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Formanten basierte Stimmenqualitätstransformation

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(Signature of Author) (Signature of Advisor)

FAKULTÄT FÜR INFORMATIK
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Formants based voice quality transformation

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to the Faculty of Informatics
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From beginning to end, sound collection took about two months. Some samples were rerecorded in order to get the correct pitches. A very warm thank you for the participation of those students. Professor Li and Professor Ying, who come from Xiamen University also helped us very much. The rating of all of the voice samples as well as the interview lasted about two days. These professors repeatedly listened to the 64 sounds patiently and also discussed the score with us.

Abstract

Vocal music, as an art of performing, which has been evaluated more and more subjectively for a long time, is mainly taught orally. But with the rapid development of modern science, music—as a vital component of culture - can no longer satisfy people at a superficial and emotional level. People are pursuing the essential understanding and knowledge of the musical phenomenon. Voice visualization involves using acoustics to measure quantitatively the quality of voice. By providing the most objective and directly measured data, the visualization of the abstract vocal performance can be realized. This provides an alternative method for the education of vocal music. Therefore, a mathematical model, which is based on the physical parameters, such as formants, pitch, voice density and can be used to quantify the voice quality into a normal score system is desired.

Contents

CHAPTER 1

Introduction

In the 16th century, the Italian composer, teacher, singer, instrumentalist and writer Giulio Romolo Caccini published his most influential work "Le nuove musiche"[1], on which he introduced an interpretation for his understanding of music as well as the introduction of music theory. It is considered the first published written collection of understanding of musical style and the music of the common vocal practice in Europe. In 1744 Giovannibasttista Manicini was an Italian soprano castrato, voice teacher. He was also the author of the book "*Practical Reflections on the Figurative Art of Singing*" [2]. He distinguished in his article the voice with four types: Tenor, Bass, Soprano and Contra alto and discussed the interaction of the intonation and diapason. This norm is still observed by people who learn vocal music.

The theory from Giovannibasttisata provided an import effect on the education of vocal performance and vocal music. It stands the test of time in the past centuries. After several centuries people still tried hard to find the essence of the vocal music and the channel for its education.

With the quickly developing technology, modern technology has been able to use the spectrum analyser to record the voice and analyse it using visualization methods. With the help of those techniques, the "singing formant" was discovered, which is produced by resonance. It is one of the most important characteristics to judge the voice's quality. For the vocal music students, a right location of his or her singing voice usually means that the upper formants are located at frequencies from about 2000 Hz to 4000 Hz. Moreover, the distance between the upper formants and also their energy have an impact on the quality levels.

With the usage of those characteristics, a transformation based on the upper formants or in other words "singing formant" is desired for evaluation of voice quality into the normal score system. It provides an intuitive feeling and enables the students to appraise the voice in a professional way, even without experienced teachers' help.

In the field of vocal music teaching, few teachers explain the level of the sound quality based on the concept of physics, but the inevitable connection between them is what we cannot ignore. From the point of view of physics, sound is composed of a fundamental tone and overtones. The human vocal tract is able to produce highly variable amplitudes of the overtones called formants. The fundamental tone determines the pitch of the voice while the overtone determines the timbre and quality of voice. Professor Johan Sunberg, whose researches will be introduced in later sections, clearly pointed out in his research report, although the voice quality may be affected by other factors, vocal tract resonance, also called formant, is the decisive factor to judge the quality of the singing-voice [11]. Thus it can be seen from the concept of the formant that we can use the physical model to study the nature of the sound.

Back to the beginning of our research purposes, we hope to find a mathematical model to explain sound quality based on formants to replace the common teaching method, which depends on the teacher's subjective judgment of the quality of a voice. This way vocal music students do not have to rely on teachers' evaluations but on the formant parameter analysis and the investigated mathematical model in order to judge for themselves.

So for an outstanding voice, how can we use the physical concept of singing formant to judge it? What is the distribution of these high frequency formants in the spectrum? If we use the 0-100 scoring system to evaluate the quality of sound, is there a certain relationship between the distribution of these formants and the scores.

CHAPTER 2

Theoretical Framework

2.1 Basis terminology of tone

2.1.1 Fundamental tone and overtone

Generally, the sound is made of a series of varying amplitude and frequency, which are emitted by the sounding-object. The lowest frequency of variation among those is called the fundamental tone, the other is called the overtone. The fundamental tone is generated by the whole body vibrating. It decides the pitch while the overtone is the sound produced by part of the body's vibration. It determines the timbre.

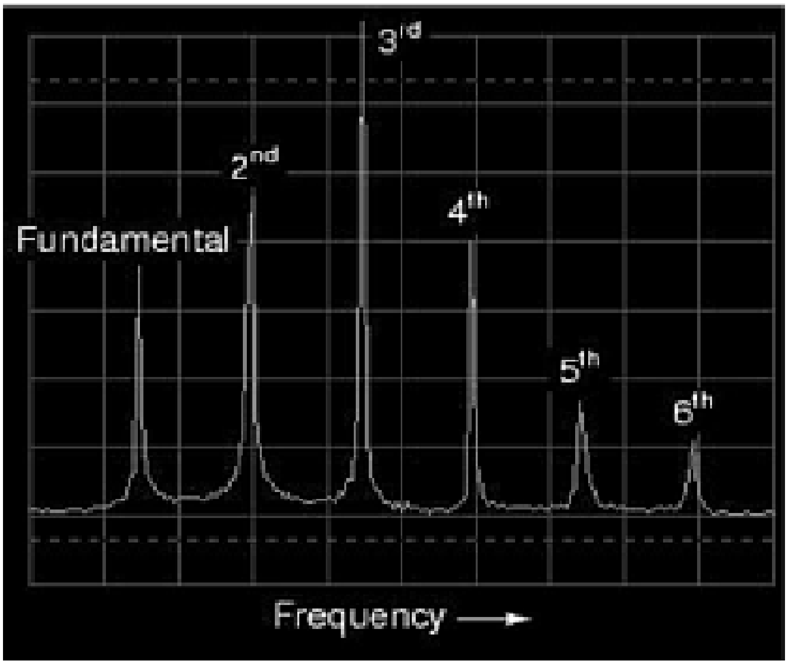

Figure 1: Spectrum of a trumpet tone.(from [14])

As the figure 1 presents above, a trumpet tone is separated into fundamental tone and the first to the fifth harmonics (which can be represented also as overtones but in another measurement). The sound which combines both the fundamental tone and the overtone is a composite tone, the sound of daily life is a composite tone.

2.1.2 Formants

Another important factor to measure an acoustic resonance of the human vocal tract are various definitions of the formant. Gunnar Fant, who is a speech researcher defines formant as “the spectral peaks of the sound spectrum", while the Acoustical Society of America defines a formant as "a range of frequencies of a complex sound in which there is an absolute or relative maximum in the sound spectrum". With the help of the visualization of the formant through a spectrum we can basically recognise the formant as the peaks which appear in the sound spectrum, an example may be like the figure 2 or the regions which are seen to have the deepest colour in the time-frequency diagram.

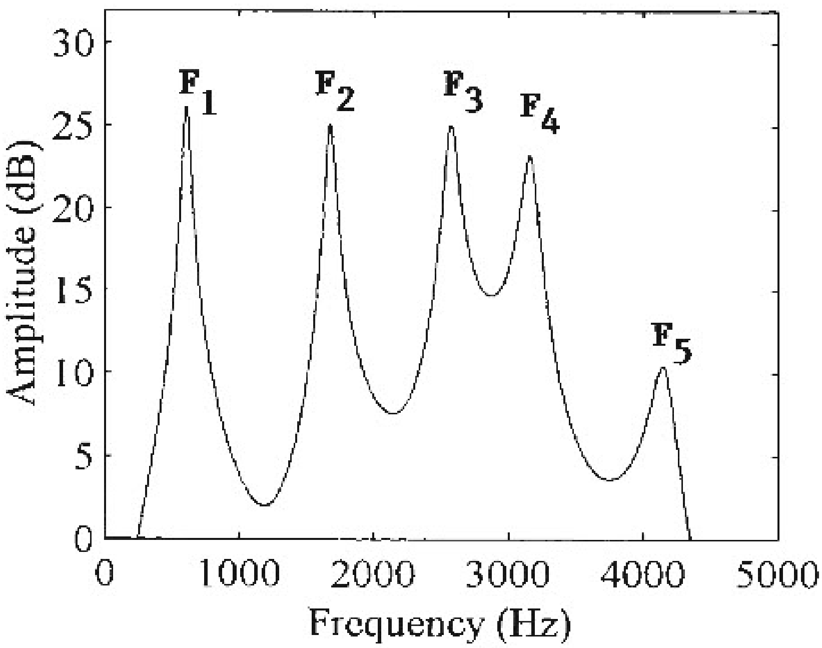

Figure 2: Impulse spectrum of the vocal tract (magnitude only)(from 15)

Further the formant can be classified from 1 to 5 degrees, which are different from the power (amplitude) and the level of its frequency. Figure 3 shows us with a time-frequency diagram the formants F1 and F2 for the English vowels ‘i’, ‘u’, and ‘a’. For the vowel ‘u’ the deepest colour appears at about 250 Hz, that is to say, the formant F1 for ‘u’ is 250 Hz.

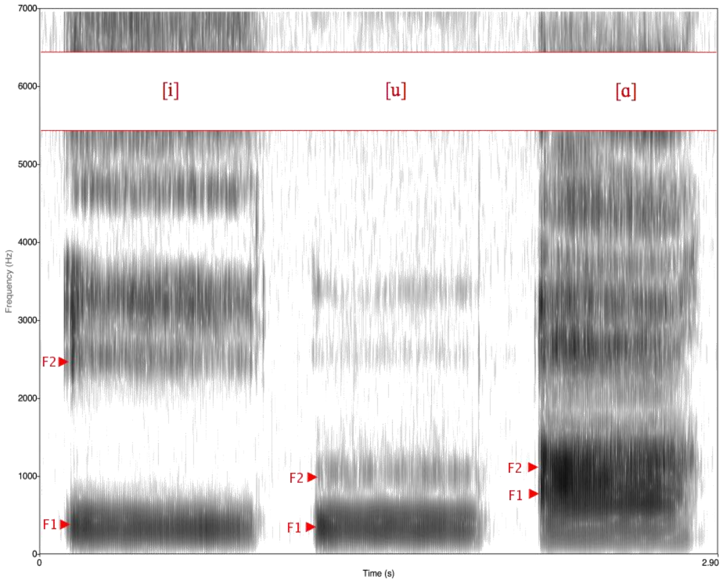

Figure 3:Spectrogram of American English vowels [i, u, a] showing the formants F_1 and F_2 (from https://en.wikipedia.org/wiki/Formant)

2.2 Researches about the formants

As the formant plays a vital role in the human vocal tract, there are various studies on the area frequency spectrum of singers. Researchers discovered that trained singers intend to present a clear formant between 2800 and 3400 Hz that is absent in speech or in the spectra of untrained singers. These formants are called singing formants. So the question is if the singing formant has great impact on the quality of the vocal and timbre. Can we judge the tone base on it? More accurately, can we transfer abstract concepts such like quality of voice and timbre in the spectrum-analyser of the singing formant, which is realized through modern visualization technology?

