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PHASE SPACE REPRESENTATIONS OF ACOUSTICAL MUSICAL SIGNALS

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Phase space representations and Poincaré sections of acoustical signals provided by woodwind musical instruments, or feedback loop physical systems designed to mimic them, are discussed. It is shown that these representations are useful in revealing interesting features of their nature, such as period doubling and tripling, and chaos, as well as multiphonic sound production.

1. INTRODUCTION

Self-sustained musical instruments, such as woodwinds or bowed strings, are conveniently divided into two different parts: the resonator which defines the pitch of the note, and which can be characterized within the frame of a linear theory by its impedance, admittance or reflection function; and the excitator, which is inherently non-linear, and feeds energy into the resonator. Although the important role of this non-linear part has been recognized for a long time [1], its detailed study is more recent [2].

The interest in non-linear dynamics of physical systems has grown recently in physics, and many interesting and very general results have been obtained in various contexts [3, 4]. Tools such as Poincaré sections in phase spaces have become widely used. They provide representations of dynamical evolution where various phenomena (e.g., period doubling or transition to chaos) become more easily visible. Since musical instruments are nothing but a particular class of dissipative non-linear mechanical systems, there is no reason why these tools should not be used in musical acoustics. The aim of the present paper is to describe some results obtained along this line.

Similar concepts have already been used by Maganza *et al.* [5]. They have shown that a clarinet-like system can in some circumstances undergo a Feigenbaum cascade of period doublings [6] before reaching chaos. It was decided to re-examine their experiment in order to investigate the results with the help of phase space representations: this is described in the first part of this paper. Also a study has been made of the non-linear dynamics of a recorder excited with an artificial blowing system. This experiment relates to the work of Castellengo [7] and Backus [8] on multiphonic sounds. In both cases, the phase space representation and the associated Poincaré sections lead not only to interesting figures in which physical phenomena appear in a much more obvious way than in the time dependent or in the Fourier transform of the signal itself, but also to beautiful diagrams. The author therefore thinks that these tools can be very useful in intricate situations such as multiphonic sound production.

PHASE SPACE REPRESENTATIONS AND POINCARÉ SECTIONS

For a general discussion of phase space representations and applications in various physical systems, the reader is referred to references [3] and [4], where several examples

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have been treated in detail. Here only a short summary of general results is to be given.

The dynamical equations of a physical system can often be written in the form

$$\mathrm{d}\,X(t)/\mathrm{d}t = F(X(t)),$$

where $X(t)$ is a n-component vector and F symbolizes n non-linear functions of these components. $X(t)$ belongs to a n-dimensional space called the phase space of the system. Representing $X(t)$ itself, which often has a large number of components (an infinite number for a continuous system, for instance woodwinds), is obviously completely out of the question. Fortunately, one can often in practice restrict the study to a finite and low number of components. Most dynamical systems involve only a small number of important degrees of freedom. Indeed, one can even limit oneself to one single component $X_1(t)$, provided that its values $x_1(t)$ are shown at different times $t=t_0$, $t=t_0+\tau_1, \ldots,$ $t=t_0+\tau_q$, or, alternatively, show the values of $X_1(t)$ and its various time derivatives at the same time. Often q is taken equal to three or even only two. These various phase space representations are topologically equivalent [9].

In a two-dimensional diagram and for a periodic system, one obtains a closed curve. In a three-dimensional diagram, a biperiodic system gives rise to a torus, a chaotic system to a strange attractor, etc. A Poincaré section is a convenient technique for reducing the dimensionality of a given representation. It is obtained by introducing sections by arbitrary planes; for example the Poincaré section of a biperiodic motion (torus) will provide a closed curve. For a detailed discussion of these general topics, the reader is referred to references [3, 4, 10].

3. THE MAGANZA EXPERIMENT

Clarinet-like systems can be obtained by replacing the excitator (the mouthpiece and the reed) of a clarinet by an electronic device which amplifies the signal provided by a microphone, modifies it in a non-linear way, and excites the resonator through a power amplifier and a loudspeaker. The authors of reference [5] have shown that this kind of system exhibits not only "normal" oscillations but also a series of period doublings (Feigenbaum cascade [6]) before reaching acoustical chaos. These phenomena were detected by monitoring the Fourier transform of the electrical signal delivered by the microphone. In what follows here it is shown that phase space representations can provide more information. This has been done first by using the data of reference [5], and then by setting up a similar experiment, which is shown schematically in Figure 1. Figure 2 was obtained from a magnetic recording provided by one of the authors of reference [5]. Figure 2(a) is a two-dimensional phase space representation of a "normal" periodic

