Practical Compositional Fairness: Understanding Fairness in Multi-Task ML Systems

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Abstract

Most literature in fairness has focused on improving fairness with respect to one single model or one single objective. However, real-world machine learning systems are usually composed of many different components. Unfortunately, recent research has shown that even if each component is "fair", the overall system can still be "unfair" [16].

In this paper, we focus on how well fairness composes over multiple components in *real systems*. We consider two recently proposed fairness metrics for rankings: exposure and pairwise ranking accuracy gap. We provide theory that demonstrates a set of conditions under which fairness of individual models does compose. We then present an analytical framework for both understanding whether a system's signals can achieve compositional fairness, and diagnosing which of these signals lowers the overall system's end-to-end fairness the most. Despite previously bleak theoretical results, on multiple data-sets—including a large-scale real-world recommender system—we find that the overall system's end-to-end fairness is largely achievable by improving fairness in individual components.

1 Introduction

Recent research has highlighted that even if two machine learning (ML) models are "fair," a combination of their predictions can still be "unfair" [16, 15]. This is known as the compositional fairness problem. The problem has been shown to hold over multiple definitions of fairness. Composing many predictive models in the final product, however, is a pervasive design pattern in real production systems [1, 9, 22, 39, 27, 40].

Most existing literature focuses on achieving fairness in the *single-task* (non-compositional) setting. [16] has studied a relatively restricted compositional setting, where the binary outputs of multiple classifiers are combined through *logical operations* to produce a single output. Regarding group fairness, [16] makes the case that classifiers that appear to satisfy group fairness properties, may not compose to also satisfy those properties. [16] also raises the concern that composed systems that do satisfy group fairness properties, may not be fair for socially meaningful sub-groups (i.e. a system "fair" across gender or race, may not be fair from the perspective of a specific gender-race sub-group).

Our high-level objective is to know what group fairness goals we *can* achieve in ranking type problems. In this case, the end score is a rank derived by composing scores produced by different components. We assume we can only control each individual component independently to create

overall system fairness. We use recently proposed ranking fairness metrics [36, 5], each capturing slightly different goals.

More specifically, we study the setting where each component outputs real values; the composition function is the multiplication of these component scores, and so also real-valued. Mathematically, we frame each component as functions $f_k, \forall k \in \{0, \ldots, K\}$ where $f_k(x) \in \mathbb{R}$. The overall system generates scores, $f(x) = \prod_{k=0}^{K} f_k(x)$. These scores are then used for ranking. This design is common in recommender systems such as cascading recommenders [22, 39] and multi-task recommenders [30, 27, 40].

The fairness metric for each component is evaluated on the order (rank) of its scores. The composed system fairness metric is evaluated on the order (rank) of the product of the component scores. Evaluating the fairness metric on rank order aligns well with most real-world multi-model recommender systems, but can also be applied to classification [24, 8, 33].

A motivating example. To better concretize the problem, consider the hypothetical example of a large-scale recommendation system for books, like the one described in [17]. The recommender system has the following components:

- $pCTR$: one that predicts click through rate on a book

- $pRating$: one that predicts the star rating given a clicked book

Let the fairness goal be demographic parity in *ranking exposure*. For example, the ranking of the composite score should not systematically differ between *white* and *non-white* authors. Each component could be made "fair" with respect to author demographics through recent mitigation methods [7, 36, 6, 5]. What does this mean for the demographic parity on the ranked composite scores?

A simple counter-example. In the example above, it may feel intuitive to assume that if each component gives equal exposure to each group, the overall system should as well. We give a simple example here showing this is not the case. Assume we have the following 4 books in each column:

Component	non-white	non-white	white	white
$pCTR$	0.1	0.4	0.2	0.3
$pRating$	0.4	0.1	0.3	0.2
$pCTR \times pRating$	0.04	0.04	0.06	0.06

Each component exposes books from each group equally: if we rank scores for $pCTR$, we get [non-white, white, white, non-white]. If we rank $pRating$, we get [non-white, white, white, non-white]. When the two components are multiplied together to form the composite score, we get the ranking: [white, white, non-white, non-white]. This composite ranking does not have demographic parity, even though all individual component rankings do.

In practice, large systems are designed with compositions; there are theoretical fairness risks to such systems. We try to ease this tension between the practical and the theoretical: we describe mathematical conditions under which compositional fairness holds, and we demonstrate how to empirically test a system for compositional fairness. Our contributions in this paper:

- **Theory:** We provide theory showing a set of conditions under which fair components can compose into fair systems.

- **System Understanding:** We provide a framework for both understanding whether a system's signals can achieve compositional fairness, and diagnosing which of these signals lowers the overall system fairness the most.

- **Empirical Analysis:** Although compositional fairness is theoretically not guaranteed, on multiple data-sets, including a large-scale real-world recommender system, we demonstrate that the overall system fairness is largely achievable by improving fairness in individual components.

2 Related Work

Fairness in Classification   The majority of the fairness metric definition literature focuses on classification. Here we cover some examples of fairness metrics in classification. *Demographic parity* [10, 38, 41] is a common way of addressing discrimination against protected attributes. It requires a decision to be independent of the protected attribute. *Equalized odds*, proposed by [21], is a fairness metric for a specified sensitive attribute in supervised learning. *Equalized odds* requires a predictor $\hat{Y}$ to be independent with respect to the sensitive attribute, conditioned on the true label Y. This metric equivalently equalizes true positive rates as well as the false positive rates across the two demographics to prevent classification models to perform well only on the majority group. *Equal opportunity*, also proposed by [21], is a relaxation of *equalized odds*. It focuses only on "advantaged" outcome $Y = 1$. More recently, metrics have been explored continuous scores from a classifier: [24, 8, 33] all break down AUC of these scores into Mann-Whitney U-tests [29].

Fairness in Ranking   Recently, there have been a few definitions proposed for fairness in the ranking setting as well [43]; in our work we focus on two recent framings. [36] proposes measuring exposure an item or group of items gets depending on what position they fall in a ranking. The work offers multiple fairness goals, such as exposure proportional to relevance, but in our usage we build on this notion of exposure to measure group representation throughout a ranked list (as we ignore any label or relevance, this is philosophically closer to the principles of demographic parity above). [5] focuses on measuring accuracy in a recommender system based on *pairwise comparisons*. The accuracy of a ranking for a pair of items is defined as the probability that the clicked item is ranked higher than the un-clicked item. In this set-up, two items from different groups are used to create a pair, and the difference in accuracy for each group is used as a fairness metric. We use this metric to capture fairness in ranking more closely aligned with equal opportunity (as it is measuring accuracy with respect to a label—clicks).

In Section 3, we will formalize the above two definitions within the same framework. For each of the ranking metrics listed, given per-component fairness, we will show conditions where the compositional fairness holds (and counter-examples where it might not hold).

Ranking diversification is a closely related area to ranking fairness. Here the goal is to diversify the ranking results to improve user satisfaction [37, 35, 19, 11, 3, 12]. In this paper, we focus on the ranking fairness goal with respect to specified demographic groups. In some cases, general-purpose diversification may not align with fairness for certain sub-groups.

