A Novel Third Harmonic Injection Method for Closed Loop Control of PMSM Motors

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Abstract **- In closed loop control using Field Oriented Control (FOC), three phase voltage outputs of FOC controller are not in a manner that the resultant stator voltage vector and rotor magnetic axis are 90° electrical apart. Calculation of the Third Harmonic Value based on rotor angle measurement can lead to phase difference between fundamental and Third Harmonic voltages, and can lead to degraded performance. This paper presents a novel methodology to dynamically calculate third harmonic value using the three phase voltage outputs from the FOC controller. Simulation and experimental results are also presented for a three phase Permanent Magnet Synchronous Motor (PMSM) in closed loop current control with FOC and with FOC plus Third Harmonic Injection to confirm the methodology.**

I. INTRODUCTION

Field Oriented Control (FOC) [1]–[4] and Third Harmonic Injection [5]-[6] are well known methods for controlling Permanent Magnet Synchronous Motors (PMSM) and Induction Motors. In the case of PMSM motors, if field weakening is not desired, FOC provides optimal performance by orthogonally aligning rotor magnetic field and torque producing stator magnetic field. For 3-phase motors, Third Harmonic Injection (THI) along with FOC enables an increase of inverter DC bus utilization by 15.47% thereby allowing better motor speed-torque performance. One of the simplest methods of inducing a third harmonic is by calculating instantaneous average of the minimum and maximum of 3-phase voltage outputs from FOC control, subtracting this value from the phase voltages, and multiplying the results by 1.1547[4]. This technique gives a third harmonic waveform in a triangular profile.

In this paper, we discuss on analytical basis for third harmonic injection justifying the magnitude of the third harmonic for best DC bus utilization. Additionally, we will present a different strategy for obtaining instantaneous values of sinusoidal third harmonic using instantaneous 3-phase voltage outputs from FOC control. Calculating the sinusoidal third harmonic value from rotor angle measurement directly in an open-loop fashion and adding it to the three-phase voltage outputs of FOC leads to degraded motor performance as the phase angle of fundamental voltage will not match with the calculated third harmonic. Determining instantaneous magnitude and phase of fundamental voltage waveforms is inevitable for correct implementation of the strategy.

II. THIRD HARMONIC INJECTION METHODOLOGY

A. Field Oriented Control

For simplicity, we consider 3-phase, 2 pole star-connected PMSM motor with isolated neutral point. Consider the dq reference frame as shown in the Figure 1. Now, in any given position of the rotor, we can resolve the magnetic fields from A, B and C phases into dq axes direction if we know the rotor position.

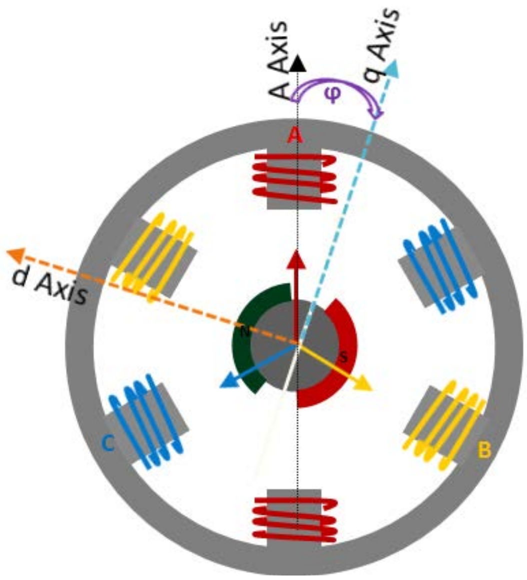

Figure 1. dq Axes Convention

Using the above convention and $IA+IB+IC = 0$, FOC equations can be written as

$$Iq = IA\left(\frac{3}{2}\cos\varphi + \frac{\sqrt{3}}{2}\sin\varphi\right) + IB\left(\sqrt{3}\sin\varphi\right) \qquad ..(1)$$

$$Id = IA\left(\frac{3}{2}\sin\varphi - \frac{\sqrt{3}}{2}\cos\varphi\right) - IB\left(\sqrt{3}\cos\varphi\right) \qquad ..(2)$$

Where,

IA = Phase A Current,

IB = Phase B Current,

IC = Phase C Current,

Iq = Quadrature Axis Current,

Id = Direct Axis Current,

φ = Angle between Phase A magnetic axis and q Axis.