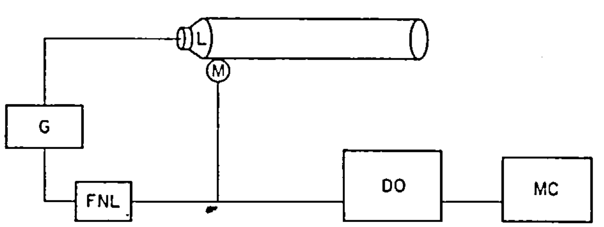


Figure 1. Schematic diagram of the experimental set-up for observing period doubling and chaos in clarinet-like systems. The acoustical resonator is part of reaction loop including a microphone M, a non-linear function $\mathrm{FNL}(x)=|x|$, an amplifier of gain G and a loudspeaker L, a digital storage oscilloscope DO and a microcomputer MC.

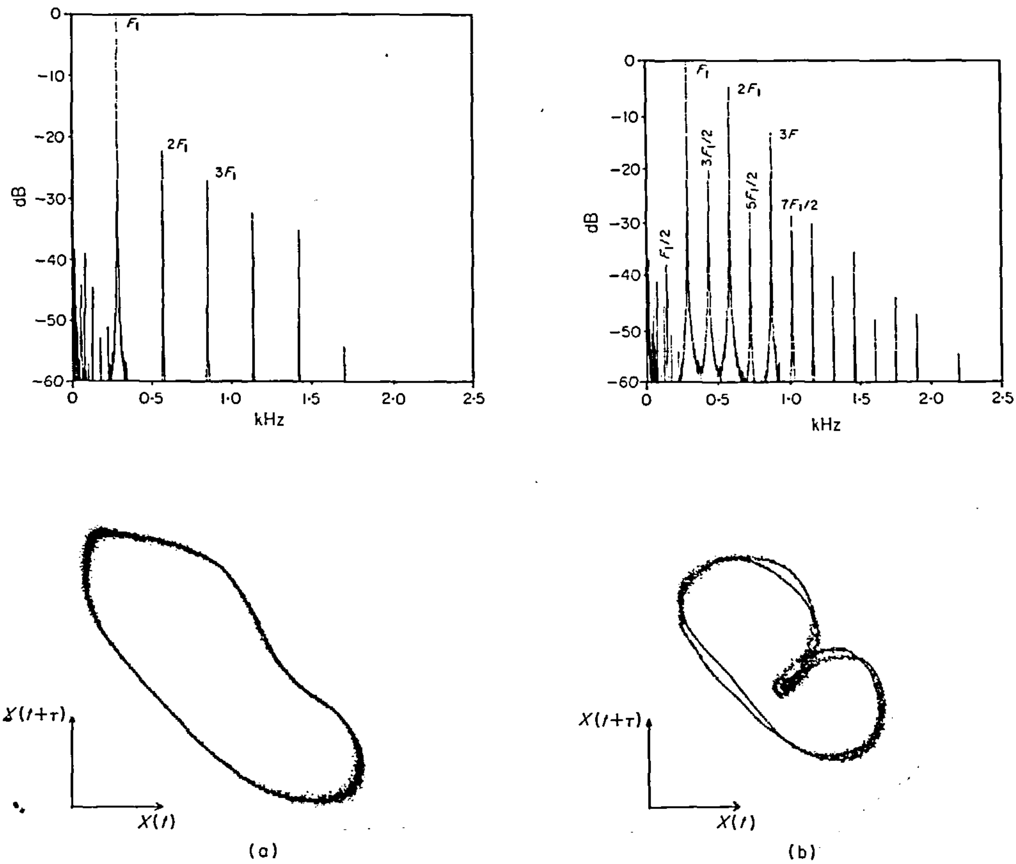
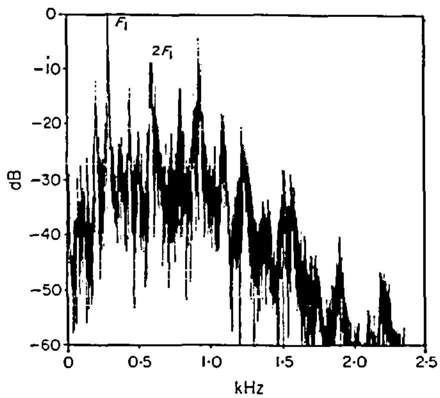


