

Lecture 21: Reinforcement Learning

Assignment 5: Object Detection

Single-stage detector

Two-stage detector

Due on Monday 12/9, 11:59pm

Assignment 6: Generative Models

Generative Adversarial Networks

Due on Tuesday 12/17, 11:59pm

So far: Supervised Learning

Supervised Learning

Data: (x, y)

x is data, y is label

Goal: Learn a *function* to map $x \rightarrow y$

Examples: Classification, regression,
object detection, semantic
segmentation, image captioning, etc.

Classification



Cat

[This image is CC0 public domain](#)

So far: Unsupervised Learning

Unsupervised Learning

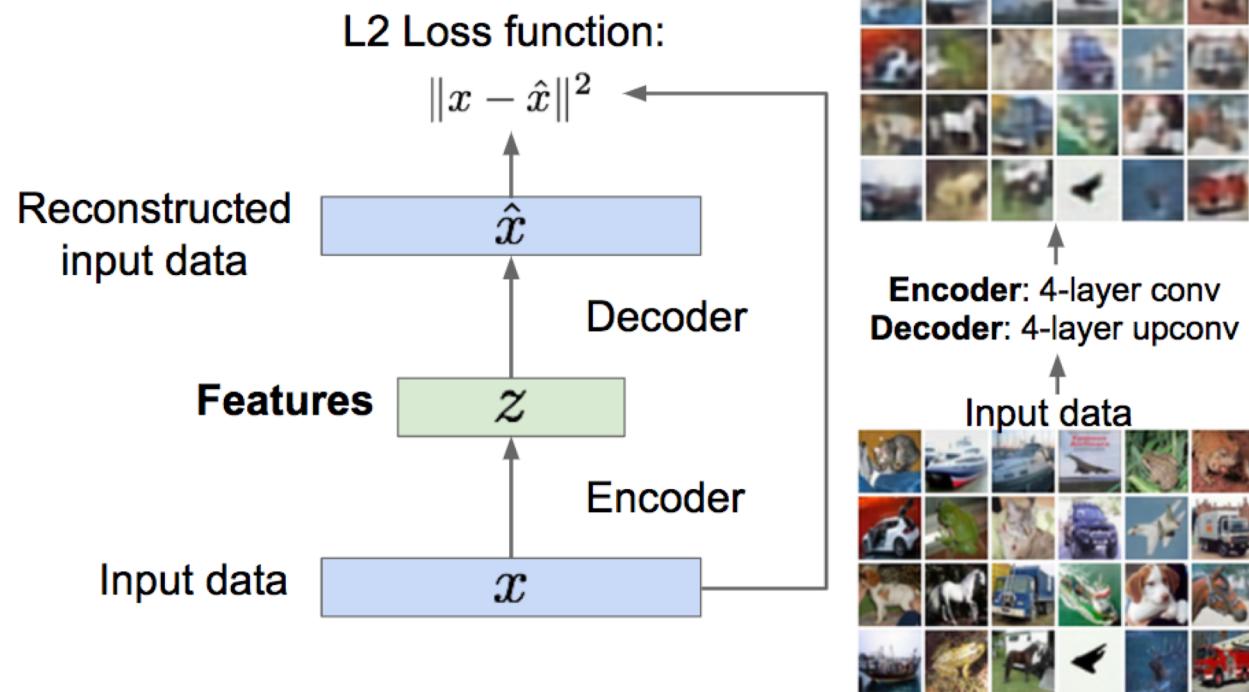
Data: x

Just data, no labels!

Goal: Learn some underlying hidden *structure* of the data

Examples: Clustering, dimensionality reduction, feature learning, density estimation, etc.

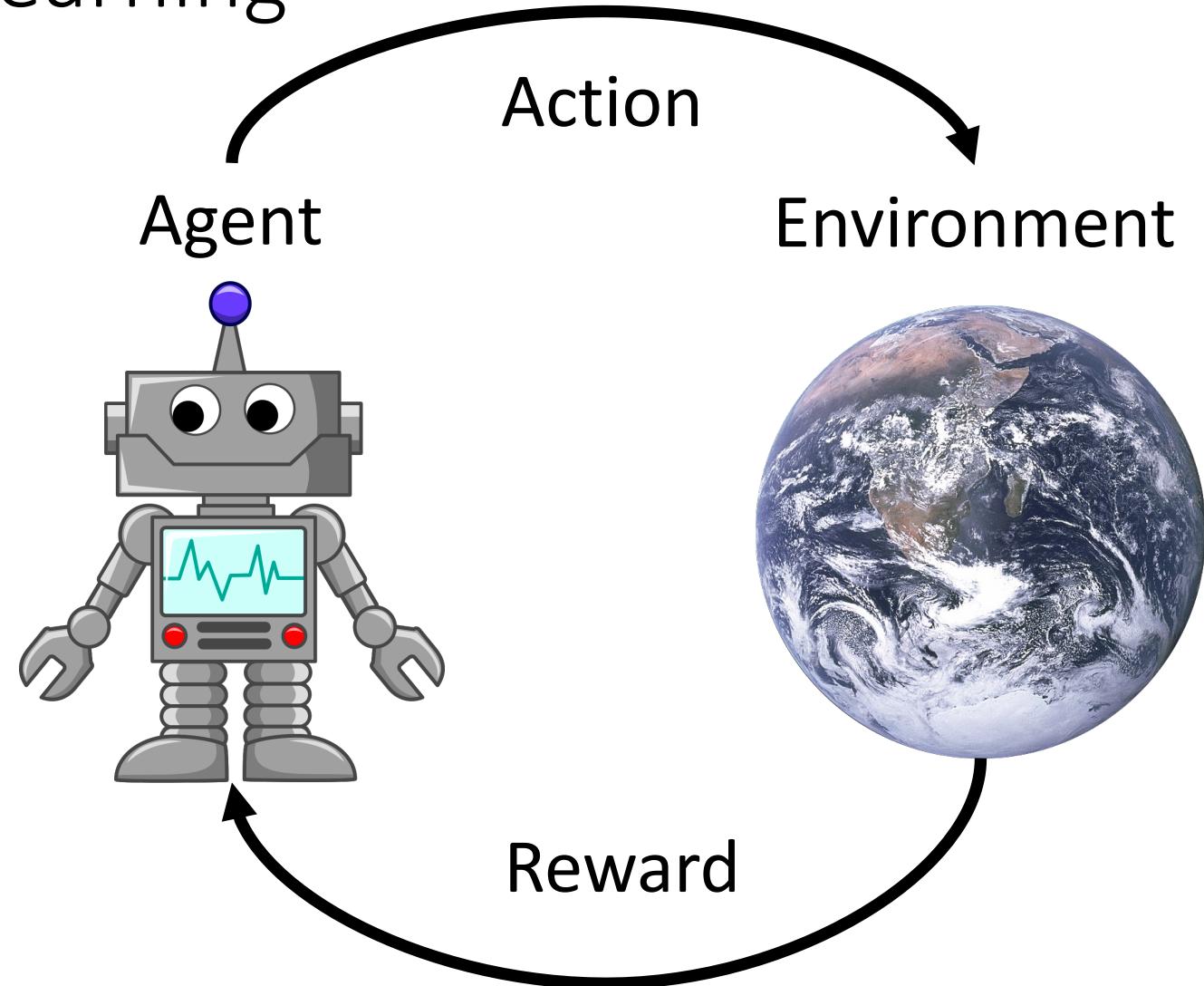
Feature Learning
(e.g. autoencoders)



Today: Reinforcement Learning

Problems where an **agent** performs **actions** in **environment**, and receives **rewards**

Goal: Learn how to take actions that maximize reward



[Earth photo](#) is in the public domain
[Robot image](#) is in the public domain

Overview

- What is reinforcement learning?
- Algorithms for reinforcement learning
 - Q-Learning
 - Policy Gradients

