Astro 121: Measuring Celestial Sizes with Interferometry

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This lab presents an analytical application of interferometry to measure the diameter of the sun. Knowledge of writing scripts to take data of the sun from sunrise to sunset, various fitting methods, and error analysis techniques are necessary for understanding all aspects of this lab. This lab report emphasizes the use of figures to represent the data analysis techniques in making measurements of the interferometer pair baselines and solar diameter. The usefulness of the equations for visibility, fringe frequency, and Bessel function fit are all thoroughly explained in the report. This lab has provided us with a strong foundation for using interferometry as a tool for probing astronomical objects.

I. INTRODUCTION

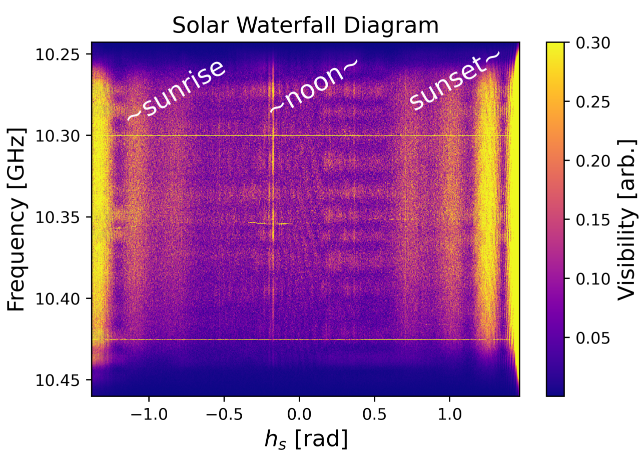

FIG. 1. Waterfall diagram of the sun. This figure was created by taking the Fourier transform of the 2-signal correlation, which outputs a measurement in units of visibility taken from sunrise to sunset with a ∼1.25s cadence. The signal chain is coupled with a bandpass filter from the range of ∼10.24-10.46 GHz, which is why there is near zero visibility (purple) outside that range. A fringe pattern is observed as the sun moves across the sky due to the solar signal's changing hour angle (h_s). The time delay between the interferometers is smallest when the sun is highest in the sky (noon, $h_s \approx 0$), resulting in the most destructive interference and low visibility (violet-orange). Alternatively, the visibilities are brightest (yellow) when the time delay is the largest (sunrise/sunset). It is towards these times when the Bessel function shapes are most apparent in the waterfall diagram

The goal of this lab to understand how celestial objects produce fringes in radio frequencies, take horizon-to-horizon visibility of the sun, obtain baseline measurements of the interferometer pair, and measure the diameter of the sun. To more generally understand radio interferometry, we also learn how to write scripts for taking data of the sun and apply various forms of data analysis (timekeeping, coordinate transformations, least-squares fitting). The figures presented in this lab apply several fitting and error propagation techniques that allow us to use interferometry as a tool for probing astronomical objects.

II. BACKGROUND ON INTERFEROMETRY

Radio interferometry is a tool that takes in an astronomical signal from multiple antennas separated by some baseline and calculates the time delay in the signal to measure properties of the astronomical object. The great distance of the object means that electromagnetic waves from the object arrive on Earth as flat plane waves. Asynchronous arrivals of the plane waves at the separate antennas result in a time delay in the signal. The time delay also changes as the object moves across the sky. Although there is intrinsic time delay in the transmission lines of the system, the primary time delay that is the focus of this paper in the geometric time delay caused by the geometric separation of the antennas. This lab uses a 2-antenna interferometer with both east-west and north-south components to capture solar data.

It is important to first understand the output of the interferometer pairing. When the two antennas capture two separate time-domain signals (e.g v_1(t), v_2(t)), the output comes in the form:

$$V = \mathcal{F}(v_1(t)) \cdot \mathcal{F}(v_2^*(t)) \tag{1}$$

where $\mathcal{F}$ represents the Fourier transform of the signals, and V is the visibility in frequency-domain. Another way to think of the visibility is the Fourier transform of the signals' correlation. Figure 1 is a Waterfall diagram that represents the evolution of solar visibilities throughout the day. It's important to note that the visibility does not measure the brightness of the signal, rather intrinsic amplitudes of the correlation's Fourier transform. A brighter color in the waterfall diagram represents larger visibilities. The visibilities periodically increasing and decreasing is the fringe pattern. Why do we see this fringe pattern?

