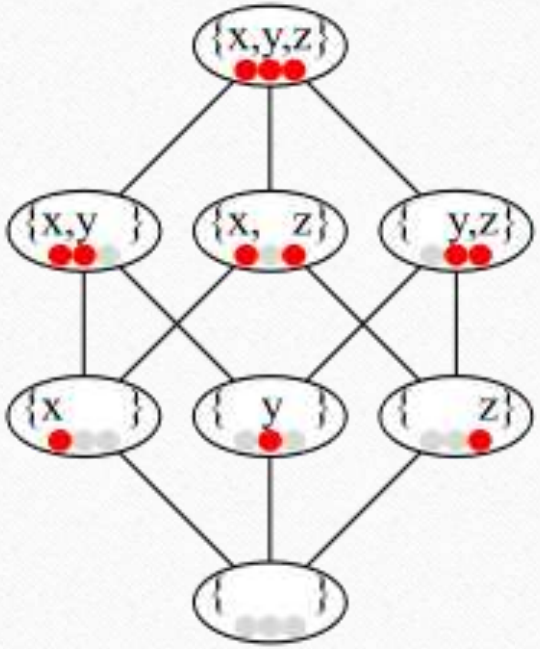
Hasse Diagram



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Partially ordered Sets (Posets)

A partial order is a binary relation "≤" over a set P which is *reflexive, anti-symmetric,* and *transitive,* i.e., which satisfies for all a, b, and c in P

- **a ≤ a (reflexivity);**
- **if a ≤ b and b ≤ a then a = b (anti-symmetry);**
- **if a ≤ b and b ≤ c then a ≤ c (transitivity).**

A set with a partial order is called partially ordered set or poset.

The power set of A = {a, b, c} consists of the family of eight subsets:
P(A): {Φ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}}
then, set inclusion relation "⊆" is a partial order on P(A)

- **Reflexive:** Clearly any set in P(A) is a subset of itself. Hence ⊆ is reflexive.
- **Anti-symmetric**: For any sets B and C in P(A) satisfying B ⊆ C and C ⊆B we have B = C. Hence ⊆ is anti-symmetric.
- **Transitive**: For any three sets B, C and D in p(A) satisfying B ⊆ C and C ⊆D we have B ⊆ D. Hence ⊆ is transitive.

Hence ⊆ is a partial order on P(A).

Hasse diagram

- In order theory, a Hasse diagram is a type of mathematical diagram used to represent partially ordered set, in the form of a drawing of its transitive reduction.
- named after Helmet Hasse (1898 – 1979);

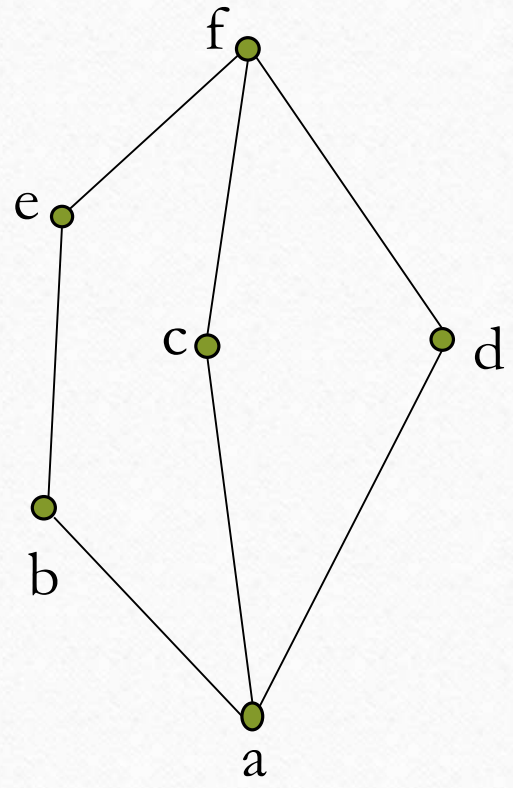
The Hasse Diagram of a finite poset P is the graph with vertices $x \in P$ and

- **If $x < y$, then y is drawn above x in the diagram;**
- **If y covers x then there is an edge between x and y in the diagram.**

