Extended Kalman Filter Based Estimation of Dynamic Quantities and Stability Indices for Bipedal Posture Control

Jorhabib Eljaik, Naveen Kuppuswamy and Francesco Nori

***Abstract*— We have designed and implemented and Extended Kalman Filter (EKF) that integrates force/torque (FT) measurements from the leg and ankle of a humanoid robot with readings from an accelerometer placed on its foot and the dynamic model of its leg to estimate different dynamic quantities. The FT measurements in the thigh can be seen as a replacement of the dynamics of the rest of the body. The main contributions are in the usage of rigid body dynamics for state update and the incorporation of control variables u within the state update, thus allowing the EKF to estimate external wrenches based based also on their direct or indirect measurement.**

***Index Terms*— Extended Kalman Filter, dynamic quantity estimation, sensor fusion, stability measuremens, balance, foot rotation indicator, center of pressure.**

I. INTRODUCTION

The evolution of a technological world built for and by humans comes with a continuing demand for machines that are also able to assist us in a more human-like fashion. To this end, bipedal humanoid robots are thought of as the perfect tool, and to be such, these machines must move as we do in our unstructured environment and *interact* with it while accounting for *whole-body postures*. The coexistence of these last two aspects implies that bipedal robots must be able to balance while in multiple contacts, which in turn means, that the wrenches due to contacts (f_c, μ_c) and those due to the movement of the body (f_o, μ_o) must balance. Basic stability criteria for bipedal robots are based on this principle and a first introduction of such concept was done in [8] by Vukobratovic et al. who defined the *zero moment point* (ZMP) as a unique point on the ground where f_c, μ_c produce zero tangential moments while the corresponding sability criterion considered that this point should stay away from the border of the convex hull of both feet in contact with the ground.

Goswami in [3] pointed out that the ZMP-based stability criterion is limited for gait planning. He therefore formulated some different statements for the characterization of the stability of planar unilateral contacts. These statements made use of the *foot rotation indicator* (FRI) which corresponds to the unique zero tipping moment point associated to f_o, μ_o and belongs to the contact plane. The FRI can however move outside the convex hull meaning that a rotation of the foot is happening and thus allowing the distinction between a marginal state of static equilibrium and loss of balance.

*This paper was supported by the FP7 EU projects CoDyCo (No. 600716 ICT 2011.2.1 Cognitive Systems and Robotics), and Koroibot (No. 611909 ICT-2013.2.1 Cognitive Systems and Robotics).

Authors are with the Department of Robotics, Brain, ad Cognitive Sciences (RBCS) at the Istituto Italiano di Tecnologia, via Morego 30, Genova Italy. (email : {francesco.nori, jorhabib.eljaik, naveen.kuppuswamy} @ iit.it

II. STATE OF THE ART AND BEYOND

Several authors have used either the ZMP or the FRI for gait planning, mostly relying on force/torque (FT) sensors at the ankles or sensor arrays at the feet soles. These sensors allow only an instantaneous measurement of the ZMP and therefore several authors have suggested the use of a Kalman filter [1] to improve the signal to noise ration and, more interestingly, to predict the ZMP position by exploiting its dynamics as it results from the integration of the whole body dynamics.

Kalman filter based ZMP estimation has been used in [5], where authors proposed a three dimensional linear inverted pendulum model to filter the measured ZMP position and to get a more reliable measurement for achieving stable walking. More recently, Park et al. [6] introduced an estimation procedure which uses camera images to filter the robot pose and to get a Kalman filter estimation of the ZMP position.

In this paper we present a framework for obtaining estimates of the dynamic quantities necessary for the computation of stability measurements (ZMP or FRI) through an Extended Kalman Filter (EKF) based data fusion approach. The main contributions are in the usage of rigid body dynamics for state update and the incorporation of control variables u within the state update, thus allowing the EKF to estimate external wrenches based also on their direct or indirect measurement. We have designed and implemented and EKF that integrates multiple F/T measurements with readings from an accelerometer placed on the foot and the dynamic model of the leg to estimate different dynamic quantities. The EKF uses an augmented state vector to include the applied wrenches and foot orientation (after local parametrization). Our approach instead uses FT sensors and accelerometers along with the dynamic model of the leg to estimate (among other quantities) contact and inertial forces that will then allow us to have a more reliable stability measurement. It is worth noting that this framework could then be extended to the rest of the body when external forces occur at points different from the feet soles.