Compositional Fairness   [16] has studied general constructions for fair composition, and showed that classifiers that are fair in isolation do not necessarily compose into fair systems, for individuals or for groups. Furthermore, systems that are fair to different types of groups, may not be fair to inter-sectional sub-groups. The authors studied the "Functional Composition" setting, where the assumption is that the binary outputs of multiple classifiers are combined through *logical operations* to produce a single output for a single task. Specifically, a notion of "OR Fairness" is proposed and relevant theory is developed in this setting.

Mitigation   Many papers have proposed different approaches for achieving fairness in the *single-task* (non-compositional) setting. The approaches can be partitioned by when they intervene in the creation of an ML system: pre-processing, training, and post-processing.

- Pre-processing the data, like obfuscation on the protected attributes, or debiasing on the input features like word embeddings [7].
- Incorporating fairness into the learning objective: Numerous approaches have been proposed to improve fairness metrics during training, including constraints [13, 14, 2], regularization [42, 6], and adversarial learning [26, 44, 4, 28]. The regularization approaches are most similar to our analysis, encouraging matching the distribution of predictions [42, 6] or representation [4, 28] across groups. [5] built on this for ranking, proposing pairwise regularization to the objective function.
- Post-processing on the predictions, e.g., [34, 25]. For example, [21] uses different classifications thresholds per group at inference time, and [36] proposes solving a linear program to achieve fair exposure in rankings.

3 Fairness Metrics and Theoretical Analysis

To formalize the problem, we denote x as the item being ranked, and for simplicity assume there are two groups being considered, Group $\mathcal{A}$ and Group $\mathcal{B}$.

We formulate the key problem we want to study in this paper as the following: We have a system composed of $0, \ldots, K$ components, where each component takes an input x and produces its own score $f_i(x)$; The overall composition function that produces the final ranking score is $f(x) = \prod_{k=0}^{K} f_k(x)$. If all K components satisfy some fairness metric by themselves $F[f_k]$, we ask whether the overall function $f(x)$ satisfies the same fairness metric $F[f]$. Restated, we would like to know whether the system achieves compositional (end-to-end) fairness given that each component has achieved fairness independently.

In the sections below, we consider two commonly-used fairness metrics in ranking: ranking exposure, and pairwise ranking accuracy. We describe each metric, and explore how the function composition affects end-to-end fairness for that metric.

3.1 Ranking exposure as the fairness metric

3.1.1 Definition and Examples

In this section, we focus on the cases where the ranking exposure [36] is the fairness metric.

Formally, we define the ranking exposure for any Group $\mathcal{G}$ as:

$$\text{Exposure}(\mathcal{G}|r) = \sum_{x \in \mathcal{G}} u[x|r],$$

under a certain ranking order r. Here u denotes the utility function, which is usually a monotonically decreasing function with respect to the rank of the item. For example, one common choice is to use an exponent $w >= 0$, and

$$u[x|r] = [\text{rank}(x)]^{-w},$$

where $\text{rank}(x)$ is the rank of item x under the ranking order r. Another common option is $u[x|r] = \frac{1}{\log(1+\text{rank}(x))}$, similar to the position discount as defined in Discounted Cumulative Gain (DCG) [23].

Now the fairness exposure metric between Group $\mathcal{A}$ and Group $\mathcal{B}$ under the ranking r is defined as:

$$\text{Gap}(\mathcal{A}, \mathcal{B}|r) = \frac{|\text{Exposure}(\mathcal{A}|r) - \text{Exposure}(\mathcal{B}|r)|}{\text{Exposure}(\mathcal{A}|r) + \text{Exposure}(\mathcal{B}|r)}, \tag{1}$$

which denotes the normalized gap between the exposure for Group $\mathcal{A}$ and $\mathcal{B}$. This gap metric ranges from 0 to 1, where $\text{Gap}(\mathcal{A}, \mathcal{B}|r) = 0$ means there is an equal exposure (50% each) of both groups, and $\text{Gap}(\mathcal{A}, \mathcal{B}|r) = 1$ means one group has all the exposure (100%) while the other group has no exposure at all.

Note, when the two groups have the same size, i.e., $|\mathcal{A}| = |\mathcal{B}|$, the ideal exposure gap should reach zero in order to be fair for the two groups. This is not the case when the two groups have different sizes, for example, if $|\mathcal{A}| = 4|\mathcal{B}|$, then one might argue that a reasonable exposure for $\mathcal{A}$ and $\mathcal{B}$ could be proportional to their sizes, i.e., $80\%, 20\%$, so the exposure gap is 0.6. In the following for simplicity we always assume the two groups have the same size.

Intuitively, this metric (here unconditioned on relevance) makes the goal providing a diverse ranking with each group being well represented throughout the ranked list. While we build on [36] for framing, similar intuitions were previously proposed in [43] and used in job search applications [18].

Counter-example Here we give a counter-example and show that under the fairness exposure metric, per-component fairness does not always guarantee end-to-end fairness.

Consider the following example with a ranking system composed of two components, two groups $\mathcal{A}, \mathcal{B}$, and each group has two items. For any $a > 0, \epsilon > 0$, suppose:

$$f_0(x) = [a + \epsilon, a + 4\epsilon], x \in \mathcal{A}; \quad f_0(x) = [a + 2\epsilon, a + 3\epsilon], x \in \mathcal{B}.$$
$$f_1(x) = [a + 4\epsilon, a + \epsilon], x \in \mathcal{A}; \quad f_1(x) = [a + 3\epsilon, a + 2\epsilon], x \in \mathcal{B}.$$

For simplicity we assume $w = 0$, i.e., each rank position contributes equally to the exposure metric, and we consider the exposure for the *first two positions*. Assume r_0, r_1 are the rankings produced by each individual component scores $f_0(x), f_1(x)$, respectively, then for each component independently, we have

$$\text{Exposure}(\mathcal{A}|r_k) = \text{Exposure}(\mathcal{B}|r_k) = 0.5, \ k \in \{0, 1\},$$

because within each component, the two highest-scored items (with scores $a + 4\epsilon$, $a + 3\epsilon$) are from $\mathcal{A}, \mathcal{B}$, respectively, in other words, each component by itself is fair. But when combined, denote r to be the ranking produced by ordering the items based on the composite score $f(x) = f_0(x) \cdot f_1(x)$, we have

$$f(x) = [a^2 + 5a\epsilon + 4\epsilon^2, a^2 + 5a\epsilon + 4\epsilon^2], x \in \mathcal{A};$$
$$f(x) = [a^2 + 5a\epsilon + 6\epsilon^2, a^2 + 5a\epsilon + 6\epsilon^2], x \in \mathcal{B},$$

i.e., Group $\mathcal{A}$ is always ranked below the Group $\mathcal{B}$. Specifically, for the first two positions, $\text{Exposure}(\mathcal{A}|r) = 0$ and $\text{Exposure}(\mathcal{B}|r) = 1$, and $\text{Gap}(\mathcal{A}, \mathcal{B}|r) = 1$.

One might think the magnitude of the scores for each component will play a role here, but the above example also suggests this is not the case, since we can make ϵ arbitrarily small to make the magnitude of the score for the two components arbitrarily close to each other, while keeping $\text{Gap}(\mathcal{A}, \mathcal{B}|r) = 1$.