Reinforcement Learning

Environment

Agent

Reinforcement Learning

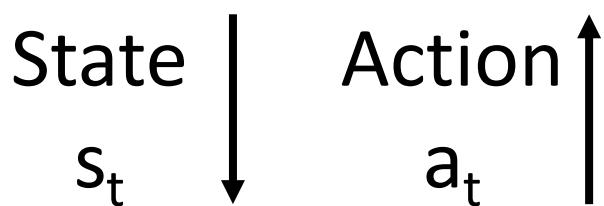
Environment

State
 s_t

The agent sees a **state**; may be noisy or incomplete

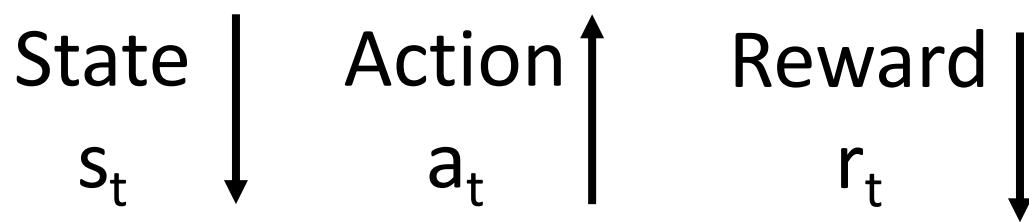
Agent

Reinforcement Learning



The makes an **action** based on what it sees

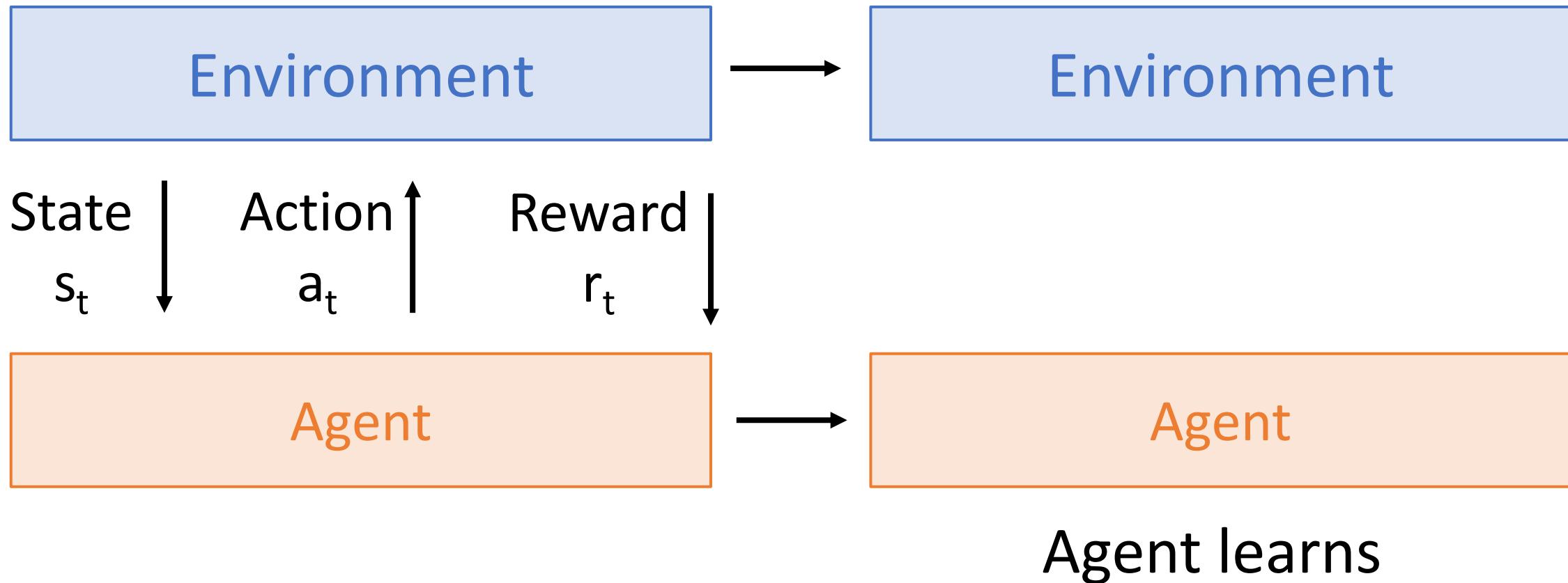
Reinforcement Learning



Reward tells the agent how well it is doing

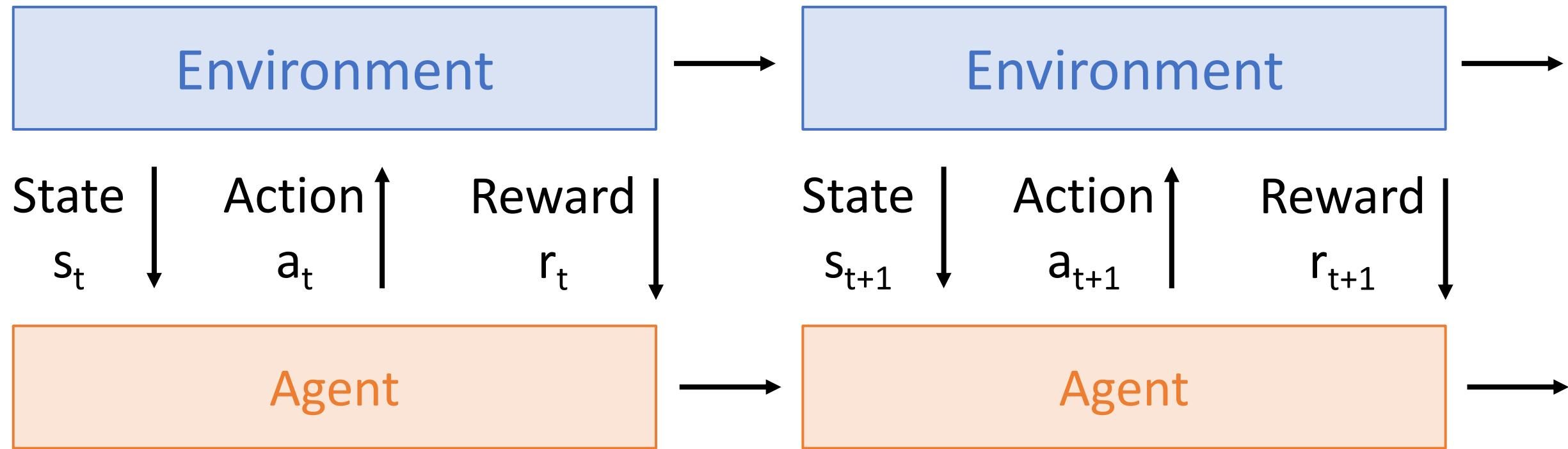
Reinforcement Learning

Action causes change
to environment

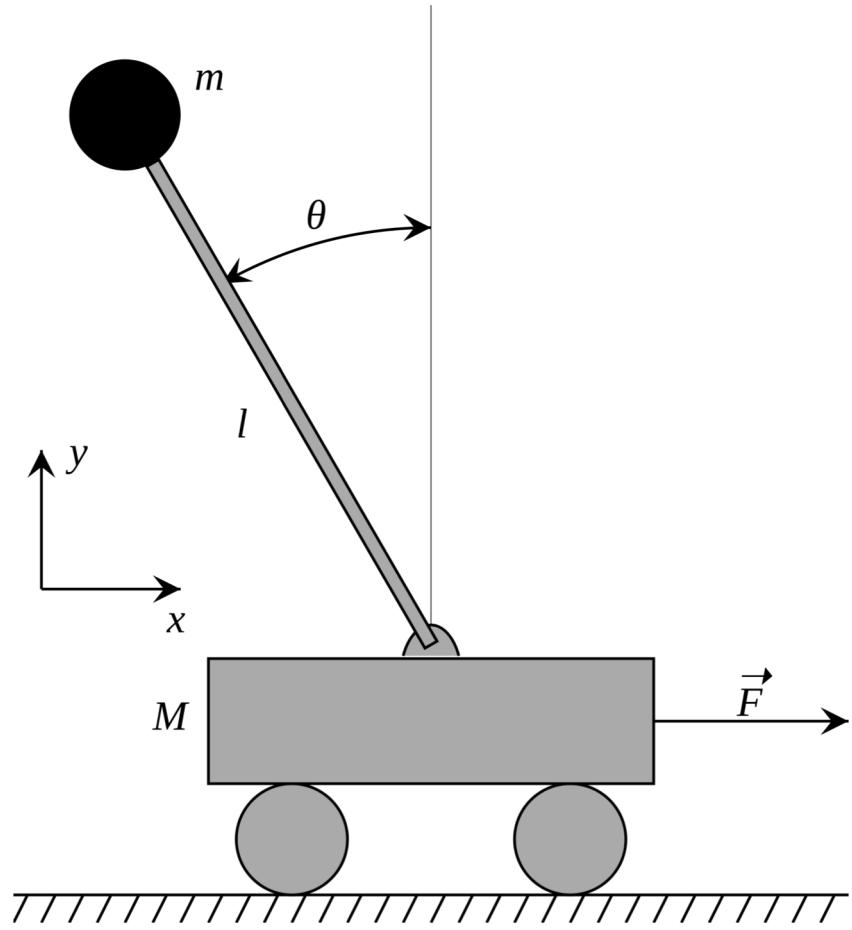


Reinforcement Learning

Process repeats



Example: Cart-Pole Problem



Objective: Balance a pole on top of a movable cart

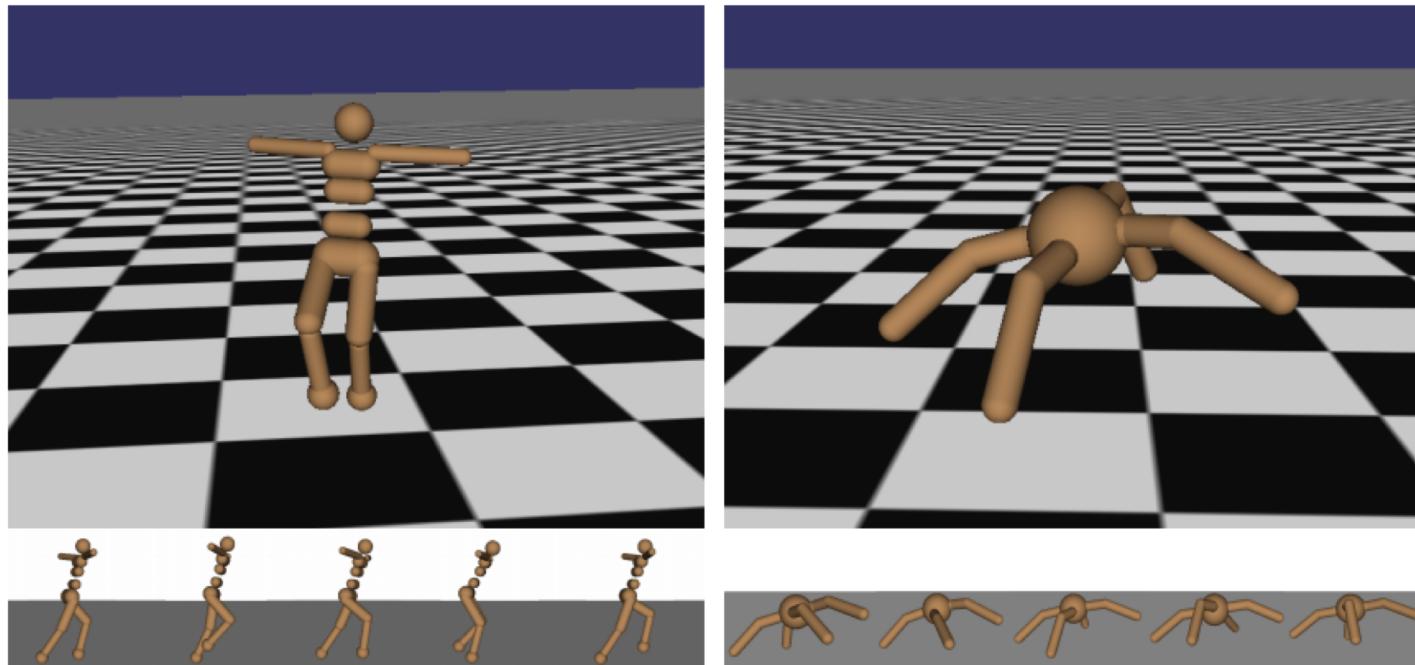
State: angle, angular speed, position, horizontal velocity

Action: horizontal force applied on the cart

Reward: 1 at each time step if the pole is upright

This image is [CC0 public domain](#)

Example: Robot Locomotion



Objective: Make the robot move forward

State: Angle, position, velocity of all joints

Action: Torques applied on joints

Reward: 1 at each time step upright + forward movement

Figure from: Schulman et al, "High-Dimensional Continuous Control Using Generalized Advantage Estimation", ICLR 2016