The structure of the paper is as follows. In Section III the proposed framework is explained, starting from the dynamic model of a general rigid body system subject to two measureable external wrenches and with a single measurement of linear acceleration. A brief introduction to the Extended Kalman Filter and later explained how the estimated external wrenches can be used for better estimation of different stability measurements, in particular, FRI and COP. Section IV contains details about the simulated and real setups as well as the specificities of the platform and the obtained results, while Section V draws some conclusions and analysis of the obtained results. Finally, Section VI elaborates on the enhancemens that can be done and how we plan to scalate the framework to estimate dynamic quantities for the entire body of the robot and ideas on how to make this efficiently.

III. PROPOSED FRAMEWORK

A. Extended Kalman Filter

The Kalman Filter is a tool for stochastic data fusion in which the state of a system is estimated, based on measurements of the system and its linear model. It minimizes in the least squares sense the inconsistencies with all pieces of information available to obtain the best estimate possible. Measurements and state estimates are expressed as Gaussian distributions thus allowing to express a desired level of confidence on the information. The model of the system consists of a *state equation*, a *measurement equation*, the measurement and process *covariance matrices* R and Q, and the *initial state estimate* $\hat{x}_0$ with its corresponding initial covariance matrix $\hat{P}_0$.

The differentiable state transition and observation models that are used in the EKF are expressed as:

$$x_k = f(x_{k-1}, u_{k-1}) + w_{k-1} \tag{1}$$
$$y_k = h(x_k) + v_k \tag{2}$$

Where x_k is the state vector at time k, f is the nonlinear state function, u the control input. y_k is the observation at time k with measurement prediction function h. The process noise w and the observation noise v have Gaussian distributions $\mathcal{N}(0, \sigma_w^2)$ and $\mathcal{N}(0, \sigma_v^2)$ respectively with corresponding covariance matrices $Q_k = \sigma_w^2 \mathbf{I}_{n\times n}$ and $R_k = \sigma_v^2 \mathbf{I}_{m\times m}$, where n is the dimension of the state vector and m of the measurement vector.

When the measurement and state equations are nonlinear, these can be linearized about the most recent state estimate in order to apply the Kalman Filter. This is known as the Extended Kalman Filter, which consists of a *prediction* and an *update* stage with the following equations:

1) Prediction:

- State prediction: $\tilde{x}_{k|k-1} = f(\hat{x}_{k-1|k-1})$
- Covariance prediction: $\tilde{P}_{k|k-1} = F_{k-1}\hat{P}_{k-1|k-1}F_{k-1}^T + Q_{k-1}$

2) Update:

- State estimate: $\hat{x}_{k|k} = \tilde{x}_{k|k-1} + K_k(y_k - h(\tilde{x}_{k|k-1}))$
- Covariance matrix estimate: $\hat{P}_{k|k} = (I - K_k H_k)\tilde{P}_{k|k-1}$

Where $\tilde{}$ stands for prediction and $\hat{}$ for estimate. The subscript $k|k-1$ indicates a computation at time k given others at time $k-1$ and similarly for $k-1|k-1$ and $k|k-1$. P and Q are the state and noise covariance matrices respectively. F_{k-1} is the state transition matrix, computed as the Jacobian of the state function evaluated at the previous state estimate, i.e. $\frac{\partial f}{\partial x}|_{\hat{x}_{k-1|k-1}}$.

In the update equations K_k is the Kalman gain and can be obtained as $K_k = \tilde{P}_{k|k-1}H_k^T S_k^{-1}$, where H is the observation matrix, computed as the Jacobian with respect to the state vector of function h, i.e. $H_k = \frac{\partial h}{\partial x}|_{\tilde{x}_{k|k-1}}$, while S is the residual covariance matrix computed as $S_k = R_k + H_k\tilde{P}_{k|k-1}H_k^T$.