Distribution Normalization   In the above example, if we normalize the distribution of f_0, f_1 for each group (i.e., $f \to \frac{f - \mu(f)}{\sigma(f)}$, where μ, σ represents the mean and standard deviation of the scores), then it is easy to show that we can achieve compositional fairness.

However, we present a slightly modified example that shows sometimes distribution normalization might not work:

$$f_0(x) = [0, 1], x \in \mathcal{A}; \quad f_0(x) = [0, 1], x \in \mathcal{B}.$$
$$f_1(x) = [0, 1], x \in \mathcal{A}; \quad f_1(x) = [1, 0], x \in \mathcal{B}.$$

After normalization, we have:

$$f_0(x) = [-1, 1], x \in \mathcal{A}; \quad f_0(x) = [-1, 1], x \in \mathcal{B}.$$
$$f_1(x) = [-1, 1], x \in \mathcal{A}; \quad f_1(x) = [1, -1], x \in \mathcal{B}.$$

Again we can see that when the scores are combined $f(x) = f_0(x) \cdot f_1(x)$, we have for Group $\mathcal{A}$, $f(x) = 1$, and for Group $\mathcal{B}$, $f(x) = -1$, i.e., Group $\mathcal{A}$ is always ranked above the Group $\mathcal{B}$.

On the other hand, this problem can simply be solved by adding any constant shift > 1 to make sure all the scores are positive, e.g., for a shift of $+2$:

$$f_0(x) = [1, 3], x \in \mathcal{A}; \quad f_0(x) = [1, 3], x \in \mathcal{B}.$$
$$f_1(x) = [1, 3], x \in \mathcal{A}; \quad f_1(x) = [3, 1], x \in \mathcal{B}.$$

Now the two items from Group $\mathcal{B}$ will be ranked in-between the two items from Group $\mathcal{A}$.

3.1.2   Condition for composition of ranking exposure

Now we present theory showing under what conditions we will achieve end-to-end fairness given we have per-component fairness, using the ranking exposure (§3.1.1) as the fairness metric.

Consider a system with two components f_0 and f_1, and two groups $\mathcal{A}, \mathcal{B}$. Let X_A^0 represent the random variable defined by $\log f_0(x), x \in \mathcal{A}$, X_B^0 represent the random variable defined by $\log f_0(x), x \in \mathcal{B}$, and similarly we define X_A^1, X_B^1 for f_1. For simplicity we assume the higher half and the lower half of the items receive different exposure values of $u[x|r]$ after sorting over the scores produced by f_0 and f_1, respectively, i.e., to achieve per-component fairness, we have $\text{median}[X_A^0] = \text{median}[X_B^0]$, and $\text{median}[X_A^1] = \text{median}[X_B^1]$. We have the following:

Theorem 1. *If $X_A^0, X_B^0, X_A^1, X_B^1$ are symmetric random variables such that $X_A^0 + X_A^1$ and $X_B^0 + X_B^1$ are also symmetric, then per-component fairness on f_0 and f_1 means we have compositional fairness for $f(x) = f_0(x) \cdot f_1(x)$.*

5

Proof. From per-component fairness on f_0 and f_1 we have: $\text{median}[X_A^0] = \text{median}[X_B^0]$ and $\text{median}[X_A^1] = \text{median}[X_B^1]$. Hence

$$
\begin{aligned}
&\underset{x \in \mathcal{A}}{\text{median}}[\log(f_0(x) \cdot f_1(x)] \\
=&\underset{x \in \mathcal{A}}{\text{median}}[\log f_0(x) + \log f_1(x)] \\
=&\text{median}[X_A^0 + X_A^1] \\
=&\text{mean}[X_A^0 + X_A^1] && \text{(by symmetry of } X_A^0 + X_A^1) \\
=&\text{mean}[X_A^0] + \text{mean}[X_A^1] && \text{(by linearity of expectation)} \\
=&\text{median}[X_A^0] + \text{median}[X_A^1] && \text{(by symmetry of } X_A^0 \text{ and } X_A^1) \\
=&\text{median}[X_B^0] + \text{median}[X_B^1] && \text{(by per-component fairness)} \\
=&\text{mean}[X_B^0] + \text{mean}[X_B^1] && \text{(by symmetry of } X_B^0 \text{ and } X_B^1) \\
=&\text{mean}[X_B^0 + X_B^1] && \text{(by linearity of expectation)} \\
=&\text{median}[X_B^0 + X_B^1] && \text{(by symmetry of } X_B^0 + X_B^1) \\
=&\underset{x \in \mathcal{B}}{\text{median}}[\log f_0(x) + \log f_1(x)] \\
=&\underset{x \in \mathcal{B}}{\text{median}}[\log(f_0(x) \cdot f_1(x))].
\end{aligned}
$$

By the monotonicity of the $\log$ function, the above equation gives

$$
\underset{x \in \mathcal{A}}{\text{median}}[f_0(x) \cdot f_1(x)] = \underset{x \in \mathcal{B}}{\text{median}}[f_0(x) \cdot f_1(x)],
$$

i.e., the compositional fairness holds for the entire system. $\square$

3.2 Pairwise ranking accuracy as the fairness metric

Recently another fairness metric in ranking has been proposed [5], where the idea is to compute the accuracy of a system ranking a pair of items correctly conditioned on the true feedback information (e.g., one being clicked and another not being clicked). The pair of items is constrained to come from two different groups, $\mathcal{A}$ and $\mathcal{B}$, through randomized experiments.

Formally, the Pairwise Ranking Accuracy is defined as:

$$
\begin{aligned}
&\text{Pairwise\_Acc}(\mathcal{A} > \mathcal{B}|r) \\
=~& P(f(x_i) > f(x_j)|y_i > y_j, x_i \in \mathcal{A}, x_j \in \mathcal{B}) \\
=~& \frac{P(f(x_i) > f(x_j), y_i > y_j|x_i \in \mathcal{A}, x_j \in \mathcal{B})}{P(y_i > y_j|x_i \in \mathcal{A}, x_j \in \mathcal{B})}
\end{aligned}
$$

Here $y_i \in \{0, 1\}$ denotes the observed label for x_i, as either being clicked: $y_i = 1$, or not-clicked: $y_i = 0$.

The empirical estimate is given by counting the item pairs $x_i \in \mathcal{A}, x_j \in \mathcal{B}$, where $f(x_i) > f(x_j), y_i > y_j$, normalized by the total number of item pairs from $\mathcal{A}, \mathcal{B}$ that satisfy $y_i > y_j$.

Correspondingly, the Pairwise Ranking Gap is defined as:

$$
\begin{aligned}
&\text{Pairwise Ranking Gap} \\
=~& |\text{Pairwise\_Acc}(\mathcal{A} > \mathcal{B}|r) - \text{Pairwise\_Acc}(\mathcal{A} < \mathcal{B}|r)|. \quad (2)
\end{aligned}
$$

In other words, given a pair of items, one from group $\mathcal{A}$ and one from group $\mathcal{B}$, conditioned on one item being clicked and the other not being clicked, we would like the system to have the same accuracy of ranking this pair of items correctly, regardless of which group the clicked item is from.