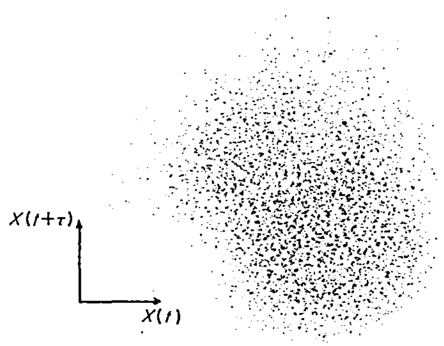
Figure 2. (a) Periodic oscillations observed in a Fourier representation (upper part) and phase space (lower part; the time delay τ is equal to 4×10^{-4} s; for more details, see the text). (b) Signal observed in the same conditions as in Figure 2(a) but with a higher gain G; a period doubling is clearly visible.

oscillation. It contains 0·8 seconds of the electrical signal sampled at a 10 kHz rate; the delay τ is 4×10^{-4} s. One notes the presence of even harmonics in the spectrum, despite the fact that the clarinet-like resonator does not have resonances at these frequencies; this phenomenon, which also occurs in real clarinets [1], is related to harmonic generation by the non-linear mechanism. Under the same conditions, Figure 2(b) shows the appearance of the first period doubling, which is obtained by increasing the gain in the reaction loop. Successive period doublings were observed in a similar way. Figure 2(c) shows the occurrence of acoustical chaos. Figure 3 was obtained with our set-up; it shows a period tripling phenomenon, observed with a real clarinet as a resonator. This "periodicity window" [11] occurs, as theory predicts [12], inside the chaotic domain. The authors of reference [5] did not report any period tripling [13]; here the phenomenon is visible on the three folds of the upper part of the attractor, as well as in its tail.

4. REPRESENTATION OF RECORDER AND MULTIPHONIC SOUNDS

The natural main control parameter of a recorder is the blowing pressure P. With a regularized source of compressed air ($0{\cdot}1\ \text{mb} < P < 2000\ \text{mb}$), it is extremely easy to artificially sound the instrument. When increasing P from zero after going beyond a certain threshold value, one first obtains a "normal" periodic oscillation, then a transition





(c)

Figure 2. (c) For still higher gain G, chaos is obtained.

to a non-periodic multiphonic sound, and then for an even higher value of P another transition to a periodic regime, of higher frequency. The phase space representation of these transitions can be discussed as follows.

The figures show phase space representations obtained from one variable, the electric signal provided by a microphone (Brüel & Kjaer 4138) placed inside the recorder just beside the edge. The signal $f(t)$ was first sampled at rates of 20 kHz and stored in the 15 872 12 bit-words memory of a Nicolet 4094 storage oscilloscope, then transferred into a microcomputer. The signal was then Fourier transformed in order to determine the main frequencies which appear in the signal. The phase space representation was computed in a 3-D space $f(t), f(t+\tau), f(t+2\tau)$, where τ is the inverse of the main frequency found in the spectrum, but this choice remains somewhat arbitrary since it does not affect the topology of the curves. The signals have zero mean value and the Poincaré section was made in the plane $f(t)=0$.

Figure 4(a) shows the representations of the acoustical signal measured inside the recorder, for a pressure just above the threshold of oscillation. The signal is periodic and its phase space diagram is a closed curve; the associated Poincaré section is a point. Figure 4(b) corresponds to the first multiphonic sound obtained when increasing P. The phase space representation is not easy to interpret, but the Poincaré section is that of a torus, indicating that two oscillators contribute to the signal. Their frequencies F_1 and

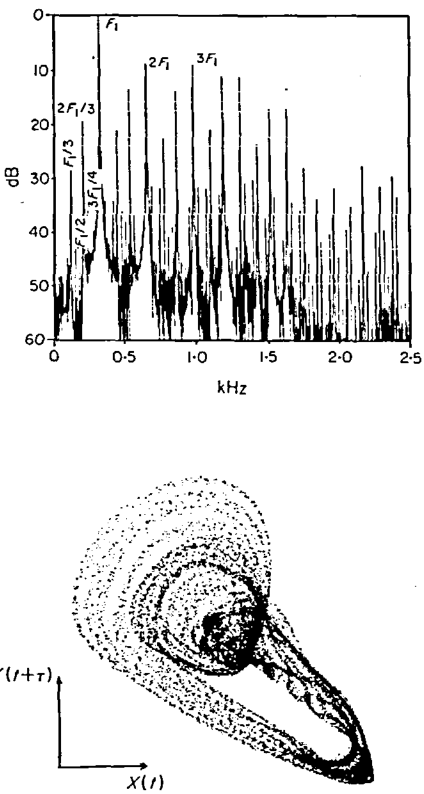


Figure 3. With a clarinet as resonator, with the set-up shown in Figure 1, one can obtain a period tripling.

