# BLOTTO-GAME COMPETITION FROM JANE STREET

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ABSTRACT:

**What is your entry?**

In Blotto-game competition from Jane Street, I propose my entry as the following: if I have $n_A$ soliders, I may assign to each castle in the way shown in the table 0.1.

**How did you go about coming up with it?**

See details in section 2. After theoretically understand algorithms (Ahmadinejad et al., 2016; Behnezhad et al., 2017; Vu et al., 2018), I impliment two two-step algorithms in (Behnezhad et al., 2017; Vu et al., 2018). The two algorithms are different from each other only in the first step of generating mixed-strategy Nash equilibriums (Li, 2018). After comparision of entry candidates, my proposed entry candidates are from (Behnezhad et al., 2017) and are summarized in table 0.1.

**How would your entry change if your were only allowed 90 soliders? What about if your received 110 soldiers? (In both cases every one else still gets 100)**

See table 0.1 and see section 2 where general procedures are implemented.

Table 0.1
*My entries depending on $n_A$, the number of soldiers I have.*

| $n_A$/castle point $v_i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 100 | 0 | 3 | 8 | 12 | 17 | 22 | 27 | 11 | 0 | 0 |
| 90 | 0 | 4 | 8 | 12 | 16 | 22 | 26 | 2 | 0 | 0 |
| 110 | 0 | 4 | 9 | 14 | 19 | 23 | 28 | 3 | 1 | 9 |

**1. Introduction.** We formulate general "N-dimensional asymmetric hetergenous Blotto game" partially referring to (Schwartz et al., 2014).

We are given $\mathbf{v} \in \mathbb{R}_+^N$, an $N$ dimensional vector as points for $N$ castles, $n_A$, $n_B$ as two postive integers, representing number of soliders owned by $A$ (me) and $B$ (rival), respectively.

$$\mathbf{x}^A \in [n_A]^N, \mathbf{x}^B \in [n_B]^N, \mathbf{1}_N^T \mathbf{x}^A = n_A, \mathbf{1}_N^T \mathbf{x}^B = n_B,$$

are partitions of $n_A, n_B$ respectively where $[M] \triangleq \{0, 1, \ldots, M\}$ for a positive integer $M$.

For $\mathbf{x}^B$ fixed, the pure strategy of finding $\mathbf{x}^A$ is

$$\max_{\mathbf{x}^A \in [n_A]^N, \mathbf{1}_N^T \mathbf{x}^A = n_A} g\left(\mathbf{x}^A, \mathbf{x}^B\right) \triangleq \sum_{i=1}^{N} v_i sgn\left(\mathbf{x}_i^A - \mathbf{x}_i^B\right), \tag{1.1}$$

where $sgn(x) \triangleq \begin{cases} 1, & x > 0; \\ 0, & x = 0; \\ -1, & x < 0. \end{cases}$.

1.1. *Mixed strategy Nash equilibrium.* One relaxation is to study continous Blotto games where they have $\mathbf{x}^A, \mathbf{x}^B \in \mathbb{R}_{\geq 0}^N$ instead of $\mathbf{x}^A \in [n_A]^N, \mathbf{x}^B \in [n_B]^N$ (Gross and Wagner, 1950; Laslier, 2002; Thomas, 2018). Since (Gross and Wagner, 1950; Laslier, 2002; Thomas, 2018) imply there is no pure-strategy Nash equilibrium with our $\mathbf{v} = [1, 2, \ldots, N]^T, N = 10$, mixed strategy Nash equilibrium is one focus of attention.

Mixed strategy Nash equilibrium is a useful concept and is essentially the first of the two-step procedure that we will mention in section 2. However, we stick to discrete version (Ahmadinejad et al., 2016; Behnezhad et al., 2017; Vu et al., 2018) although it is possible to come up with entry candidates from solutions to continuous Blotto games.

**2. Implementations of two research work.** Besides investigation into theoretical work (Thomas, 2018; Roberson, 2006; Hart, 2008; Hortala-Vallve and Llorente-Saguer, 2012), this solution implements Python version of (Vu et al., 2018) and Julia version of (Behnezhad et al., 2017). Codes are updated on (Li, 2018).

2.1. *Two-step procedure.* We can use the two-step procedure in (Vu et al., 2018) to summarize our two-step procedure, since (Behnezhad et al., 2017) (as an improvement of (Ahmadinejad et al., 2016)) is just an alternative to algorithm I (DIU strategy) in Vu et al. (2018).