Example: Atari Games



Objective: Complete the game with the highest score

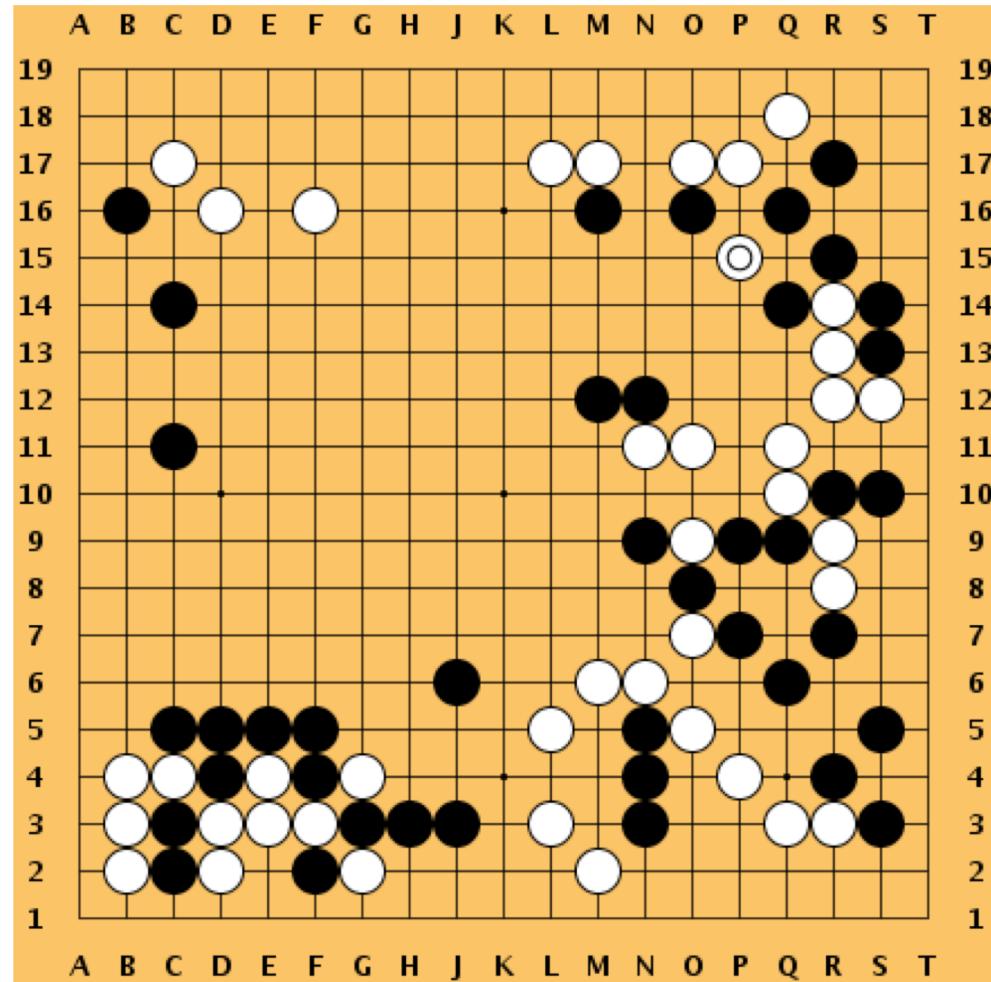
State: Raw pixel inputs of the game screen

Action: Game controls e.g. Left, Right, Up, Down

Reward: Score increase/decrease at each time step

Mnih et al, "Playing Atari with Deep Reinforcement Learning", NeurIPS Deep Learning Workshop, 2013

Example: Go



Objective: Win the game!

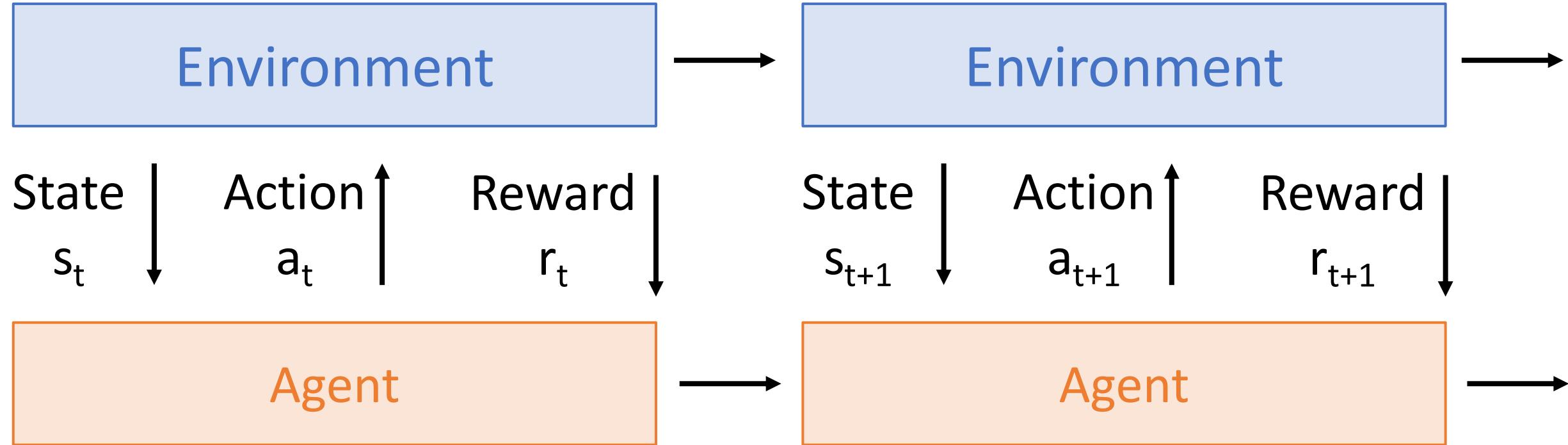
State: Position of all pieces

Action: Where to put the next piece down

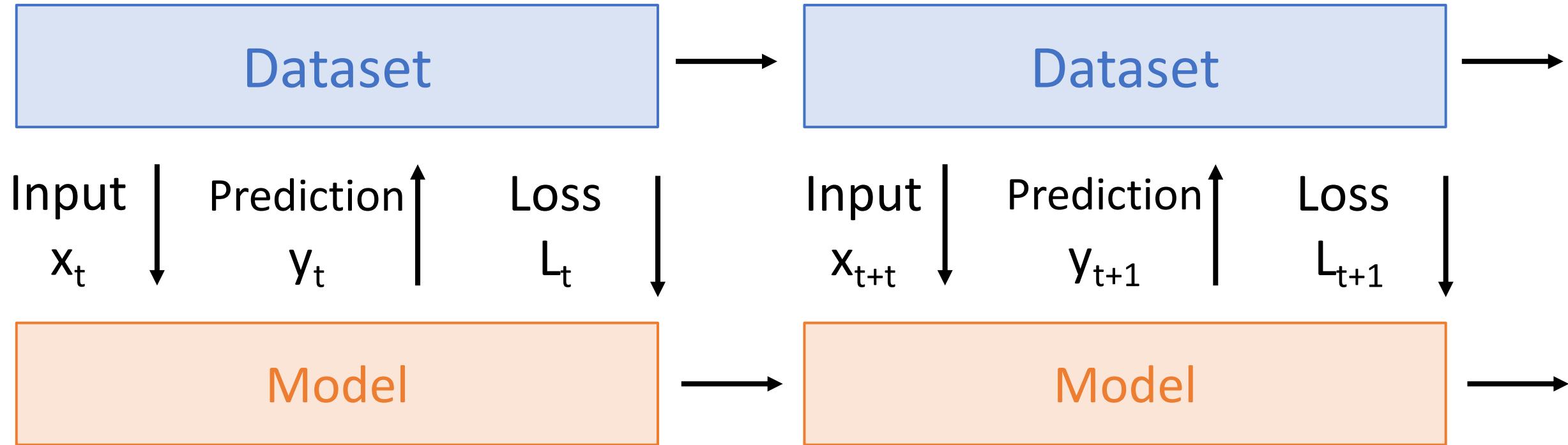
Reward: On last turn: 1 if you won, 0 if you lost

This image is CC0 public domain

Reinforcement Learning vs Supervised Learning

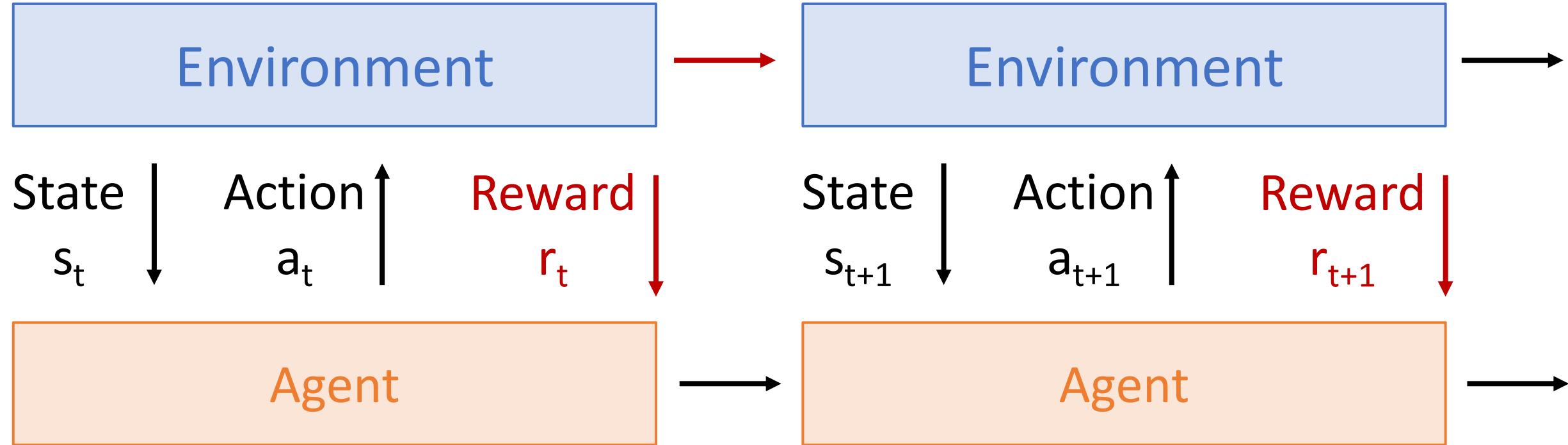


Reinforcement Learning vs Supervised Learning



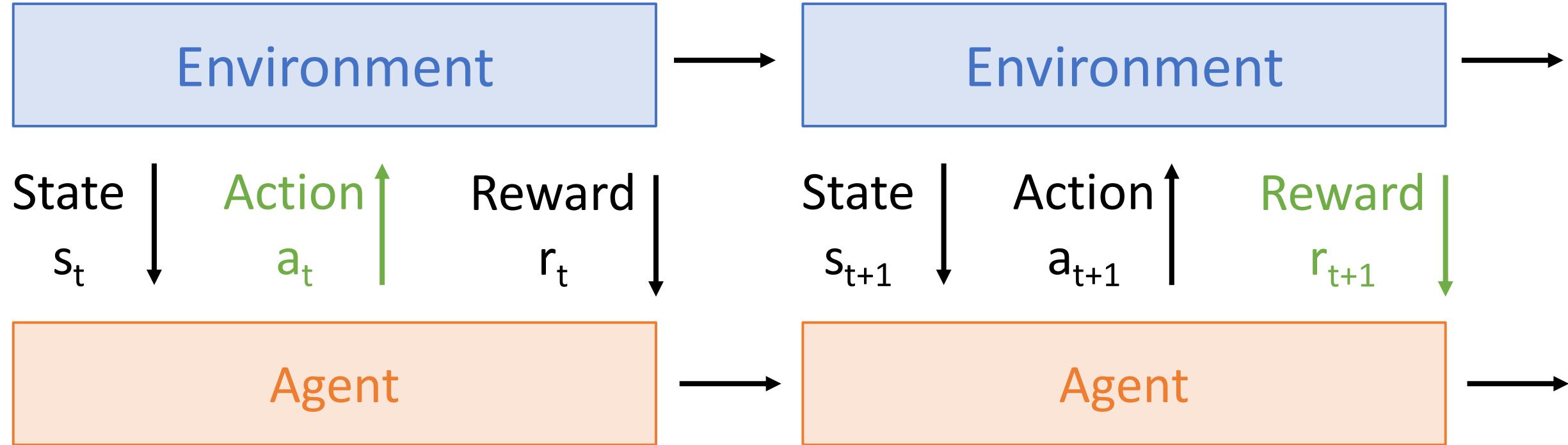
Why is RL different from normal supervised learning?