The transition and observation models will come from the dynamic model of a rigid foot in contact with the ground as described next.

B. Dynamic model

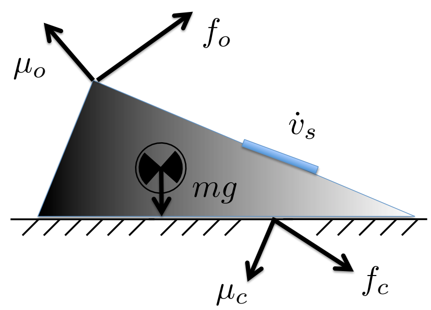

Fig. 1. The foot of a bipedal robot as a rigid body. Two external wrenches can be considered - those applied at the ankle from the rest of the robot dynamics and the contact forces due to ground reaction.

Consider the foot in contact with the ground as a single rigid body (see Fig. 1) subject to external wrenches with the following dynamics,

$$\begin{aligned} m\dot{v}^B + \omega^B \times (mv^B) &= {f_o}^B + {f_c}^B + mg^B \\ I^B\dot{\omega}^B + \omega^B \times (I^B\omega^B) &= {\mu_o}^B + {\mu_c}^B \\ \dot{\phi}^B &= T_\phi^{-1}\omega^B \end{aligned} \tag{3}$$

where linear and angular velocities are given by v^B and ω^B. The external wrenches affecting the body are $({f_c}^B \quad {\mu_c}^B)$ and $(f_o^B \quad \mu_o^B)$. The first corresponds to the total wrench due to the body and control dynamics while the latter is due to contacts (as seen in Fig. 1). The superscript $*^B$ denotes quantities that are expressed in the body reference frame. The foot can be considered to have mass m and inertia I^B. In the dynamics we have included also the orientation of the foot ϕ, expressed in ZYZ Euler angles, or roll (α), pitch (υ) and yaw (ψ), since it will also be estimated. Its dynamics come from the relationship between the angular velocity ω and the rotational velocity $\dot{\phi}$ [7] through the matrix

$$T_\phi = \begin{bmatrix} 0 & -sin(\alpha) & cos(\alpha)sin(\upsilon) \\ 0 & cos(\alpha) & sin(\alpha)sin(\upsilon) \\ 1 & 0 & cos(\upsilon) \end{bmatrix} \tag{4}$$

This local parametrization of the orientation is acceptable to reduce the complexities of using more consistent representations for the orientation, such as quaternions, since for the interest of the authors and the scope of the experiments later presented, the foot pitch is kept within $0 < \upsilon < \pi$ thus avoiding singularities in the inversion of T_ϕ.

From the dynamic model described in (3), the state of our system is given by:

$$x' = \begin{bmatrix} v^B \\ \omega^B \\ \phi^B \end{bmatrix} \tag{5}$$

Inputs to this system are $u = [f_0^B \quad \mu_0^B \quad f_c^B \quad \mu_c^B]$. For the EKF, however, the state vector will be augmented to include these inputs and provide their estimates, so that (5) turns into:

$$x = [v^B \quad \omega^B \quad f_o^B \quad \mu_o^B \quad fc^B \quad \mu_c^B \quad \phi^B]^T \tag{6}$$

We will then assume the following measurement vector:

$$y = [f_o^B \quad \mu_o^B \quad f_c^B \quad \mu_c^B \quad \dot{v}^B]^T \tag{7}$$

With the previously defined state and measurement vectors, the EKF can be computed as explained in Section III-A in a recursive way, thus having at each time step k an estimate of the external wrenches acting on the rigid body and an estimate of their covariances which can later be used to compute stability indices such as FRI and CoP. The way do this is described next.