These are standard FOC equations of Clark transformation with ABC to αβ transformation, and then Park transformation from αβ to dq transformation, as found in literature. Eq. (1) can be represented in matrix form as follows

$$\begin{bmatrix} Iq \\ Id \end{bmatrix} = \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} \frac{3}{2} & 0 \\ -\frac{\sqrt{3}}{2} & -\sqrt{3} \end{bmatrix} \begin{bmatrix} IA \\ IB \end{bmatrix} \quad ..(3)$$

In order to calculate the phase voltages from the FOC controller output quadrature axis and direct axis voltages, we use the following equation

$$\begin{bmatrix} VA \\ VB \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & 0 \\ \frac{-1}{3} & \frac{-1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} Vq \\ Vd \end{bmatrix} \quad ..(4)$$

and,

$$VC = -(VA + VB) \quad ..(5)$$

Where,
VA = Phase A Voltage,
VB = Phase B Voltage,
VC = Phase C Voltage,
Vq = Quadrature Axis Voltage,
Vd = Direct Axis Voltage.

B. Third Harmonic Injection - Sinusoidal

For a well-controlled system, under steady operating conditions, phase voltages extracted from Eq. (4) & (5) are sinusoidal.

Now, for injecting a Sinusoidal Third Harmonic signal, we have to consider a key point, which is the optimum amplitude of Third Harmonic wave. The best DC bus utilization can be obtained using injected amplitude which brings down the resultant waveform's peak amplitude the most [5][6]. To find this amplitude let us consider a simple fundamental sine wave along with a third harmonic of unknown amplitude, x as shown below.

$$F(\theta) = Sin(\theta) + xSin(3\theta) \quad ..(6)$$

Finding the extremum of the above equation using $\frac{d}{d\theta}F(\theta) = 0$ gives,

$$\theta = Cos^{-1}\left(\sqrt{\frac{9x-1}{12x}}\right) \quad ..(7)$$

Substituting the above θ in Eq. (6) gives

$$P(x) = F(\theta) = (3x+1)\sqrt{\frac{(3x+1)}{(12x)}} - 4x\left(\frac{3x+1}{12x}\right)^{3/2} \quad ..(8)$$

Finding the extremum of the above function using $\frac{d}{dx}P(x) = 0$, gives two solutions for x, -1/3 and 1/6.

At x = 1/6,

$$\frac{d^2}{dx^2}P(x) = 10.3923,$$

which shows this is a minimum. The other solution at x = -1/3 is infeasible. Using x = 1/6 and Eq. (8), we get θ = 60 deg. At θ = 60 deg,

$$\frac{d^2}{dx^2}F(\theta) = -2,$$

which shows this is the maximum of the function F(θ).

From the above analysis, it is clear that the amplitude of the third harmonic should be 1/6th of the amplitude of fundamental for best modulation factor and thereby for the best DC bus utilization. Substituting x = 1/6 and θ = 60 in Eq. (6) gives 0.866, which is the peak of resultant waveform with third harmonic. This gives us a modulation factor of 1/0.866, i.e, 1.1547, giving 15.47% more DC bus utilization.

Now, for a given instantaneous Vq and Vd output combination of FOC control, we need to find the instantaneous third harmonic value to be induced. For that we need to find the phase and the magnitude of the fundamental phase waveform. For a balanced 3-Phase system, phase voltages can be expressed as

$$\begin{bmatrix} VA \\ VB \\ VC \end{bmatrix} = \begin{bmatrix} VSin(\theta) \\ VSin(\theta - 120) \\ VSin(\theta - 240) \end{bmatrix} \quad ..(9)$$

Where,
V = Instantaneous Magnitude of Fundamental,
θ = Instantaneous Phase of the Fundamental.