First step is to generate mixed-strategy Nash equilibrium and is essentially the difference between two research works (that is, two algorithms I used to generate my entry candidates). (Vu et al., 2018) uses its DIU strategy where the idea essentially originates from the uniform marginal distribution proved in (Gross and Wagner, 1950; Laslier, 2002). (Behnezhad et al., 2017) solves an improved LP (relative to (Ahmadinejad et al., 2016)) and actually has no randomness.

Second step is a minor adjustment to algorithm II in (Vu et al., 2018). The necessary adjustment is to have marginal distribution $G_i(j-1)$ instead of $G_i(j)$ in fourth line of the pseudocode since if $x_i^A = x_i^B$, we assign reward to neither player $A$ nor player $B$, while (Behnezhad et al., 2017) has its corresponding "$\alpha$-tie-breaking rule". This second step corresponds to *best_response.py* in (Li, 2018).

2.2. *Entry candidates.* While (Vu et al., 2018) has randomness due to sampling in DIU strategy (algorithm I in (Vu et al., 2018)), Julia version of (Behnezhad et al., 2017) solves a LP and actually has no randomness. At a glimpse, a selection of entry candidates are recorded in *document.csv* and are shown in figure 1; by "selection", I mean I definitely can generate more; here the paper is just for demonstration.

Now let's compare the entry candidates. Comparision in (Vu et al., 2018) does not focus on our scenario, but emphasizing the cases when $N$, the number of battles is large. In addition to the fact that (Behnezhad et al., 2017) has more theoretical guarantee than (Vu et al., 2018)'s strategy, there is one simple way to heuristically show that (Behnezhad et al., 2017) is better: with the candidates in figure 1, we do head-to-head tournaments. It is obviously meaningful for $n_A = 100$; to be honest, for $n_A = 90, 110$, the tournaments are less intepretable since the mixed strategy Nash equilibriums are found in games with rival $n_B = 100$. Whatever the case, I show three tournaments in figure 2, 3, 4. One can immediately realize "Julia" in last two rows (the two are identical in figure 1 and showing two rows is just one of the ways to double check my analysis and codes all the way), win the most. As a result, we fill table 0.1 by (Behnezhad et al., 2017)'s simulation.

**3. Conclusion.** After theoretically understand algorithms (Ahmadinejad et al., 2016; Behnezhad et al., 2017; Vu et al., 2018), I impliment two two-step algorithms in (Behnezhad et al., 2017; Vu et al., 2018). The two algorithms are different from each other only in the first step of generating mixed-strategy Nash equilibriums (Li, 2018). After comparision of entry candidates, my proposed entry candidates are from (Behnezhad et al., 2017) and are summarized in table 0.1.

FIG 1. *Selected results (by "selected", I mean I definitely can generate more; here the paper is just for demonstration) in document.csv where yellows are for* $n_A = 100$*, greens are for* $n_A = 90$ *and orange are for* $n_A = 110$*. Column names include "number of DIUs" as well as "number of castles", "rival's soldiers", "my soliders" that are literally understandable. "number of DIUs" has attribution values of the following (1) numbers, indicating number of simulations of DIU in (Vu et al., 2018) or (2) "Julia" indicating the mixed strategy Nash equilibrium is generated from Behnezhad et al. (2017).*