Let X_{A_0} represent the random variable defined by $f(x_i)$, for $y_i = 0, x_i \in \mathcal{A}$; X_{A_1} represent the random variable defined by $f(x_i)$ for $y_i = 1, x_i \in \mathcal{A}$, and X_{B_0}, X_{B_1} are defined similarly. We can simplify the Pairwise Ranking Gap metric as

$$
\begin{aligned}
&\text{Pairwise Ranking Gap} \\
=~& |\text{Pairwise\_Acc}(\mathcal{A} > \mathcal{B}|r) - \text{Pairwise\_Acc}(\mathcal{A} < \mathcal{B}|r)| \\
=~& |P(X_{A_1} > X_{B_0}) - P(X_{B_1} > X_{A_0})|
\end{aligned}
$$

This pairwise ranking accuracy has a nice connection with the Mann-Whitney U-test [29], and aligns well with the equality gap metric [8] and the xAUC metric [24] for classification.

Counter-example. In the following we present a simple example that shows per-component fairness might not lead to compositional fairness, using the pairwise ranking gap as the fairness metric.

Consider the following system with two components, two groups, and two items within each group:

Component	X_{A_1}	X_{B_0}	X_{B_1}	X_{A_0}
f_0	$[1, 4]$	$[2, 3]$	$[1, 4]$	$[2, 3]$
f_1	$[4, 1]$	$[3, 2]$	$[1, 4]$	$[3, 2]$
f	$[4, 4]$	$[6, 6]$	$[1, 16]$	$[6, 6]$
Pairwise Ranking Acc	0.0		0.5	

For X_{A_1} and X_{B_0}, i.e., we have two clicked items from group $\mathcal{A}$ and two un-clicked items from group $\mathcal{B}$, the Pairwise Ranking Accuracy under the composite function $f(x) = f_0(x) \cdot f_1(x)$ is 0.0, because both clicked items from $\mathcal{A}$ receive a lower prediction score ($[4, 4]$) than un-clicked items from $\mathcal{B}$ ($[6, 6]$). On the other hand, for X_{B_1} and X_{A_0}, i.e., when we have two clicked items from group $\mathcal{B}$ and two un-clicked items from group $\mathcal{A}$, the Pairwise Ranking Accuracy is much higher, 0.5. In other words, the predictor f does not have equal treatment for ranking the items from $\mathcal{A}$ and B.

4 Analytical Framework

As we can see in the theoretical results, it is not the case that improving the fairness of individual components never effects the fairness of the composite score, but rather it is dependent on the components and the relationship between them when we can expect compositional fairness to hold. Therefore we ask: if we have a multi-component system where we observe fairness issues, how much will improving the fairness for each component help the overall system's fairness?

Taking this data-driven view of the problem, we find there are multiple questions that we can tractably answer:

1. How much would "fixing" a particular component improve the combined system's fairness?
2. Given a system with a fairness issue, improving which components would yield the greatest benefit?
3. If all components were independently "fixed," what would be the resulting fairness metrics for the combined system?

We describe below our analytical framework for answering these questions.

4.1 Per-Component Fixes

First we consider what are realistic classes of methods for improving the fairness of a model? For example, while multiplying all model predictions by zero will result in good fairness metrics, it is also unrealistic in that it will destroy the usefulness of the system. Rather, we consider the two methods, which we believe are realistic as we explain below.

4.1.1 Distribution Matching

A significant amount of academic literature [20, 31] and publications on what is used practice [6, 5], takes the perspective of regularizing the model such that the distribution of predictions from each group (sometimes conditioned on the label) is matching. Under different formulations this has been done by comparing the covariance [42], correlation [6, 5], and maximum mean discrepancy [20, 32] between the distributions. Therefore, we consider whether matching the groups' distributions of predictions for each model has the desired effect on the combined fairness metrics.

As this is an analytical framework, in contrast to a training framework, we can easily do this offline by directly changing the predictions over our dataset. We consider distribution matching for a component f_k. In order to match the distributions, we sort all examples in each group by their scores f_k. We define by $\mathbf{a}^{(k)}$ a sorted vector of scores for examples in Group $\mathcal{A}$ and by $\phi_{a,k}$ the mapping of examples

to positions in this sorted list, i.e. $\mathbf{a}^{(k)}_{\phi_{a,k}(x)} = f_k(x)$ and $\mathbf{a}^{(k)}_j \leq \mathbf{a}^{(k)}_{j+1}$ for all j; we similarly define $\mathbf{b}^{(k)}$ and $\phi_{b,k}$ for examples from Group $\mathcal{B}$. For simplicity, we assume the number of examples from each group is equal, $|\mathcal{A}| = |\mathcal{B}|$. Therefore, when matching the distributions, we define the "fixed" component $\tilde{f}_k$ as follows:

$$
\tilde{f}_k(x) = \begin{cases} f_k(x) & \text{if } x \in \mathcal{A} \\ \mathbf{a}^{(k)}_{\phi_{b,k}(x)} & \text{if } x \in \mathcal{B} \end{cases} \tag{3}
$$

That is, for examples in $\mathcal{B}$, $\tilde{f}_k$ returns the score for the similarly ranked item from $\mathcal{A}$ such that the empirical distribution over $\mathcal{A}$ and $\mathcal{B}$ exactly matches. Note, $\mathbf{a}$ and $\mathbf{b}$ are the empirical cumulative distribution function (CDF) for f_k over $\mathcal{A}$ and $\mathcal{B}$ respectively, and as such if $|\mathcal{A}| \neq |\mathcal{B}|$ then simple interpolation to match the empirical CDFs can be used.

Theorem 2. *$\tilde{f}_k$ as defined by Eq. (3) has a exposure gap (defined by Eq. (1)) of zero, assuming the ranking order r based on $\tilde{f}_k(x)$ gives the exact same rank of x given the same $\tilde{f}_k(x)$.*

Proof. Given the definition in Eq. (1), it is easy to see that

$$
\begin{aligned}
\text{Gap}(\mathcal{A}, \mathcal{B}|r) &= \frac{|\text{Exposure}(\mathcal{A}|r) - \text{Exposure}(\mathcal{B}|r)|}{\text{Exposure}(\mathcal{A}|r) + \text{Exposure}(\mathcal{B}|r)} \\
&= \frac{|\sum_{x \in \mathcal{A}}[\text{rank}(x)]^{-w} - \sum_{x \in \mathcal{B}}[\text{rank}(x)]^{-w}|}{\text{Exposure}(\mathcal{A}|r) + \text{Exposure}(\mathcal{B}|r)} \\
&= 0.
\end{aligned}
$$

Because there is an exact one-to-one correspondence of $\tilde{f}_k(x_i), x_i \in \mathcal{A}$ and $\tilde{f}_k(x_j), x_j \in \mathcal{B}$ that gives exactly one pair of $\text{rank}(x_i) = \text{rank}(x_j)$, which cancels each other given $|\mathcal{A}| = |\mathcal{B}|$ and thus results in a zero gap. □

Note in real applications a tie-breaking strategy is still needed, and assume the tie-breaking strategy is random, then the above approach should achieve an exposure gap close to zero.

