TD2

Exercise 1 (CTL$^+$)**.** CTL$^+$ extends CTL by allowing boolean connectives on path formulæ, according to the following abstract syntax:

$$f ::= \top \mid a \mid f \wedge g \mid \neg f \mid \mathsf{E}\,\varphi \mid \mathsf{A}\,\varphi \qquad \text{(state formulæ } f, g)$$
$$\varphi ::= \varphi \wedge \psi \mid \neg\varphi \mid \mathsf{X}\,f \mid f \,\mathsf{U}\, g \qquad \text{(path formulæ } \varphi, \psi)$$

where a is an atomic proposition. The associated semantics is that of CTL*.

We want to prove that, for any CTL$^+$ formula, there exists an equivalent CTL formula.

1. Give an equivalent CTL formula for
$$\mathsf{E}((a_1 \,\mathsf{U}\, b_1) \wedge (a_2 \,\mathsf{U}\, b_2)) \; .$$

2. Generalize your translation for any formula of form
$$\mathsf{E}\Big(\bigwedge_{i=1,\dots,n} (\psi_i \,\mathsf{U}\, \psi'_i) \wedge \mathsf{G}\,\varphi\Big) \; . \tag{1}$$
What is the complexity of your translation?

3. Give an equivalent CTL formula for the following CTL$^+$ formula:
$$\mathsf{E}(\mathsf{X}\,a \wedge (b \,\mathsf{U}\, c)) \; .$$

4. Using subformulæ of form (1) and E modalities, give an equivalent CTL formula to
$$\mathsf{E}\Big(\mathsf{X}\,\varphi \wedge \bigwedge_{i=1,\dots,n} (\psi_i \,\mathsf{U}\, \psi'_i) \wedge \mathsf{G}\,\varphi'\Big) \; . \tag{2}$$
What is the complexity of your translation?

5. We only have to transform any CTL$^+$ formula into (nested) disjuncts of form (2). Detail this translation for the following formula:
$$\mathsf{A}((\mathsf{F}\,a \vee \mathsf{X}\,a \vee \mathsf{X}\,\neg b \vee \mathsf{F}\,\neg d) \wedge (d \,\mathsf{U}\, \neg c)) \; .$$

Exercise 2 (Fair CTL)**.** We consider *strong* fairness constraints, which are conjunctions of formulæ of form

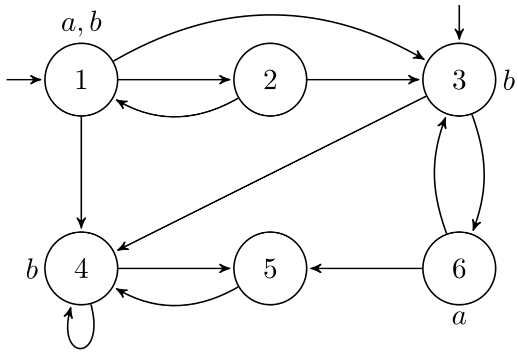
$$\mathsf{GF}\psi_1 \Rightarrow \mathsf{GF}\psi_2 \ .$$

We want to check whether the following Kripke structure fairly verifies

$$\varphi = \mathsf{A}_f \mathsf{G}\, \mathsf{A}_f \mathsf{F}\, a$$

under the fairness requirement e defined by

$$\begin{aligned}\psi_1 &= b \wedge \neg a\\ \psi_2 &= \mathsf{E}(b\, \mathsf{U}\, (a \wedge \neg b))\\ e &= \mathsf{GF}\, \psi_1 \Rightarrow \mathsf{GF}\, \psi_2 \ .\end{aligned}$$



1. Compute $[\![\psi_1]\!]$ and $[\![\psi_2]\!]$.
2. Compute $[\![\mathsf{E}_f\, \mathsf{G}\, \top]\!]$.
3. Compute $[\![\varphi]\!]$.