Example: If P= {a, b, c, d, e, f} and

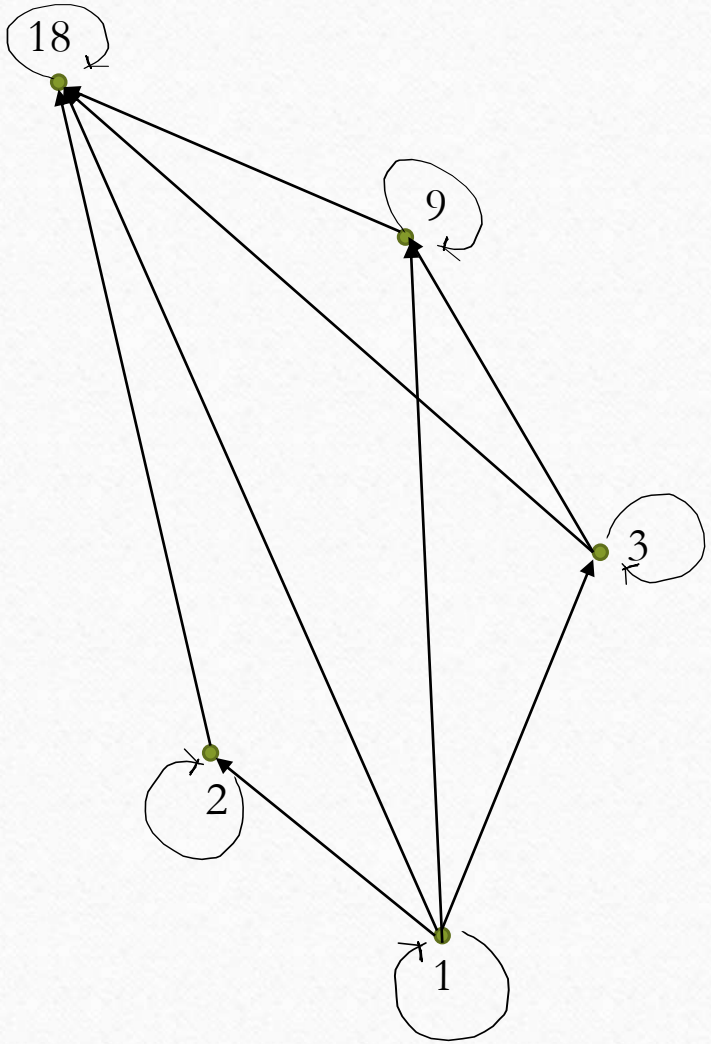
a<b, a<c, a<d, b<e, e<f, c<f, d<f

Then the Hasse diagram will be…….



Q. Let A = {1, 2, 3, 9, 18} and consider the 'divides' relation A: For all a, b ∈ A, a | b ⇔ b = ka for some integer k.

The directed graph for the given relation is → → →


18
9
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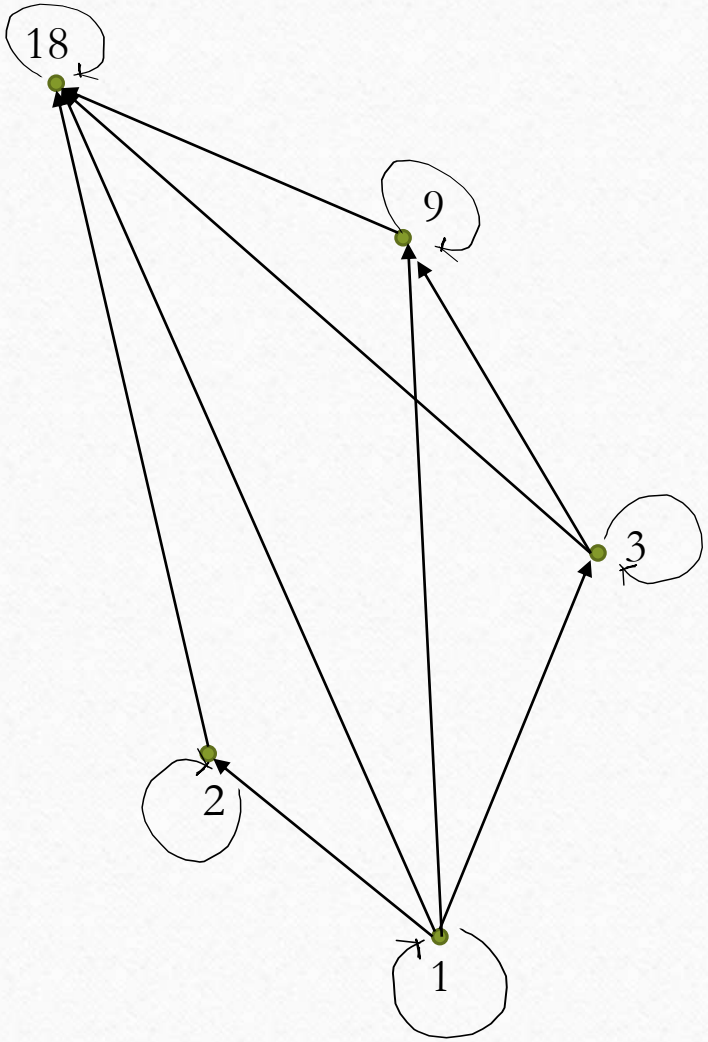
1

