CSE 15: Discrete Mathematics
Homework 6

Fall 2020

Preliminary Notes

- **This homework must be solved individually.** You can discuss your ideas with others, but when you prepare your solution you must work individually. Your submission must be yours and yours only. No exceptions, and be reminded that the CSE academic honesty policy discussed in class will be enforced.
- Your solution must be exclusively submitted via CatCourses. Pay attention to the posted deadline because **the system automatically stops accepting submissions when the deadline passes. Late submissions will receive a 0**. You only need to submit the PDF and you have to use the template file provided in CatCourses. Please note that the system does not allow to submit any other file format. Do not submit the LaTeXsource of your solution.
- By now you should have become somewhat familiar with LaTeX. You still will not be penalized for poor typesetting, but it is in your own interest to prepare your submission in a way that is easy to understand, so try using the appropriate LaTeXsymbols. If you do not know how to type a certain math symbol, search on the Internet and you will quickly find the answer.[1] **If in your LaTeXsubmission you embed screenshots or scans of your handwritten solution those will not be graded.** You are encouraged to collaborate with other students to determine how to best format your submission or improve your LaTeXskills.
- Start early.

1 Recursively defined functions

Find $f(1)$, $f(2)$, $f(3)$, $f(4)$, $f(5)$ if $f(n)$ is defined recursively by $f(0) = 3$ and for $n = 1, 2, \ldots$

a) $f(n+1) = -2f(n)$

b) $f(n+1) = 3f(n) + 7$

c) $f(n+1) = f(n)^2 - 2f(n) - 2$

d) $f(n+1) = 3^{f(n)/3}$

[1] see `https://www.caam.rice.edu/~heinken/latex/symbols.pdf` for example.

2 Recursively defined sequences

In this exercise we consider sequences defined over the positive natural numbers $1, 2, 3, \ldots$ The n-th element in the sequence is denoted as a_n and therefore the elements in the sequence are $a_1, a_2, a_3, \ldots$ Each of the following sequences is defined using a closed formula that directly gives a_n for any positive natural number n. For each sequence, give an equivalent recursive definition, i.e., a basis step and an inductive step defining the n-th element in the sequence as a function of elements already in the sequence (either the previous one or some other element preceding a_n.)

a) $a_n = 4n - 2$

b) $a_n = 1 + (-1)^n$

c) $a_n = n(n - 1)$

d) $a_n = n^2$

Suggestion: it may be convenient to first tabulate the values of the sequence for a few values of n, observe the pattern, and then guess the basis and inductive steps. Then, make sure that the basis and inductive steps give the same elements you tabulated.
Note: to be fully correct, one should formally prove that the inductive definition of the sequences generate all and only the elements in the sequence. This would require some additional steps, but we omit them for brevity.

3 Recursively defined sets

Recall the definition of alphabet Σ and string over an alphabet. Let ε be the empty string. The following rules recursively define a set S of strings over the alphabet $\Sigma = \{0, 1\}$:

Basis step : the empty string in S, i.e., $\varepsilon \in S$.

Inductive step : if $x \in S$ is a string in S, then $0x1$ is a string in S, i.e., $0x1 \in S$.

Describe without using recursion, the set of strings in S. The suggested way is to first give a few examples of strings in S, and then describe in plain English the structure of all and only the strings in S.
Important: your description must separate the strings in S from the strings not in S. If you say something like, "S is a set of strings made of 0s and 1s," you are not making an incorrect statement, but the answer is not acceptable because there are many strings made of 0s and 1s that are not in S.