As we mentioned about singing formant, there is a person, who should never be ignored—Johann Sundberg, who began his comprehensive research on the vocalization phenomenon in 1981. He used analytical methods of frequency spectrum to describe resonator organs for the voice.

2.2.1 A perceptual function of the "singing formant"

On this research, which was published in 1972 by J. Sunberg, the perceptual function of the singing formant was discovered [3]. He wondered why the singer's sound is even audible when sounding together with a loud orchestral accompaniment. He explored this phenomenon by answering the two questions. First, what is the spectral nature of the orchestral sound, and second, what are the masking effects of singer's sound?

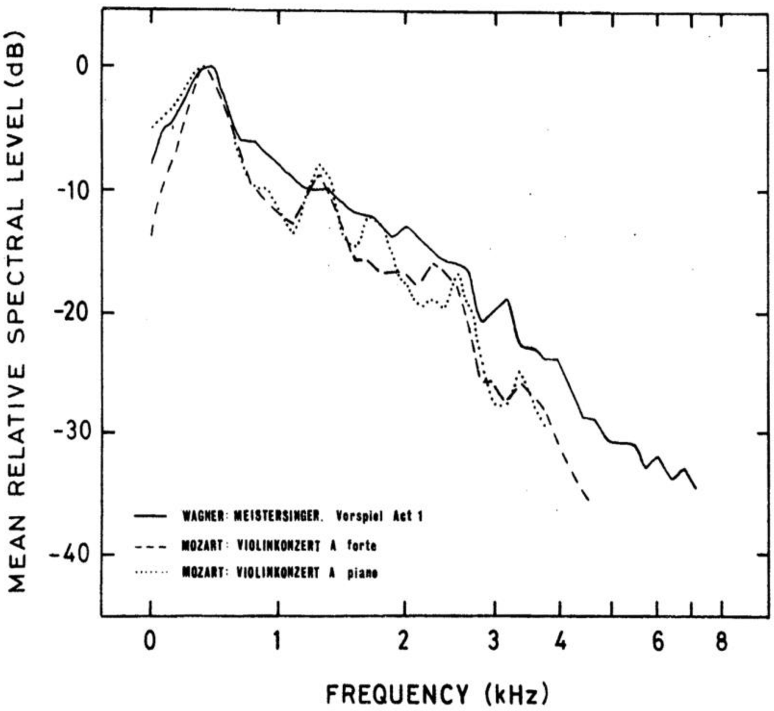

Figure 4: Average spectra of various types of gram-

mophone recorded orchestra music. (from [3])

First question was answered by a simulation with a spectrum-analyzer as a visualization method for given various types of gramophone recorded music. The result is shown below Fig. 4. He concluded that the orchestra provides a peak at 450 Hz and an average slope of about 9 dB/octave above this frequency.

As the next step he compared average spectrum differences between normal speech and orchestral music with and without a solo singer.

Obviously in the figure 5 there is a peak in the graph along the line which was displayed as the orchestra with solo singer. The singing formant is therefore recognized as this only point in which the orchestra with and without singing differ substantially. At 3 kHz the sound level of the orchestra dropped about 25 dB, speech dropped about 15 dB or more, but orchestra with singer dropped only about 6 dB. It explains with a more scientific point of view, why an opera singer on the stage without a microphone is able to overpower the whole orchestra and transmit his voice to the audiences' ears. The energy of singing formant is evident

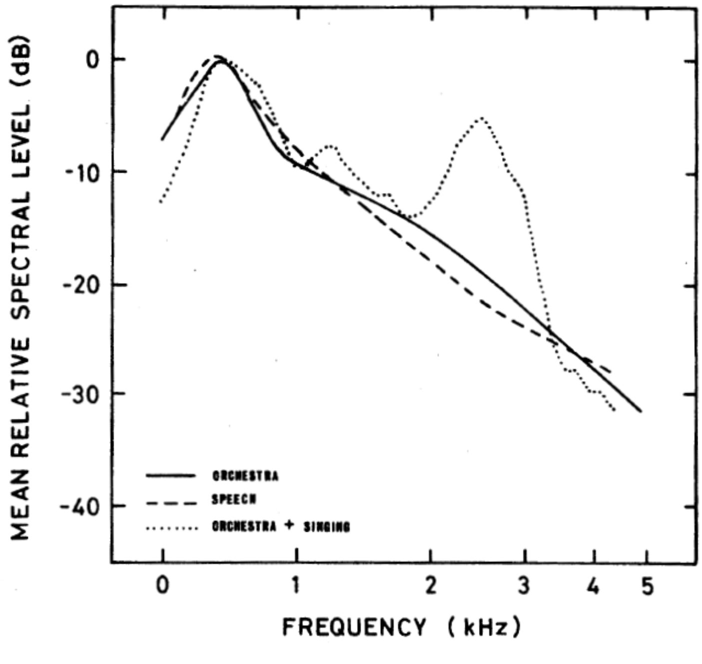

Figure 5: Idealized average spectra of normal speech and orchestra music.

The dotted curve shows the average spectra of Jussi Björling singing with a loud orchestra accompaniment. (from [3])

2.2.2 Observations of the higher formant structure in the male operatic vowel

Some of the research from J. Sunberg has shown that “singing formant” can be interpreted as a formant cluster consisting of the third, fourth and fifth formants. Those formants decide the timbre, formant 1 and 2 on the contrary determine the pitch. We now concentrate on the higher formant structure and attempt to find how it effects the vocal timbre.

The experiment from Thomas J. Millhouse in 2012 indicated that operatic singers maintain an even vocal timbre throughout their vocal range [4].He invited six male singers who are shown in table 1. All of them were professional operatic artists. 4.1 means that the subject is a professional national opera singer and 3.1 indicates a regional opera singer. Sub classifications “a”, “b”, and “c” differentiate between Major Principal (a), Minor Principal (b) and Chorus Artist (c). The participants were asked to both sing and speak the given pitch, which was based on the A major scale. The formants F3, F4, and F5 of all subjects were recorded both for singing and speaking.

The visualization method chosen here is a diagram with help of the frequency-spectrum as analyser. In the converted diagram pitch was represented on the x-axis und frequency on the y-axis. The comparison between speaking and singing was visualized as a black and red line respectively.

Singer	Age (years)	Voice Type	Taxonomy
AL	27	Tenor	4.1c
JP	26	Tenor	4.1b
EE	47	Tenor	3.1b / 4.1a
TM	27	Baritone	4.1b
DG	33	Bass	4.1c
PA	28	Bass	3.1b / 4.1a

Table 1: Taxonomic classification of participants (from [4])

The result in figure 6 indicated that in most participants the upper formants fall and cluster in the 2.4-3.2 kHz region. Notably for the tenors, AL and JP, F3 rises in the centre frequency across each vowel. However, both F4 and F5 for these two participants fell. In other words, the falling and clustering of the opera singers' vowels upper formants structure are indicated. This discovery also explains why the "singing formant" from the above research of J. Sundberg is located at about 2 kHz to 3kHz.

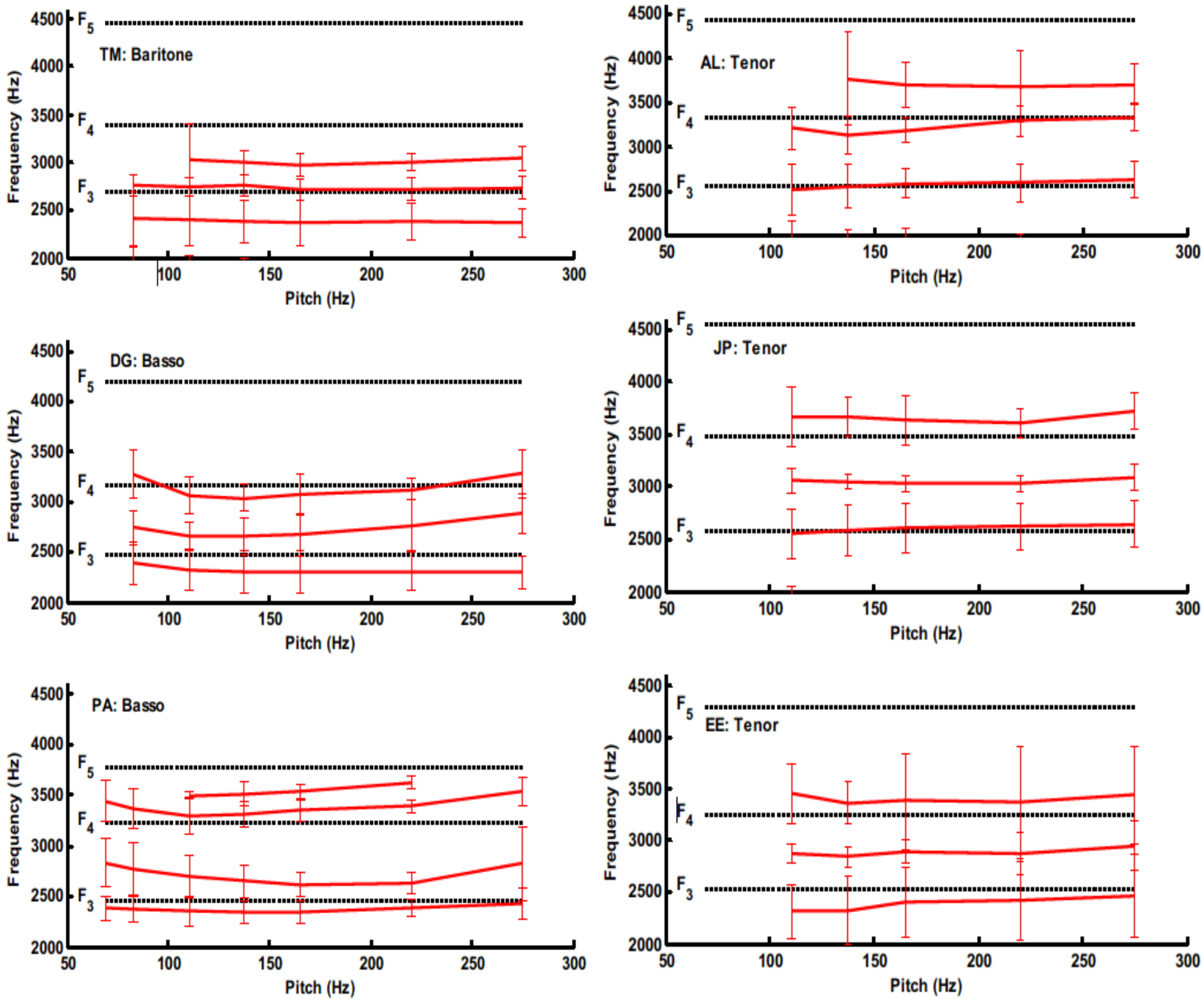

Figure 6: Centre Frequencies and CI (95%) of the upper formant structure for spoken and sung vowels for all participants on a pitch by pitch basis averaged for all vowels. Black dotted lines indicate the spoken F3, F4 and F5 whilst sung formants are represented by straight lines and the error bars provide 95% confidence intervals for the data analysed. (from [4])

2.2.3 The level of the ‘singing formant’ and the source spectra of professional bass singers

After discussing the existence of singing formant and the lowering and clustering effect of third, fourth, and fifth formants for singers, professor Sunberg J. wanted to illustrate the relationship between the first formant and singing formant as well as loudness, sound pressure level and pitch [7].