Reinforcement Learning vs Supervised Learning



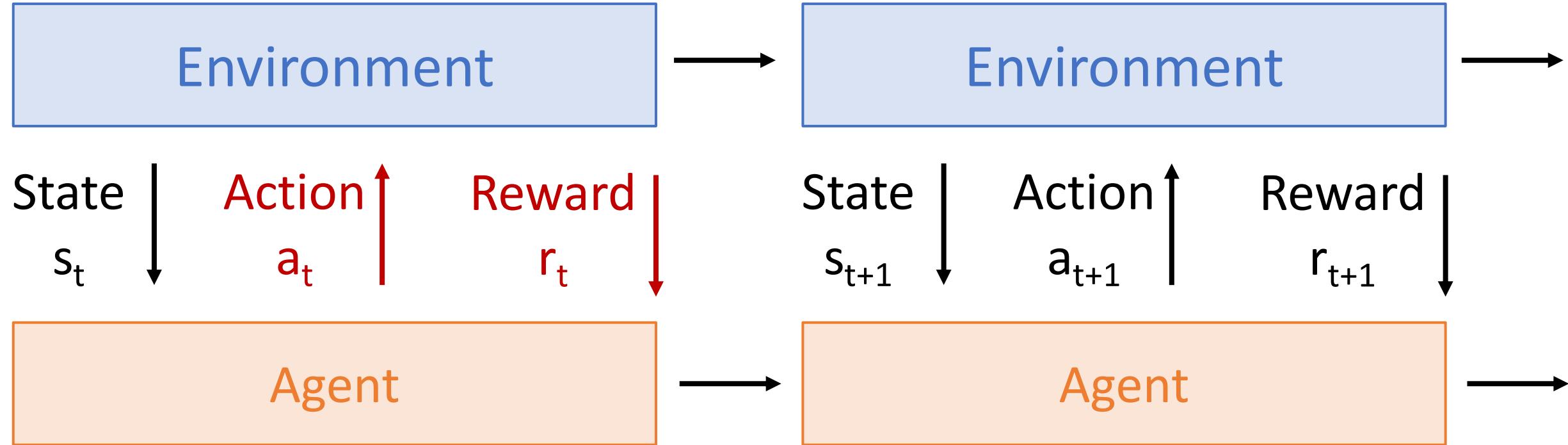
Stochasticity: Rewards and state transitions may be random

Reinforcement Learning vs Supervised Learning



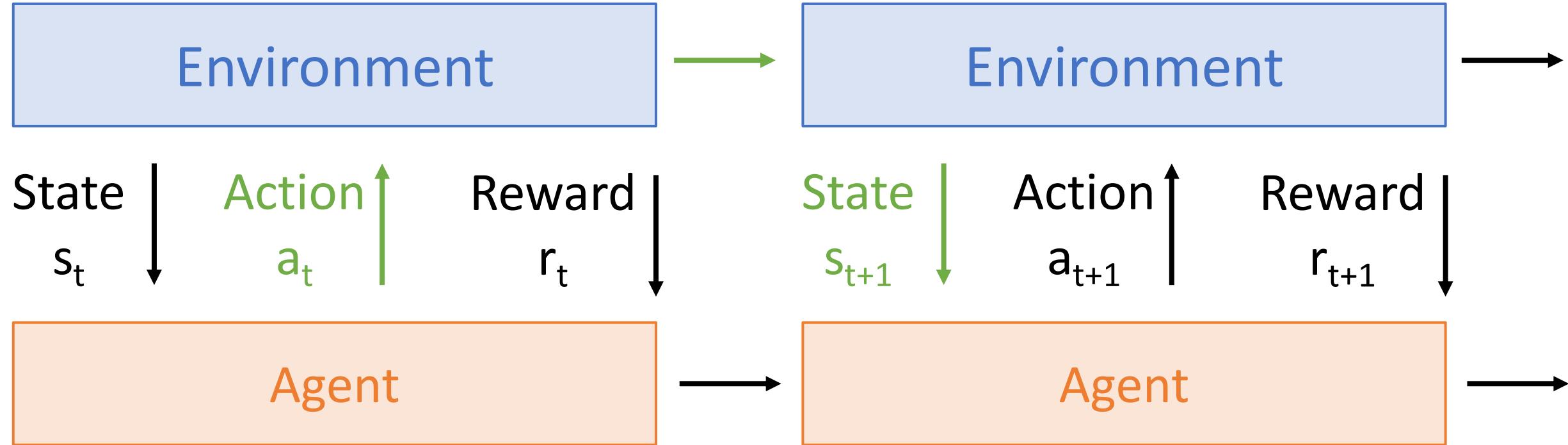
Credit assignment: Reward r_t may not directly depend on action a_t

Reinforcement Learning vs Supervised Learning



Nondifferentiable: Can't backprop through world; can't compute dr_t/da_t

Reinforcement Learning vs Supervised Learning



Nonstationary: What the agent experiences depends on how it acts

Markov Decision Process (MDP)

Mathematical formalization of the RL problem: A tuple (S, A, R, P, γ)

S: Set of possible states

A: Set of possible actions

R: Distribution of reward given (state, action) pair

P: Transition probability: distribution over next state given (state, action)

γ : Discount factor (tradeoff between future and present rewards)

Markov Property: The current state completely characterizes the state of the world. Rewards and next states depend only on current state, not history.

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Goal: Find policy π^* that maximizes cumulative discounted reward: $\sum_t \gamma^t r_t$

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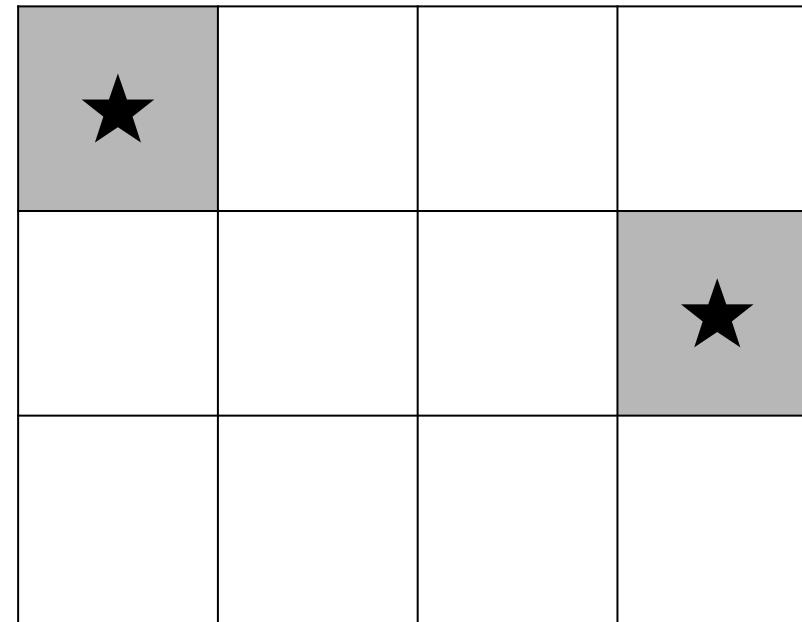
- At time step $t=0$, environment samples initial state $s_0 \sim p(s_0)$
- Then, for $t=0$ until done:
 - Agent selects action $a_t \sim \pi(a | s_t)$
 - Environment samples reward $r_t \sim R(r | s_t, a_t)$
 - Environment samples next state $s_{t+1} \sim P(s | s_t, a_t)$
 - Agent receives reward r_t and next state s_{t+1}

A simple MDP: Grid World

Actions:

1. Right
2. Left
3. Up
4. Down

States



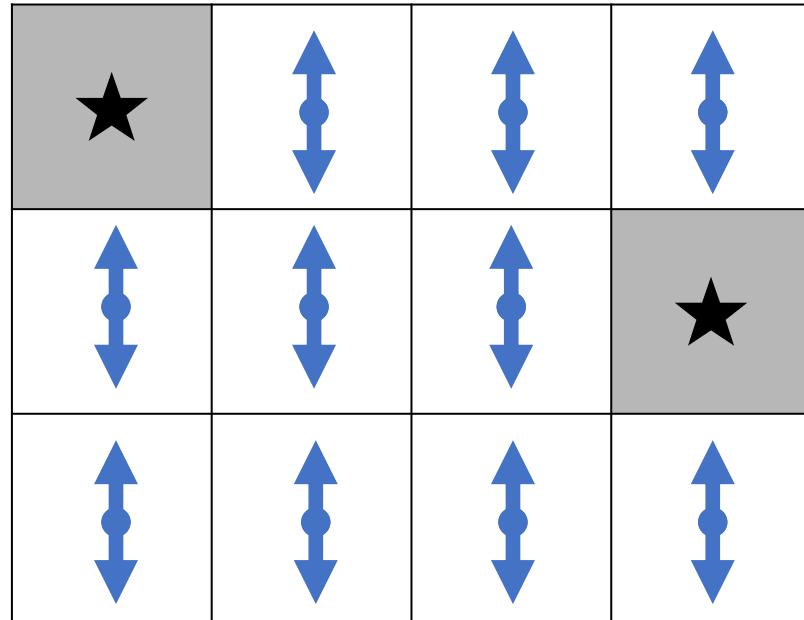
Reward

Set a negative “reward” for each transition
(e.g. $r = -1$)

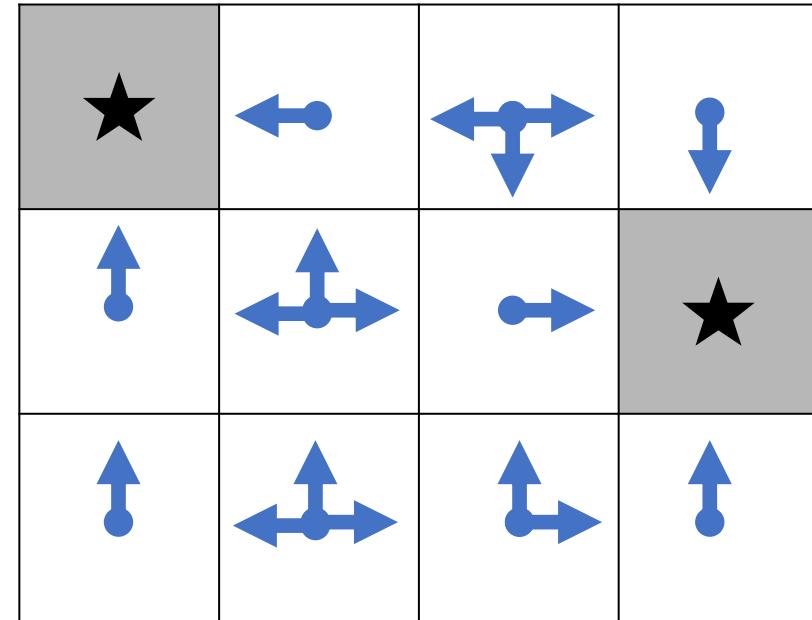
Objective: Reach one of the terminal states in as few moves as possible

A simple MDP: Grid World

Bad policy



Optimal Policy



Finding Optimal Policies

Goal: Find the optimal policy π^* that maximizes (discounted) sum of rewards.