If we know instantaneous V and θ, we can easily calculate the instantaneous third harmonic value $\frac{V}{6}\sin(3\theta)$. Using Eq. (4), (5) and (9), and trigonometric identities, we can prove that instantaneous magnitude of fundamental waveform is

$$V = \frac{\sqrt{(Vq^2+Vd^2)}}{1.5} \quad ..(10)$$

With the knowledge of instantaneous magnitude, we can find instantaneous Sin(θ) from Eq. (9) as,

$$Sin(\theta) = \frac{VA}{V} \quad ..(11)$$

From Eq. (10) and (11), instantaneous third harmonic value can be calculated as

$$V_{ref} = \frac{V}{6}Sin(3\theta) = \frac{V}{6}(3Sin(\theta) - 4Sin^3(\theta)) \quad ..(12)$$

Using the above instantaneous Sinusoidal Third harmonic value, the net phase voltage with THI can be expressed as,

$$VA_{THI} = (VA + V_{ref}) * 1.1547$$
$$VB_{THI} = (VB + V_{ref}) * 1.1547$$
$$VC_{THI} = (VC + V_{ref}) * 1.1547 \quad ..(13)$$

C. Triangular Third Harmonic Injection

For triangular third harmonic injection [4], instantaneous value of third harmonic is obtained as follows

$$V_{ref} = \frac{[min(VA,VB,VC) + max(VA,VB,VC)]}{2} \quad ..(14)$$

Using the above instantaneous triangular third harmonic value, phase voltages with THI are calculated as,

$$VA_{THI} = (VA - V_{ref}) * 1.1547$$

$$VB_{THI} = (VB - V_{ref}) * 1.1547$$

$$VC_{THI} = (VC - V_{ref}) * 1.1547 \quad ..(15)$$

III. Simulation Results and Analysis

To understand the difference between Sinusoidal THI and Triangular THI, a simulation model has been made. A snapshot of the model is shown in Fig. 2. The model consists of algorithms for both Sinusoidal and Triangular THI with open-loop test vector inputs for FOC controller outputs Vq & Vd and Ψ. Ψ varies at a rate of 60 rpm. Vq and Vd are in per-unit, and Vd is considered zero for PMSM motor. Simulation results are shown in Figures 3 to 5.