| number of DIUs | number of castles | rival's soldiers | my soldiers | my entry | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10000 | 3 | 5 | 5 | 0 | 2 | 3 | | | | | | | |
| 10000a | 10 | 100 | 100 | 1 | 4 | 6 | 9 | 12 | 13 | 16 | 18 | 21 | 0 |
| 10000b | 10 | 100 | 100 | 1 | 4 | 7 | 7 | 11 | 13 | 17 | 18 | 22 | 0 |
| 10000c | 10 | 100 | 100 | 1 | 4 | 6 | 9 | 11 | 14 | 15 | 19 | 21 | 0 |
| 100000a | 10 | 100 | 100 | 1 | 4 | 6 | 9 | 11 | 14 | 16 | 18 | 21 | 0 |
| 100000b | 10 | 100 | 100 | 1 | 4 | 6 | 9 | 11 | 13 | 16 | 19 | 21 | 0 |
| 200000c | 10 | 100 | 100 | 1 | 4 | 6 | 9 | 11 | 13 | 16 | 19 | 21 | 0 |
| 200000d | 10 | 100 | 100 | 1 | 4 | 6 | 9 | 11 | 13 | 16 | 19 | 21 | 0 |
| 400000a | 10 | 100 | 100 | 1 | 4 | 6 | 9 | 11 | 14 | 16 | 18 | 21 | 0 |
| 400000b | 10 | 100 | 100 | 1 | 4 | 6 | 9 | 11 | 13 | 16 | 18 | 22 | 0 |
| Julia_a | 10 | 100 | 100 | 0 | 3 | 8 | 12 | 17 | 22 | 27 | 11 | 0 | 0 |
| Julia_b | 10 | 100 | 100 | 0 | 3 | 8 | 12 | 17 | 22 | 27 | 11 | 0 | 0 |
| 10000a | 10 | 100 | 90 | 1 | 4 | 7 | 10 | 13 | 16 | 18 | 21 | 0 | 0 |
| 10000b | 10 | 100 | 90 | 1 | 4 | 7 | 10 | 13 | 15 | 17 | 0 | 23 | 0 |
| 100000 | 10 | 100 | 90 | 1 | 4 | 7 | 10 | 13 | 16 | 18 | 21 | 0 | 0 |
| 200000a | 10 | 100 | 90 | 1 | 4 | 7 | 10 | 13 | 15 | 19 | 21 | 0 | 0 |
| 200000b | 10 | 100 | 90 | 1 | 4 | 7 | 10 | 13 | 16 | 18 | 21 | 0 | 0 |
| 400000a | 10 | 100 | 90 | 1 | 4 | 7 | 10 | 13 | 16 | 18 | 21 | 0 | 0 |
| 400000b | 10 | 100 | 90 | 1 | 4 | 7 | 10 | 13 | 16 | 18 | 21 | 0 | 0 |
| Julia_a | 10 | 100 | 90 | 0 | 4 | 8 | 12 | 16 | 22 | 26 | 2 | 0 | 0 |
| Julia_b | 10 | 100 | 90 | 0 | 4 | 8 | 12 | 16 | 22 | 26 | 2 | 0 | 0 |
| 10000a | 10 | 100 | 110 | 1 | 4 | 6 | 10 | 11 | 16 | 16 | 19 | 1 | 26 |
| 10000b | 10 | 100 | 110 | 1 | 4 | 7 | 9 | 12 | 15 | 16 | 21 | 24 | 1 |
| 100000 | 10 | 100 | 110 | 1 | 4 | 7 | 9 | 12 | 15 | 17 | 20 | 24 | 1 |
| 200000a | 10 | 100 | 110 | 1 | 4 | 7 | 9 | 12 | 15 | 17 | 21 | 23 | 1 |
| 200000b | 10 | 100 | 110 | 1 | 4 | 7 | 9 | 12 | 15 | 18 | 20 | 23 | 1 |
| 400000a | 10 | 100 | 110 | 1 | 4 | 7 | 9 | 12 | 15 | 17 | 21 | 23 | 1 |
| 400000b | 10 | 100 | 110 | 1 | 4 | 7 | 9 | 12 | 15 | 18 | 20 | 23 | 1 |
| Julia_a | 10 | 100 | 110 | 0 | 4 | 9 | 14 | 19 | 23 | 28 | 3 | 1 | 9 |
| Julia_b | 10 | 100 | 110 | 0 | 4 | 9 | 14 | 19 | 23 | 28 | 3 | 1 | 9 |

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FIG 2. *Pairwise results in the tournament of yellow entry candidates from document.csv where* $n_A = 100$. *Each row represents my entry while each column represents the rival's. Correponding entry assignments are in figure 1.*