Two main conceptions should be declared here:

1. The sound pressure level of a vowel is mainly determined by the amplitude of the first formant.

2. The level of the singing formant depends on two factors, namely, the density of the third, fourth and the fifth formants, i.e., how close they are in frequency, and the other is the amount of sound energy generated by the voice at this frequency.

For this experiment two professional bass singers B1 and B4 were involved. B1 had a dark voice that ranged from C2 to F4, and B4 had a light voice that ranged from F2 to G4. The two subjects sang three sustained vowels ‘u’, ‘i’, and ‘a’ at four levels of loudness (piano ‘p’, mezzopiano ‘mp’, mezzoforte ‘mf’, and forte ‘f’) and at four pitches about equally spaced within their respective pitch ranges.

In order to visualise the measured data a spectrum analysis was performed by a Rodhe & Schwarz FNA spectrograph. With the help of a calibrated microphone about 17 cm in front of the mouth the voice of both candidates were recorded in an anechoic room.The main purpose here was to determine how the relative and absolute levels of “singing formant” vary when pitch and degree of loudness change.

Figure 7 shows that the sound pressure level is pitch-dependent also in singing. The higher the pitch, the higher the SPL. The rate of increase is as rapid as about 9 dB/ octave*. The result also shows that the singers use the same source strength for the different vowels on the same pitch.

* This is related to Equal Loudness Gurves (ref. Phon Scale) [Barkhausen, 1925]

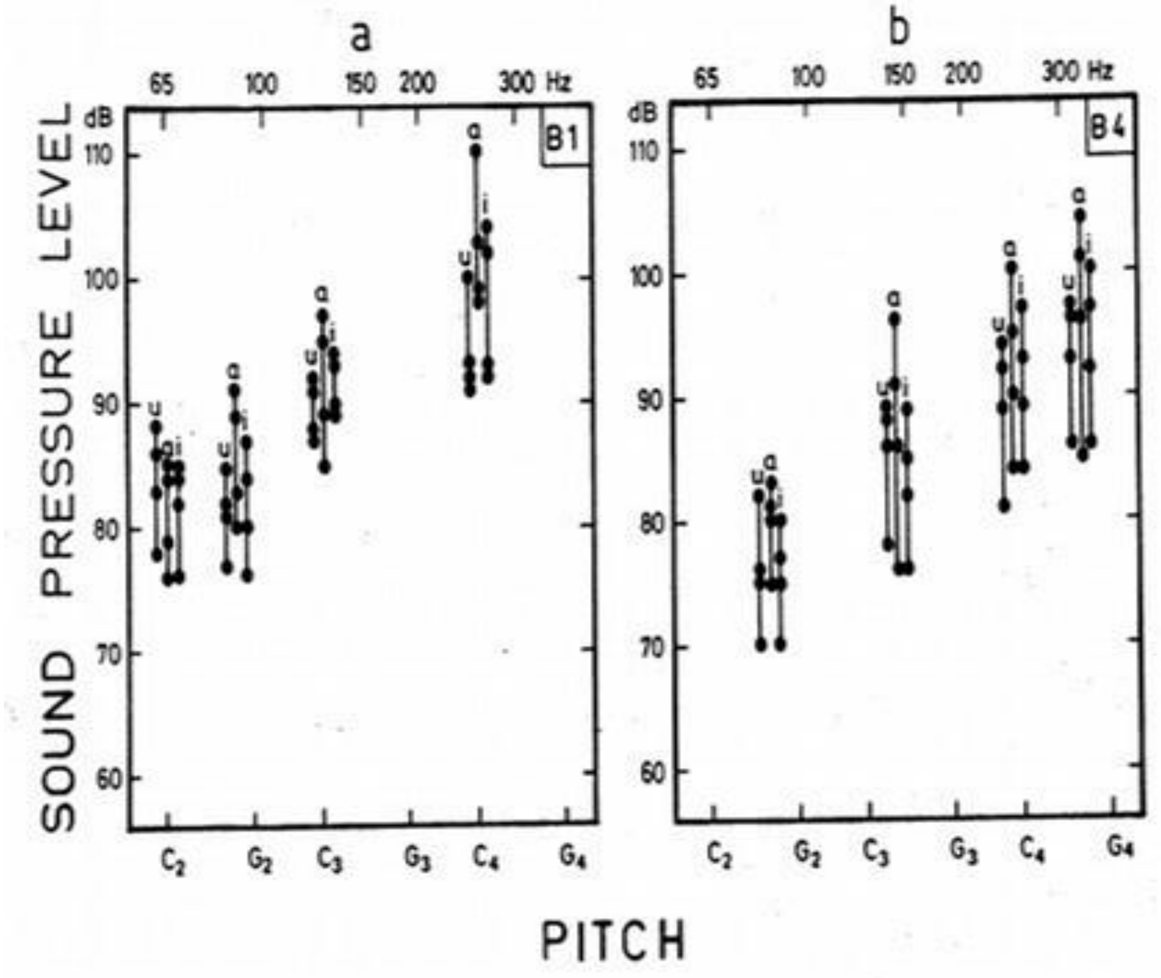

Figure 7: Sound pressure level measured in four degrees of vocal intensity (p, mp, mf, f) and different pitch, a: dark voice, b: light voice (from [7])

Another result is demonstrated in Fig. 8, the figure showing the mean of relative level difference between the singing formant and the first formant in the three vowels sung in four pitches. The relative level of the singing formant (level difference between the singing formant and the first formant) rises as the degree of vocal intensity becomes stronger. The relative level of the singing formant concerning the difference between the degrees of loudness gradually decreases as loudness grows. The relative level of the singing formant is on the average higher for the light voice than for the dark one.

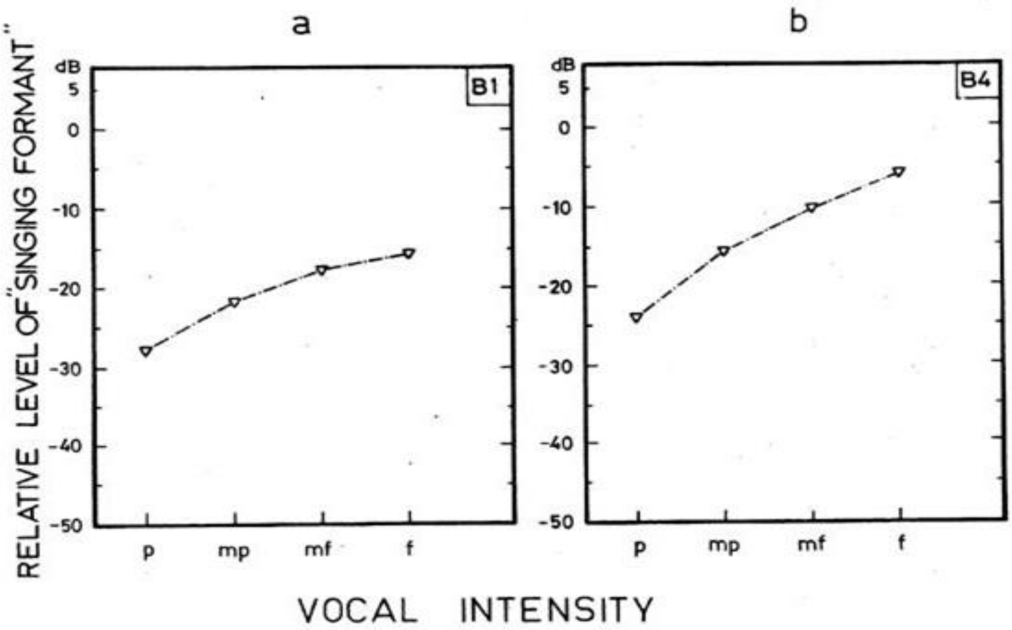

Figure 8: Mean level difference between the "singing formant" and the first formant in three vowels sung in four pitches. (from [7])

2.2.4 Summary of the research

According to the research, which was discussed above, we found what is so special about singers. The most essential reason is the existence of the "singing formant" [8]. We also discovered that the appearance of "singing formant" is actually the clustering and falling effect of upper formants F3, F4, and F5, which was clearly displayed through the formant visualization with plots. Another crucial factor is the location of the "singing formant" in the plot. It should be located at about 2500 Hz to 4500 Hz.

The second research above also shows that among the tenor group, the major principal singer presents a smaller range of the "singing formant" than the minor principal singer, the chorus artist presents a bigger range than the minor principal singer. With no doubt, the solo singer should be better trained than the chorus artist, even though they are all in the nation level. That is to say, the closer the third, fourth and fifth formant get to each other, the better skill processed by the singer and also the greater the quality of the singer's voice. As we can see in Thomas's research [4] the given evidences were not enough to prove this assumption. In our latter experiment we will investigate for this question through the given subjective score from professional vocal music teachers and compare the score with the range of the "singing formant" to find if there is a relationship between these two factors.

With the experiment from Sundberg J "The level of the 'singing formant' and the source spectra of professional bass singers" [7], he discovered a positive linear relationship between the pitch and sound pressure level, or better said, the bigger the vocal intensity the closer the first formant gets to the "singing formant". The difference between a dark voice and light voice are also found in his research.