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Problem: Lots of randomness! Initial state, transition probabilities, rewards

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Solution: Maximize the expected sum of rewards

$$\pi^* = \arg \max_{\pi} \mathbb{E} \left[\sum_{t \geq 0} \gamma^t r_t \mid \pi \right]$$

$$\begin{aligned} s_0 &\sim p(s_0) \\ a_t &\sim \pi(a \mid s_t) \\ s_{t+1} &\sim P(s \mid s_t, a_t) \end{aligned}$$

Value Function and Q Function

Following a policy π produces **sample trajectories** (or paths) $s_0, a_0, r_0, s_1, a_1, r_1, \dots$

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How good is a state? The **value function** at state s , is the expected cumulative reward from following the policy from state s :

$$V^\pi(s) = \mathbb{E} \left[\sum_{t \geq 0} \gamma^t r_t \mid s_0 = s, \pi \right]$$

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How good is a state-action pair? The **Q function** at state s and action a , is the expected cumulative reward from taking action a in state s and then following the policy:

$$Q^\pi(s, a) = \mathbb{E} \left[\sum_{t \geq 0} \gamma^t r_t \mid s_0 = s, a_0 = a, \pi \right]$$

Bellman Equation

Optimal Q-function: $Q^*(s, a)$ is the Q-function for the optimal policy π^*

It gives the max possible future reward when taking action a in state s :

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Bellman Equation: Q^* satisfies the following recurrence relation:

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Where $r \sim R(s, a), s' \sim P(s, a)$

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Intuition: After taking action a in state s , we get reward r and move to a new state s' . After that, the max possible reward we can get is $\max_{a'} Q^*(s', a')$

Solving for the optimal policy: Value Iteration

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Idea: If we find a function $Q(s, a)$ that satisfies the Bellman Equation, then it must be Q^* .

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Idea: If we find a function $Q(s, a)$ that satisfies the Bellman Equation, then it must be Q^* . Start with a random Q , and use the Bellman Equation as an update rule:

$$Q_{i+1}(s, a) = \mathbb{E}_{r, s'} \left[r + \gamma \max_{a'} Q_i(s', a') \right]$$

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Solution: Approximate $Q(s, a)$ with a neural network, use Bellman Equation as loss!

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Use the Bellman Equation to tell what Q should output for a given state and action:

$$y_{s,a,\theta} = \mathbb{E}_{r,s'} \left[r + \gamma \max_{a'} Q(s', a'; \theta) \right]$$

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Use this to define the loss for training Q : $L(s, a) = (Q(s, a; \theta) - y_{s,a,\theta})^2$

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Problem: How to sample batches of data for training?

Case Study: Playing Atari Games



Objective: Complete the game with the highest score

State: Raw pixel inputs of the game screen

Action: Game controls e.g. Left, Right, Up, Down

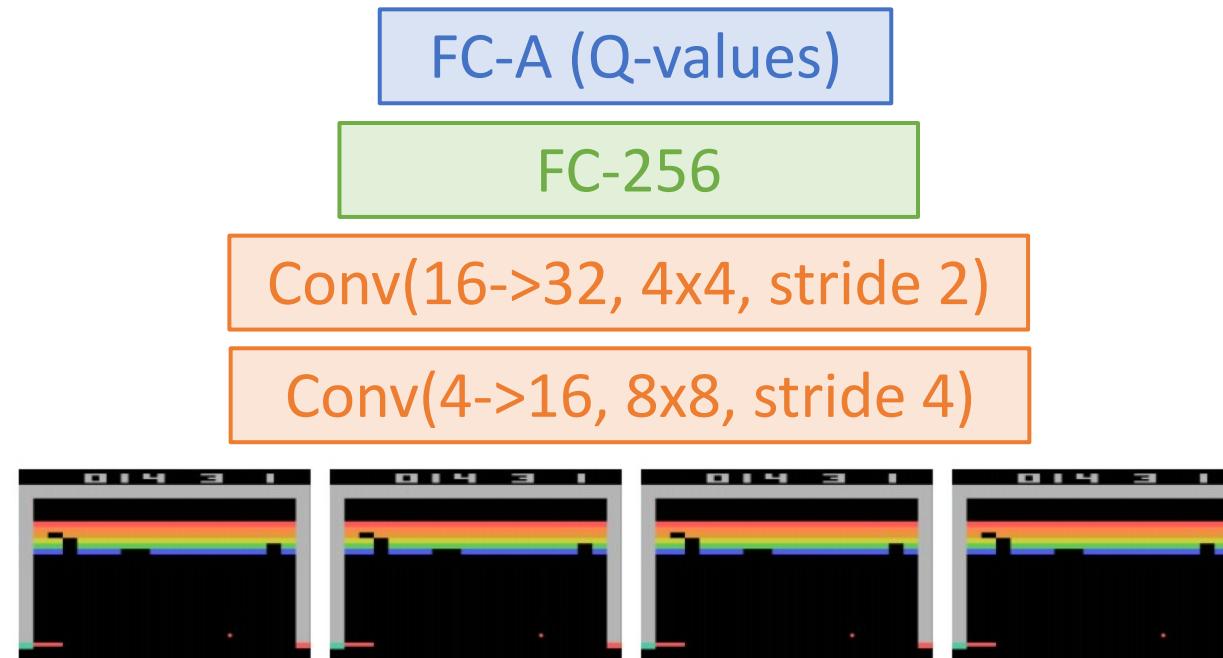
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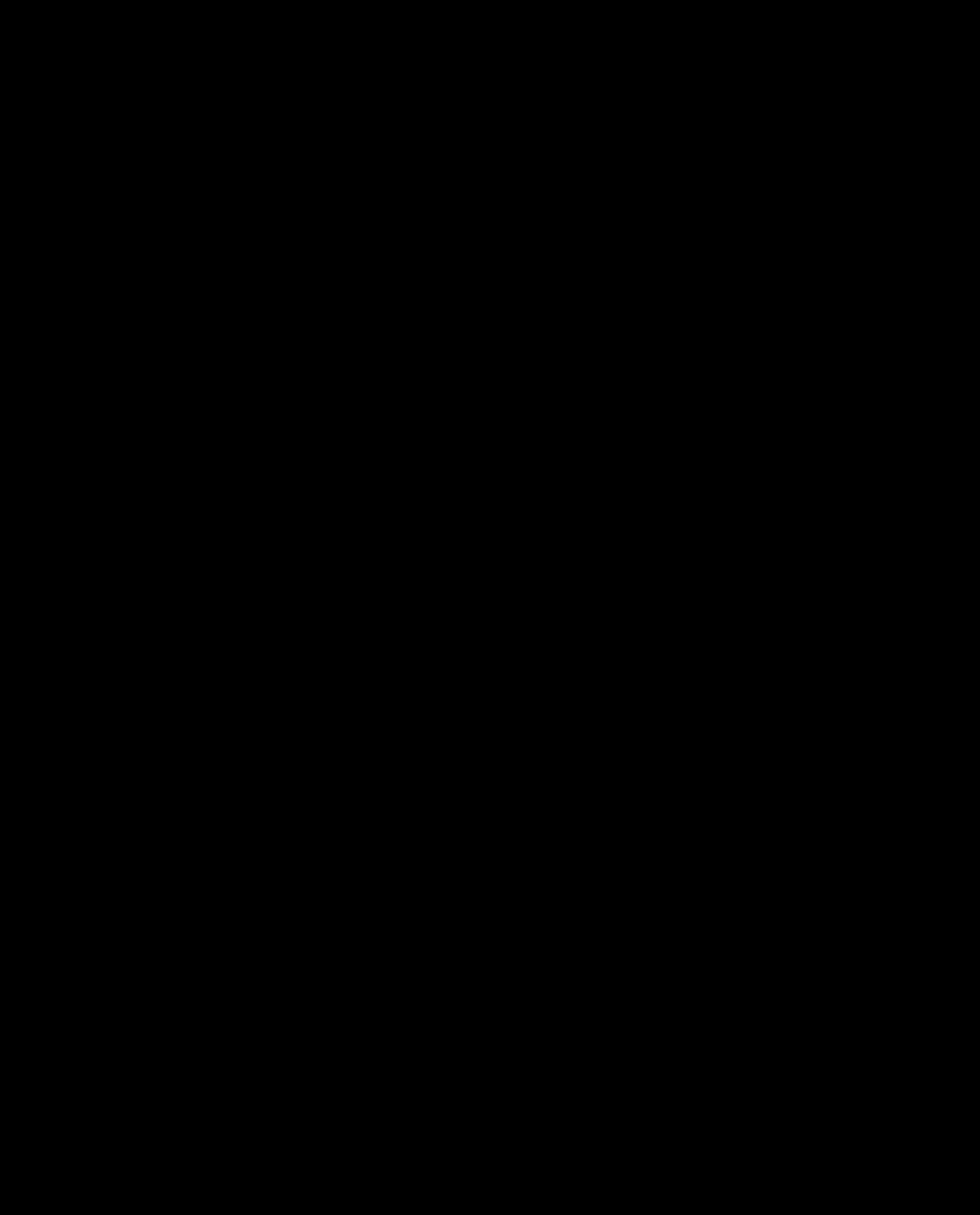
$Q(s, a; \theta)$
Neural network
with weights θ

Network output:
Q-values for all actions



With 4 actions: last
layer gives values
 $Q(s_t, a_1), Q(s_t, a_2),$
 $Q(s_t, a_3), Q(s_t, a_4)$

Network input: state s_t : 4x84x84 stack of last 4 frames
(after RGB->grayscale conversion, downsampling, and cropping)



<https://www.youtube.com/watch?v=V1eYniJ0Rnk>

Q-Learning

Q-Learning: Train network $Q_\theta(s, a)$ to estimate future rewards for every (state, action) pair

Problem: For some problems this can be a hard function to learn.