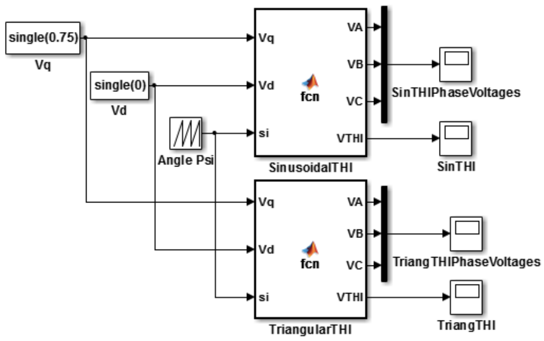

Figure 2. Simulation Models

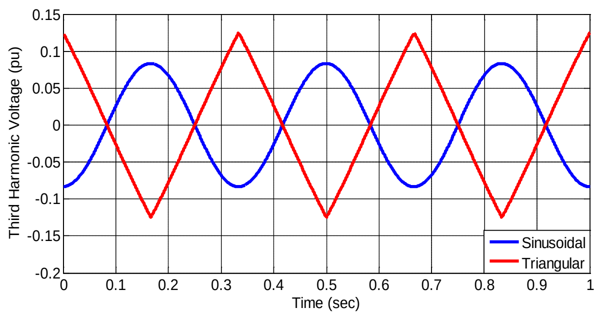

Figure 3. Simulated Third Harmonic Voltages

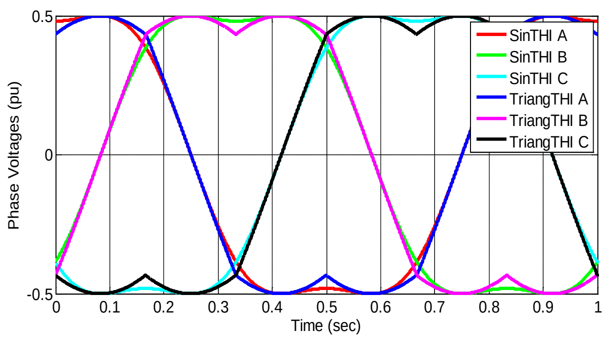

Figure 4. Phase Voltages with Third Harmonics

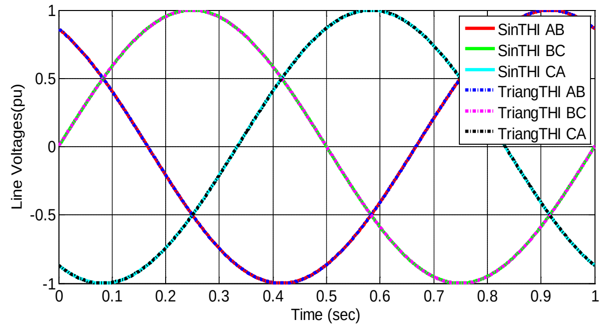

Figure 5. Line Voltages

From Figure 3, we can clearly see the difference between Sinusoidal and Triangular Third Harmonics. A phase shift of 180 deg between the two is due to the fact that Sinusoidal TH is added to phase voltages and Triangular TH is subtracted from the phase voltages. Despite the difference between the two Third Harmonic waves, the maximum value of phase voltage with a modulation factor of 1.1547 is same, and is evident from Fig. 4. From Figure 5, we can also see that line voltages are exactly the same in both the cases.

VI. Experimental Verification

The test setup consists of a PMSM motor connected to a fully controllable loading DC motor. A torque sensor and an encoder are mounted between the unit under test and the loading motor. PMSM motor is controlled through Freescale's PowerPC based motor control processor tailored for motor control. Loading motor can be controlled precisely in speed control mode or current control mode. The loading torque on the test motor can be controlled by controlling the current through the loading motor. The torque supplied by the test motor can be measured through the torque sensor. The picture of the test setup is shown below.

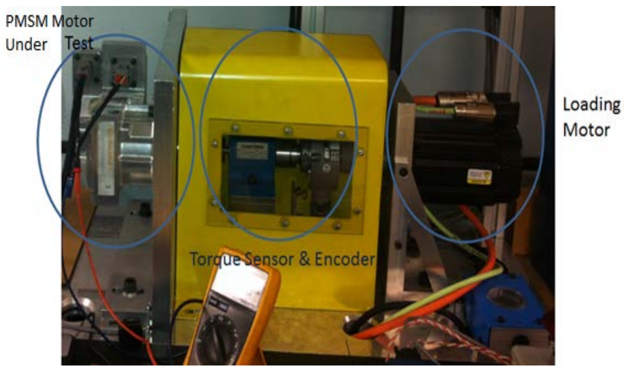

Figure 6. Experimental Setup

For comparing Sinusoidal and Triangular THIs, PMSM motor is run in speed control mode and the Loading motor is run in current control mode. Fig. 7 shows experimental data collected at 500 rpm and around 2.96 Nm load torque. As can be seen clearly, the phase voltage profiles are matching with the simulation results.