One comparison that helps with understanding the premise of interferometry is the double slit experiment. When an electromagnetic wave is sent through slits, they disperse and produce a fringe pattern from the waves interfering with one another. An interferometer is the inverse of the double slit: the fringes come from the sky and the slits are the antennas.

Interferometers project a non-uniform sine wave in the sky (SWS), also known as the fringe. The amplitude and frequency of the fringe vary as a function of source hour angle (h_s) and baseline lengths. The full visibility equation goes as follows:

$$V = e^{-2\pi i w} \int A(l,m)\ S(l,m)\ e^{2\pi i(ul+vm)}\ dl\ dm \quad (2)$$

where the first exponent is a phase term, w is the vertical component, A is the antenna response, S is the source intensity, l is the east-westward baseline component, m is the north-south component, and u-v are the spatial-frequency Fourier-equivalents of l-m. The evolution of the fringe strongly depends on the projections of u-v-w as the sun goes across the sky.

The solar disk is represented in the spatial l-m plane and the Fourier transform (enacted by Eq. 1) converts the disk into a Bessel function in the u-v plane. The Bessel function shapes are characterized by the noticeable patches of yellow in Figure 1.

To perform measurements of the sun, we need to measure the east-west (b_{ew}) and north-south baseline (b_{ns}) and then characterize the Bessel function in our observations.

III. DATA ANALYSIS

The first step to measure the baselines is to take one channel (i.e. frequency) of the visibility evolution. This is represented in Figure 2, where I took the average of nine local channels centered at 10.34GHz to increase the signal-to-noise ratio of our visibility. I didn't include more than nine channels because non-local variations in the visibility can strongly affect the fringe characteristics described in the previous section. The Bessel function shape is similar to the beat frequency shapes we see at sunrise and sunset.

Zooming into Figure 2 reveals a visibility oscillating at different frequencies. This is because the voltages at various positions in the SWS oscillate between modes and define the fringe pattern we see in the internal visibility oscillations of Figure 1 and 2.

Since the SWS provides us with a fringe pattern dependent on the baselines, we can measure the fringe frequency locally (in time) to fit for a baseline with the relationship:

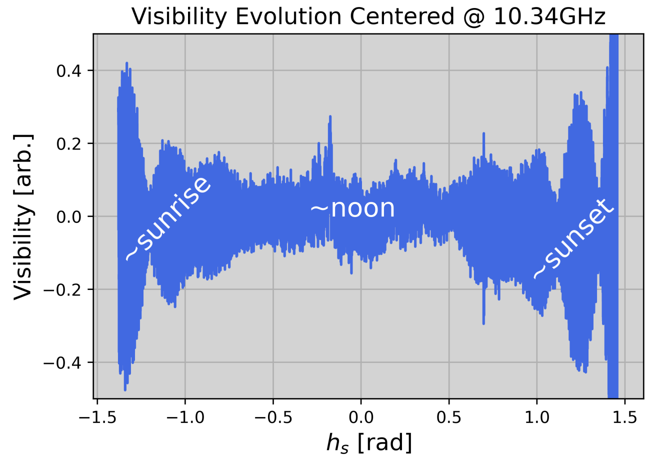

FIG. 2. Visibility evolution of the sun. This figure uses the average of 9 frequency channels centered around 10.34 GHz to increase the visibility signal-to-noise ratio. Sunrise and sunset most clearly show the Bessel function shape (similar to a beat frequency), which is expected because the signals at these times have the largest delay and least destructive interference. Interestingly, the visibility is much larger at sunset than sunrise possibly because the western Berkeley hills prevent early-sunrise observations. The visibility around noon loses the Bessel shape and appears more noise-like; this could be due to a variety of factors like interferometer positioning, wind/clouds, signal chain noise, building interference, etc.. The visibility evolution most importantly allows us to measure the solar radius.

$$\frac{f_f}{w_e} = \frac{b_{ew}}{\lambda}\ \mathbf{cos}\delta\ \mathbf{cos}h_s - \frac{b_{ns}}{\lambda}\ \mathbf{sin}L\ \mathbf{cos}\delta\ \mathbf{sin}h_s \quad (3)$$

where f_f is the local fringe frequency, w_e is the rotation rate of the earth, λ is the wavelength of the visibility channel, δ is the declination, and L is latitude.