For some problems it is easier to learn a mapping from states to actions

Q-Learning vs Policy Gradients

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Objective function: Expected future rewards when following policy π_θ :

$$J(\theta) = \mathbb{E}_{r \sim p_\theta} \left[\sum_{t \geq 0} \gamma^t r_t \right]$$

Find the optimal policy by maximizing: $\theta^* = \arg \max_\theta J(\theta)$ **(Use gradient ascent!)**

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Policy Gradients: REINFORCE Algorithm

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$$\frac{\partial J}{\partial \theta} = \frac{\partial}{\partial \theta} \mathbb{E}_{x \sim p_\theta}[f(x)] = \frac{\partial}{\partial \theta} \int_X p_\theta(x)f(x)dx$$

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$$\frac{\partial}{\partial \theta} \log p_\theta(x)$$

Policy Gradients: REINFORCE Algorithm

General formulation: $J(\theta) = \mathbb{E}_{x \sim p_\theta}[f(x)]$ Want to compute $\frac{\partial J}{\partial \theta}$

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$$\frac{\partial}{\partial \theta} \log p_\theta(x) = \frac{1}{p_\theta(x)} \frac{\partial}{\partial \theta} p_\theta(x)$$

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$$\frac{\partial J}{\partial \theta} = \int_X f(x) p_\theta(x) \frac{\partial}{\partial \theta} \log p_\theta(x) dx = \mathbb{E}_{x \sim p_\theta} \left[f(x) \frac{\partial}{\partial \theta} \log p_\theta(x) \right]$$

Approximate the expectation via sampling!

Policy Gradients: REINFORCE Algorithm

Goal: Train a network $\pi_\theta(a | s)$ that takes state as input, gives distribution over which action to take in that state

Define: Let $x = (s_0, a_0, s_1, a_1, \dots)$ be the sequence of states and actions we get when following policy π_θ . It's random: $x \sim p_\theta(x)$

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Sequence of states
and actions when
following policy π_θ

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Reward we get from
state sequence x

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Gradient of predicted action scores with respect to model weights. Backprop through model π_θ !

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1. Initialize random weights θ

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1. Initialize random weights θ
2. Collect trajectories x and rewards $f(x)$ using policy π_θ
3. Compute $dJ/d\theta$

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1. Initialize random weights θ
2. Collect trajectories x and rewards $f(x)$ using policy π_θ
3. Compute $dJ/d\theta$
4. Gradient ascent step on θ
5. GOTO 2

Policy Gradients: REINFORCE Algorithm

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Intuition:

When $f(x)$ is high: Increase the probability of the actions we took.

When $f(x)$ is low: Decrease the probability of the actions we took.

So far: Q-Learning and Policy Gradients

Q-Learning: Train network $Q_\theta(s, a)$ to estimate future rewards for every (state, action) pair

Use Bellman Equation to define loss function for training Q:

$$y_{s,a,\theta} = \mathbb{E}_{r,s'} \left[r + \gamma \max_{a'} Q(s', a'; \theta) \right] \quad \text{Where } r \sim R(s, a), s' \sim P(s, a)$$
$$L(s, a) = (Q(s, a; \theta) - y_{s,a,\theta})^2$$

Policy Gradients: Train a network $\pi_\theta(a | s)$ that takes state as input, gives distribution over which action to take in that state. Use REINFORCE Rule for computing gradients:

$$J(\theta) = \mathbb{E}_{x \sim p_\theta} [f(x)] \quad \frac{\partial J}{\partial \theta} = \mathbb{E}_{x \sim p_\theta} \left[f(x) \sum_{t \geq 0} \frac{\partial}{\partial \theta} \log \pi_\theta(a_t | s_t) \right]$$

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Improving policy gradients: Add **baseline** to reduce variance of gradient estimator

Other approaches: Model Based RL

Actor-Critic: Train an actor that predicts actions (like policy gradient) and a critic that predicts the future rewards we get from taking those actions (like Q-Learning)

Sutton and Barto, "Reinforcement Learning: An Introduction", 1998; Degris et al, "Model-free reinforcement learning with continuous action in practice", 2012; Mnih et al, "Asynchronous Methods for Deep Reinforcement Learning", ICML 2016

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Inverse Reinforcement Learning: Gather data of experts performing in environment; learn a reward function that they seem to be optimizing, then use RL on that reward function

Ng et al, "Algorithms for Inverse Reinforcement Learning", ICML 2000

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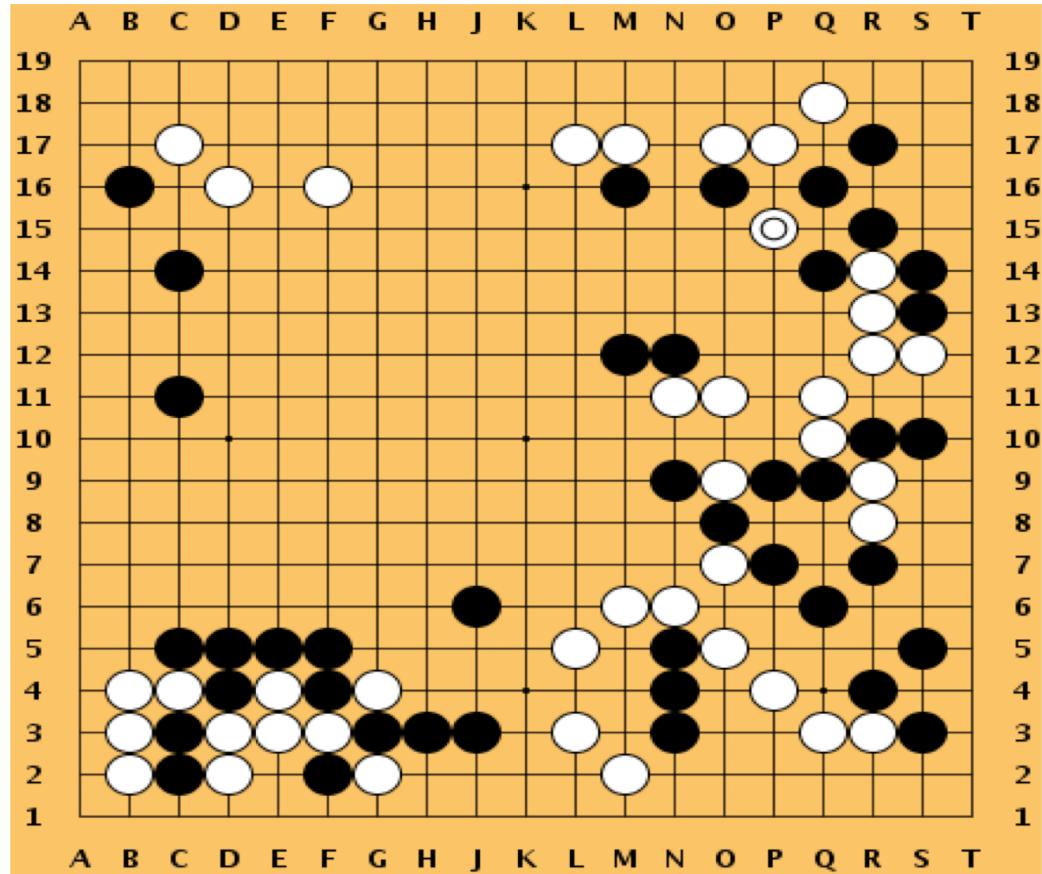
Adversarial Learning: Learn to fool a discriminator that classifies actions as real/fake

Ho and Ermon, "Generative Adversarial Imitation Learning", NeurIPS 2016

Case Study: Playing Games

AlphaGo: (January 2016)

- Used imitation learning + tree search + RL
- Beat 18-time world champion Lee Sedol



Silver et al, "Mastering the game of Go with deep neural networks and tree search", Nature 2016