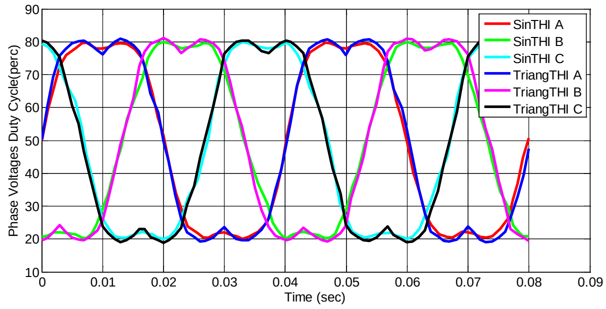

Figure 7. Phase Voltages Duty Cycles (with Third Harmonics)

To understand the performance of motor for both Sinusoidal and Triangular THI, Speed-Torque characterization is conducted for the PMSM motor using the two techniques. PMSM motor is controlled in speed control mode and loading motor is controlled in current control mode. PMSM motor is commanded with a desired target speed and the loading torque is increased by increasing the loading motor current command. PMSM motor will regulate the speed to the commanded speed till the load torque is within the capacity at that speed. As the load torque increase beyond the capability of the motor, the speed drops. This behavior is used to extract the speed torque characteristics. Figure 8 shows Speed-Torque characteristics of the PMSM motor with different techniques.

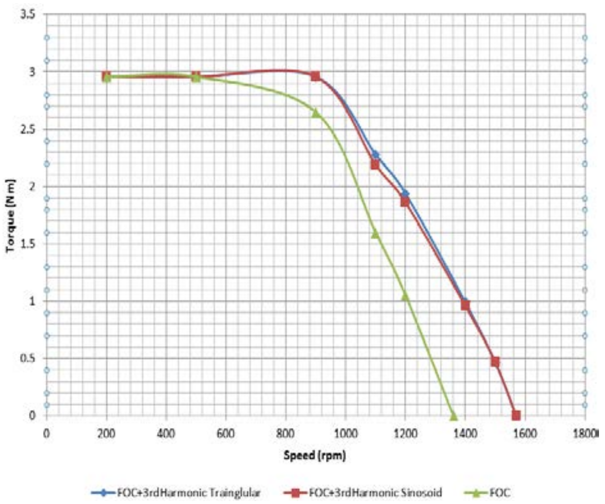

Figure 8. PMSM Motor Speed-Torque Characteristics with different techniques

From Figure 8, we can see that motor Speed-Torque performance is higher with FOC+THI techniques compared to just FOC control. This is from the fact that THI enables 15.47% more DC bus utilization compared to just FOC control. Also, we can see that Sinusoidal and Triangular THI performance is similar proving the fact that Third Harmonic gets cancelled in the line voltages.

VII. Conclusions

In this paper, theoretical/analytical basis for Third Harmonic Injection has been presented along with a new sinusoidal Third harmonic injection technique. Simulation and experimental results for the comparison of conventional Triangular THI and the proposed Sinusoidal THI have also been presented. The results show that the motor performance with the proposed Sinusoidal THI technique is at par with the conventional Triangular THI technique, thereby increasing the DC utilization by 15.47% and improving motor Speed-Torque performance compared to FOC control.

VIII. Acknowledgement

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IX. References

[1] R. Krishnan, "Electric Motor Drives – Modeling, Analysis, and Control," Prentice Hall, 2001.

[2] Bimal. K. Bose, "Modern Power Electronics and AC Drives," Prentice Hall, 2002.

[3] Freescale, "Beyond Bits – Motor Control Edition', Issue 8, www.freescale.com.

[4] Microsemi User Guide, "Field Oriented Control of Permanent Magnet Synchronous Motors," www.microsemi.com.

[5] Jose. J., G. N. Goyal, M. V. Aware, "Improved Inverter Utilization Using Third Harmonic Injection", In Power Electronics, Drives and Energy Systems(PEDES) & 2010 Power India, 2010 Joint International Conference on, pp. 1-6, IEEE, 2010.

[6] A. Iqbal, E. Levi, M. Jones and S. N. Vukosavic, "Generalized sinusoidal PWM with harmonic injection for multi-phase VSIs," in Proc. IEEE Power Elec. Spec. Conf. PESC, Jeju, Korea, 2006, pp. 2871-2877. [6]