Since this relationship is inherently a function of time, we can calculate the local fringe frequency by taking the Fourier transform of small segments in the visibility evolution centered around some h_s. I used `scipy.signal.stft` for this calculation and fitted for the baselines by using the gradient descent method. `scipy.signal.stft` takes in a small segment of the fringe and takes a short time Fourier transform (STFT) with an integration time equal to the time between visibility observations (~1.25s). This fitting method takes in a guess for the baselines and calculates all of the errors in each iteration. Although one has to be cautious with not to fall in a false minimum (i.e. an unreasonable best fit value whose error is a local minima), I was able to retrieve a best fit that gave me the values (in meters) b_{ew} = 15.12 ± 0.029 and b_{ns} = 1.58 ± 0.052. Figure 3 shows the resulting local fringe frequency fit and χ^2 in all of the baseline combinations.

The final part of this lab to the measure diameter of the sun, which is done by fitting the characteristic Bessel

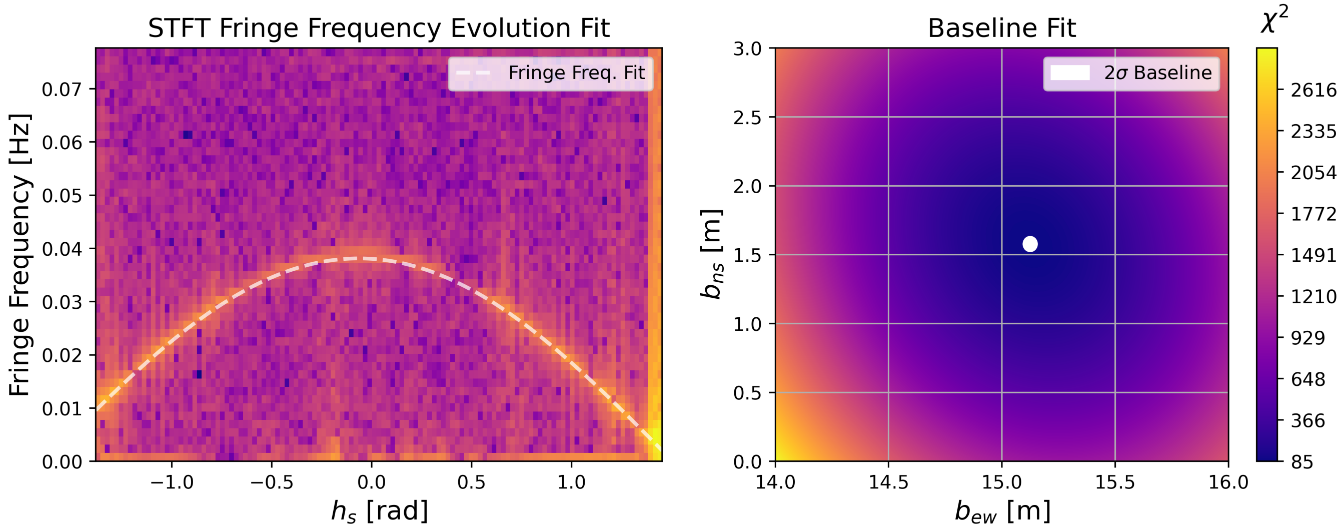

FIG. 3. Interferometer baseline fitting. **LEFT:** The fringe frequencies were taken with the short-time Fourier Transform (STFT) tool `scipy.signal.stft`, which takes in time-series segments of the visibilities and outputs the local fringe frequency centered around the each segment's hour angle. The local fringe frequency is plotted as the yellow upside-down parabola structure. When fitting Equation 3 with the best baseline measurements from the right plot we get the fringe frequency fit as the dashed white line. The χ^2 values from the right plot are calculated using the squared-difference of the STFT output and the fringe frequency fit. **RIGHT:** Measurements of the east-west (b_{ew}) and north-south baseline (b_{ns}) were taken by modelling the fringe frequency as a function of hour angle from Equation 3. The best fits (in meters) were b_{ew} = 15.12 ± 0.029 and b_{ns} = 1.58 ± 0.052. The gradient of the plot represents the fit's χ^2 values and white circle visualizes the 2σ significance of the baseline measurements.

function to the visibility. The Bessel fit follows the functions form:

$$\text{Fit: } A\frac{|J_1(2\pi u_r \cos(h_s+\phi)R_\odot)|}{2\pi u_r \cos(h_s+\phi)R_\odot} + B \tag{4}$$

$$u_r = \frac{\sqrt{b_{ew}^2 + b_{ns}^2}}{\lambda} \tag{5}$$

where J_1 is the first order Bessel function, ϕ is a phase offset applied to h_s, and $R_\odot$ is the radius of the sun. Aside from minimizing the χ_r^2 of the Bessel fit, I will elaborate on the significance of ϕ in the next section.

