Forward kinematics of one finger in our design

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1 DH table and Link Frames

DH table:

i	θ	d	a	α
1	θ_1	d_1	0	$90°$
2	θ_2	0	L_2	0
3	θ_3	0	L_3	0
4	θ_4	0	L_4	0
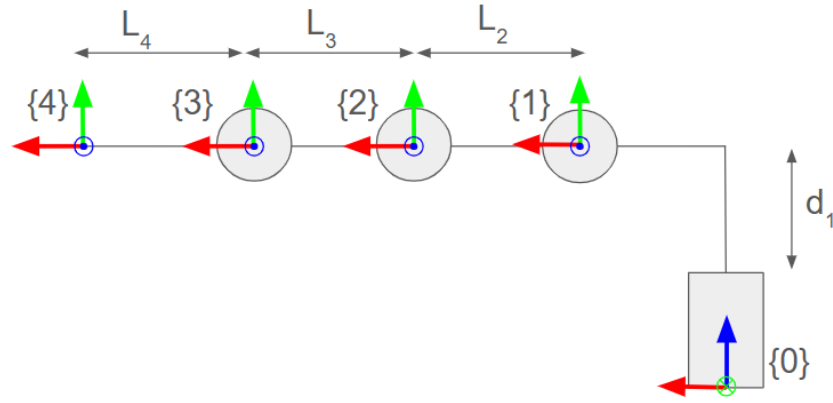
2 Forward Kinematics

Using the general formula for each transformation matrix:

$$^{i-1}T_i(\theta_i) = \begin{bmatrix} cos\theta_i & -cos\alpha_i sin\theta_i & sin\alpha_i sin\theta_i & a_i cos\theta_i \\ sin\theta_i & cos\alpha_i cos\theta_i & -sin\alpha_i cos\theta_i & a_i sin\theta_i \\ 0 & sin\alpha_i & cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

$$^0T_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$^1T_2 = \begin{bmatrix} c_2 & -s_2 & 0 & L_2c_2 \\ s_2 & c_2 & 0 & L_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$^2T_3 = \begin{bmatrix} c_3 & -s_3 & 0 & L_3c_3 \\ s_3 & c_3 & 0 & L_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$$^3T_4 = \begin{bmatrix} c_4 & -s_4 & 0 & L_4c_4 \\ s_4 & c_4 & 0 & L_4s_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

$$^0T_1 \cdot{}^1T_2 = \begin{bmatrix} c_1c_2 & -s_2c_1 & s_1 & L_2c_1c_2 \\ s_1c_2 & -s_1s_2 & -c_1 & L_2s_1c_2 \\ s_2 & c_2 & 0 & L_2s_2+d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

$$^0T_1 \cdot{}^1T_2 \cdot{}^2T_3 = \begin{bmatrix} c_1c_2c_3 - c_1s_2s_3 & -c_1c_2s_3 - c_1s_2c_3 & s_1 & L_3c_1c_2c_3 - L_3c_1s_2s_3 + L_2c_1c_2 \\ s_1c_2c_3 - s_1s_2s_3 & -s_1c_2s_3 - s_1s_2c_3 & -c_1 & L_3s_1c_2c_3 - L_3s_1s_2s_3 + L_2s_1c_2 \\ s_2c_3 + c_2s_3 & -s_2s_3 + c_2c_3 & 0 & L_3s_2c_3 + L_3c_2s_3 + L_2s_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\tag{7}$$

$$^0T_1 \cdot{}^1T_2 \cdot{}^2T_3 \cdot{}^3T_4 = {}^0T_4 = \tag{8}$$

Column 1:

$$\begin{bmatrix} (c_1c_2c_3 - c_1s_2s_3)c_4 + (-c_1c_2s_3 - c_1s_2c_3)s_4 \\ (s_1c_2c_3 - s_1s_2s_3)c_4 + (-s_1c_2s_3 - s_1s_2c_3)s_4 \\ (s_2c_3 + c_2s_3)c_4 + (-s_2s_3 + c_2c_3)s_4 \\ 0 \end{bmatrix} \tag{9}$$

Column 2:

$$\begin{bmatrix} (c_1c_2c_3 - c_1s_2s_3)(-s_4) + (-c_1c_2s_3 - c_1s_2c_3)c_4 \\ (s_1c_2c_3 - s_1s_2s_3)(-s_4) + (-s_1c_2s_3 - s_1s_2c_3)c_4 \\ (s_2c_3 + c_2s_3)(-s_4) + (-s_2s_3 + c_2c_3)c_4 \\ 0 \end{bmatrix} \tag{10}$$