Many other studies also show this. F1 and F2 are mostly located in the low frequency range (0-500 Hz) of the frequency-spectrum [5] while the upper formant is about 2kHz to 3kHz for better singers. The voices with F1 and F2 are powerful and plump and those with F3, F4, and F5 sound bright, shiny and penetrating [6].

2.3 Interview with two vocal music professors

In order to clarify the purposes of our experiment and the voice parameters to be measured during the experiment, we seek the help from two experts in vocal music field Zhaodan Li, director of the Department of vocal music at the Xiamen University, and Hua Ying, professor in the field of the vocal music in Xiamen University. We made a brief summary of the interview as follows.

The two professors said there are many factors that need to be considered in order to judge voice quality, such as resonance effect, timbre, singer's pronunciation, breathing and so on. But the two main criterion to judge voice quality is timbre and acoustic resonance. In vocal music training the most important and also the most difficult part is precisely learning of various resonance skills, namely, the cavity resonance and nasal resonance. Students need years or even decades to learn these skills. It is mainly because of the influence of individual differences that there is no fixed method for the resonance technique. Due to differences in the structure of the various organs in the body, students must try to find a most accurate resonance position, which is suitable for them and can make their voice most penetrating. Teachers have to teach students in accordance with their strengths in order to help them find the right position. The brilliant resonance can also positively affect the timbre.

When we asked why resonance is so important in vocal music teaching, professors thought that wonderful songs cannot do without resonance. The fundamental tone which is produced by the vibration of the vocal cords as a sound source is so faint that it cannot be heard. The contribution of the resonance is greater than that of the breathing, vocal cord size and sound power. Resonance expands the sound and makes the voice mellow and calm. This is a physical phenomenon. Bel Canto is a product of the Steam Engine Era. In order to meet volume requirements, vocalists attempt to utilize the human megaphone as much as possible—resonance. This resulted in various traditional vocal skills. Pop singing is a product of the age of electricity. Singers used the power amplifier—the microphone. It is the consequence of diverse methods of resonance and different demands. In vocal pedagogy the most essential thing is to explore resonance. Bel Canto requires a broad vocal range, unified sound, smooth breathing and a coherent voice. All those requirements need to make full use of resonance with the help of the body's organs. Resonance often vivifies the voice with volume and quality, and strengthens its loudness. Relying on resonance technology is more effective than any other vocal technique.

Body's resonance chambers are mostly without fixed volume and having an appreciable adaptable. The artificially adjusted resonance chambers are larynx, pharynx (hypopharynx, oropharynx, nasopharynx), chest, mouth and nasal cavity.

Professor Li described for us briefly how people train sinus resonance. He said "sinus resonance including the forehead (above the brow), maxillary sinus (on both sides of the nose), sphenoid sinus (temples), and ethmoid sinus (two inside corners of the eyes), together form four pairs of resonance. These are the places to which the nasopharyngeal cavity resonance can spread. We need to strive to open them, so that they can sufficiently vibrate under the influence of breathing. To keep the nose cavity empty and nasopharynx cavity wide is important. It makes the voice smoothly pass through the mask or head chamber. Keep in mind where to sing. It is necessary to sing vowels clearly and find the core point of resonance otherwise the resonance is inevitable cloudy."

Professor Li indicated that he had not heard of "formant", a physical concept, but he guessed formant was probably the resonance, which he mentioned several time already. This assumption is correct. Singing resonance is the result of the vibration generated by other organs of the body, and it is driven by vocal fold vibration. This vibration produces the overtones. They can be represented in the spectrum with the help of the visualization of the upper formants also called the singing-formant.

CHAPTER 3

Experiment

3.1 Introduction to the experiments

According to the previous studies, we know that the first two formants determine the quality of vowels, more accurately, they allow us to distinguish between different vowels [10]. Third, fourth and fifth formants are singing formants that decide the resonance level and then determine the timbre and quality of sound, whether or not the sound is penetrating and moving. That is to say, it is the high frequency formants that decide the voice quality.

Until now, the 'quality' from all formant based researches above is a general concept. They give us a common standard of voice judgement. With the help of them, we can identify if the voice generally reached at the requirements. If the singer does resonate, the "singing formant" can be caught in the frequency spectrum. If the resonance is made in the right position or to say in the right frequency-range, the "singing formant" should be located at about 2500-3500Hz. Yet still, we cannot compare the quality of two different voices and we are not able to quantify the voice quality. Therefore, our research purpose is to discovery the relationship between the upper-formants' structure and the subjective voice quality judgement given by the vocal music professor. Certainly the score given by the professor is the best data to quantify the abstract voice quality.

In our following experiments, we will first discuss whether the individual distribution of upper-formants differs in pitch and loudness. Secondly, we will mainly investigate the relationship between the distribution of upper-formants and the subjective ratings of the vocal music teachers, then try to fit the potential relationship with a mathematical function model.

3.2 Participants and collection of voice sample

The samples involve four records from master students. The abbreviations M1, M2, M3 and M4 will be used for the later figures and descriptions. M1 and M2 are two master student, who just start their study this year. M3 and M4 are sophisticated and better skilled, for they will soon finish their master study. Also, there are four records from bachelor students, abbreviated as B1, B2, B3 and B4. They all study now at the Musik und Kunst Privatuniversität der Stadt Wien. The voices are recorded by the recording-pen OLYMPUS LS-14 linear PCM Recorder.

All participants were asked to sing eight given pitches, which are based on Middle C and have a frequency from 130 Hz to 263 Hz. (For each pitch an error from +5 Hz to -5 Hz is allowed.) The variables below were be recorded:

- Formant3 (Hz), Formant4 (Hz), Formant5 (Hz)
- Upper formants' distance (Hz) between F3 and F5
- Total amplitude (dB)
- Pitch ($\pm$5Hz)

The collected voice samples' parameters were listed in the Appendix A. The abbreviation 'X' presents the corresponding name abbreviation of all students, 'F3', 'F4' and 'F5' are third formant, forth formant and fifth formant. 'D' is the frequency range between third formant an fifth formant. 'P' is the frequency of the pitch, 'E' shows the loudness of the voice, 'S' is set for the score, which was given by the vocal music teacher.

Professor Zhaodan Li and Hua Ying, who are both vocal music teachers from Xiamen University (China) judged those recorded voice samples. All recordings were played for two professors at same time. They discussed the score for each pitch and gave the final score decision together for all samples. The score has been given from 0 to 100 points based on their subjective evaluation of voice quality or to be exact the resonance effect of sound. (See Appendix A, column S)

3.3 Software analysis platform and data analysis algorithm

3.3.1 Praat

Praat is a professional computer software package for voice analysis. It has many powerful (and amazing) functionalities such as acquisition, analysis and annotation of signals, synthesizing voice and sound, statistical analysis of voice data and assistance with the teaching test. 3-dimensional graph, spectrum graph and formant curve provide us with the intuitive feeling of the voice. It also supports the voice experiment.

In our experiment it will be used to determine the exact frequencies of the upper formants, the pitches and the intensities. Figure 9 shows us a spectrogram of a voice sample. Each pitch was carefully cut from the spectrogram and analysed.

By selecting from the function "Formant" in the menu list, we can easily get the formants' frequencies from formant three to formant five.

Pitch gives us the possibility to get the pitch's frequencies. Formants' energy analysis is acquirable through selection of intensity in the menu list.

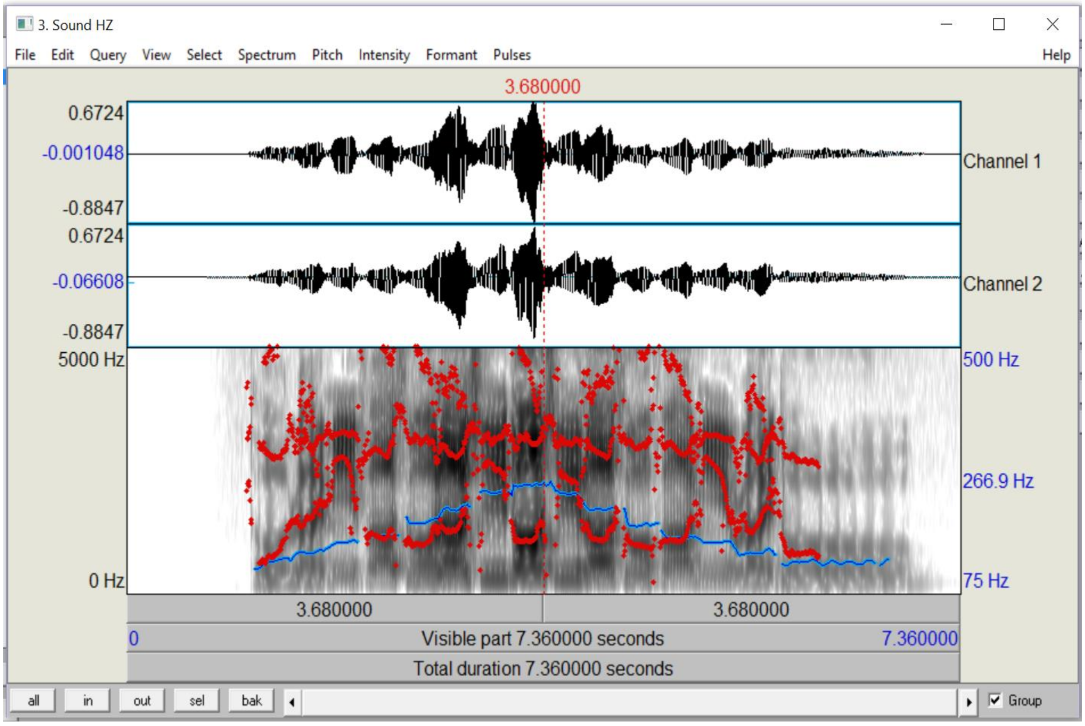

Figure 9: Using Praat to analyse voice sample

3.3.2 R

R is a language and environment for statistical computing and graphics. R provides a wide variety of statistical (i.e. linear and nonlinear modelling, classical statistical tests, time-series analysis, classification, clustering, etc.) and graphical techniques, and is highly expandable.

How R will be used in our experiment, we will describe in section 4.4.

3.4 Purposes, methods and results

Through the interview with the two professors from Xiamen University, we further clarified the purposes of our experiment. Modern vocal music teaching theory is in fact essentially all about learning and exploring a variety of human body resonances. Since the studies we mentioned before, we know that third, fourth and fifth formants determine the overtone of the sound and dictate the resonance effect, thereby defining the quality of the voice. According to the above studies, we can say that the analysis and study of upper formants is the main key to judge voice quality.

