Landau Levels in Graphene

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Introduction and Overview

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- Hall effect: experiment in 1879
- Quantum Hall effect: quantization of Hall resistance, experiment in 1959
- Both effects involve a charged particle in a magnetic field
- We will cover
    1. Classical particles in magnetic field
    2. Quantum particles in magnetic field
    3. Dirac fermions in magnetic field

Classical particle in Magnetic field

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- Consider a free particle in xy plane and magnetic field in z direction
- The velocity as a function of time is given by $v(t) = (v\cos\omega_B t, -\frac{|q|}{q} v \sin\omega_B t)$, where $\omega_B \equiv eB/m$ is the cyclotron frequency
- This is nothing but circular motion!
- $\mathbf{F}\cdot\mathbf{v} = 0 \implies E =$const

Classical Hall effect

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- Consider a setup in fig 1
- Current in x direction, $\mathbf{B}$ in z direction
- $\mathbf{F_B} = qv_dB$ in y direction
- Accumulation of charges at the edges resulting an electric field in y direction
- Force will be balanced when $E_y = v_dB$
- For a constant current, resistance in y direction will be $\frac{V_y}{I} = \frac{Bv_dL_y}{I}$
- ρ_{xy} vs B: linear plot

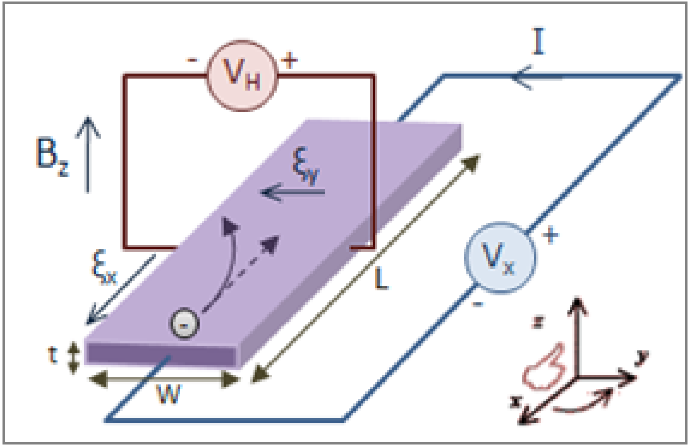

Figure: Hall Experiment setup

Quantum Particle in Magnetic field

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- The Hamiltonian for a free particle is given by $H = \frac{1}{2m}(\hat{p}_x^2 + \hat{p}_y^2)$
- Consider the magnetic field in the z-direction, using minimal coupling, $\hat{p}_{0i} = \hat{p}_i - \frac{e\hat{A}_i}{c}$
- We work in Landau gauge: **A** = (-By, 0), in this gauge $[H, p_x] = 0$ and hence $p_x = \hbar k_x$ is conserved
- Periodic boundary conditions $\implies k_x = \frac{2\pi n_x}{L_x}$
- The eigen-function therefore has the form $\Psi(x, y) = \frac{1}{\sqrt{L_x}} e^{ik_x x}\chi(y)$
- $\chi(y)$ satisfies $\left[\frac{p_y^2}{2m} + \frac{1}{2}m(\frac{eB}{m})^2(y - y_0)^2\right]\chi(y) = \mathcal{E}\chi(y), y_0 = \frac{p_x}{eB}$

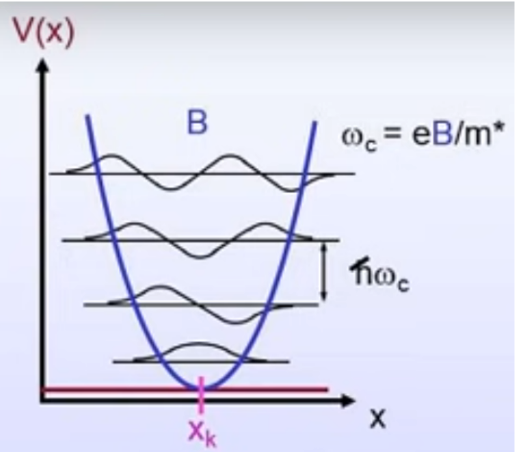

Figure: Landau Levels for 2DEG

- The equation looks similar to that of a harmonic oscillator along $\hat{y}$ direction
- The energy levels are thus given by $E_n = \hbar\omega_c(n+\frac{1}{2})$, $\omega_c = \frac{e_o B}{mc}$
- $\chi(y) = A_n e^{-\frac{-eB(y-y_0)}{\hbar}} H_n(\frac{eB(y-y_0)}{\hbar})$, where A_n is normalisation constant and H_n is nth Hermite polynomial
- These are called Landau levels (see fig 2)
- $y_0 = \frac{\hbar k_x}{eB} = \frac{\hbar 2\pi n_x}{eBL_x}$
- $y_0 \leq L_y \implies (n_x)_{\text{max}} = \frac{eBL_xL_y}{2\pi\hbar} = \frac{AB}{(h/e)}$

