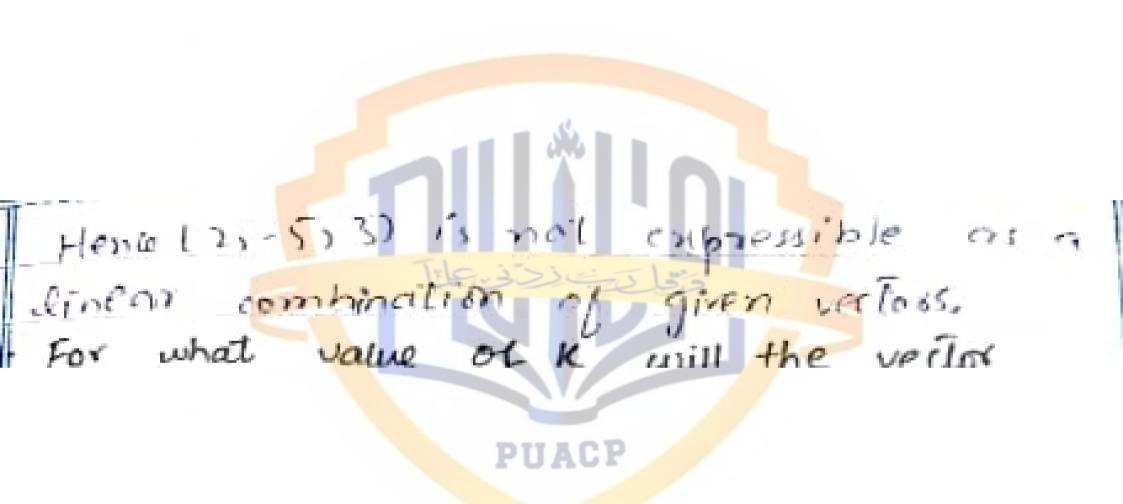
Express the vector (2,-5,3) in Rs as a linear combination of the vectors (1,-3,27, (2,-4)-1) and (1,-5,7) sol= let (2,-5,3)= 9(1)-3,2)+6(2,-4,-1)+(11,-5,7) (21-513) = (a+2b+()-3a-4b-5c) 2a-b+7c) 1+26+(=2 -7/2 -39-48-50=5 -113 29-6+70=3 -1(4) multiply 2 by 3, and add the result with Thon 2b-2c=1 i.e b-c=1-15 2 and subtract the Mulliphy (2) by result from 4 i.e b-(= 1 -16). -5b+5c=-1 Equations (3) Ep 6) are inconsistent. Thus it is not possible to find values of arb Epc To satisfy given system.



1		Date:
	Day:	
Lanning		
	(cz +c3+c4) a+3	
	at3	
-	ar3 1 a	
·	a+3 1 1 a 1.	
<u></u>		
	(2+3)	
_		
	1 - 1 - a .	
n2n	ويُقا لِهِ بِي الْحِيدِ عِلَمًا الْمِ	
R3-R1	(a+3) 0 a-1 0 0	
Ru-RI	0 0 0 0 0 0	
	(a+3) a-IPUACP o	
	(a+s)	
		bounding from
	a+3)(a-1)3/ 0 0/	
	0 0 1	
	2 (a+3) (a-1)3 Hence proved.	
#		

Example 20. What conditions must
$$a$$
, b , c and d satisfy so that the matrices
$$\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 3 \\ -2 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

in M22 are linearly dependent ?

Solution. Since the matrices are to be linearly dependent, there must exist scal c1, c2, c3, not all zero, such that

$$c_{1}\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} + c_{2}\begin{bmatrix} 2 & 3 \\ -2 & 1 \end{bmatrix} + c_{3}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

This implies

$$\begin{bmatrix} c_1 + 2c_2 + ac_3 & 2c_1 + 3c_2 + bc_3 \\ -c_1 - 2c_2 + cc_3 & c_2 + dc_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

i.e., the system of homogeneous equations

$$c_{1} + 2c_{2} + ac_{3} = 0$$

$$2c_{1} + 3c_{2} + bc_{3} = 0$$

$$-c_{1} - 2c_{2} + cc_{3} = 0$$

$$c_{2} + dc_{3} = 0$$

must have a nontrivial solution.