Figure 4 shows the resulting Bessel fit with its residual plotted during sunrise. There appears to be structure that isn't noise-like in the residual and I suspect that is either due to noise or spectral leakage around the node of the visibility. Considering this fit had a $\chi_r^2 \approx 1.03$ (with 1 being a perfect fit), I trust my measurement of $D_\odot = 0.47 \pm 0.0026°$. My diameter measurement is close to prediction since an order-of-magnitude estimate is that the sun makes up about half of a degree in the sky.

IV. DISCUSSION ON ERROR

There were a few questions and interesting points I gathered while working on this lab. Firstly, why did my fit look better with the ϕ phase term than without it? The fit without ϕ wasn't worth including because it visually didn't fit well to the envelop. The phase term in my sunrise Bessel fit was on the order of 5°, which could imply an error of in h_s. Although the interferometer's pointing/timing may not be perfect, I wouldn't expect h_s to be off by 5°. I'm also curious if winds and clouds play a role with ϕ.

Another contribution in the error of our Bessel fit could be in u_r; since I am averaging around nine local channels, the value of λ may not be totally accurate. I tried to account for this by measuring the diameter using single channels, but I still needed ϕ to minimize my χ_r^2. Additionally, these single channel diameter measurements hovered around 0.47°.

My last guess for the necessity of ϕ is due to the contribution of w (i.e. the vertical component) around sunrise and sunset. Since the the vertical component of the baseline is more prominent when the sun is near the horizon, ϕ could represent an additional term in the low-mode frequency orientation mapped in the center of the uv-plane. This could also imply that b_{ns} becomes more dominant in the fringe during sunrise and sunset. The ϕ is an interesting contribution to the science of this interferometry lab because it could imply a degeneracy in all of the parameters in our baseline and Bessel fits.

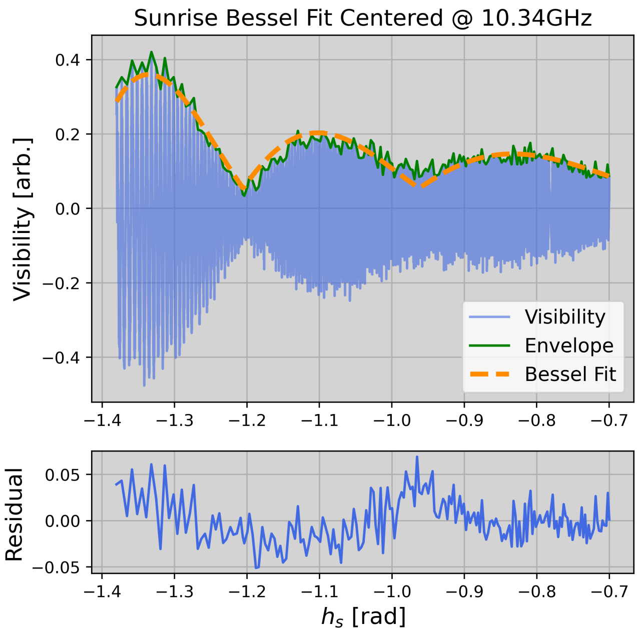

FIG. 4. Visibility Bessel function fit centered around 10.34GHz plotted with the residual. The inner sinusoidal structure of the visibility is the fringe (i.e. phasor) from the projected sine wave in the sky and the outer envelop is the Bessel function. This Bessel function is a result of Fourier transforming the solar disk and taking a slice of the Bessel function in the uv-plane. The Bessel fit represented by Equation 4 is plotted over this outer envelop. The best fit for the solar radius was $R_\odot = .23 \pm .0013°$. The residual doesn't ideally look like noise, and I suspect that this is due to intrinsic noise or spectral leakage at the nodes of the visibility. The Bessel function was best fitted during sunrise and sunset because the signal-to-noise ratio in the visibility was significantly higher than visibilities around noon, which is why I cut off the fit before it got too close to noon.

V. CONCLUSIONS

This interferometry lab is the most complex lab I've done. The start of this lab was very daunting, especially with writing the data taking script and learning about the basics of the interferometry. I found it fascinating how this work is related to very long baseline interferometry that allowed us to image M87 central supermassive black hole and Sag. A*. Overall, this lab was very rewarding and I'm excited to apply my newfound understanding of interferometry to future labs!