C. Linearized stability indices

Two kinds of stability indices were used and compared in this paper: FRI and CoP. The way to compute them used in this paper is described in [2]. The equations for these measurements come from the static equilibrium equations of the foot at the desired stationary reference points (i.e. FRI or CoP), assuming that the dynamics of the rest of the body can be replaced by a F/T measurement at the ankle. For the CoP (P) the equation is

$$\mu_c + PG \times mg - \mu_o - PO \times f_o = 0 \tag{8}$$

Where f_c and μ_c are assumed to be at the CoP of the single foot. G is the reference frame of the foot center of mass (COM) and O the ankle reference frame. Considering the tangential components $(.)_t$ of the previous equation yields the expression for the CoP,

$$(\mu_o + PO \times f_o - PG \times mg)_t = 0 \tag{9}$$

Changing the stationary reference point to one outside the support polygon and calling this point F, we obtain the expression for the FRI,

$$(\mu_0 + FO \times f_o - FG \times mg)_t = 0 \tag{10}$$

At the point F, the resultant moment of the force and torque at the ankle plus the weight of the foot (different from the reaction forces) is normal to the surface.

To compute FRI and CoP after estimates of the external wrenches acting on the rigid body along with their corresponding variances, we simply express both external wrenches in the rigid-body frame (foot frame) and eliminate PG from the equations. Solving for the x and y components of P and F, the following simpler nonlinear relationships can be obtained.

$$\begin{aligned} P_x &= \frac{-\mu_c^x}{f_c^z}, \quad P_y = \frac{\mu_c^y}{f_c^z}, \\ F_x &= \frac{-\mu_o^x}{f_o^z}, \quad F_y = \frac{\mu_o^y}{f_o^z}, \end{aligned} \tag{11}$$

NAVEEN Don't forget to add a few lines about how to fuse the estimates to get the corresponding estimate of the FRI and CoP along with their variances[jeg]

IV. EXPERIMENTS AND RESULTS

A. Simulation of a rigid body system

The first set of experiments were carried out in simulation. The simulated rigid body was subjected to two wrenches. In order for these wrenches not to put the body in orientation singularity, an inverse dynamics algorithm was used to obtain suitable control inputs given a desired orientation trajectory. Afterwards, forward dynamics were integrated to simulate the evolution of the state vector x and later the simulated outputs y. Noise was then added to implement the EKF. The parameters in Table I have been used for the simulations in Fig. 2-5.

TABLE I

SIMULATION PARAMETERS

Parameter	Value	Description
T	1.5 s	Simulation time span
σ_{v_f}	0.025	Forces meas. error variance
σ_{v_u}	0.025	Torques meas. error variance
σ_{v_a}	0.01	Lin. acc. meas. error variance
σ_{v_ω}	0.01	Ang. vel. meas. error variance
σ_{w_f}	0.04	Forces error variance
σ_{w_u}	0.04	Torques error variance
σ_{w_a}	0.001	Lin. acc. error variance
σ_{w_ϕ}	0.001	Orientation error variance
m	7 Kg	Mass
g	9.8 m/s^2	Gravity
Δt_{EKF}	0.01 s	Time interval used in EKF
I_{xx}	0.05 $Kg \cdot m^2$	Main moment of inertia along x
I_{yy}	0.02 $Kg \cdot m^2$	Main moment of inertia along y
I_{zz}	0.03 $Kg \cdot m^2$	Main moment of inertia along z

The *EKF/UKF Toolbox for Matlab* detailed in [4] was used for the EKF algorithms.

B. Implementation on the real platform

The iCub robot was used for the experiments described in this section. We used two sets of force/torque measurements and an external measurement of acceleration and angular velocity (using a Vicon tracker system). The scenario tested was that of externally induced toppling. The aim was to estimate accurately the variation in the stability indices (CoP, FRI) during the momments preceding and at toppling.

The F/T sensors used for the experiment are depicted in Fig. 6.

The scenario tested was to predict the behaviour of the stability indices, particularly that of the CoP as the robot reaches a toppling condition. F