The main purpose of our experiment is mathematically to explore a function, so that we can take advantage of the data of the F3, F4 and F5. Also, with the help of said investigated function, we can evaluate the quality of the sound with a 0-100 rating system, or more exactly, to evaluate the quality of the resonance effect.

Therefore, we decided to ignore the test parameters of low frequency formants and only measure high frequency formants.

In addition, Thomas J. Millhouse [4] found in his experiments that trained singers tend to gather their upper formants and make F5 lower when they are singing. That is to say, trained singers have a narrow singing-formant range. As a hypothesis, we suspect that the frequency distribution of singing-formant is related to pitches and voice intensities. Thus, our investigated questions are explicitly listed as the following three points:

Question 1: The narrower the singing-formant range is the better the grade given by the professors

To identify the answer, we use the following data and methods.

1. Input data: all voice sample

- Frequencies of F3 and F5;
- given grades from professor Li and professor Ying.

2. Method:

- Pearson product-moment correlation coefficient

According to the degree of the correlation, we can classify the relation as either totally relevant, not entirely relevant and completely irrelevant. Depending on the relevant direction, they can be categorized as either having a positive correlation or a negative correlation. So we need the correlation coefficient to describe these kinds of relationships mathematically.

There are many definitions of the correlation coefficient. Commonly we use the Pearson correlation coefficient. The Pearson correlation coefficient is represented by the Greek letter ρ, the formula is:

$$\rho_{X,Y} = \frac{cov(X,Y)}{\sigma_X \sigma_Y} \qquad (1)$$

Where 'cov' is the covariance, σ_X is the standard deviation of X and σ_Y is the standard deviation of Y.

We have a dataset $\{x_1, \dots, x_n\}$ now that contains n values for the varying singing-formant ranges, which can be calculated with

$$X = F5 - F3 \qquad (2)$$

and another dataset $\{y_1, \dots, y_n\}$ for the varying scores. The coefficient can then be defined as:

$$\rho_{X,Y} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}((x_i - \bar{x}))^2}\sqrt{\sum_{i=1}^{n}((y_i - \bar{y}))^2}} \qquad (3)$$

where n is the total amount of elements in the dataset, and x_i and y_i are defined as above. $\bar{x}$ is the mean of the variable X, and it can be calculated with following formula:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad (4)$$

$\bar{y}$ can be calculated analogously.

- Significance test of correlation—Z-test

After obtaining the correlation coefficient between two variables, whether the results can pass the significance test should also be considered. In other words, if the correlation coefficient is significant or not.

We should establish a hypothesis test:

Null hypothesis H_0: Variables X and Y are not related ($\rho_{X,Y} = 0$).
Alternative hypothesis H_1: Variables X and Y are related ($\rho_{X,Y} > 0 \; or \; \rho_{X,Y} < 0$).

We can construct the test statistics as follows:

$$\text{Test statistic: T=} \frac{\rho_{X,Y}\sqrt{n-2}}{\sqrt{1-\rho_{X,Y}{}^2}} \qquad (5)$$

If T>$t_{1-a}(n-2)$ the null hypothesis is rejected. This means that variables X and Y are significantly related.

In R we use the function:

cor.test(X,Y)   (6)

to calculate the Pearson coefficient and perform the T- test.

3. Result

As declared before, singers have the ability to gather the energy which is distributed at different high frequency domains. To be exact, singers accumulate energy from F3, F4 and F5 at one central point, so that the voice seems more powerful.

Using the methods and collected data, R presents the results in table 2:

t	-9.9968
p-value	1.805e-14
95 percent confidence interval	[-0.867 -0.671]
sample estimates correlation coefficient	-0.788

Table 2: R statistic of Pearson's product-moment correlation for singing-formant range and score

Looking at table 2, we find that p-value is smaller than 0.05, so with 95% confidence we say the null hypothesis H_0: Variables X and Y are not related ($\rho_{X,Y} = 0$) should be thrown out and instead we accept the alternative hypothesis: Variables X and Y are related ($\rho_{X,Y} \neq 0$).

A negative correlation between upper-formant range and score was demonstrated. The coefficient interval is said with 95% confidence to be [-0.867, -0.671]. That is to say the smaller the range is then the better the score of the voice sample that can be given. In other words, a narrow singing-formant range means high quality of voice.

Question 2: F3, F4 and F5 vary at different pitch and voice intensity.

The motivation to investigate this question serves question 3. This way we can decide whether we should add interaction terms to our regression model.

Assume that higher pitch expects higher frequencies of F3, F4 and F5. Because of the interaction, the effect of higher upper-formant frequencies is different if the singing pitch is high or low. Another way of saying this is that the slopes of regression lines between score and upper-formant frequencies are different for the different pitches.

As we know, abnormal data is the data that should not normally be accepted, for those observed values differ from their expected values based on the whole population from which the statistical unit was chosen randomly. Using of this kind of data often lead to an imprecise result.

In order to get rid of unnecessary abnormal data due to lack of resonance training, we will only use the voice samples from M3 and M4. M3 and M4 are both almost finished with their Master studies.

1. Input data: collected data from Master students M3 and M4

- Frequencies of F3, F4 and F5
- Pitches
- Intensities (Amplitudes)

2. Methods

The methods used here are similar to those mentioned in question 1, but the variables for X and Y are different. New variables for X and Y are listing in table 3

3. Result

We drew the line chart as shown in figure 10 and figure 11 (square for Formant 3, circle for Formant 4 and triangle for Formant 5) for both pitch and intensity. Obviously, M3 and M4 present stable formant lines. By means of the observations of figure 9 and figure 10, we did not notice a relation between singing-formant and pitch as well as singing-formant and loudness on the whole.

The test statistics are showed in table 4. Since all p-values in table 4 are bigger than 0.05, with 95% confidence we can accept null hypotheses for all tests. That means pitches and voice intensities do no impact to the distribution of the singing-formant.

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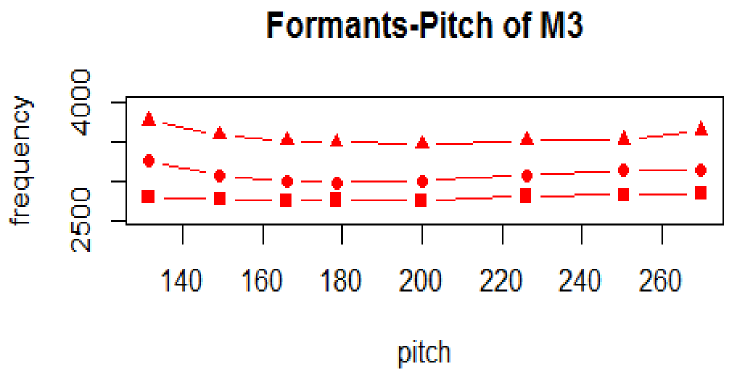
Test	X	Y	H_0	H_1
1	pitch	F3	$\rho_{X,Y}$=0	$\rho_{X,Y} \neq 0$
2	pitch	F4	$\rho_{X,Y}$=0	$\rho_{X,Y} \neq 0$
3	pitch	F5	$\rho_{X,Y}$=0	$\rho_{X,Y} \neq 0$
4	Voice intensity	F3	$\rho_{X,Y}$=0	$\rho_{X,Y} \neq 0$
5	Voice intensity	F4	$\rho_{X,Y}$=0	$\rho_{X,Y} \neq 0$
6	Voice intensity	F5	$\rho_{X,Y}$=0	$\rho_{X,Y} \neq 0$

Table3: list of the test pair(X,Y) for question2



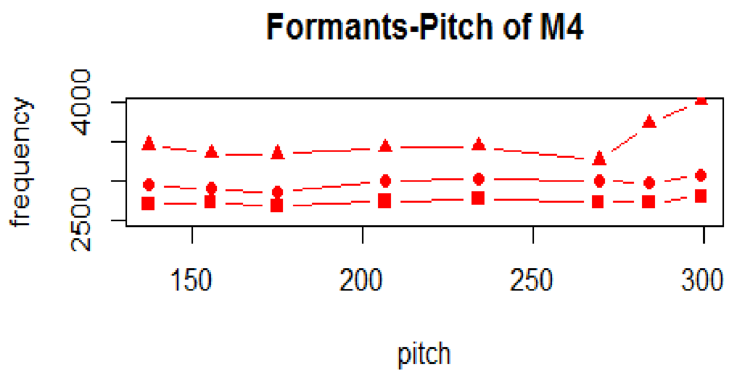
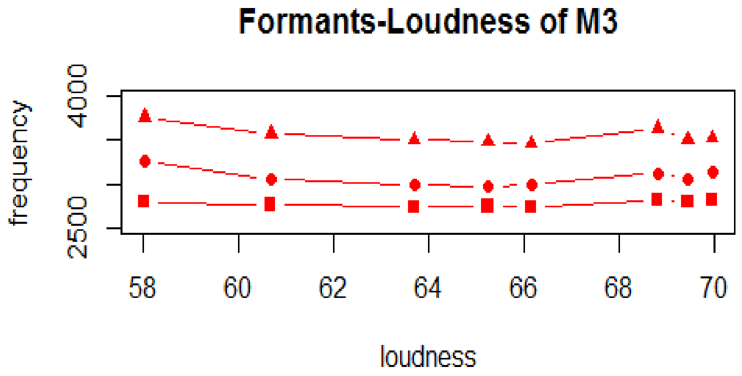


Figure 10: F3, F4 and F5 distribution among master students M3 and M4 in different pitches

(Square: F3, Circle: F4, Triangle: F5)



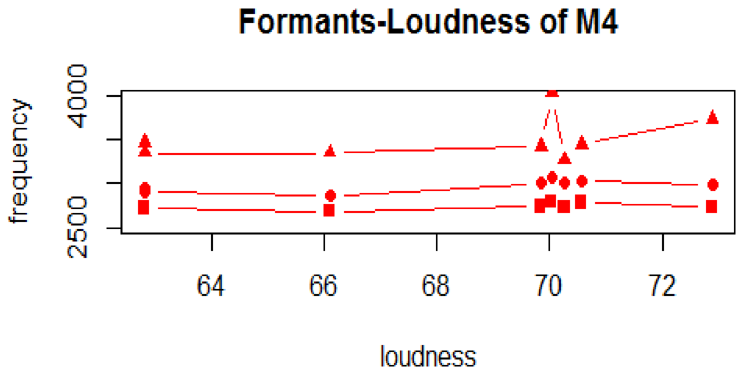


Figure 11: F3, F4 and F5 distribution among master students M3 and M4 at different levels of loudness.