Let's construct a Hasse Diagram……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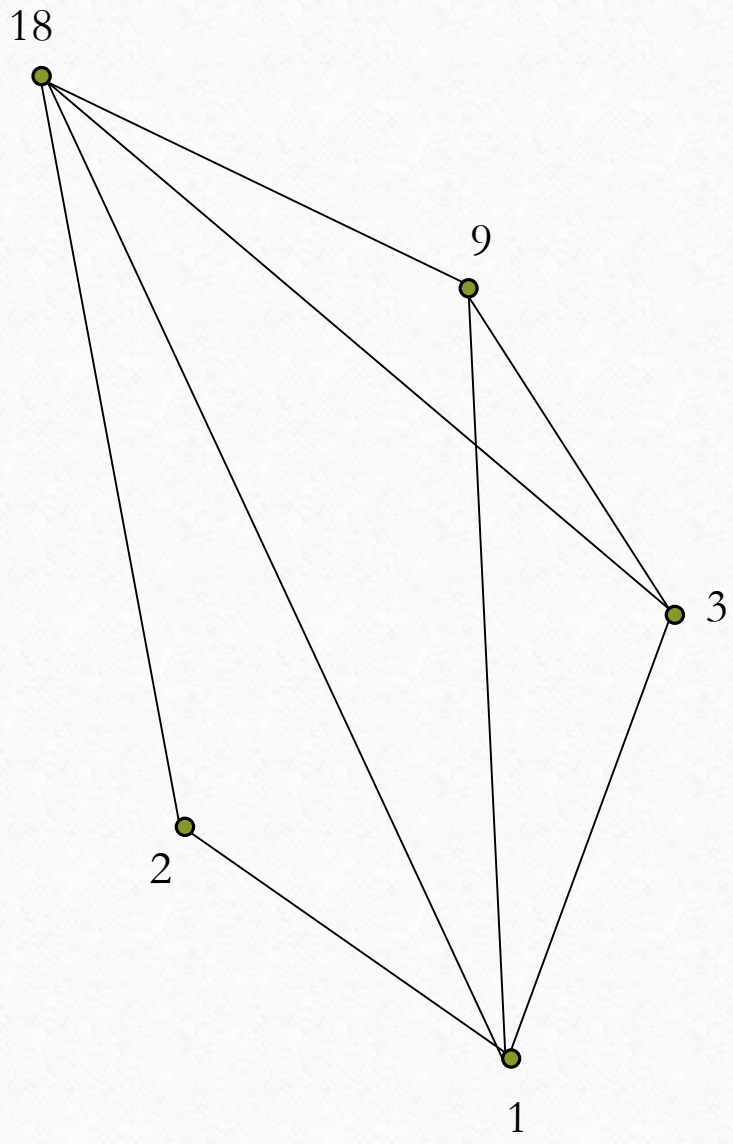
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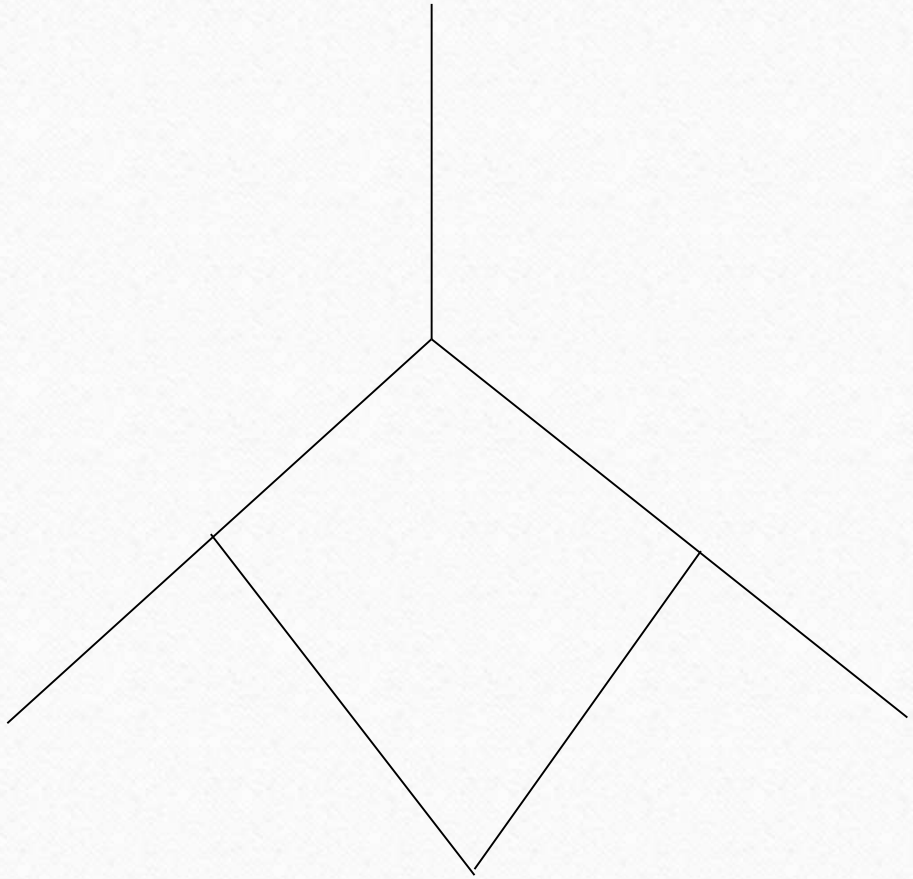
