that

$$\theta_2 = \cos^{-1}\left(\frac{r_1^2 - L_1^2 - L_2^2}{2L_1 L_2}\right)$$

Case 1

$$\frac{r_1^2 - L_1^2 - L_2^2}{2L_1 L_2} > 1, \text{ 0 solutions for } \theta_2$$

Case 2

$$\frac{r_1^2 - L_1^2 - L_2^2}{2L_1 L_2} = 0, \cos(\theta_2) = 1$$

Therefore

$$a = (L_1 + L_2)c_1, \quad b = (L_1 + L_2)s_1$$

Rearranging:

$$c_1 = \frac{a}{L_1 + L_2} s_1 = \frac{b}{L_1 + L_2}$$

$$\tan(\theta_1) = \frac{s_1}{c_1}$$

$$\theta_1 = \text{Atan2}(b, a)$$

Case 3

When

$$-1 < \frac{r_1^2 - L_1^2 - L_2^2}{2L_1L_2} < 1,$$

$$\theta_2 = \cos^{-1}\left(\frac{r_1^2 - L_1^2 - L_2^2}{2L_1L_2}\right) \text{ and } \theta_2 = 2\pi - \cos^{-1}\left(\frac{r_1^2 - L_1^2 - L_2^2}{2L_1L_2}\right)$$

Case 4

When

$$\frac{r_1^2 - L_1^2 - L_2^2}{2L_1L_2} = -1$$

So

$$\theta_2 = \pi$$

Case 5

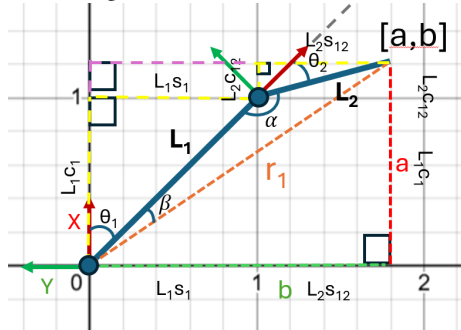
When

$$\frac{r_1^2 - L_1^2 - L_2^2}{2L_1L_2} < -1$$

So 0 solutions for when ≤ -1 .

The solution we require is when elbow is up only because the finger doesn't bend backwards, and also only in half of the workspace because there is a palm, but i just put all the different cases here. Need to apply constraints

2. Finding θ_1



The line r_1 makes an angle $\theta_1 + \beta$ with the x-axis. This makes a right angled triangle with sides a and b, which gives:

$$\tan(\theta_1 + \beta) = \frac{b}{a}, \text{ where } b = L_1 s_1 + L_2 s_{12} \text{ and } a = L_1 c_1 + L_2 c_{12}.$$

$$\theta_1 = \tan^{-1}\left(\frac{b}{a}\right) - \beta \quad (1)$$

And using the law of cosines

$$\cos(\beta) = \frac{r_1^2 + L_1^2 - L_2^2}{2r_1 L_2} \quad (2)$$

$$\beta = \cos^{-1}\left(\frac{r_1^2 + L_1^2 - L_2^2}{2r_1 L_2}\right) \quad (3)$$

Meaning:

$$\theta_1 = \tan^{-1}\left(\frac{b}{a}\right) - \cos^{-1}\left(\frac{r_1^2 + L_1^2 - L_2^2}{2r_1 L_2}\right) \quad (4)$$

As 3 link manipulator

For end effector position $[a, b, c]^T$

$$a = L_1 c_1 + L_2 c_{12} + L_3 c_{123} \quad (5)$$

$$b = L_1 s_1 + L_2 s_{12} + L_3 s_{123} \quad (6)$$

$$c = 0 \quad (7)$$