F_2 (as well as several linear combinations) are easily visible in the Fourier spectrum shown in the figure. Figure 4(c) was obtained with an even higher pressure P. The phase space representation and the Poincaré section show that the signal is neither periodic nor biperiodic, but rather chaotic. Figure 4(d) characterizes the signal at immediately higher blowing pressures. The signal is again biperiodic, but F_1 is higher than in Figure 4(b) and close to the frequency of the second regime of the instrument. When increasing P even more, one finally reaches the second periodic regime.

The results presented here are characteristic of all the recorder fingerings the author has studied (known or not to give rise to multiphonics). One can infer from these results that the usual description of the transition between two regimes for the flute family [14] is oversimplified. What one sees is, first, a periodic signal of frequency F_1 close to the first resonance of the recorder, and then a biperiodic-multiphonic regime where two frequencies are non-linearly coupled: one is still close to F_1 and the second F_2 is easily related to the travel time of a perturbation along the air jet between the flue and the edge of the instrument [15-17]. Then a chaotic-multiphonic regime occurs, immediately followed by a new biperiodic-multiphonic regime, the frequency F_1 of which is now close to the second resonance frequency of the instrument. Finally, the signal sounds periodic with the frequency of the second regime.

The author has tried this method on various other multiphonic sounds: flute, clarinet and oboe. The results are similar. Multiphonics (at least those that the author has studied) are essentially biperiodic sounds. They have closed curves as Poincaré sections; it is easy to verify that their spectra are only linear combinations of two frequencies. As an example