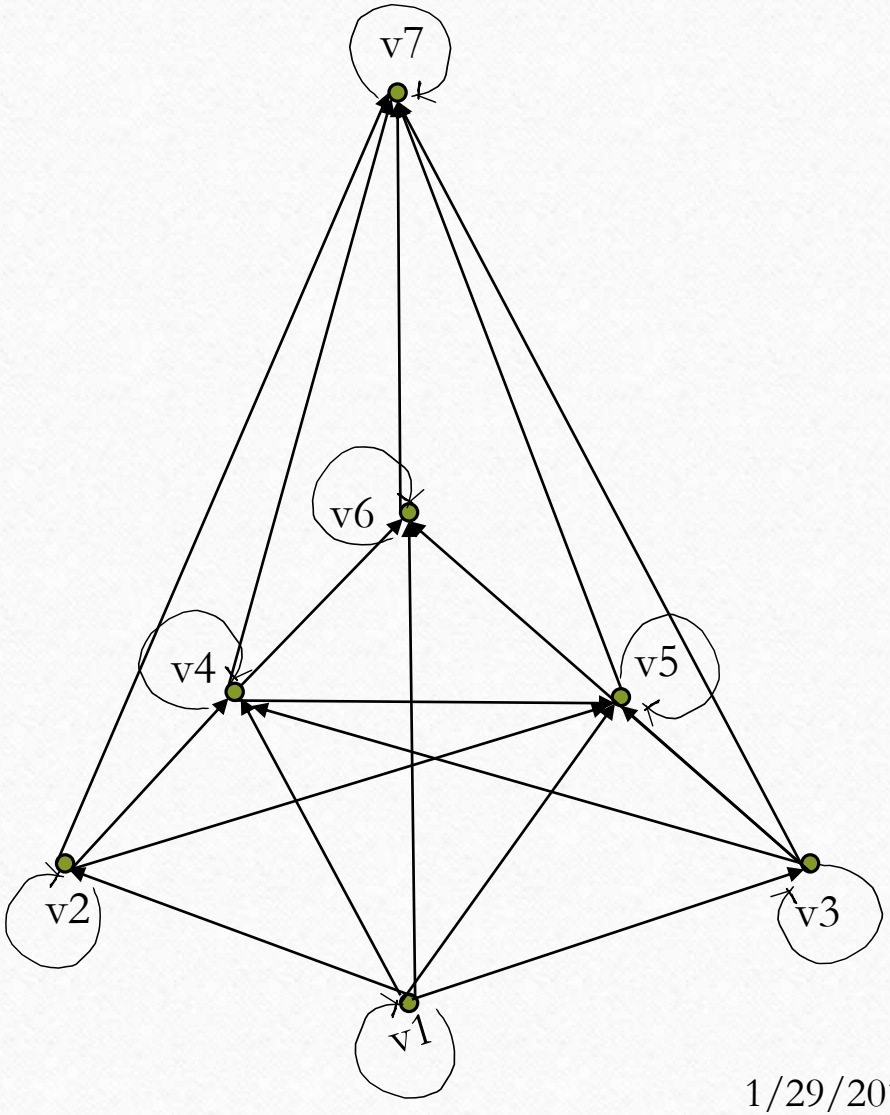
Question: A partial order relation R has the following Hasse diagram. Find the directed graph of R.




v7
v6
v4
v5
v2
v3
v1