| mine/rival | 10000a | 10000b | 10000c | 100000a | 100000b | 200000c | 200000d | 400000a | 400000b | Julia_a | Julia_b |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 10000a | 0 | 9 | 12 | 5 | 5 | 5 | 5 | 5 | 5 | 20 | 20 |
| 10000b | 19(win) | 0 | 19(win) | 19(win) | 19(win) | 19(win) | 19(win) | 19(win) | 10(win) | 20 | 20 |
| 10000c | 14(win) | 18 | 0 | 8(win) | 6 | 6 | 6 | 8(win) | 14 | 20 | 20 |
| 100000a | 6(win) | 10 | 7 | 0 | 6 | 6 | 6 | 0 | 6 | 20 | 20 |
| 100000b | 8(win) | 12 | 7(win) | 8(win) | 0 | 0 | 0 | 8(win) | 8 | 20 | 20 |
| 200000c | 8(win) | 12 | 7(win) | 8(win) | 0 | 0 | 0 | 8(win) | 8 | 20 | 20 |
| 200000d | 8(win) | 12 | 7(win) | 8(win) | 0 | 0 | 0 | 8(win) | 8 | 20 | 20 |
| 400000a | 6(win) | 10 | 7 | 0 | 6 | 6 | 6 | 0 | 6 | 20 | 20 |
| 400000b | 9(win) | 4 | 16(win) | 9(win) | 9(win) | 9(win) | 9(win) | 9(win) | 0 | 20 | 20 |
| Julia_a | 25(win) | 25(win) | 25(win) | 25(win) | 25(win) | 25(win) | 25(win) | 25(win) | 25(win) | 0 | 0 |
| Julia_b | 25(win) | 25(win) | 25(win) | 25(win) | 25(win) | 25(win) | 25(win) | 25(win) | 25(win) | 0 | 0 |

FIG 3. *Pairwise results in the tournament of yellow entry candidates from document.csv where* $n_A = 90$. *Each row represents my entry while each column represents the rival's. Correponding entry assignments are in figure 1.*

| mine/rival | 10000a | 10000b | 100000 | 200000a | 200000b | 400000a | 400000b | Julia_a | Julia_b |
|---|---|---|---|---|---|---|---|---|---|
| 10000a | 0 | 21(win) | 0 | 6 | 0 | 0 | 0 | 9 | 9 |
| 10000b | 9 | 0 | 9 | 9 | 9 | 9 | 9 | 10 | 10 |
| 100000 | 0 | 21(win) | 0 | 6 | 0 | 0 | 0 | 9 | 9 |
| 200000a | 7(win) | 15(win) | 7(win) | 0 | 7(win) | 7(win) | 7(win) | 9 | 9 |
| 200000b | 0 | 21(win) | 0 | 6 | 0 | 0 | 0 | 9 | 9 |
| 400000a | 0 | 21(win) | 0 | 6 | 0 | 0 | 0 | 9 | 9 |
| 400000b | 0 | 21(win) | 0 | 6 | 0 | 0 | 0 | 9 | 9 |
| Julia_a | 25(win) | 33(win) | 25(win) | 25(win) | 25(win) | 25(win) | 25(win) | 0 | 0 |
| Julia_b | 25(win) | 33(win) | 25(win) | 25(win) | 25(win) | 25(win) | 25(win) | 0 | 0 |

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FIG 4. *Pairwise results in the tournament of yellow entry candidates from document.csv where* $n_A = 110$. *Each row represents my entry while each column represents the rival's. Correponding entry assignments are in figure 1.*

| mine/rival | 10000a | 10000b | 100000 | 200000a | 200000b | 400000a | 400000b | Julia_a | Julia_b |
|---|---|---|---|---|---|---|---|---|---|
| 10000a | 0 | 20 | 20 | 20 | 20 | 20 | 20 | 19 | 19 |
| 10000b | 25(win) | 0 | 8(win) | 9(win) | 17(win) | 9(win) | 17(win) | 18 | 18 |
| 100000 | 32(win) | 7 | 0 | 9(win) | 9(win) | 9(win) | 9(win) | 18 | 18 |
| 200000a | 32(win) | 7 | 8 | 0 | 8(win) | 0 | 8(win) | 18 | 18 |
| 200000b | 32(win) | 7 | 7 | 7 | 0 | 7 | 0 | 18 | 18 |
| 400000a | 32(win) | 7 | 8 | 0 | 8(win) | 0 | 8(win) | 18 | 18 |
| 400000b | 32(win) | 7 | 7 | 7 | 0 | 7 | 0 | 18 | 18 |
| Julia_a | 25(win) | 35(win) | 35(win) | 35(win) | 35(win) | 35(win) | 35(win) | 0 | 0 |
| Julia_b | 25(win) | 35(win) | 35(win) | 35(win) | 35(win) | 35(win) | 35(win) | 0 | 0 |