(Square: F3, Circle: F4, Triangle: F5)

Test	t	p-value	Sample estimated cor
1	1.6108	0.1295	0.3954093
2	0.51609	0.6138	0.1366371
3	1.3152	0.2096	0.3316015
4	0.57607	0.5737	0.1521683
5	-0.51995	0.6112	-0.1376391
6	0.11935	0.9067	0.03188261

Table 4: test statistics for question2

Question 3: Explore a suitable mathematical score transformation model based on singing-formant.

Score transformation model is just a mathematical function which takes frequencies of F3, F4 and F5 as input and score as output. Each set of inputs is related to exactly one output and the output is a number between 0-100.

We can see from question 1 that range of singing-formant does impact the quality of voice. Based on the observation of the sample data we discovered that students tend to have a different start frequency F3, even though they have the same range of upper-formants. Also, the given scores are quite different.

1. Input data: all voice samples

- Frequencies of F3, F4 and F5
- Given score from professors

2. Methods

- Regression analysis

Regression analysis is the core of statistics. It usually refers to those with one or more predictor variables for predicting the response variables method. There are various regression models such as simple linear, non-linear and time series. R provides more than 200 functions for fitting regression models. We will focus on the OLS regression. It includes simple linear regression, polynomial regression and multiple linear regression.

In R, the basic function for the linear regression is

Myfit<- lm(formula, data)   (7)

'Formula' is the mathematical model, and 'data' is the data frame. It contains data, which are used to fit the model. Myfit stores all the information for the model.

Hence we learned from question 1 and question 2 that score depends on the locations of F3 and F5 and F3, F4 and F5 do not differ in pitch and voice intensity. So there is no need to add an interaction term (somehow like: S=F3*E or S=F3*P) in our model. The below formula is assumed:

$$\text{Score} \sim F3 + F5 \text{ or } \text{Score} \sim F3 + F4 + F5 \qquad (8)$$

OLS regression is based on the least squares method as shown in figure 11—regression model residuals. Least squares method aims to find a line, which makes the sum of the square of the distance between all points and the line as small as possible.

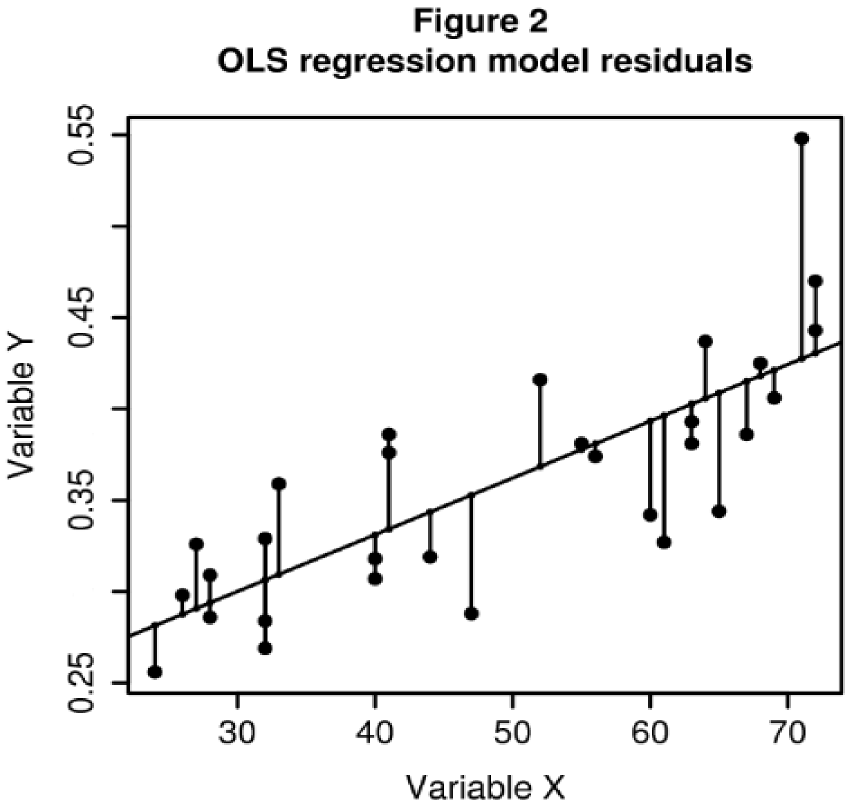


Figure12: OLS regression model residuals

(from [13])

According to our assumption, the multiple linear regression model looks like this:

$$S = \beta_1 + \beta_2 F3 + \beta_3 F5 \qquad (9)$$

or

$$S = \beta_1 + \beta_2 F3 + \beta_3 F4 + \beta_4 F5 \qquad (10)$$

and our task is to find the suitable $\beta_{1,}$ $\beta_{2,}$ β_3 and even for β_4

- F-Test

For our multiple regression model with y-intercept, we want to test the following null hypothesis and alternative hypothesis:

H_0: $\beta_1 = \beta_2 = \ldots = \beta_i = 0$ (i=3 or 4)

H_1: $\beta_j \neq 0$, for at least one value of j

F-Test statistic computes as:

(11)

SSR presents the sum of squares for regression and is equal to

$$SSR = \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2 \qquad (12)$$

SSE stands for the sum of squares for error and is calculated as

$$SSE = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \qquad (13)$$

T is the number of observations and k is the number of regression parameters.

If

F>$F_{\alpha(k-1,T-K)}$ (α is the significant level)

we reject the null hypothesis and otherwise we accept it.

- Regression Diagnostics

To be able to properly explain the OLS model, the data must meet the following statistical hypothesis

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1. Normally distributed: for the fixed values of the independent variable, the dependent variable is normally distributed.

2. Independence: all observations should be independent from each other.

3. Linearity: There is a linear correlation between dependent and independent variables.

4. Homoskedasticity: variables do not change depending on the levels of the independent variable.

The common way to test if the model is suitable using the function plot() is to draw four pictures: "Residuals vs Fitted", "Normal Q-Q", "Scale-Location" and "Residuals vs Leverage".

If the data is normally distributed, then all the points in the Q-Q plot should form a straight line at a 45-degree angle.

If the independent variable and dependent variable are linearly related, then the points in the plot "Residuals vs Fitted" are randomly distributed.

If the data fulfils the homoskedasticity in the picture „Scale-Location", then the points around the horizontal line should be randomly distributed.

3. Result

Question 1 told us that the singing-formant range is related to the score. At first we simulated F3 and F5 and ignored the effect of F4. Because we only take F3 and F5 into account, we can already define the distance between F5 and F3. The test statistics are shown in table 5.

The test result was not ideal. F3 was not significant. This means β_2 is possibly equal to zero. Although the F-Test statistics have a p-value smaller than 0.001, which means the F-test was successfully being done. We would rather to explore another mathematical model which may better suit for it.

Then we examined the model now considering F4, the results of which can be found in table 5. This time the p-value for β_1, β_2, β_3 and β_4 are all significant. With 99.5% confidence we can accept the coefficient of F3 and with 99.9% confidence we can trust the coefficient for F4, F5 and the y-intercept.

	S~F3+F5	S~F3+F4+F5
p-value of intercept	6.47e-11	5.03e-15
p-value of F3	0.368	0.00227
p-value of F4	----	5.32e-06
p-value of F5	1.47e-13	2.25e-06
Estimate intercept	153.563692	178.575104
Estimate coefficient of F3	0.004977	0.016539
Estimate coefficient of F4	------	-0.027711
Estimate coefficient of F5	-0.021510	-0.013246
Residual standard error	6.384 on 60 degrees of freedom	5.394 on 59 degrees of freedom
F-statistic	57.19 on 2 and 60 DF	61.77 on 3 and 59 DF
p-value of F-test	1.257e-14	2.2e-16

Table 5: regression summary from R for two regression models S ~ F3 + F5 and S ~ F3 + F4 + F5

Also, the F-Test presents a satisfying result. The p-value for the F-Test is 2.2e-16, which is quite small, thus, we can reject the null hypothesis ($\beta_1 = \beta_2 = ... = \beta_i = 0$ (i=3 or 4)) and accept the alternative hypothesis ($\beta_j \neq 0$, for at least one value of j).

This model should be suitable for the transformation.

We know that the data for regression analysis should be normally distributed, and we want to find out if there are any abnormal values in our sample. Therefore, we draw the plots below, figure 13 to figure 16, to demonstrate that our data of samples are suitable for the OLS-regression model.

