9. Express the vector $(2, -5, 3)$ in R^3 as a linear combination of the vectors $(1, -3, 2)$, $(2, -4, -1)$ and $(1, -5, 7)$.

Sol: let

$(2, -5, 3) = a(1, -3, 2) + b(2, -4, -1) + c(1, -5, 7)$

$(2, -5, 3) = (a+2b+c, -3a-4b-5c, 2a-b+7c) \rightarrow (1$

$a + 2b + c = 2 \rightarrow (2$

$-3a - 4b - 5c = -5 \rightarrow (3$

$2a - b + 7c = 3 \rightarrow (4$

Multiply 2 by 3 and add the result (in 3)

Then

$2b - 2c = 1$ i.e $b - c = \frac{1}{2} \rightarrow (5$

Multiply (2) by 2 and subtract the result from (4)

$-5b + 5c = -1$ i.e $b - c = \frac{1}{5} \rightarrow (6)$.

Equations (5) & (6) are inconsistent. Thus it is not possible to find values of a, b & c to satisfy given system.

Hence (2, -5, 3) is not expressible as a linear combination of given vectors.

For what value of K will the vector

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$C_1+(C_2+C_3+C_4)$ $\begin{vmatrix} a+3 & 1 & 1 & 1 \\ a+3 & a & 1 & 1 \\ a+3 & 1 & a & 1 \\ a+3 & 1 & 1 & a \end{vmatrix}$

$(a+3)\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & a & 1 & 1 \\ 1 & 1 & a & 1 \\ 1 & 1 & 1 & a \end{vmatrix}$

R_2-R_1
R_3-R_1
R_4-R_1

$(a+3)\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & a-1 & 0 & 0 \\ 0 & 0 & a-1 & 0 \\ 0 & 0 & 0 & a-1 \end{vmatrix}$

$(a+3)\begin{vmatrix} a-1 & 0 & 0 \\ 0 & a-1 & 0 \\ 0 & 0 & a-1 \end{vmatrix}$. Expanding from

$(a+3)(a-1)^3\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$= (a+3)(a-1)^3$ Hence proved.

Example 20. What conditions must a, b, c and d satisfy so that the matrices
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$$\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 3 \\ -2 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

in M_{22} are linearly dependent ?

Solution. Since the matrices are to be linearly dependent, there must exist scal c_1, c_2, c_3, not all zero, such that

$$c_1\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} + c_2\begin{bmatrix} 2 & 3 \\ -2 & 1 \end{bmatrix} + c_3\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

This implies

$$\begin{bmatrix} c_1 + 2c_2 + ac_3 & 2c_1 + 3c_2 + bc_3 \\ -c_1 - 2c_2 + cc_3 & c_2 + dc_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

i.e., the system of homogeneous equations

$$\left.\begin{aligned} c_1 + 2c_2 + ac_3 &= 0 \\ 2c_1 + 3c_2 + bc_3 &= 0 \\ -c_1 - 2c_2 + cc_3 &= 0 \\ c_2 + dc_3 &= 0 \end{aligned}\right\}$$

must have a nontrivial solution.

By Theorem 4.12, the system (2) will have a nontrivial solution if r matrix

$$A = \begin{bmatrix} 1 & 2 & a \\ 2 & 3 & b \\ -1 & -2 & c \\ 0 & 1 & d \end{bmatrix}$$

is less than 3 i.e., Rank $A = 2$ at the most.

Now,

$$A \overset{R}{\sim} \begin{bmatrix} 1 & 2 & a \\ 0 & -1 & b-2a \\ 0 & 0 & c+a \\ 0 & 1 & d \end{bmatrix} \quad \begin{array}{l} \text{by} \quad R_2 - 2R_1 \\ \text{and} \quad R_3 + R_1 \end{array}$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & 2 & a \\ 0 & -1 & b-2a \\ 0 & 0 & c+a \\ 0 & 0 & b+d-2a \end{bmatrix} \quad \text{by} \quad R_4 + R_2.$$

The last matrix has rank 2 at the most only if

$$c + a = 0 \qquad \text{and} \qquad b + d - 2a = 0.$$

Thus the required conditions on a, b, c, d are

$$a + c = 0 \qquad \text{and} \qquad b + d - 2a = 0.$$

Note that, for $a = b = d = 1$, $c = -1$ and $c_1 = 1$, $c_2 = -1$, $c_3 = 1$, the equation (1) satisfied.

Define $T: R^3 \to R^3$ by $T(x_1, x_2, x_3) = (-x_3, x_1, x_1+x_3)$

Find $N(T)$ Is T one to one?

Given Transformation is

$$T(x_1, x_2, x_3) = (-x_3, x_1, x_1+x_3).$$

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$N(T) = \{(x_1, x_2, x_3) \in R^3 : T(x_1, x_2, x_3) = (0,0,0)\}$.

Now $T(x_1, x_2, x_3) = (0,0,0)$.

$(-x_3, x_1, x_1+x_3) = (0,0,0)$.

$-x_3 = 0.$ —(1

$x_1 = 0.$ —(2

$x_1 + x_3 = 0$ —(3.

$x_3 = 0, \quad x_1 = 0.$

which shows that $N(T)$ will consist of all vectors of the form $(0, x_2, 0)$ which x_2-axis

i.e, $N(T) = \{(0, x_2, 0) \in R^3 : x_2 \in R\}$.

Since $N(T) = (0, x_2, 0) \neq (0,0,0)$ so T is not one to one.