LIML 推定量の導入

以下の同時方程式を考える。

$$\begin{cases} y_1 = \alpha_1 + \gamma_1 y_2 + \delta_1 x_1 + \epsilon_1 \\ y_2 = \alpha_2 + \gamma_2 y_1 + \delta_2 x_2 + \epsilon_2 \end{cases} \tag{1}$$

この式を y_1, y_2 について解くと、

$$\begin{cases} y_1 = \frac{1}{1-\gamma_1\gamma_2}(\alpha_1 + \gamma_1\alpha_2 + \delta_1 x_1 + \gamma_1\delta_2 x_2 + \epsilon_1 + \gamma_1\epsilon_2) \\ y_2 = \frac{1}{1-\gamma_1\gamma_2}(\alpha_2 + \gamma_2\alpha_1 + \gamma_2\delta_1 x_1 + \delta_2 x_2 + \epsilon_2 + \gamma_2\epsilon_1) \end{cases} \tag{2}$$

となる。さらに、行列を用いて表すと、

$$\vec{y} = \begin{pmatrix} \lambda \\ \Gamma \end{pmatrix} X + e \tag{3}$$

where

$$\begin{aligned} \vec{y} &= \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \\ \lambda &= \begin{pmatrix} \frac{\alpha_1+\gamma_1\alpha_2}{1-\gamma_1\gamma_2} & \frac{\delta_1}{1-\gamma_1\gamma_2} & \frac{\gamma_1\delta_2}{1-\gamma_1\gamma_2} \end{pmatrix} \\ \Gamma &= \begin{pmatrix} \frac{\alpha_2+\gamma_2\alpha_1}{1-\gamma_1\gamma_2} & \frac{\gamma_2\delta_1}{1-\gamma_1\gamma_2} & \frac{\delta_2}{1-\gamma_1\gamma_2} \end{pmatrix} \\ X &= \begin{pmatrix} 1 & x_1 & x_2 \end{pmatrix}' \\ e &= \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} \epsilon_1 + \gamma_1\epsilon_2 \\ \epsilon_2 + \gamma_2\epsilon_1 \end{pmatrix} \end{aligned}$$

これをさらに

$$\vec{y} = \Pi_0' + \Pi_1' x_1 + \Pi_2' x_2 + e \tag{4}$$

where

$$\begin{aligned} \Pi_0 &= \begin{pmatrix} \frac{\alpha_1+\gamma_1\alpha_2}{1-\gamma_1\gamma_2} & \frac{\alpha_2+\gamma_2\alpha_1}{1-\gamma_1\gamma_2} \end{pmatrix} \\ \Pi_1 &= \begin{pmatrix} \frac{\delta_1}{1-\gamma_1\gamma_2} & \frac{\gamma_2\delta_1}{1-\gamma_1\gamma_2} \end{pmatrix} \\ \Pi_2 &= \begin{pmatrix} \frac{\gamma_1\delta_2}{1-\gamma_1\gamma_2} & \frac{\delta_2}{1-\gamma_1\gamma_2} \end{pmatrix} \end{aligned}$$

と書き直す。このとき、Π_2 に対して

$$\Pi_2 \gamma = 0 \qquad \text{where} \quad \gamma = \begin{pmatrix} 1 \\ -\gamma_1 \end{pmatrix} \tag{5}$$

が成り立つ。