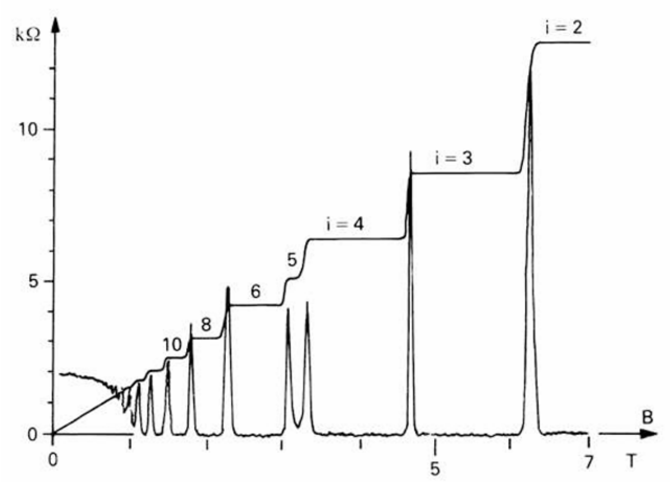

Figure: Integer Quantum Hall Effect

Graphene: Review

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- A hexagonal unit cell with a basis
- Basis vectors are given by $\delta_{\mathbf{1}} = \frac{a}{2}(\sqrt{3}\hat{x} + \hat{y})$ and $\delta_{\mathbf{2}} = \frac{a}{2}(\sqrt{3}\hat{x} - \hat{y})$
- The tight binding Hamiltonian is given by

$$H = \begin{pmatrix} 0 & 1 + e^{i\mathbf{k}\cdot\mathbf{a_1}} + e^{i\mathbf{k}\cdot\mathbf{a_2}} \\ 1 + e^{-i\mathbf{k}\cdot\mathbf{a_1}} + e^{-i\mathbf{k}\cdot\mathbf{a_2}} & 0 \end{pmatrix}$$

energy spectrum: $E(\mathbf{k}) = \pm\sqrt{3 + 2\cos\sqrt{3}k_y a + 4\cos\frac{\sqrt{3}k_y a}{2}\cos\frac{3k_x a}{2}}$

- $E = 0$ at 6 points: $\pm\frac{2\pi}{3a}(1, \frac{1}{\sqrt{3}}), \pm\frac{2\pi}{3a}(1, -\frac{1}{\sqrt{3}})$ and $\pm\frac{4\pi}{3\sqrt{3}a}(0, 1)$
- In Brillouin zone, e.g. $\frac{4\pi}{3\sqrt{3}a}(0, 1) + \mathbf{b_2} = \frac{2\pi}{3a}(1, -\frac{1}{\sqrt{3}})$
- Only two independent points: K, K'

- Low energy dispersion near K, K' points, let $\mathbf{q} = \mathbf{k} - \mathbf{K}(\mathbf{K}')$
- Consider the off-diagonal term $f(\mathbf{q})$ and $f'(\mathbf{q}) = \frac{\partial f(\mathbf{k})}{\partial k_x}\Big|_{\mathbf{K}(\mathbf{K}')}(k_x - K_x(K'_x)) + \frac{\partial f(\mathbf{k})}{\partial k_y}\Big|_{\mathbf{K}(\mathbf{K}')}(k_y - K_y(K'_y)) = \frac{3at}{2}(q_x + (-)iq_y)$
- The low energy Hamiltonian is $H_{\mathbf{K},\mathbf{K}'} = \hbar v_F \mathbf{q} \cdot \sigma$, where $\mathbf{q} = (q_x, q_y)$, $\sigma = (\sigma_x, \sigma_y)$ and $v_F = \frac{3at}{\hbar}$

Dirac Fermions in Magnetic field

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- Consider Dirac fermions in graphene in a strong magnetic field
- Electrons have relativistic dispersion law near K and K' points, which strongly modifies the Landau quantization of the energy and the position of the levels
- The Hamiltonian for Dirac fermions

$$H = v_F \begin{pmatrix} 0 & -(p_{0x} - ip_{0y}) & 0 & 0 \\ -(p_{0x} + ip_{0y}) & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{0x} - ip_{0y} \\ 0 & 0 & p_{0x} + ip_{0y} & 0 \end{pmatrix}$$