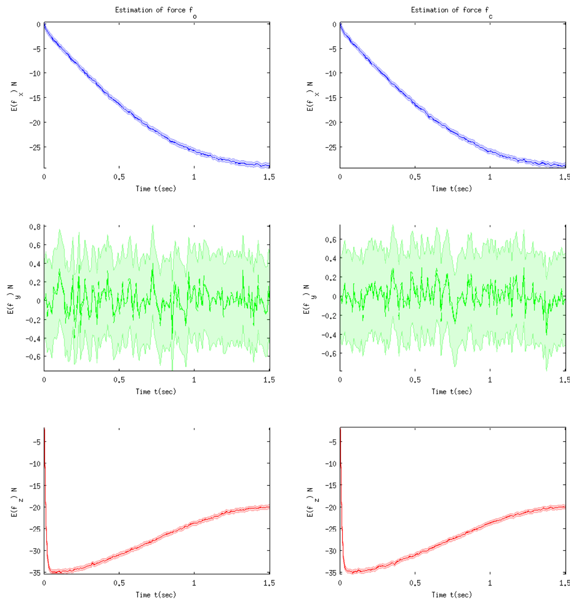

Fig. 2. Estimated external forces acting on the rigid body system after simulation.

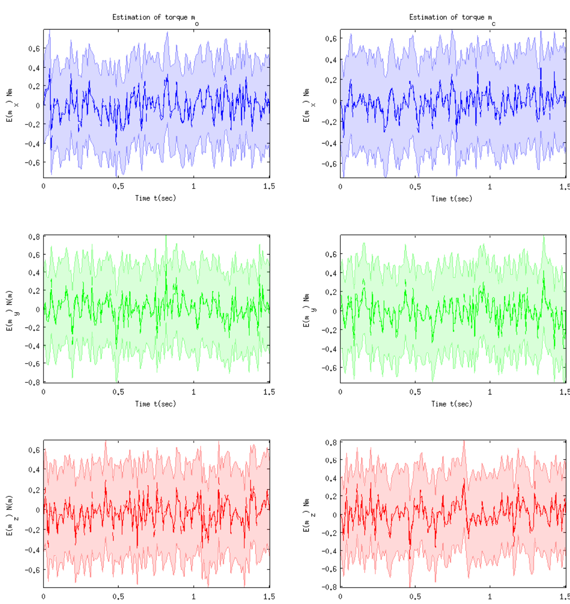

Fig. 3. Estimated external torques acting on the rigid body system after simulation.

The robot was placed in a standing-upright position with knees Toppling was induced by rotating the torso of the robot in the saggital plane, i.e. bending forward. The topling condition expected is that the CoP exits the support polygon at the momment of toppling.

The parameters used in the experiments with the robot can be found in Table II

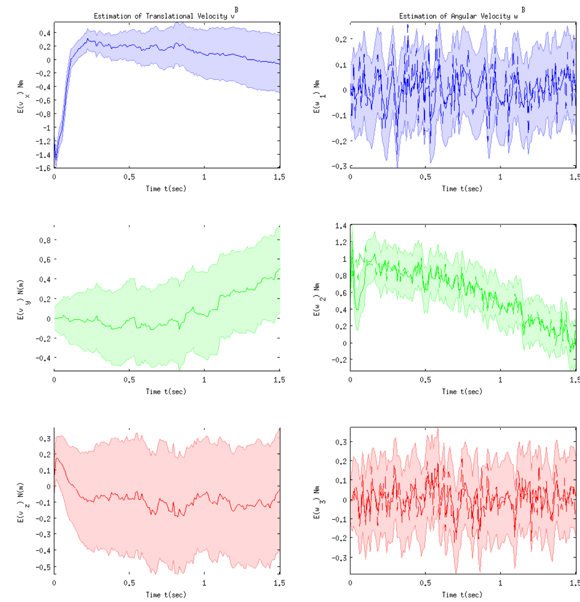

Fig. 4. Estimated angular and linear velocities of the rigid body system after simulation.

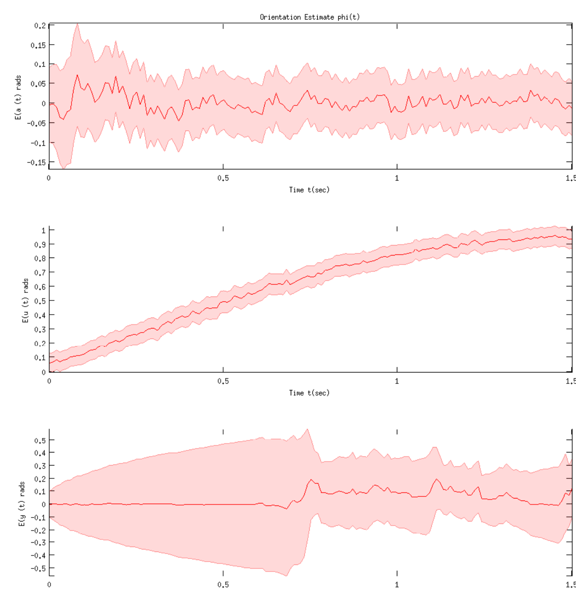

Fig. 5. Estimated orientation $\phi(t) = [\alpha \quad \upsilon \quad \psi]^T$ during simulation.

