Hydrogen-Deuterium Spectrum Experiment

1. *Introduction*

In this experiment, we used the wavelength differences between hydrogen and deuterium Balmer series to calculate the mass ratio of hydrogen and deuterium.

First, we use a monochromator to scan over the sodium spectrum at a constant rate. Since the wavelength difference between the two lines of the yellow sodium doublet is known (sodium D lines), it can be used to calculate a conversion factor between wavelength difference and time difference that depends on the scanning rate we choose. Then we measured the wavelength differences between hydrogen and deuterium Balmer series (namely: α, β, γ, δ, and ϵ lines). Using the expression of effective mass and orbital energy difference

$$\mu = \frac{mM}{M+m}, \quad E_i - E_f \propto \mu\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

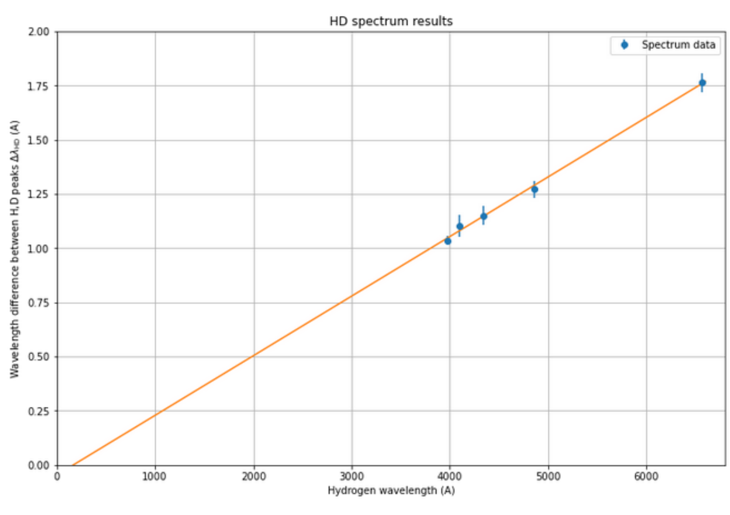
we find that

$$\frac{\Delta\lambda_{HD}}{\lambda_H} = \frac{\lambda_H - \lambda_D}{\lambda_H} = \frac{1/\mu_H - 1/\mu_D}{1/\mu_H} = \frac{1 - M_H/M_D}{1 + M_H/m}$$

Since M_H/m and λ_H is known and we can measure $\Delta\lambda_{HD}$, we then can solve for the desired quantity M_H/M_D. To improve accuracy, we plot $\Delta\lambda_{HD}$ for all lines in the Balmer series, and fit the data to find the slop (i.e $\frac{\Delta\lambda_{HD}}{\lambda_H}$) instead of just using one single spectral line.

2. *Result for the mass ratio*

The data plot for $\Delta\lambda_{HD}$ with respect to hydrogen wavelength is shown below (note that the fitted line is included also)



The fitted line gives a slop of 0.000257 ± 0.000009. Using the equation given above, the mass ratio is:

$$\frac{M_H}{M_D} = 0.49 \pm 0.02$$

The accepted value of hydrogen-deuterium mass ration is $M_H/M_D = 0.500248$, which is only about 0.01 off from our measured value, so lies within the uncertainty (0.02). This concluded that our measured value agree very well with the accepted value.

3. *Possible error*

Assume we made a mistake in measurements by pausing the wavelength scanning for 2 seconds between two sodium D lines, how would it effects our mass ratio final result?

First, we consider how this mistake will effect the scaling factor $K_{Na} = \frac{\Delta\lambda_D}{\Delta t_D}$. Since the time interval between sodium peaks is lengthened by our mistake

$$K_{Na} = \frac{\Delta\lambda_D}{\Delta t_D} \to \frac{\Delta\lambda_D}{\Delta t_D + 2}$$

The scaling factor will become smaller, namely, it is scaled by $\frac{\Delta t_D}{\Delta t + 2} = 0.94026 \pm 0.00004$ (with $\Delta t_D = 31.476 \pm 0.025$ from our analysis).

Since we calculate $\Delta\lambda_{HD}$ values using K_{Na} as follow

$$\Delta\lambda_{HD} = K_{Na}\Delta t_{HD}$$

the new $\Delta\lambda_{HD}$ will also be scaled by the same amount (i.e $\Delta\lambda_{HD} \to 0.94026\Delta\lambda_{HD}$). So the slope ($\frac{\Delta\lambda_{HD}}{\lambda_H}$) will also decrease by a factor of 0.94026. Solving for the mass ratio in the equation on the first page, with the known value $M_H/m = 1836.15$, we get

$$\frac{M_H}{M_D} = 1 - 1837.15 \times \frac{\Delta\lambda_{HD}}{\lambda_H}$$

this means that a 0.94026 decrease in slope will result in a new mass ratio of 0.525 ± 0.015 (original slope is 0.000275 ± 0.000009, detailed calculation is done in the analysis notebook). So, by taking the ratio of the "correct mass ratio" and the "incorrect mass ratio"

$$\frac{(M_H/M_D)_{incorrect}}{(M_H/M_D)_{correct}} = \frac{0.525 \pm 0.015}{0.49 \pm 0.02} = 1.061 \pm 0.004$$

So the mistake we made causes us to measure a mass ratio that is 106.1% of the mass ratio without making the mistake.