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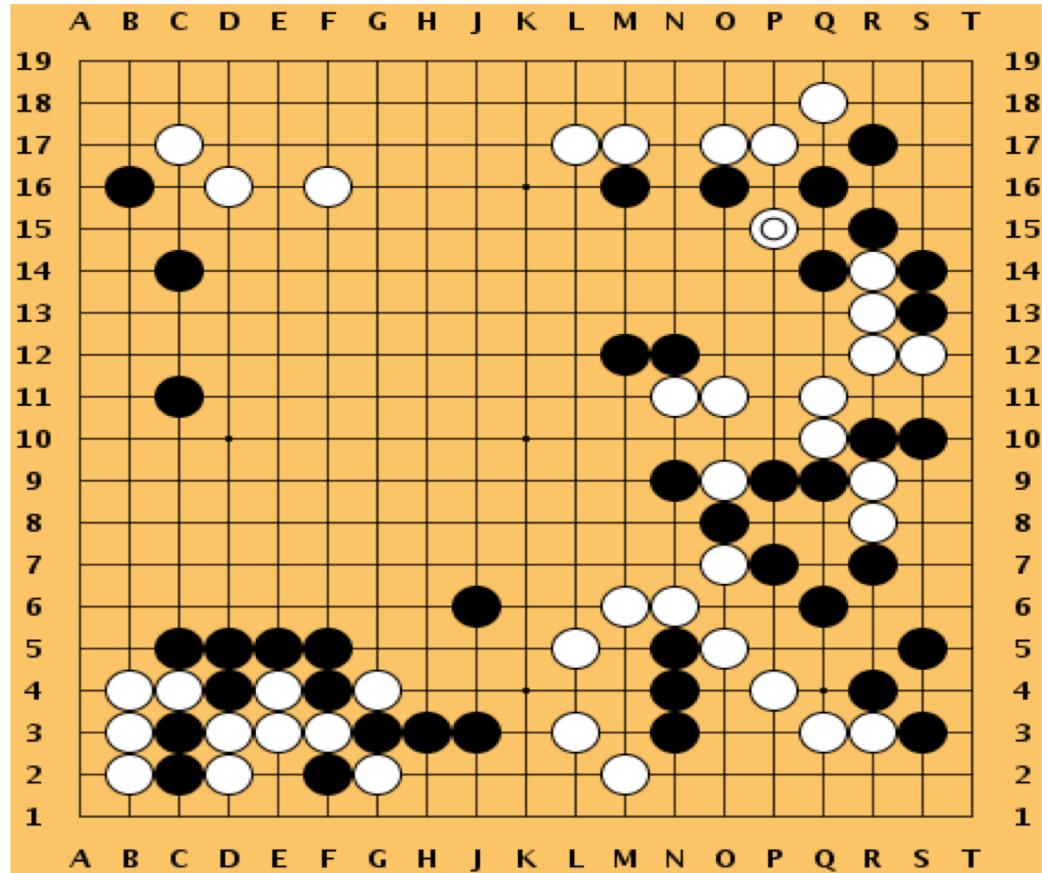
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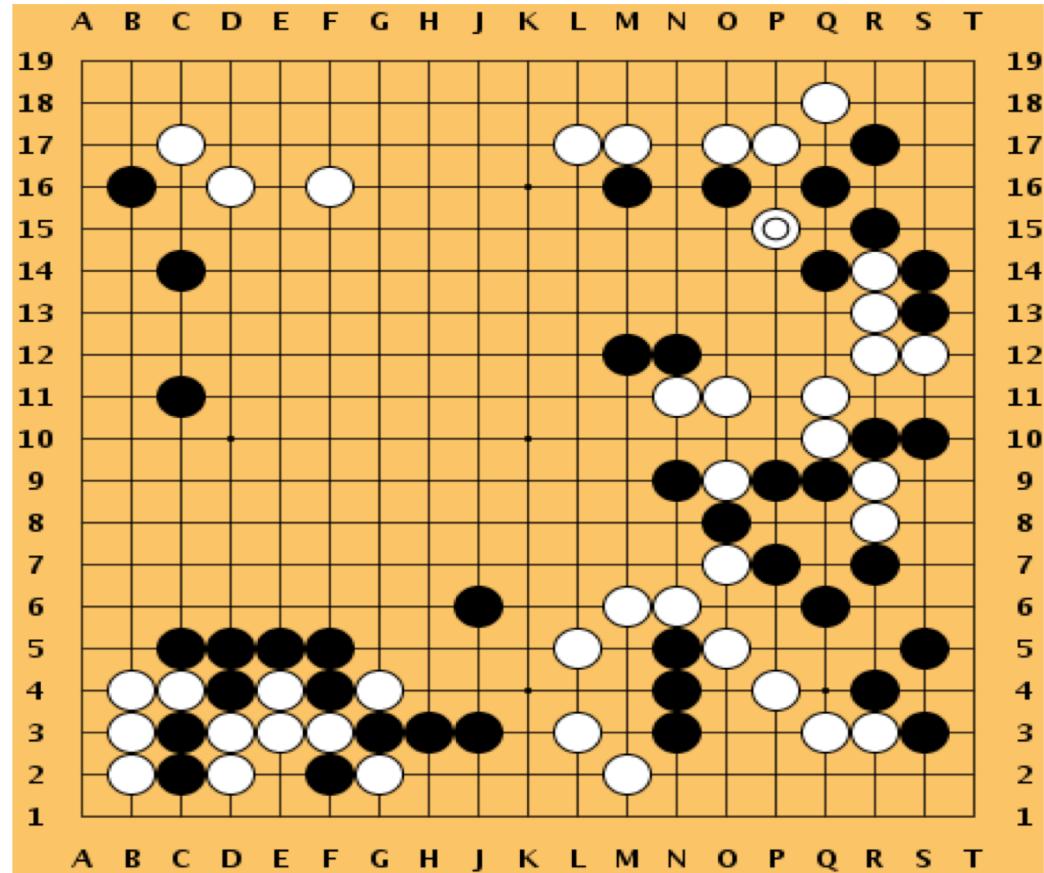
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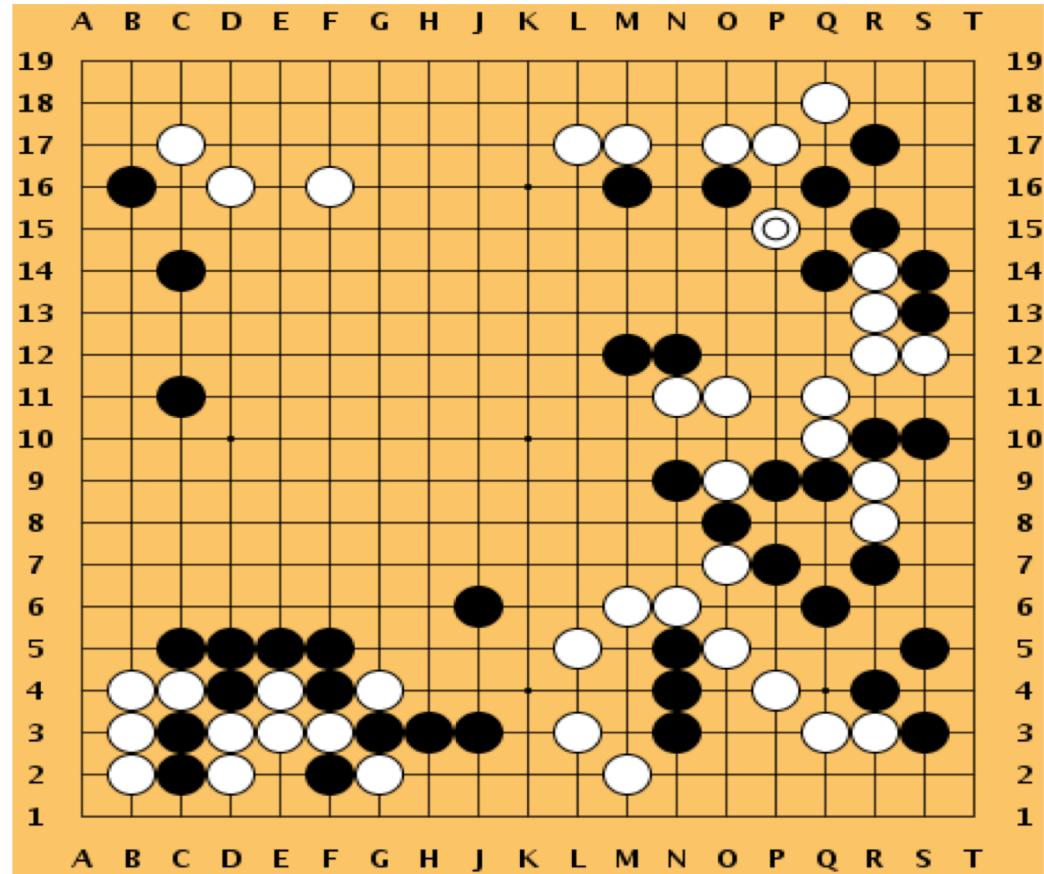
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November 2019: Lee Sedol announces retirement



“With the debut of AI in Go games, I've realized that I'm not at the top even if I become the number one through frantic efforts”

“Even if I become the number one, there is an entity that cannot be defeated”

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Quotes from: <https://en.yna.co.kr/view/AEN20191127004800315>
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More Complex Games

StarCraft II: AlphaStar

(October 2019)

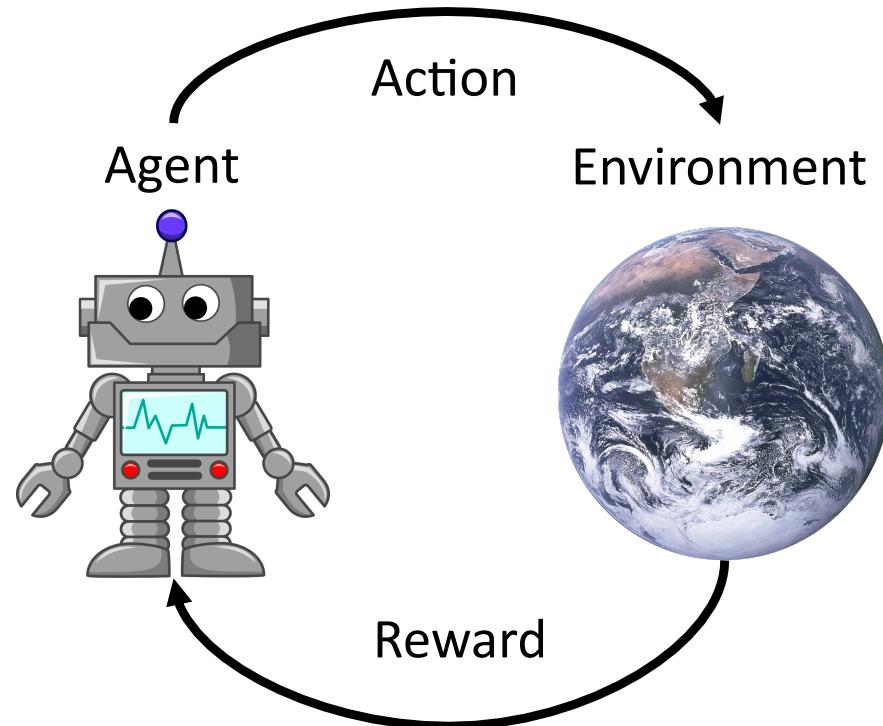
Vinyals et al, “Grandmaster level in StarCraft II using multi-agent reinforcement learning”, Science 2018

Dota 2: OpenAI Five (April 2019)

No paper, only a blog post:

<https://openai.com/five/#how-openai-five-works>

Reinforcement Learning: Interacting With World



Normally we use RL to train
agents that interact with a (noisy,
nondifferentiable) **environment**

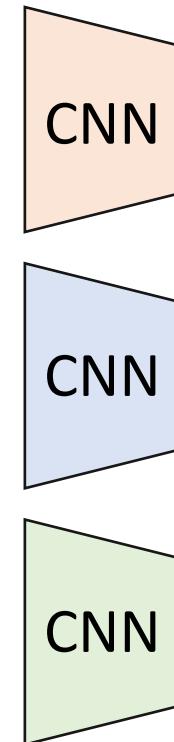
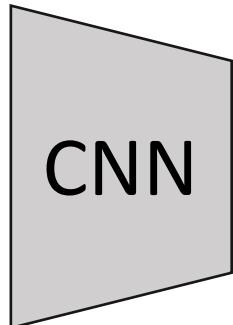
Reinforcement Learning: Stochastic Computation Graphs

Can also use RL to train neural networks with **nondifferentiable** components!