4.1.2 Label-Conditioned Distribution Matching

The above approach only recalibrates the predictions by group but does not necessarily align with any labels for the task. As such, for the pairwise fairness metric Eq. (2) the method as described is not guaranteed to give per-component pairwise fairness. For that, we provide a slight modification of the algorithm above. We consider $\mathcal{A}_0$ and $\mathcal{A}_1$ to be the set of examples in $\mathcal{A}$ with a negative and positive label, respectively; we similarly define $\mathcal{B}_0$ and $\mathcal{B}_1$.

We define a delta term between all pairs from $\mathcal{A}_1$ and $\mathcal{B}_0$ and similarly between all pairs from $\mathcal{B}_1$ and $\mathcal{A}_0$, i.e.,

$$
\begin{aligned}
\Delta_k(\mathcal{A}_1, \mathcal{B}_0) &= \{f_k(x_i) - f_k(x_j) \mid x_i \in \mathcal{A}_1, x_j \in \mathcal{B}_0\}; \\
\Delta_k(\mathcal{B}_1, \mathcal{A}_0) &= \{f_k(x_i) - f_k(x_j) \mid x_i \in \mathcal{B}_1, x_j \in \mathcal{A}_0\}.
\end{aligned}
$$

In the following we propose a method that exactly matches the empirical distribution between $\Delta_k(\mathcal{A}_1, \mathcal{B}_0)$ and $\Delta_k(\mathcal{B}_1, \mathcal{A}_0)$, which aligns with the regularization proposed in [5]. We also show that it suffices to match $\Delta_k(\mathcal{A}_1, \mathcal{B}_0)$ and $\Delta_k(\mathcal{B}_1, \mathcal{A}_0)$ to achieve fairness for each component k.

We define by $\mathbf{c}^{(k,\mathcal{A}_1,\mathcal{B}_0)}$ a sorted vector of scores in $\Delta_k(\mathcal{A}_1, \mathcal{B}_0)$, and by $\phi_{c,k,\mathcal{A}_1,\mathcal{B}_0}$ the mapping of examples to positions in this sorted list, i.e. $\mathbf{c}^{(k,\mathcal{A}_1,\mathcal{B}_0)}_{\phi_{c,k,\mathcal{A}_1,\mathcal{B}_0}(x)} = f_k(x)$ and $\mathbf{c}^{(k,\mathcal{A}_1,\mathcal{B}_0)}_j \leq \mathbf{c}^{(k,\mathcal{A}_1,\mathcal{B}_0)}_{j+1}$ for all j; we similarly define $\mathbf{d}^{(k,\mathcal{A}_0,\mathcal{B}_1)}$ and $\phi_{d,k,\mathcal{A}_0,\mathcal{B}_1}$ for examples from $\mathcal{A}_0, \mathcal{B}_1$.

We again for simplicity, assume the number of examples from each group is equal, i.e., $|\mathcal{A}_\ell| = |\mathcal{B}_\ell|, \ell = \{0,1\}$. Note the definition of the pairwise ranking accuracy implies that the number of examples from $\mathcal{A}_1$ and $\mathcal{B}_0$ is the same in order to form pairs, hence $|\mathcal{A}_1| = |\mathcal{B}_0|$, and similarly $|\mathcal{A}_0| = |\mathcal{B}_1|$. Given the above assumption we essentially have the same number of examples for all quadrants, i.e., $|\mathcal{A}_0| = |\mathcal{A}_1| = |\mathcal{B}_0| = |\mathcal{B}_1|$.

Therefore, to exactly match on the delta terms, we keep the scores for the deltas on one pair of the groups (e.g., $\Delta_k(\mathcal{A}_1, \mathcal{B}_0)$), and fix the scores on the other pair of groups (e.g., $\Delta_k(\mathcal{B}_1, \mathcal{A}_0)$, which can be achieved by either changing the scores for $\mathcal{B}_1$ or $\mathcal{A}_0$). For example, suppose we only fix the scores for $\mathcal{B}_1$, we define the "fixed" component $\hat{f}_k$ as follows:

$$
\begin{aligned}
&\langle \hat{f}_k(x_i), \hat{f}_k(x_j) \rangle \\
&= \begin{cases} \langle f_k(x_i), f_k(x_j) \rangle, x_i \in \mathcal{A}_1, x_j \in \mathcal{B}_0 \\ \langle [f_k(x_j) + \mathbf{c}^{(k,\mathcal{A}_1,\mathcal{B}_0)}_{\phi_{d,k,\mathcal{A}_0,\mathcal{B}_1}(x_i)}], f_k(x_j) \rangle, x_i \in \mathcal{B}_1, x_j \in \mathcal{A}_0 \end{cases}
\end{aligned} \tag{4}
$$

Theorem 3. $\hat{f}_k$ *as defined by Eq.* (4) *has a pairwise fairness gap, defined in Eq.* (2) *of 0.*

Proof. It is easy to see that the Pairwise_Ranking_Accuracy$(\mathcal{A} > \mathcal{B}|r)$ is given by

$$
\frac{\sum_{x_i \in \mathcal{A}_1, x_j \in \mathcal{B}_0} I[\hat{f}_k(x_i) > \hat{f}_k(x_j)]}{|\mathcal{A}_1| \cdot |\mathcal{B}_0|},
$$

i.e., it is equal to the percentage of positive deltas in $\hat{\Delta}_k(\mathcal{A}_1, \mathcal{B}_0) = \{\hat{f}_k(x_i) - \hat{f}_k(x_j) \mid x_i \in \mathcal{A}_1, x_j \in \mathcal{B}_0\}$ by definition. Similarly, the Pairwise_Ranking_Accuracy$(\mathcal{B} > \mathcal{A}|r)$ is

$$
\frac{\sum_{x_i \in \mathcal{B}_1, x_j \in \mathcal{A}_0} I[\hat{f}_k(x_i) > \hat{f}_k(x_j)]}{|\mathcal{B}_1| \cdot |\mathcal{A}_0|},
$$

and is equal to the percentage of positive deltas in $\hat{\Delta}_k(\mathcal{B}_1, \mathcal{A}_0) = \{\hat{f}_k(x_i) - \hat{f}_k(x_j) \mid x_i \in \mathcal{B}_1, x_j \in \mathcal{A}_0\}$.

Given we exactly matched the delta terms in $\Delta_k(\mathcal{A}_1, \mathcal{B}_0)$ and $\Delta_k(\mathcal{A}_1, \mathcal{B}_0)$ by Eq. (4), and since $|\mathcal{A}_0| = |\mathcal{A}_1| = |\mathcal{B}_0| = |\mathcal{B}_1|$, we have Pairwise_Acc$(\mathcal{A} > \mathcal{B}|r) =$ Pairwise_Acc$(\mathcal{B} > \mathcal{A}|r)$, i.e., the pairwise fairness gap, defined as in Eq. (2) is 0. $\qquad\square$

4.1.3 Distribution Normalization

While the above procedure is provably guaranteed to achieve per-component fairness, under the definitions given previously, in practice we would want to use a regularization on the model for this goal, which will be noisier. As such, we consider a lighter-weight approach: per-group normalization:

Definition 1. *Per-Group Normalization For groups $\mathcal{A}$ and $\mathcal{B}$, we modify component f_k to incorporate per-group normalization by:*

$$
\bar{f}_k(x) = \begin{cases} \frac{f_k(x) - \mu_{x \in \mathcal{A}}[f_k(x)]}{\sigma_{x \in \mathcal{A}}[f_k(x)]} & \text{for } x \in \mathcal{A}, \\ \frac{f_k(x) - \mu_{x \in \mathcal{B}}[f_k(x)]}{\sigma_{x \in \mathcal{B}}[f_k(x)]} & \text{for } x \in \mathcal{B}, \end{cases} \tag{5}
$$

where μ, σ is the empirical mean and standard deviation on $f_k(x)$, for $x \in \mathcal{A}$ or $x \in \mathcal{B}$, respectively. While $\bar{f}$ is not guaranteed to provide even per-component fairness under either definition, we find in practice it too can significantly improve end-to-end fairness.

