







Regularized Regression

- Want to build a parsimonious but interpretable model
- Shrink the number of predictors and/or size by imposing penalties on the estimated coefficients (L1, L2)
  - LASSO:
    - picks one correlated variable, others discarded. Sparse.
  - Ridge:
    - correlated variables coefficients are pushed to the same value
  - Elastic Net:
    - sparse solution, correlated variables grouped, enter/ leave the model together

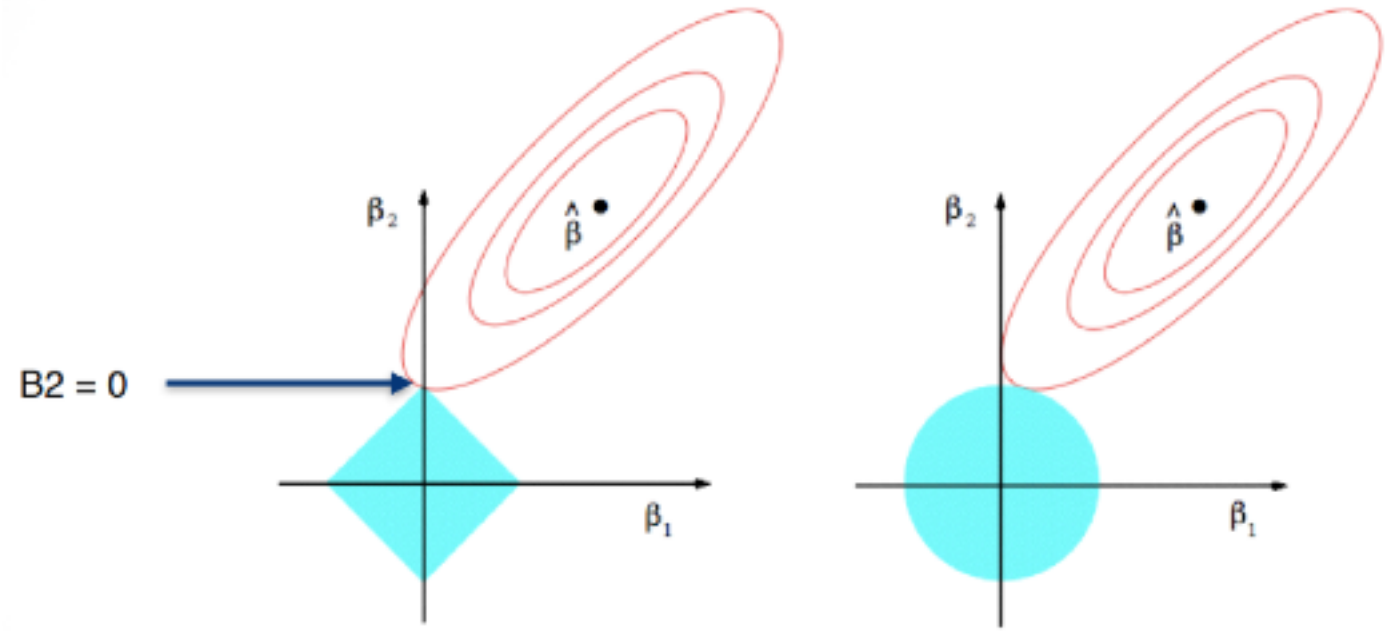
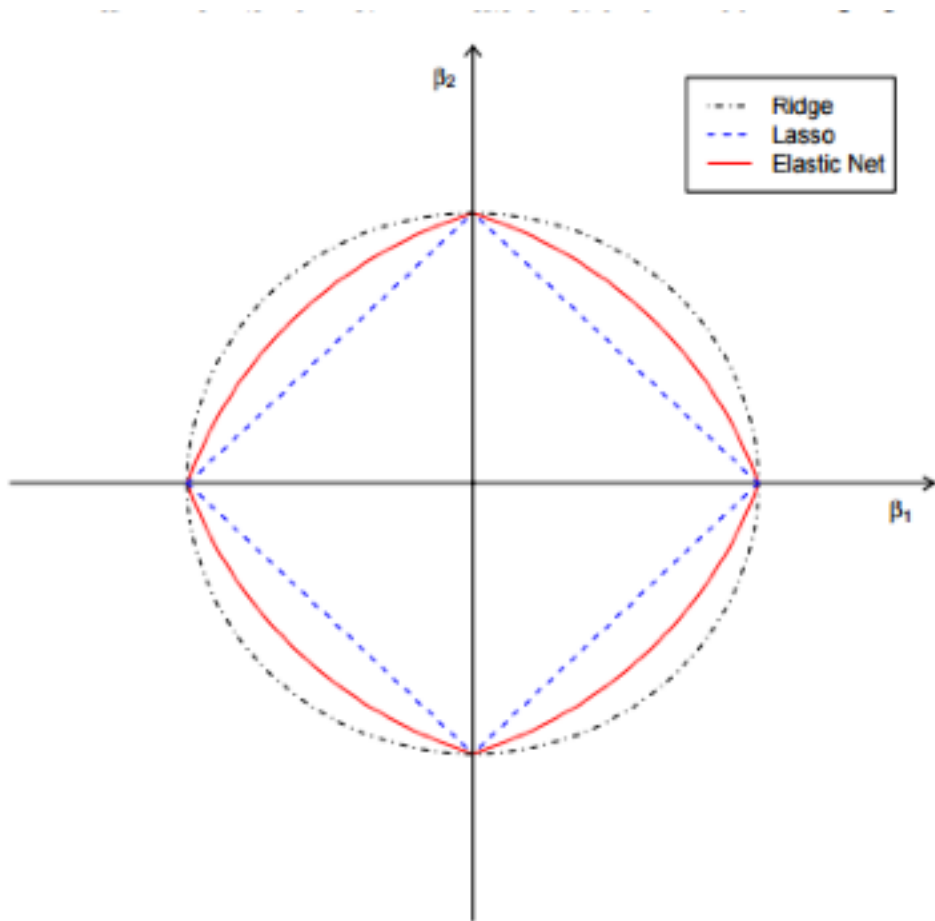
$$\min_{\beta_0, \beta} \left\{ \frac{1}{N} \sum_{i=1}^N (y_i - \beta_0 - x_i^T \beta)^2 \right\} \text{ subject to } \sum_{j=1}^p |\beta_j| \leq t.$$