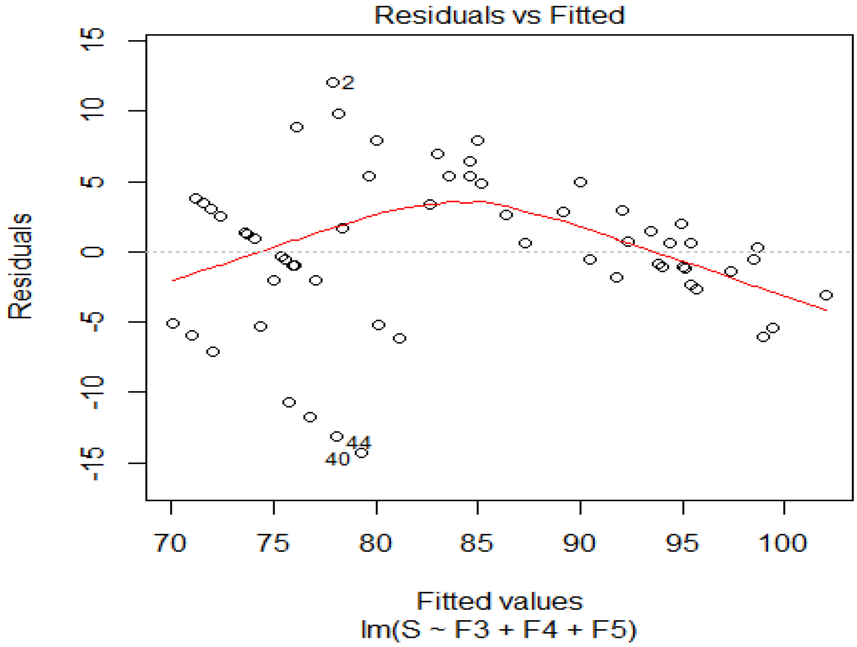

Figure 13: Residuals vs Fitted for regression model S~F3+F4+F5

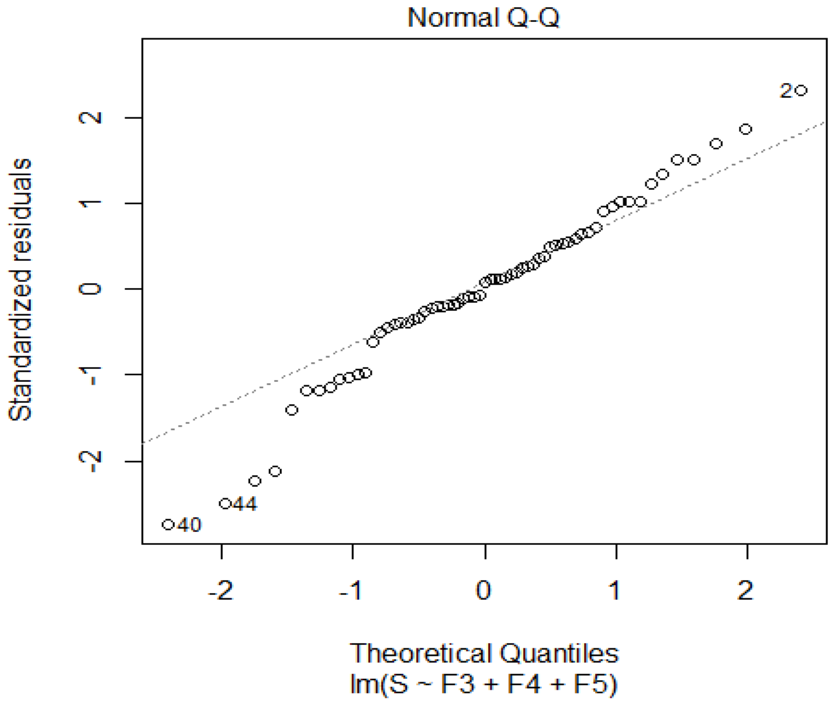

Figure 14: Normal Q-Q plot for regression model S~F3+F4+F5

In figure 13 and figure 15 we find that all points in the plots are approximately arbitrarily distributed. That means the data satisfied the requirements of linearity and Homoskedasticity. From the Q-Q plot in figure 14, we can see that all the data approximately form a straight line at a 45-degree angle to the x-axis. From this we determine that the data are normally distributed. Figure 15 shows that sample 2, 40 and 44 are potentially abnormal values, for they are marked in the figure 14 as the points, which have long distance to reach the 45-degree straight line and in also marked in the figure 13 and the figure 15 as the values, which have longest distance from red lines.

We now delete those three potentially abnormal data points from the data frame and repeat the regression in R again. The output summary for the selected data compared to the previous data from R is listed in table 6.

The p-value of the coefficient of F3 is now smaller than before, and all coefficients can be accepted with 99.9% confidence. Clearly we also discovered that the residual standard error and p-value for the F-test are smaller than before.

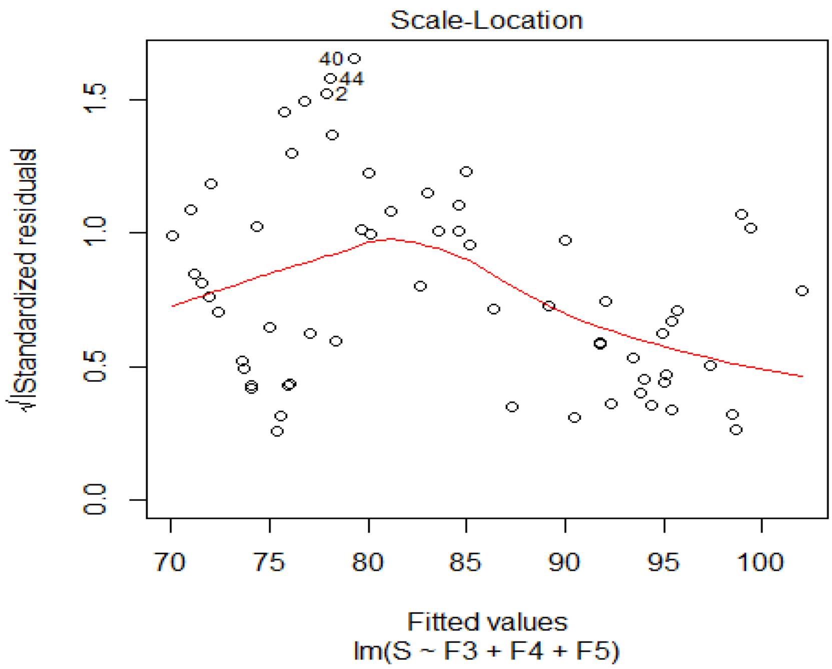

Figure 15: Scale-Location plot for regression model S~F3+F4+F5

In conclusion our singing-formant based voice quality score transformation is defined as:

$$S=0.0203*F3-0.0301*F4-0.0114*F5+169.2743 \qquad (14)$$

$$\text{where } 2000<F3<F4<F5$$

This mathematical equation is suitable to judge how good the resonance effect is when people sing.

Further, we printed the independent and dependent variables separately for F3-Score (figure 16), F4-Score (figure 17) and also F5-Score (figure 18).

	Old data	New data
p-value of intercept	5.03e-15	< 2e-16
p-value of F3	0.00227	3.39e-05
p-value of F4	5.32e-06	9.53e-08
p-value of F5	2.25e-06	4.02e-06
Estimate intercept	178.575104	169.274343
Estimate coefficient of F3	0.016539	0.020290
Estimate coefficient of F4	-0.027711	-0.030075
Estimate coefficient of F5	-0.013246	-0.011367
Residual standard error	5.394 on 59 degrees of freedom	4.571 on 56 degrees of freedom
F-statistic	61.77 on 3 and 59 DF	82.45 on 3 and 56 DF
p-value of F-test	2.2e-16	<2.2e-16

Table 6: regression summary with and without abnormal values from data frame

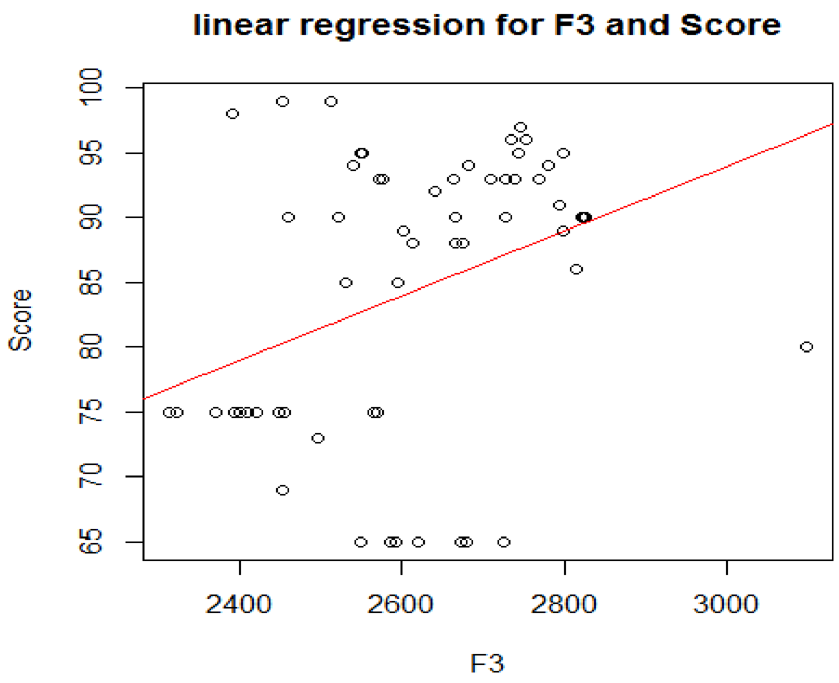

Figure 16: Linear regression for F3 and Score

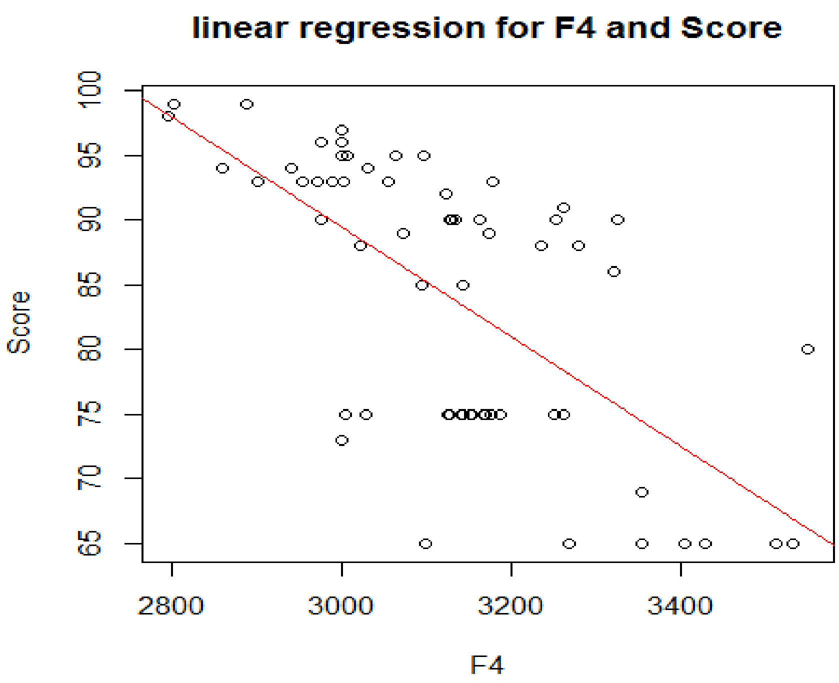

Figure 17: Linear regression for F4 and Score

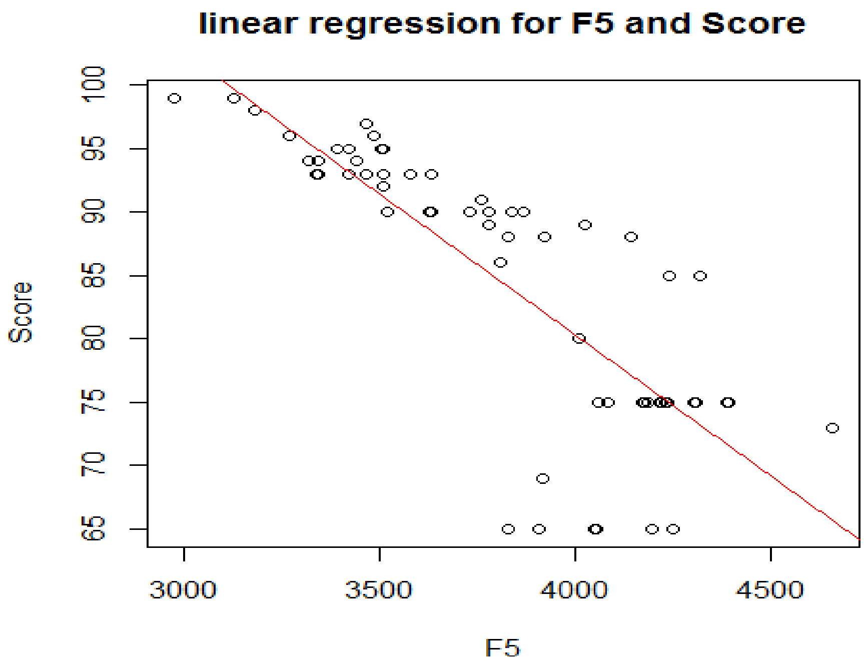

Figure 18: Linear regression for F5 and Score

Unfortunately, no one knows what exactly the best voice in the world is. Professor Li and Professor Ying scored our collected voice samples based on their personal tastes. The voice that is assigned 100 points is only the most wonderful voice for these professors. It is possible that a voice, which has an even better singing-formant distribution, can get a score of more than 100. To solve this problem, additional conditions to use this model should be defined as:

If S > 100 then S =100, similarly if S < 0 then S = 0.