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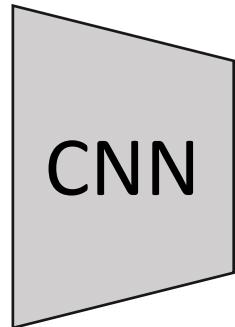
Example: Small “routing” network sends image to one of K networks



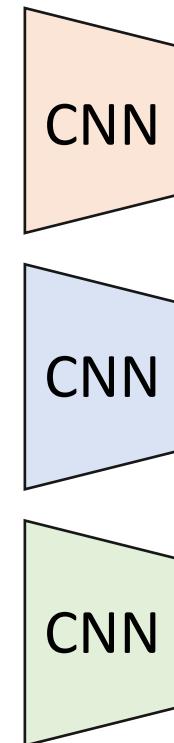
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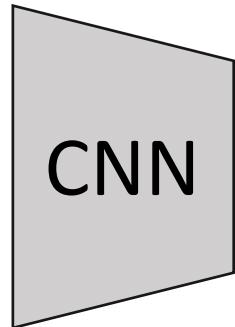
Which network
to use?
 $P(\text{orange}) = 0.2$
 $P(\text{blue}) = 0.1$
 $P(\text{green}) = 0.7$



Reinforcement Learning: Stochastic Computation Graphs

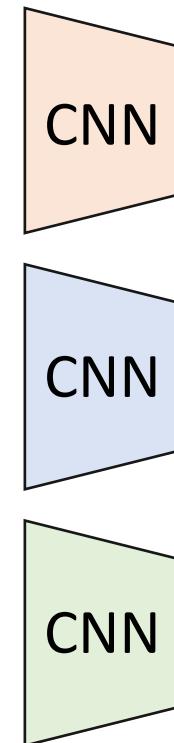
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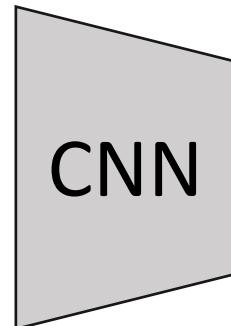
Sample:
Green



Reinforcement Learning: Stochastic Computation Graphs

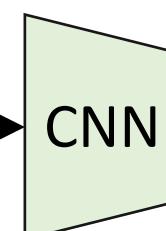
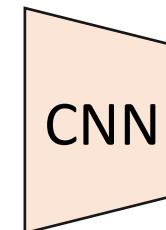
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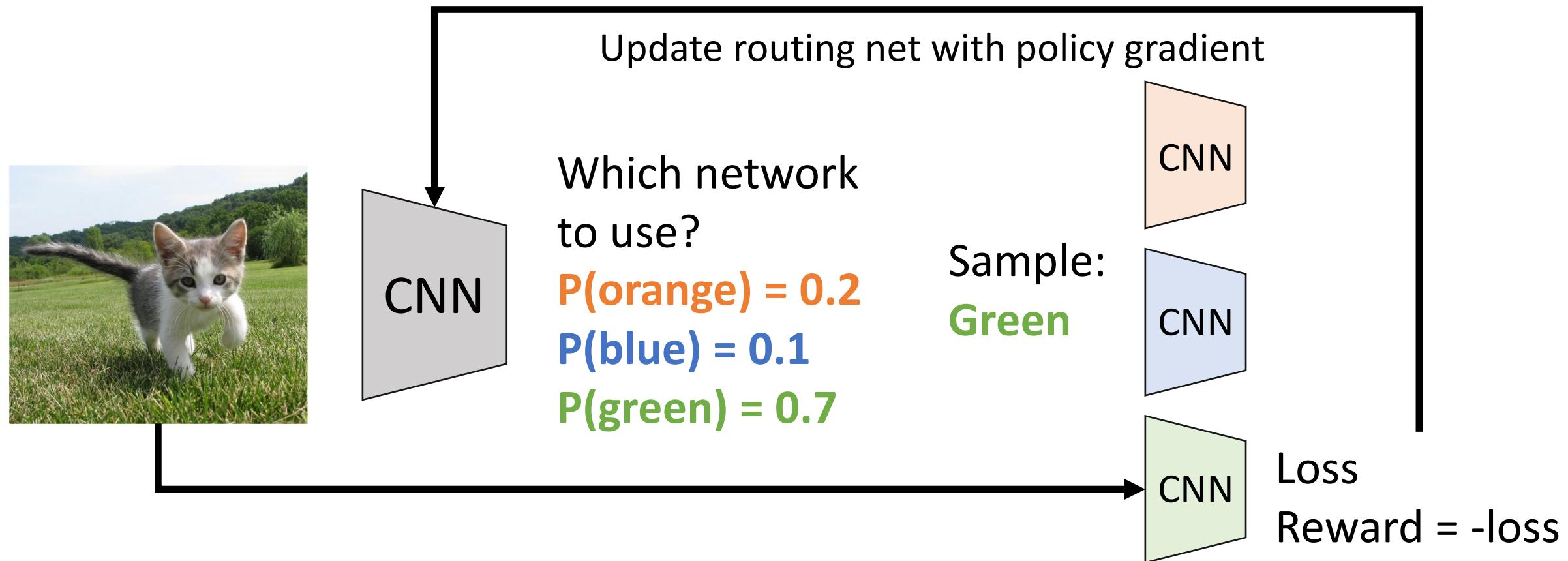


Loss
Reward = -loss

Reinforcement Learning: Stochastic Computation Graphs

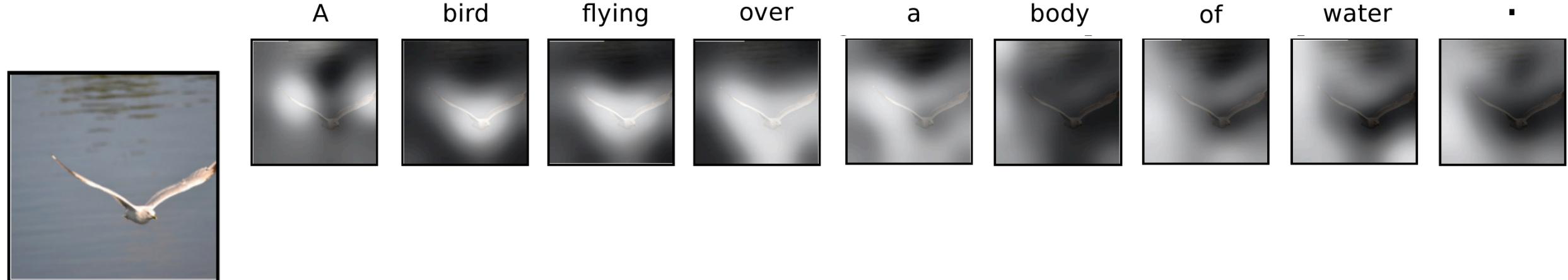
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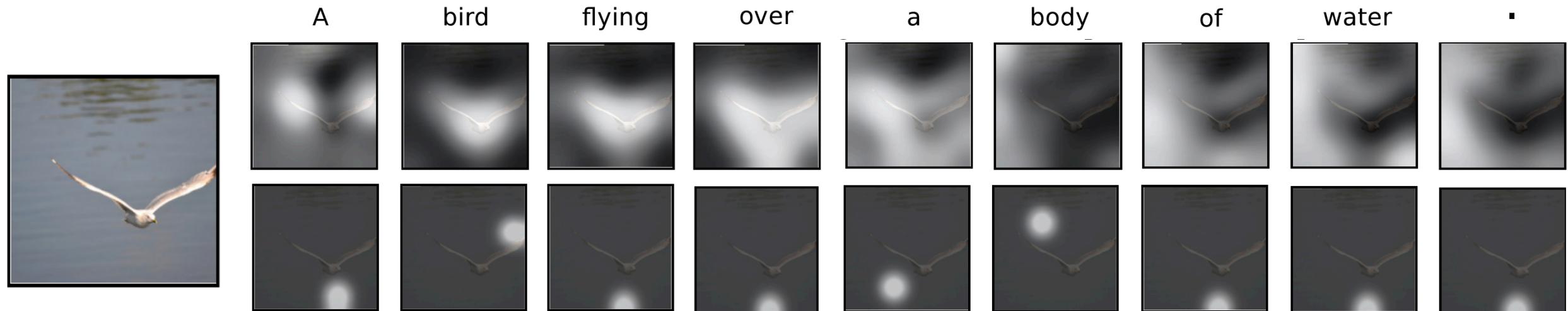
Stochastic Computation Graphs: Attention

Recall: Image captioning with attention. At each timestep use a weighted combination of features from different spatial positions
(Soft Attention)



Stochastic Computation Graphs: Attention

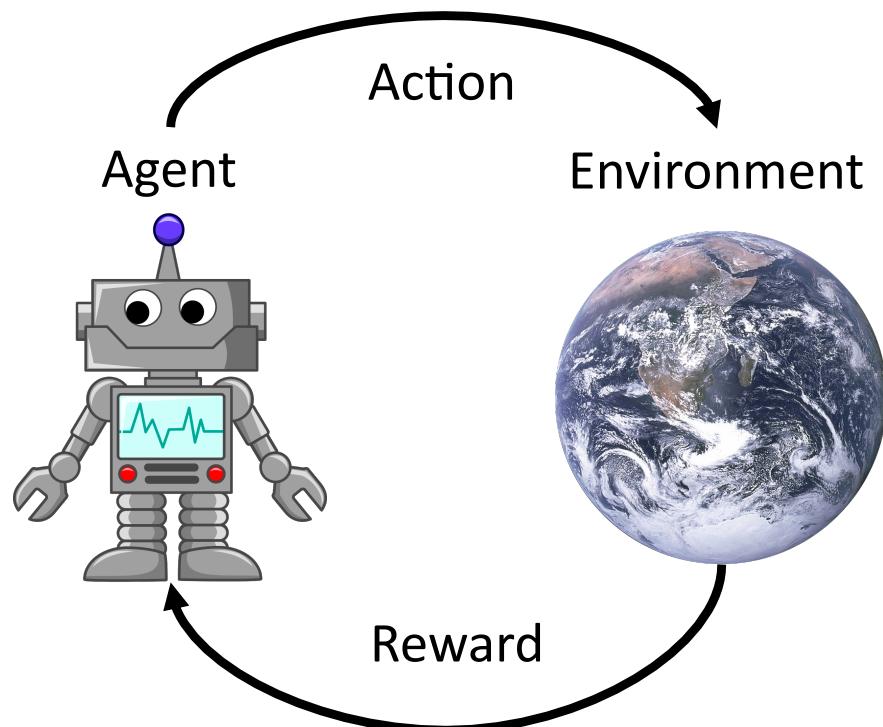
Recall: Image captioning with attention. At each timestep use a weighted combination of features from different spatial positions
(Soft Attention)



Hard Attention: At each timestep, select features from exactly one spatial location. Train with policy gradient.

Summary: Reinforcement Learning

RL trains **agents** that interact with an **environment** and learn to maximize **reward**



Q-Learning: Train network $Q_\theta(s, a)$ to estimate future rewards for every (state, action) pair. Use Bellman Equation to define loss function for training Q

Policy Gradients: Train a network $\pi_\theta(a | s)$ that takes state as input, gives distribution over which action to take in that state. Use REINFORCE Rule for computing gradients

Next Time: Course Recap Open Problems in Computer Vision