4.2 Counterfactual Testing

How can we use the modified functions described above to understand the system's end-to-end fairness properties? All of the questions given at the beginning of this section are counterfactual questions: what would happen if we succeeded in fixing a component or set of components? With the above methods for simulating a fixed component (without actually changing the model training), we can do this headroom analysis.

Per-Component Effect As before, we assume we have K components which are multiplied together such that the overall score given to an example by the system is $f(x) = \prod_{k=0}^{K} f_k(x)$. Even when improving the fairness of one component, it is not guaranteed to improve the fairness of the overall system. For example, two components could be equally biased in opposite directions such that improving only one actually worsens the end-to-end fairness metrics.

Therefore, we use the above per-component modifications to test the effect of independently improving individual components. We will use g_κ to characterize a modified component as described above, i.e., $g_\kappa \in \{\tilde{f}_\kappa, \hat{f}_\kappa, \bar{f}_\kappa\}$. With this we can simulate how the system would behave if we improve a given component κ:

Definition 2 (κ-Improved System). *Given a system f with K components f_k, and a simulated improved component g_κ for component κ, we define the improved system as:*

$$f^{(\bar{\kappa})}(x) = g_\kappa(x) \prod_{k \neq \kappa} f_k(x) = f(x) \frac{g_\kappa(x)}{f_\kappa(x)}. \tag{6}$$

With this we can measure the fairness of system $f^{(\bar{\kappa})}$, using either Eq. 1 or Eq. 2, to understand this counterfactual – if we improved the fairness of component κ, what would be the resulting end-to-end fairness?

Given that we can now answer this counterfactual question concretely, we now ask: which components should I prioritize improving? First, we define the degree to which improving a given component helps the end-to-end fairness:

Definition 3 (Fairness Improvement). *Given a system f and a κ-improved system $f^{(\bar{\kappa})}$, we define the fairness improvement by*

$$\text{FI}_\kappa = Fairness(f^{(\bar{\kappa})}) - Fairness(f). \tag{7}$$

Finally, we can measure the fairness improvement FI_κ for all components κ and sort them in decreasing order to find the components for which an improvement would have the largest effect.

Overall System Effect   While the procedure described above is valuable for understanding which components are more important for improving the end-to-end system, they do not tell us how much improving each component independently will ultimately improve the overall system's fairness. For that, we build on the counterfactual testing above but now across all of the components. That is we define the per-component improved system as follows:

Definition 4 (All-Components Improved System). *Given a system f with K components f_k, and for each component we have a simulate improved version g_k, we define the improved system as:*

$$g(x) = \prod_{k=0}^{K} g_k(x). \tag{8}$$

Note as before we assume $g_k \in \{\tilde{f}_k, \hat{f}_k, \bar{f}_k\}$. Finally, with this, we can test how the improved system where each component is fair performs on the fairness metrics.

5   Experiments

We now use a combination of synthetic and real-world experiments to explore how well fairness composes in different settings.

5.1   Synthetic Data

We begin with presenting experiments on synthetic datasets to demonstrate the relationship between per-component fairness and compositional fairness. Again we assume the system has two components f_0 and f_1, and we evaluate the fairness metrics with respect to two groups $\mathcal{A}$ and $\mathcal{B}$.

Dataset with independent Gaussian distributions.   Assume

$$f_0(x) \sim \mathcal{N}(10, 0.5), x \in \mathcal{A}; \quad f_0(x) \sim \mathcal{N}(9, 0.5), x \in \mathcal{B}.$$
$$f_1(x) \sim \mathcal{N}(5, 0.5), x \in \mathcal{A}; \quad f_1(x) \sim \mathcal{N}(4, 0.1), x \in \mathcal{B}.$$

We draw 1000 examples from each group. Figure 1 (left) shows the distribution of this synthetic dataset, with x-axis representing the scores from component 0, $f_0(x)$ and y-axis representing component 1, $f_1(x)$. The two different colors show the two groups, respectively.

Fixed Component(s)	Group $\mathcal{A}$	Group $\mathcal{B}$	Overall Gap
None (baseline)	0.7640	0.2360	0.5281
Distribution Matching			
Component 1	0.7433	0.2567	0.4865
Component 2	0.6856	0.3144	0.3712
Both	0.4818	0.5182	-0.0365[1]
Distribution Normalization			
Component 1	0.5472	0.4528	0.0943
Component 2	0.5470	0.4530	0.0940
Both	0.4858	0.5142	-0.0285

Table 1: Effect on compositional fairness on Synthetic Dataset 1, with independent distributions.

Fixed Component(s)	Group $\mathcal{A}$	Group $\mathcal{B}$	Overall Gap
None (baseline)	0.7699	0.2301	0.5398
Distribution Matching			
Component 1	0.7602	0.2398	0.5205
Component 2	0.7318	0.2682	0.4636
Both	0.6262	0.3738	0.2524
Distribution Normalization			
Component 1	0.6156	0.3844	0.2312
Component 2	0.5765	0.4235	0.1529
Both	0.6950	0.3050	0.3899

Table 2: Effect on compositional fairness on Synthetic Dataset 2, with anti-correlated distributions between components.

Table 1 shows the fairness metric (in terms of the ranking exposure, as defined in Section 3.1.1, with a $w = 0.65$). We see that in the original data, Group $\mathcal{A}$ gets significantly more exposure ($> 50\%$ more). We start by applying the fix on each component by distribution matching (as defined by Eq. 3). From the table we see that fixing only one component has very limited effect on overall system's fairness, and the end-to-end fairness can only be achieved by fixing both components. Second, we apply the fix on each component by distribution normalization (as defined by Eq. 5). Compared to the distribution matching approach, this is much more effective in reducing the gap between the two groups while fixing only one component at a time.
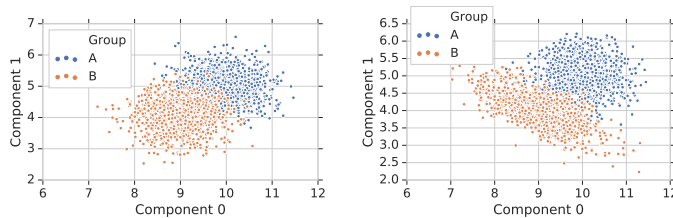

Figure 1: Data distribution for the two components on synthetic dataset 1 (left) and 2 (right).