- Equations for K and K' valley are decoupled
- Basis for four-component wave function

$$\Psi = \begin{pmatrix} \Phi^A_{K'} \\ \Phi^B_{K'} \\ \Phi^A_{K} \\ \Phi^B_{K} \end{pmatrix}$$

such that Φ^A_K and Φ^B_K are envelope wavefunctions of A and B sites for the valley K, and $\Phi^A_{K'}$, $\Phi^B_{K'}$ for the valley K'

- Consider magnetic field orthogonal to the graphene layer
- We work in Landau gauge: $A = (-By, 0)$

- For the solutions of the K valley:

$$\epsilon\Phi_A^K = v_F(p_{0x} - ip_{0y})\Phi_B^K$$
$$\epsilon\Phi_B^K = v_F(p_{0x} + ip_{0y})\Phi_A^K$$

- Solving these equations with minimal coupling and Landau gauge $p_x \to p_x + eBy$ gives (solving for Φ_B^K first)
- From there we observe that the right side of the equation has the form of Hamiltonian for the harmonic oscillator, from that we get

$$E = \hbar\tilde{\omega}\text{sgn}(n)\sqrt{|n|}, n = 0, \pm 1...$$

- Similarly, solving for Φ_A^K gives

$$\epsilon = \hbar\tilde{\omega}\text{sgn}(n)\sqrt{|n| + 1}, n = 0, \pm 1, ...$$

- The Landau level index, n, can be positive or negative. The positive values correspond to electrons (conduction band), while the negative values correspond to holes (valence band).
- Furthermore, they are not equidistant as in conventional case and the largest energy separation is between the zero and the first Landau level. This large gap allows one to observe the quantum Hall effect in graphene, even at room temperature

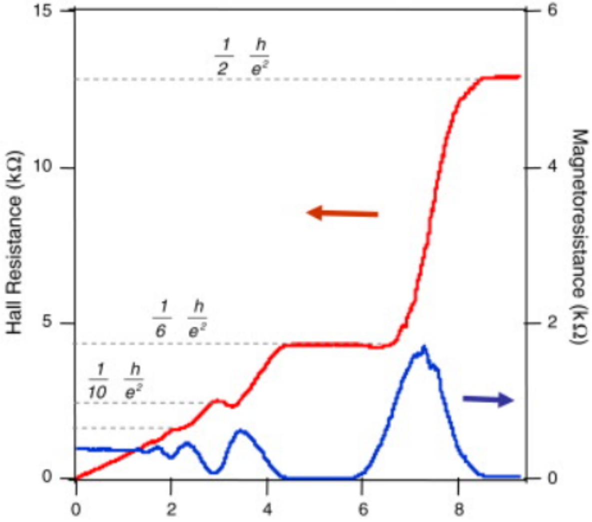
- We see that energy levels for site A from a non-zero value (when we solve for K points)
- So, for zero energy value, the electrons are completely localised on B sublattice for valley K and A sublattice for valley K'
- The asymmetry between A and B sublattice originates from asymmetry in positions of the nearest neighbors for atoms at A and B sublattice. This property of the wavefunctions for the Landau levels in graphene makes the $n = 0$ level very special for different magnetic applications of graphene

- $$\Psi^{K}_{n,k} = \frac{C_n}{\sqrt{L}} e^{-ikx} \begin{pmatrix} 0 \\ 0 \\ \mathrm{sgn}(n)(-i)\phi_{|n|-1,k} \\ \phi_{|n|,k} \end{pmatrix}$$
- $$\Psi^{K'}_{n,k} = \frac{C_n}{\sqrt{L}} e^{-ikx} \begin{pmatrix} \phi_{|n|,k} \\ \mathrm{sgn}(n)(-i)\phi_{|n|-1,k} \\ 0 \\ 0 \end{pmatrix}$$
- Here L^2 is the area of system, C_n is normalisation and $\phi_{n,k} \propto \exp(-\frac{1}{2}\frac{(y-kl_B^2)^2}{l_B^2})H_n(\frac{(y-kl_B^2)}{l_B})$, $H_n(x)$ are Hermite polynomials
- $\phi_{n,k}$ is the wavefunction for a non-relativistic electron at the nth Landau level.

- The Landau levels are obtained in Bilayer Graphene in Magnetic field and the levels are given by:

$$E_n = \text{sgn}(n)\hbar\omega_c\sqrt{|n|(|n|-1)}$$

- This is formed due to interlayer coupling; This sequence is linear in field, similar to the standard case, but it contains an additional zero-energy level, which is independent of the field.


15
10
5
0
6
4
2
0
0
2
4
6
8
Hall Resistance (kΩ)
Magnetoresistance (kΩ)
$\frac{1}{2}\ \frac{h}{e^2}$
$\frac{1}{6}\ \frac{h}{e^2}$
$\frac{1}{10}\ \frac{h}{e^2}$

Landau Levels in Graphene

Thank you for listening!
Any questions?