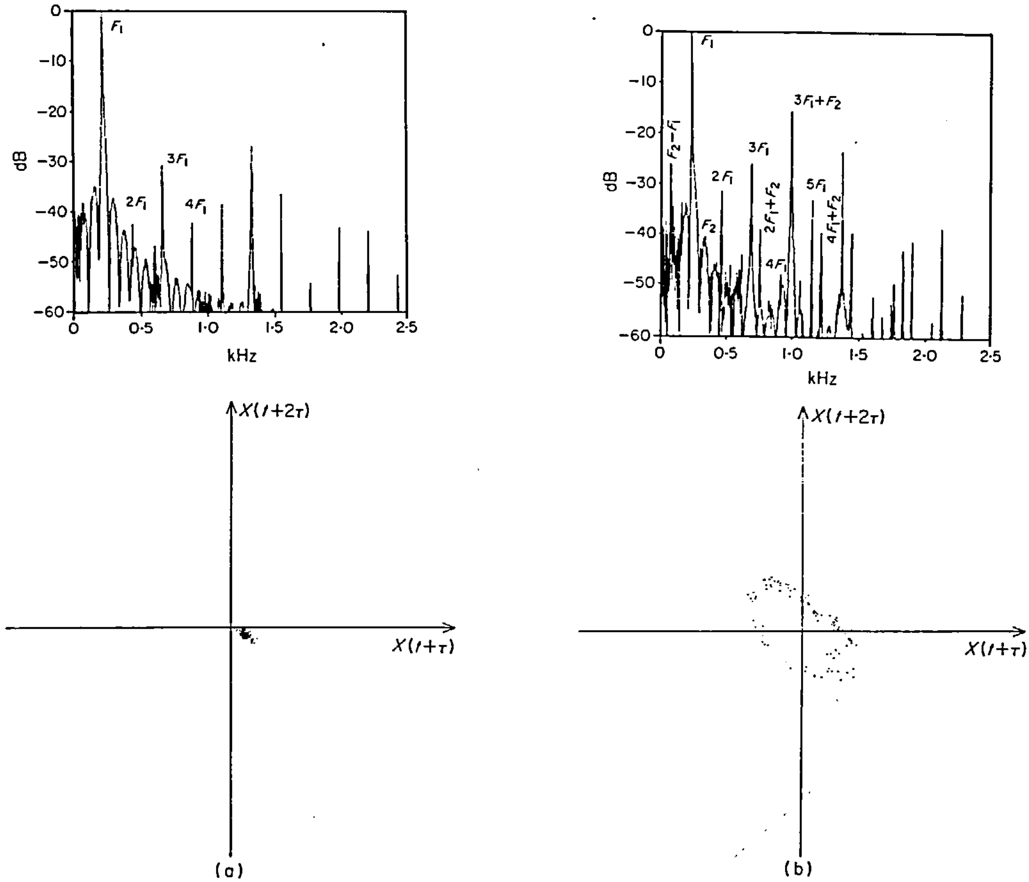


Figure 4. Signals observed with a recorder artificially blown (pressure $P=0{\cdot}37$ mb). The upper part is a Fourier representation. The lower part is a Poincaré section, showing $X(t+\tau)$ (horizontal axis) and $X(t+2\tau)$ (vertical axis) when $X(t)=0$ and $\mathrm{d}X(t)/\mathrm{d}t<0$ (conditions defining the section). (a) The Fourier spectrum shows a periodicity at a frequency $F_1=221$ Hz; τ is therefore chosen as $\tau=45\times10^{-4}\mathrm{s}\cong 1/F_1$. The total duration of the signal is 1·6 s; (b) signals observed in the same conditions but with a higher blowing pressure (0·72 mb), leading to a biperiodic oscillation. Here τ is chosen equal to 44×10^{-4} s ($\simeq F_1^{-1}$ where $F_1=227$ Hz). In the upper part of the figure, a second frequency $F_2=313$ Hz is visible, as well as linear combination of F_1 and F_2. The Poincaré section is typical of a torus.

Figure 5 shows the Poincaré section of an oboe multiphonic sound previously studied in reference [7] (ref. HB12).

5. CONCLUSION

In the situations that the author has studied, the use of phase space representations gives information that is not easily visible otherwise. In the case of the recorder, it suggests a different scenario for the transition between the regimes than those previously considered. In the case of clarinet-like systems, it reveals phenomena (period tripling) that had escaped the attention of previous authors. It also introduces more coherence in the description of multiphonic sounds.

The author therefore thinks that these phase space techniques and those related can be extremely useful in the study of non-linear problems in musical acoustics.