Dataset with anti-correlated Gaussian distributions. In this experiment, we follow the exact same setting as the previous experiment except changing $f_0(x) = \mathcal{N}(13, 0.5) - f_1(x)$ for $x \in \mathcal{B}$ (we choose $\mu = 13$ of the first Gaussian such that $\mu[f_0(x)] = 9$, same as the first dataset) to create some anti-correlation between f_0 and f_1 for group $\mathcal{B}$. Again 1000 examples are sampled for each group. Figure 1 (right) shows the distribution of this synthetic dataset, compared to the first dataset we can clearly see this anti-correlation showing up as we have a very different shape for group $\mathcal{B}$.

Table 2 shows the fairness metrics, compared with Table 1, we can see that the anti-correlation makes the end-to-end fairness metric much harder to achieve. Figure 2 and 3 show the histogram of the final ranking scores by distribution matching (left), and distribution normalization (right), on synthetic

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[1]The gap cannot be exactly 0 from discretization effects at the top of the list.
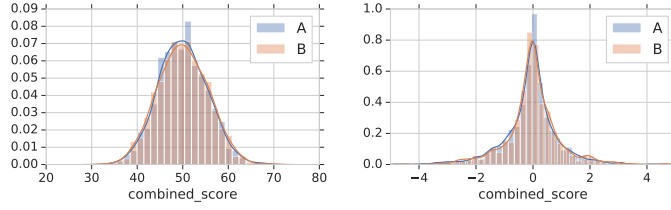
Figure 2: Histogram of the final ranking scores by distribution matching (left), and distribution normalization (right), on Synthetic Dataset 1.
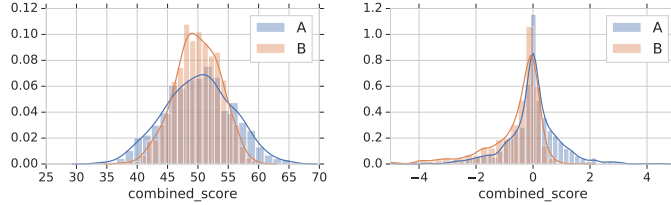

Figure 3: Histogram of the final ranking scores by distribution matching (left), and distribution normalization (right), on Synthetic Dataset 2.

data 1 and 2, respectively. We can also observe that the scores are matched much better when there is no anti-correlation between the component scores.

5.2 German Credit Data

In this section, we demonstrate our analytical framework on a public academic dataset: the German Credit data[2], as another example to illustrate the effect of score composition on the end-to-end fairness. This dataset provides a set of attributes for each person, including credit history, credit amount, installment rate, personal status, gender, age, etc., and the corresponding credit risk.

We assume the final score for assessing credit risk is composed by the following four attributes: 1) credit amount; 2) age; 3) Number of existing credits at this bank (denoted as "num_credits" in the following), 4) Number of people being liable to provide maintenance for (denoted as "num_liable"). We consider the problem of ranking all people in this dataset by the above score composition, and

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[2]https://archive.ics.uci.edu/ml/datasets/statlog+(german+credit+data)

Fixed Component(s)	Male Rep.	Female Rep.	Overall Gap
None (baseline)	0.6081	0.3919	0.2162
credit amount	0.5852	0.4148	0.1704
age	0.5865	0.4135	0.1731
num_credits	0.5986	0.4014	0.1972
num_liable	0.5953	0.4047	0.1907
credit amount & age	0.5652	0.4348	0.1304
credit amount & num_credits	0.5810	0.4190	0.1621
credit amount & age & num_credits	0.5572	0.4428	0.1145
credit amount & age & num_liable	0.5392	0.4608	0.0783
All components	0.5352	0.4648	0.0705

Table 3: Effect on end-to-end fairness by distribution matching for each component on the German Credit dataset.

we consider the end-to-end fairness metric to be the ranking exposure with respect to *gender*: male, female[3]. The fairness metric we consider is the ranking exposure, as defined in Section 3.1.1, again with a $w = 0.65$.

In the first setting, we assume the group size to be the same, i.e., $|\mathcal{A}| = |\mathcal{B}|$, which means the top N people within each gender group should receive the same ranking exposure (we restrict the larger group to be of the same size as the smaller group, N). In this case the ideal exposure gap should reach zero. In Table 3, we show the effect on the end-to-end fairness, in terms of the percentage of male and female representation in the end ranking, as well as the gap between them. The method we use for improving the system is *distribution matching*, as defined in Eq. 3, and we use the counterfactual testing (Section 4.2) to test the effect of fixing each component alone, and the effect of fixing different combinations of the components. For any combination with multiple components, we sampled some combinations and show the results in Table 3 to save space.

From Table 3 we can see that distribution matching for each component independently can help on the compositional fairness (Column "Overall Gap") to some extent. Fixing multiple components simultaneously better helps on the compositional fairness, and the overall gap is reduced most when all components are fixed. In addition, fixing different combination of the components help on the compositional fairness by different degrees, for example, fixing "credit amount" plus "age", and fixing "credit amount" plus "age" plus "num_liable", reduce the gap much further when compared to other 2/3-component fixes. The headroom analysis can provide us guidance on which components should be prioritized for improving end-to-end fairness.

Figure 4: Gap between gender groups with respect to each position, by single-component distribution normalization, on the German Credit Dataset.

In the second setting, we do not assume the same group size, and we rank $\mathcal{A}$ and $\mathcal{B}$ proportional to their respective sizes ($|\mathcal{A}| = 690$ for male, $|\mathcal{B}| = 310$ for female on this dataset). We vary the number of top positions t and plot the exposure gap metric with respect to the positions. As a reference, we also plot the exposure gap under random ordering (denoted as "Gap (random)" in the figures, by averaging over 100 runs), which ranks each person regardless of their gender. In Figure 4, we show the end-to-end fairness (in terms of exposure gap) by applying *distribution normalization* on each component. In Figure 5, we show the results by distribution normalization on multiple components simultaneously. The title of each sub-figure indicates the components that we have applied distribution normalization on. We can see that doing per-component fixes can help on the gap metric in most of the cases, and similar as the first setting, fixing different combinations of the components lead to improvements with various degrees on the end-to-end fairness metric.

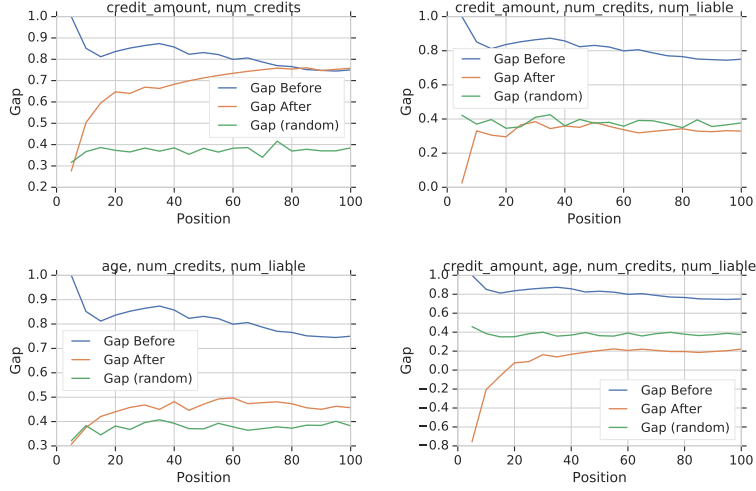[3]As that is how gender is categorized in the dataset.

Figure 5: Gap between gender groups with respect to each position, by multi-component distribution normalization, on the German Credit Dataset.