$$\text{minimize} \sum_{i=1}^n (y_i - \beta^T z_i)^2 \text{ s.t. } \sum_{j=1}^p \beta_j^2 \leq t$$

$$\arg \min_{\beta} ||\mathbf{y} - \mathbf{X}\beta||^2 + \lambda_2 ||\beta||^2 + \lambda_1 ||\beta||_1$$



# Constraints for L1, L2 and L1&L2



LASSO Regression vs. Ridge Regression

# Regularized Regression

- Want to build a parsimonious but interpretable model
- Shrink the number of predictors and/or size by imposing penalties on the estimated coefficients (L1, L2)
  - LASSO:  $\min_{\beta_0, \beta} \left\{ \frac{1}{N} \sum_{i=1}^N (y_i - \beta_0 - x_i^T \beta)^2 \right\}$  subject to  $\sum_{j=1}^p |\beta_j| \leq t$ .
    - picks one correlated variable, others discarded. Sparse.
  - Ridge:  $\text{minimize } \sum_{i=1}^n (y_i - \beta^T \mathbf{z}_i)^2 \text{ s.t. } \sum_{j=1}^p \beta_j^2 \leq t$ 
    - correlated variables coefficients are pushed to the same value
  - Elastic Net:  $\arg \min_{\beta} \|\mathbf{y} - \mathbf{X}\beta\|^2 + \lambda_2 \|\beta\|^2 + \lambda_1 \|\beta\|_1$ 
    - sparse solution, correlated variables grouped, enter/ leave the model together