Examples of problems for the oral examination EV2

(1) Let $u_0 > 0$ be a real number and let (a_n) be a sequence of strictly positive real numbers. Define the sequence (u_n) by:

$$u_{n+1} = u_n + \frac{a_n}{u_n}.$$

Show that (u_n) is convergent if and only if $\sum a_n < \infty$.

(2) Let E be a real vector space of dimension n. Find all endomorphisms f of E which satisfy $f \circ f = Id_E$.

(3) Let $f : [0,1] \to \mathbb{R}$ be a function of class C^1 such that $f(0) = 0$ and there exists $a \in]0,1[$ with $f(a)f'(a) < 0$. Show that there exists $b \in]0,1[$ with $f'(b) = 0$.

(4) a) Find all functions of class C^2, $f : \mathbb{R}^2 \to \mathbb{R}$ such that

$$\frac{\partial^2 f}{\partial x \partial y} = 0.$$

b) Find all functions of class C^2, $f : \mathbb{R}^2 \to \mathbb{R}$ such that

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial y^2}.$$