Column 3:

$$\begin{bmatrix} s_1 \\ -c_1 \\ 0 \\ 0 \end{bmatrix} \tag{11}$$

2

Column 4:

$$
\begin{bmatrix}
(c_1c_2c_3 - c_1s_2s_3)L_4c_4 + (-c_1c_2s_3 - c_1s_2c_3)L_4s_4 + (L_3c_1c_2c_3 - L_3s_1s_2s_3 + L_2c_1c_2) \\
(s_1c_2c_3 - s_1s_2s_3)L_4c_4 + (-s_1c_2s_3 - s_1s_2c_3)L_4s_4 + (L_3s_1c_2c_3 - L_3s_1s_2s_3 + L_2s_1c_2) \\
(s_2c_3 + c_2s_3)L_4c_4 + (-s_2s_3 + c_2c_3)L_4s_4 + L_3s_2c_3 + L_3c_2s_3 + L_2s_2 + d_1 \\
1
\end{bmatrix}
\tag{12}
$$

Now, using the trigonometric identities:

$$
sinAcosB \pm cosAsinB = sin(A \pm B) \tag{13}
$$

$$
cosAcosB \mp sinAsinB = cos(A \pm B) \tag{14}
$$

each column can be simplified to the following:
Column 1:

$$
\begin{bmatrix}
c_1(c_4c_{23} - s_4s_{23}) \\
s_1(c_4c_{23} - s_4s_{23}) \\
s_{23}c_4 + c_{23}s_4 \\
0
\end{bmatrix}
\tag{15}
$$

Column 2:

$$
\begin{bmatrix}
-c_1(s_4c_{23} + c_4s_{23}) \\
-s_1(s_4c_{23} + c_4s_{23}) \\
-s_4s_{23} + c_4c_{23} \\
0
\end{bmatrix}
\tag{16}
$$

Column 3 (same as above):

$$
\begin{bmatrix}
s_1 \\
-c_1 \\
0 \\
0
\end{bmatrix}
\tag{17}
$$

Column 4:

$$
\begin{bmatrix}
c_1(L_4(c_{23}c_4 - s_{23}s_4) + L_3c_{23} + L_2c_2) \\
s_1(L_4(c_{23}c_4 - s_{23}s_4) + L_3c_{23} + L_2c_2 \\
L_4(s_{23}c_4 + c_{23}s_4) + L_3s_{23} + L_2s_2 + d_1 \\
1
\end{bmatrix}
\tag{18}
$$

Simplfying further using:

$$
sin(a + b + c) = sin(a + b)cosc + cos(a + b)sin(c) \tag{19}
$$

and

$$
cos(a + b + c) = cos(a + b)cos(c) - sin(a + b)sin(c) \tag{20}
$$

Column 1:

$$
\begin{bmatrix}
c_1c_{234} \\
s_1c_{234} \\
s_{234} \\
0
\end{bmatrix}
\tag{21}
$$

Column 2:

$$\begin{bmatrix} -c_1 s_{234} \\ -s_1 s_{234} \\ c_{234} \\ 0 \end{bmatrix} \tag{22}$$

Column 3 (same as previously):

$$\begin{bmatrix} s_1 \\ -c_1 \\ 0 \\ 0 \end{bmatrix} \tag{23}$$

Column 4:

$$\begin{bmatrix} L_4 c_1 c_{234} + L_3 c_{23} + L_2 c_2 \\ L_4 s_1 c_{234} + L_3 c_{23} + L_2 c_2 \\ L_4 s_{234} + L_3 s_{23} + L_2 s_2 + d_1 \\ 1 \end{bmatrix} \tag{24}$$

So full forward kinematics is given by:

$$\begin{bmatrix} c_1 c_{234} & -c_1 s_{234} & s_1 & L_4 c_1 c_{234} + L_3 c_{23} + L_2 c_2 \\ s_1 c_{234} & -s_1 s_{234} & -c_1 & L_4 s_1 c_{234} + L_3 c_{23} + L_2 c_2 \\ s_{234} & c_{234} & 0 & L_4 s_{234} + L_3 s_{23} + L_2 s_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{25}$$