|  | $\mathbf{u}_0$ | $\mathbf{u}_1$ | $\mathbf{u}_2$ | $\mathbf{u}_3$ | $\mathbf{u}_4$ | $\mathbf{u}_5$ | $\mathbf{u}_6$ |
|---|---|---|---|---|---|---|---|
| $\mathbf{i}_0$ | 5 | 5 | 2 | 0 | 1 | ? | ? |
| $\mathbf{i}_1$ | 3 | ? | ? | 0 | ? | 3 | ? |
| $\mathbf{i}_2$ | ? | 4 | 1 | ? | ? | 1 | 2 |
| $\mathbf{i}_3$ | 2 | 2 | 3 | 4 | 4 | ? | 4 |
| $\mathbf{i}_4$ | 2 | 0 | 4 | ? | ? | ? | 5 |
|  | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ |
| $\bar{u}_j$ | 3 | 11/4 | 5/2 | 4/3 | 5/2 | 2 | 11/3 |

a) Original rating matrix $\mathbf{Y}$ and mean user ratings.

|  | $\mathbf{u}_0$ | $\mathbf{u}_1$ | $\mathbf{u}_2$ | $\mathbf{u}_3$ | $\mathbf{u}_4$ | $\mathbf{u}_5$ | $\mathbf{u}_6$ |
|---|---|---|---|---|---|---|---|
| $\mathbf{i}_0$ | 2 | 9/4 | -1/2 | -4/3 | -3/2 | 0 | 0 |
| $\mathbf{i}_1$ | 0 | 0 | 0 | -4/3 | 0 | 1 | 0 |
| $\mathbf{i}_2$ | 0 | 5/4 | -3/2 | 0 | 0 | -1 | -5/3 |
| $\mathbf{i}_3$ | -1 | -3/4 | 1/2 | 8/3 | 3/2 | 0 | 1/3 |
| $\mathbf{i}_4$ | -1 | -11/4 | 3/2 | 0 | 0 | 0 | 4/3 |

b) Normalized rating matrix $\bar{\mathbf{Y}}$.

|  | $\mathbf{u}_0$ | $\mathbf{u}_1$ | $\mathbf{u}_2$ | $\mathbf{u}_3$ | $\mathbf{u}_4$ | $\mathbf{u}_5$ | $\mathbf{u}_6$ |
|---|---|---|---|---|---|---|---|
| $\mathbf{u}_0$ | 1 | 0.85 | -0.55 | -0.67 | -0.87 | 0 | -0.31 |
| $\mathbf{u}_1$ | 0.85 | 1 | -0.87 | -0.40 | -0.55 | -0.23 | -0.72 |
| $\mathbf{u}_2$ | -0.55 | -0.87 | 1 | 0.27 | 0.32 | 0.47 | 0.97 |
| $\mathbf{u}_3$ | -0.67 | -0.40 | 0.27 | 1 | 0.87 | -0.29 | 0.13 |
| $\mathbf{u}_4$ | -0.87 | -0.55 | 0.32 | 0.86 | 1 | 0 | 0.11 |
| $\mathbf{u}_5$ | 0 | -0.23 | 0.47 | -0.29 | 0 | 1 | 0.55 |
| $\mathbf{u}_6$ | -0.31 | -0.72 | 0.97 | 0.13 | 0.11 | 0.55 | 1 |

c) User similarity matrix $\mathbf{S}$.

|  | $\mathbf{u}_0$ | $\mathbf{u}_1$ | $\mathbf{u}_2$ | $\mathbf{u}_3$ | $\mathbf{u}_4$ | $\mathbf{u}_5$ | $\mathbf{u}_6$ |
|---|---|---|---|---|---|---|---|
| $\mathbf{i}_0$ | 2 | 9/4 | -1/2 | -4/3 | -3/2 | -1/2 | -0.60 |
| $\mathbf{i}_1$ | 0 | -0.21 | 0.15 | -4/3 | -4/3 | 1 | 0.56 |
| $\mathbf{i}_2$ | 5/4 | 5/4 | -3/2 | -1.55 | -1.54 | -1 | -5/3 |
| $\mathbf{i}_3$ | -1 | -3/4 | 1/2 | 8/3 | 3/2 | 0.41 | 1/3 |
| $\mathbf{i}_4$ | -1 | -11/4 | 3/2 | 1.45 | 1.46 | 1.41 | 4/3 |

d) $\hat{\mathbf{Y}}$

|  | $\mathbf{u}_0$ | $\mathbf{u}_1$ | $\mathbf{u}_2$ | $\mathbf{u}_3$ | $\mathbf{u}_4$ | $\mathbf{u}_5$ | $\mathbf{u}_6$ |
|---|---|---|---|---|---|---|---|
| $\mathbf{i}_0$ | 5 | 5 | 2 | 0 | 1 | 1.5 | 3.06 |
| $\mathbf{i}_1$ | 3 | 2.54 | 2.65 | 0 | 1.17 | 3 | 4.22 |
| $\mathbf{i}_2$ | 4.25 | 4 | 1 | -0.22 | 0.96 | 1 | 2 |
| $\mathbf{i}_3$ | 2 | 2 | 3 | 4 | 4 | 2.41 | 4 |
| $\mathbf{i}_4$ | 2 | 0 | 4 | 2.78 | 3.96 | 3.41 | 5 |

e) Full $\mathbf{Y}$