V. CONCLUSIONS

VI. FUTURE WORK

Equations 3 can of course be generalied for rigid bodies with N external wrenches

REFERENCES

[1] J. De Schutter, J. De Geeter, T. Lefebvre, and H. Bruyninckx. Kalman filters: A tutorial. *Journal A*, 40(4):52–59, 1999.

TABLE II

PARAMETERS FOR EXPERIMENT ON THE REAL PLATFORM NAVEEN DON'T FORGET TO FILL IN THIS TABLE.JEG

Parameter	Value	Description
T	s	Simulation time span
σ_{v_f}		Forces meas. error variance
σ_{v_u}		Torques meas. error variance
σ_{v_a}		Lin. acc. meas. error variance
σ_{v_ω}		Ang. vel. meas. error variance
σ_{w_f}		Forces error variance
σ_{w_u}		Torques error variance
σ_{w_a}		Lin. acc. error variance
σ_{w_ϕ}		Orientation error variance
m	Kg	Mass
g	9.8 m/s^2	Gravity
Δt_{EKF}	s	Time interval used in EKF
I_{xx}	$Kg \cdot m^2$	Main moment of inertia along x
I_{yy}	$Kg \cdot m^2$	Main moment of inertia along y
I_{zz}	$Kg \cdot m^2$	Main moment of inertia along z

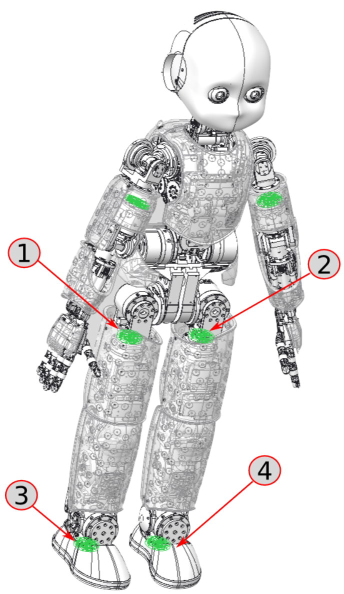

Fig. 6. Schematic of iCub and the force/torque sensors used for the experiments.

[2] A. Goswami. Foot rotation indicator (fri) point: A new gait planning tool to evaluate postural stability of biped robots. In *IEEE International Conference on Robotics and Automation*, pages 47–52, 1999.

[3] A. Goswami. Postural stability of biped robots and the foot-rotation indicator (fri) point. *The International Journal of Robotics Research*, 18(6):523–533, 1999.

[4] J. Hartikainen, A. Solin, and S. Särkkä. Optimal filtering with kalman filters and smoothers. *Department of Biomedica Engineering and Computational Sciences, Aalto University School of Science, 16th August*, 2011.

[5] S. Park, Y. Han, and H. Hahn. Kalman filter based zmp estimation scheme for balance control of a biped robot. In *Proceedings of The 4th International Conference on Ubiquitous Robots and Ambient Intelligence (URAI 2007)*, 2007.

[6] S. Park, Y. Han, and H. Hahn. Balance control of a biped robot using camera image of reference object. *International Journal of Control, Automation and Systems*, 7(1):75–84, 2009.

[7] B. Siciliano, L. Sciavicco, L. Villani, and G. Oriolo. *Robotics: Modelling, Planning and Control*. Advanced Textbooks in Control and Signal Processing. Springer, 2009.

[8] M. Vukobratovic and D. Juricic. Contribution to the synthesis of biped gait. *Biomedical Engineering, IEEE Transactions on*, BME-16(1):1–6, 1969.