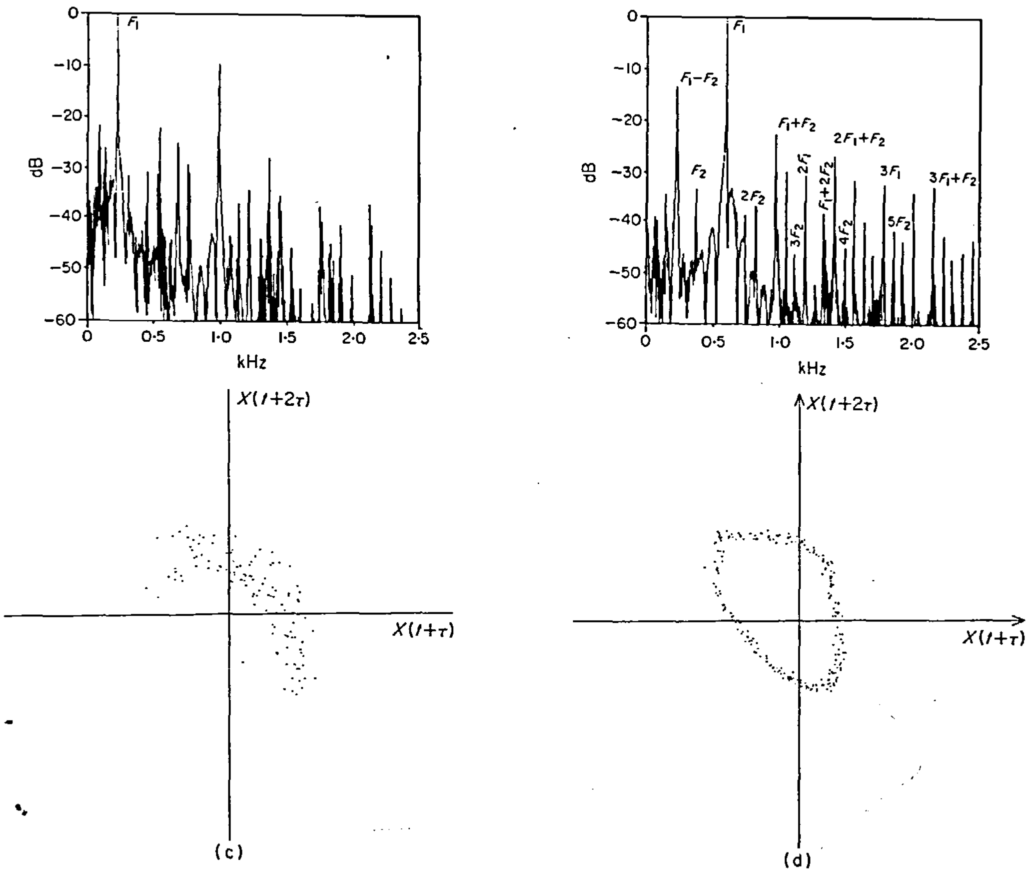


Figure 4. (c) Low-dimension chaotic signals observed at higher blowing pressures (0·75 mb): same τ as in Figure 4(b). (d) At even higher blowing pressures (·84 mb), the signal returns to a biperiodic regime. $F_1 = 591$ Hz, $F_2 = 366$ Hz and $\tau = 17 \times 10^{-4}$ s $\simeq (588 \text{ Hz})^{-1}$.

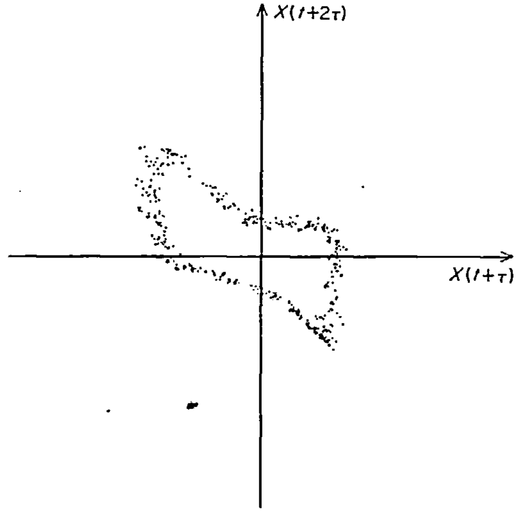


Figure 5. Example of an oboe multiphonic sound: the Poincaré section is that of a torus.

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REFERENCES

1. H. HELMHOLTZ 1885 *On the Sensations of Tone*. New York: Dover.
2. M. E. MCINTYRE, R. T. SCHUMACHER and J. WOODHOUSE 1983 *Journal of the Acoustical Society of America* **74**(5), 1325–1345. On the oscillations of musical instruments.
3. P. CVITANOVIC 1984 *Universality in Chaos*. Bristol: Adam Hilger.
4. P. BERGE, Y. POMMEAU and CH. VIDAL 1984 *L'ordre dans le chaos*. Paris; Hermann.
5. C. MAGANZA, R. CAUSSE and F. LALOE 1986 *Europhysics Letters* **1**, 295–302. Bifurcations, period doublings and chaos in clarinet-like systems.
6. M. FEIGENBAUM 1983 *Physica* **D7**, 16–39. Universal behaviour in non-linear systems.
7. M. CASTELLENGO 1982 *Rapport IRCAM* 34/82, Paris. Sons multiphoniques aux instruments à vents.
8. J. BACKUS 1978 *Journal of the Acoustical Society of America* **63**, 591–599. Multiphonic tones in the woodwind instruments.
9. N. H. PACKARD, J. P. CRUTCHFIELD, J. D. FARMER and R. S. SHAW 1980 *Physical Review Letters* **45**, 712–716. Geometry from a time series.
10. J. P. CRUTCHFIELD, J. D. FARMER, N. H. PACKARD and R. S. SHAW 1986 *Scientific American* **255**, 38–48. Chaos.
11. J. TESTA, J. PEREZ and C. JEFFERIES 1982 *Physical Review Letters* **48**, 714–717. Evidence for universal chaotic behaviour of a driven nonlinear oscillator.
12. N. METROPOLIS, M. L. STEIN and P. R. STEIN 1973 *Journal of Combinational Theory* **15**, 25–44. On finite set for transformations on the unit interval.
13. C. MAGANZA 1985 *Thèse de docteur-ingénieur, université du Maine, Le Mans*. Excitations non-linéaires d'un conduit cylindrique. Observations de doublements de période précédant un comportement chaotique. Application à la clarinette.
14. M. CASTELLENGO 1976 *Thèse d'Université*. Contribution à l'étude expérimentale des tuyaux à bouche.
15. J. W. COLTMAN 1976 *Journal of the Acoustical Society of America* **60**, 725–733. Jet drive mechanisms in edge tones and organ pipes.
16. J. W. COLTMAN 1968 *Journal of the Acoustical Society of America* **44**, 983–992. Sounding mechanism of the flute and organ pipe.
17. N. H. FLETCHER and S. THWAITES 1979 *Acustica* **42**, 323–334. Wave propagation on an acoustically perturbed jet.