CHAPTER 4

Summary

From the above experimental study, we know that the singing-formant distribution is not affected by pitch and loudness. Also, the narrower the range of the singing-formant, the better the singing resonance effect. However, specific evaluation of resonance quality still depends on the upper formants F3, F4 and F5. In our experiments we found a relatively reasonable mathematical function that can evaluate voice quality, or better said, the quality of the resonance effect. To use this function, some conditions must be met. For example, the singers must have singing-formant including F3, F4 and F5. If the singers do not have singing-formant, then this means there is no singing resonance or resonance is so weak it cannot be detected. In this case, it is unnecessary to judge if the resonance is good or bad. Only when singers do resonate do we have a reason to use our mathematical model to judge the quality of the resonance, to quantify it into the score system and receive the end score. Because the quality of resonance has no absolute evaluation criteria our model has a flaw. After obtaining the upper formants data and setting them into the model, it is possible to get a score of more than 100, which is due to the relative subjective ratings. In order to overcome this problem, when the grade is greater than 100, we give a score of 100 points, and when the grade is less than 0, the we give a score of 0 points.

CHAPTER 5

Discussion

Due to the lack of teacher involvement and an inadequate size of sound samples, our experiments and results have limitations and inaccuracies. In regards to the sound rating, the professors issued scores according to his or her personal standard. In order to make our results more accurate, we need more sound samples as well as help and participation from vocal music teachers.

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Appendix

A: Estimated data collected voice samples

	Name	F3	F4	F5	D	P	E	S
1	M1	2530.70	3142.16	4318.47	1787.77	131.41	63.89	85
2	M1	2460.69	3253.45	3865.68	1404.99	147.33	66.93	90
3	M1	2640.18	3123.59	3509.76	869.58	164.56	67.90	92
4	M1	2665.27	3235.37	4141.83	1476.56	176.08	69.35	88
5	M1	3098.31	3550.26	4007.43	909.12	198.50	69.01	80
6	M1	2613.83	3021.02	3830.17	1216.34	219.55	77.99	88
7	M1	2601.13	3173.76	3779.84	1178.71	251.26	72.83	89
8	M1	2815.09	3320.83	3810.88	995.79	262.49	80.05	86
9	M2	2495.50	3000.26	4657.52	2162.02	129.87	60.72	73
10	M2	2522.15	3161.54	3626.10	1103.95	147.34	67.30	90
11	M2	2552.12	3097.44	3391.12	839.00	164.40	66.11	95
12	M2	2674.70	3278.04	3921.06	1246.36	172.06	69.24	88
13	M2	2665.64	3128.72	3836.61	1170.97	221.36	75.00	90
14	M2	2576.96	3178.31	3631.75	1054.79	247.57	71.59	93
15	M2	2823.65	3324.19	3781.31	957.66	262.66	80.02	90
16	M3	2793.05	3261.19	3759.87	966.82	131.59	58.01	91
17	M3	2769.23	3055.71	3577.61	808.38	149.40	60.67	93
18	M3	2742.12	3000.45	3505.78	763.66	166.12	63.68	95
19	M3	2751.45	2976.83	3486.80	735.35	178.55	65.24	96
20	M3	2746.23	3000.26	3464.40	718.17	199.97	66.14	97
21	M3	2798.37	3063.70	3507.59	709.22	226.12	69.44	95
22	M3	2819.85	3132.96	3518.70	698.85	250.57	69.95	90
23	M3	2826.35	3128.39	3633.86	807.51	269.91	68.80	90
24	M4	2707.95	2952.79	3463.51	755.56	137.26	62.81	93
25	M4	2727.07	2900.54	3345.29	618.22	155.63	62.81	93
26	M4	2681.53	2859.38	3343.48	661.95	175.08	66.10	94
27	M4	2738.94	3001.17	3423.08	684.14	206.67	69.85	93
28	M4	2779.67	3029.72	3440.04	660.37	234.07	70.57	94
29	M4	2734.61	3000.34	3270.90	536.29	269.36	70.27	96
30	M4	2727.42	2976.15	3729.01	1001.59	284.13	72.89	90
31	M4	2797.72	3072.24	4025.01	1227.29	299.44	70.04	89
32	B1	2595.08	3093.55	4237.05	1641.97	125.85	74.85	85
33	B1	2390.71	2795.53	3180.78	790.07	145.99	83.17	98
34	B1	2511.49	2887.76	3128.56	617.07	163.03	81.58	99
35	B1	2548.85	3006.52	3423.61	874.76	168.49	83.17	95
36	B1	2571.38	2989.83	3336.29	764.91	196.98	83.19	93
37	B1	2540.20	2939.68	3320.06	779.86	219.50	86.02	94
38	B1	2453.18	2802.50	2976.11	522.93	247.76	84.53	99
39	B1	2664.03	2970.55	3508.03	844.00	262.59	84.69	93
40	B2	2593.33	3098.94	4249.16	1655.83	125.85	74.70	65
41	B2	2453.12	3354.34	3915.20	1462.08	141.59	73.61	69
42	B2	2680.27	3267.43	4194.31	1514.04	163.72	75.59	65

43	B2	2548.46	3403.25	3826.38	1277.92	168.07	75.63	65
44	B2	2673.40	3353.78	3906.06	1232.66	191.84	75.63	65
45	B2	2618.98	3511.41	4047.52	1428.54	214.01	77.03	65
46	B2	2586.72	3428.55	4248.18	1661.46	247.39	76.44	65
47	B2	2726.13	3532.01	4055.13	1329.00	258.86	77.42	65
48	B3	2400.67	3142.23	4170.16	1769.49	125.13	72.80	75
49	B3	2313.75	3152.21	4220.12	1906.37	142.61	72.94	75
50	B3	2564.96	3169.20	4236.60	1671.64	161.66	70.59	75
51	B3	2454.41	3186.98	4176.62	1722.21	165.84	76.65	75
52	B3	2393.21	3005.00	4057.54	1664.33	192.90	74.89	75
53	B3	2421.38	3251.06	4302.83	1881.45	211.06	78.19	75
54	B3	2401.16	3167.63	4385.62	1984.46	242.82	73.98	75
55	B3	2370.77	3127.21	4309.50	1938.73	251.14	75.80	75
56	B4	2400.59	3141.40	4169.58	1768.99	125.13	73.07	75
57	B4	2323.09	3154.84	4216.23	1893.14	142.57	73.07	75
58	B4	2569.25	3175.44	4228.49	1659.24	161.70	70.76	75
59	B4	2448.25	3186.53	4183.57	1735.32	166.08	76.60	75
60	B4	2392.69	3028.27	4080.19	1687.50	193.41	74.67	75
61	B4	2422.01	3260.59	4306.47	1884.46	211.17	78.12	75
62	B4	2409.89	3187.19	4389.20	1979.31	243.38	73.69	75
63	B4	2370.08	3125.27	4309.77	1939.69	251.10	75.78	75

(F3: Third Formant; F4: Forth Formant; F5: Fifth Formant; D: Frequency range between F3 and F5; P: Pitch; E: Energy of Voice; S: Score)

B: R-code

```
data<-read.csv(file.choose())
cor.test(data[,5],data[,8])
m3<-data[16:23,]
m4<-data[24:31,]
par(mfrow=c(2,1))
plot(m3[,6],m3[,2],type="b",pch=15,col="red",ylim=c(2500,4000),xlab="pitch",ylab="frequency",main="Formants-Pitch of M3")
lines(m3[,6],m3[,3],type="b",pch=16,col="red")
lines(m3[,6],m3[,4],type="b",pch=17,col="red")
plot(m4[,6],m4[,2],type="b",pch=15,col="red",ylim=c(2500,4000),xlab="pitch",ylab="frequency",main="Formants-Pitch of M4")
lines(m4[,6],m4[,3],type="b",pch=16,col="red")
lines(m4[,6],m4[,4],type="b",pch=17,col="red")
par(mfrow=c(2,1))
plot(m3[,7],m3[,2],type="b",pch=15,col="red",ylim=c(2500,4000),xlab="loudness",ylab="frequency",main="Formants-Loudness of M3")
lines(m3[,7],m3[,3],type="b",pch=16,col="red")
lines(m3[,7],m3[,4],type="b",pch=17,col="red")
plot(m4[,7],m4[,2],type="b",pch=15,col="red",ylim=c(2500,4000),xlab="loudness",ylab="frequency",main="Formants-Loudness of M3")
lines(m4[,7],m4[,3],type="b",pch=16,col="red")
lines(m4[,7],m4[,4],type="b",pch=17,col="red")
m3m4<-data[16:31,]
cor.test(m3m4[,2],m3m4[,6])
cor.test(m3m4[,3],m3m4[,6])
cor.test(m3m4[,4],m3m4[,6])
cor.test(m3m4[,2],m3m4[,7])
cor.test(m3m4[,3],m3m4[,7])
cor.test(m3m4[,4],m3m4[,7])
test1<-lm(S~F3+F5,data)
summary(test1)
test2<-lm(S~F3+F4+F5,data)
summary(test2)
```

```
plot(test2)
newdata<-data[- 44,]
newdata<-newdata[-40,]
newdata<-newdata[-2,]
test3<-lm(S~F3+F4+F5,newdata)
summary(test3)
plot(data[,8]~data[,2],xlab="F3",ylab="Score",main="linear regression for F3 and Score")
abline(lm(data[,8]~data[,2]),col="red")
plot(data[,8]~data[,3],xlab="F4",ylab="Score",main="linear regression for F4 and Score")
abline(lm(data[,8]~data[,3]),col="red")
plot(data[,8]~data[,4],xlab="F5",ylab="Score",main="linear regression for F5 and Score")
abline(lm(data[,8]~data[,4]),col="red")
```