Fixed component(s)	Gap (CTR)	Gap (S1)	Gap (S2)	Group $\mathcal{A}$ Acc.	Group $\mathcal{B}$ Acc.	Overall Gap
None (baseline)	0.1024	0.1781	0.1078	0.5862	0.7408	0.1546
Matching on Marginal Distributions						
CTR	**0.0049**	0.1781	0.1078	0.6198	0.7084	0.0886
Satisfaction 1	0.1024	**0.0103**	0.1078	0.6292	0.7054	0.0762
Satisfaction 2	0.1024	0.1781	**0.0202**	0.6021	0.7270	0.1248
All	0.0049	0.0103	0.0202	0.6781	0.6546	0.0236
Matching on Conditional Distributions						
CTR	**0.0057**	0.1781	0.1078	0.6164	0.7048	0.0884
Satisfaction 1	0.1024	**0.0092**	0.1078	0.6258	0.7039	0.0781
Satisfaction 2	0.1024	0.1781	**0.0197**	0.6003	0.7258	0.1255
All	0.0057	0.0092	0.0197	0.6697	0.6472	0.0225
Matching on Delta Distributions						
CTR	**0.0000**	0.1781	0.1078	0.6473	0.7408	0.0935
Satisfaction 1	0.1024	**0.0000**	0.1078	0.6669	0.7408	0.0739
Satisfaction 2	0.1024	0.1781	**0.0000**	0.6197	0.7408	0.1211
All	0.0000	0.0000	0.0000	0.7630	0.7408	0.0222

Table 4: Effect on end-to-end fairness by distribution matching within each component, on a large-scale real-world recommender system.

5.3 Case Study on A Real Production System

In this section, we describe the results on a large-scale real-world recommender system. On an abstract level, the system mainly consists of three different components, one predicting the probability of click (denoted as "CTR"), and two other components predicting different signals of user satisfaction, denoted as "Satisfaction 1" and "Satisfaction 2".

In the following, we present results by fixing each individual component using *distribution matching* to

- Match the marginal distribution of $f_k(x), x \in \mathcal{A}$ and $f_k(x), x \in \mathcal{B}$, as in Eq. (3).

- Match the conditional distributions of X_{A_0} and X_{B_0}, as well as the conditional distributions of X_{A_1} and X_{B_1}, building on Eq. (3).

- Match the distribution on the delta terms: $\Delta_k(\mathcal{A}_1, \mathcal{B}_0)$ and $\Delta_k(\mathcal{B}_1, \mathcal{A}_0)$, as in Eq. (4).

Fixed Comp.	Group $\mathcal{A}$ Acc.	Group $\mathcal{B}$ Acc.	Overall Gap
None (baseline)	0.5862	0.7408	0.1546
$p = 0.51$			
CTR	0.5050	0.6767	0.1717
Satisfaction 1	0.6411	0.7508	0.1096
Satisfaction 2	0.6111	0.7590	0.1479
All	1.0000	1.0000	0.0000
$p = 0.9$			
CTR	0.9702	0.9891	0.0189
Satisfaction 1	0.9690	0.9882	0.0192
Satisfaction 2	0.9439	0.9787	0.0348
All	1.0000	1.0000	0.0000
$p = 0.99$			
CTR	0.9993	0.9996	0.0003
Satisfaction 1	0.9991	0.9994	0.0003
Satisfaction 2	0.9964	0.9990	0.0026
All	1.0000	1.0000	0.0000

Table 5: Effect on end-to-end fairness by setting $(p, 1 - p)$ values on the (clicked, unclicked) pairs, on a large-scale real-world recommender system.

- As a reference, to test the fairness on the extreme end, we set a constant value $(p, 1 - p)$ for all the (clicked, un-clicked) pairs from each component. This experiment is to explore what other conditions might help end-to-end fairness when we have per-component fairness.

The results are shown in Table 4, the first column denotes the "fixed" component(s), and column 2-4 show the Pairwise Ranking Gap for each component (abbreviated to "CTR", "S1", "S2"), respectively. Column 5-7 show the overall (compositional) Pairwise Ranking Accuracy (as defined in Section 3.2) for Group $\mathcal{A}$ and $\mathcal{B}$, as well as the overall (compositional) Pairwise Ranking Gap. From the table there are a few interesting observations:

- Compared to matching on marginal/conditional distributions, matching on the delta distributions is the only method that achieves zero gap on the per-component gap metric (Column "Gap(CTR), Gap(S1), Gap(S2)" in Table 4). This is consistent with our theory (Theorem 3).

- Although marginal/conditional distribution matching does not provably ensure per-component fairness, empirically they still lead to a good amount of gap reduction (all close to zero), and effectively help on the compositional fairness (Column "Overall Gap" in Table 4).

- Compositional fairness is better achieved when all the components are fixed, and fixing per-component alone helps to different extents on the compositional fairness. On this dataset, fixing "CTR" or "Satisfaction 1" has a larger effect on reducing the overall gap, while fixing "Satisfaction 2" has a relatively smaller effect.

In addition, for the set of experiments that set $(p, 1 - p)$ values on the (clicked, un-clicked) pairs, Table 5 shows the effect on the overall (compositional) gap metric. Note any $p > 0.5$ achieves a zero per-component pairwise ranking gap since all pairs are ordered correctly, but the overall compositional fairness varies with different values of p. For example, by setting $p = 0.51, p = 0.9$, and $p = 0.99$ for any single component, the pairwise ranking accuracy for both group $\mathcal{A}$ and $\mathcal{B}$ is 1.0 and the pairwise ranking gap is 0.0 for that single component, but choosing a larger p (e.g., $p = 0.9$ or $p = 0.99$), clearly helps the end-to-end fairness ("Overall Gap" in Table 5) much better. Again, this suggests an interesting interplay between the prediction values, beyond the component's fairness, and the effect on the overall system's fairness.

6 Conclusion

In this paper, we study the problem of compositional fairness in ranking, i.e, given a multi-component system, where the end ranking score is the product of scores from each component, does making each

component fair independently improve the system's end-to-end fairness? We formalize this problem in two recently proposed fairness metrics for ranking, *fairness of exposure*, and *pairwise ranking accuracy gap*, and present examples where compositional fairness might not hold, aligned with prior work [16].

While these lack of guarantees can be disheartening, we also present theory showing conditions under which we *can* achieve end-to-end fairness from achieving per-component fairness. Because the theory shows that composition is distribution-dependent, we propose taking a data-driven approach to this problem. We offer an analytical framework for diagnosing which components are most damaging end-to-end fairness and measuring how much improving per-component fairness will improve end-to-end fairness. By applying our analytical framework to multiple datasets, including a large real-world recommender system, we are able to identify the signals that are lowering the end-to-end fairness the most and observe that in practice most of the end-to-end exposure or accuracy gaps can be addressed through applying independently per-component improvements!

As most real-world ML systems are composed of many models and tasks, understanding how and when fairness composes is crucially important to enabling the application of fairness principles in practice. Our results highlight that while guarantees don't hold in the worst-case, there is more nuance over realistic data distributions. As a result, we believe there is a lot of potential in generalizing both the theory and empirical frameworks to different applications, covering different data distributions, compositional functional forms, and fairness metrics.

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