By Theorem 4.12, the system (2) will have a nontrivial solution if r matrix

$$A = \begin{bmatrix} 1 & 2 & a \\ 2 & 3 & b \\ -1 & -2 & c \\ 0 & 1 & d \end{bmatrix}$$

is less than 3 i.e., Rank A = 2 at the most.

Now,

$$A^{R} \begin{bmatrix} 1 & 2 & a \\ 0 & -1 & b-2a \\ 0 & 0 & c+a \\ 0 & 1 & d \end{bmatrix} \quad \begin{array}{c} by & R_{2}-2R_{1} \\ and & R_{3}+R_{1} \end{array}$$

$$R \begin{bmatrix} 1 & 2 & a \\ 0 & 1 & d \\ 0 & -1 & b-2a \\ 0 & 0 & c+a \\ 0 & 0 & b+d-2a \end{bmatrix} \quad \begin{array}{c} by & R_{4}+R_{2}. \\ by & R_{4}+R_{2}. \end{array}$$

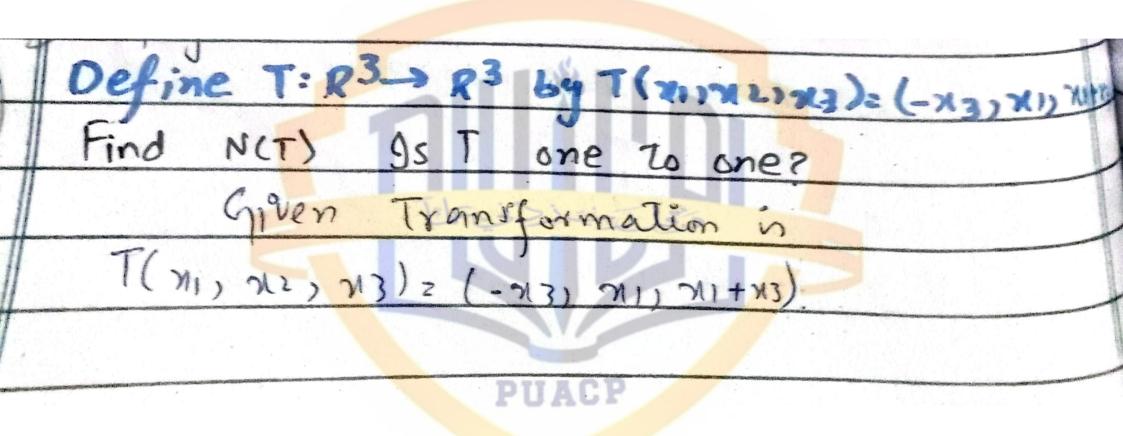
The last matrix has rank 2 at the most only if

$$c+a=0$$
 and $b+d-2a=0$.

Thus the required conditions on a. b. c. d are

$$a+c=0 \quad \text{and} \quad b+d-2a=0.$$

Note that, for a = b = d = 1, c = -1 and $c_1 = 1$, $c_2 = -1$, $c_3 = 1$, the equation (1) satisfied.



DATE: __/_ N(T)= 3 (MI) X2) N3) ER3: T(MI) N2 ins)= (00000) T(N1) NZ) 2 (0)010) NOW - x3, x1, x1+x3)= (01616) = 0. X1+X3=0 N3 = 0, M1=0. which shows that NCTI will Consist of all of the form (ornzio) which ni-anis vectors NLT) 2 { (0) X 2) 0) ER 3 : X2 E R }. 1.01 N(T) = (0) x2) 0) + (0,000) 50 T6 Since one to one. not