CQF Exam One

January 2023 Cohort

Instructions

All questions must be attempted. Requested mathematical and full computational workings must be provided to obtain maximum credit. Books and lecture notes may be referred to. Help from others is not permitted.

- Submit an exam report as a pdf file and name it E1 YOURNAME REPORT.pdf. Example: E1_JOHN SMITH_REPORT.pdf. Use the same name as you gave on CQF Portal.
- If your report is a Python notebook or Excel with handwritten inserts – remove unnecessary output and print as ONE PDF.
  Submission of exam report as Word/Excel documents or Python notebook only will receive a deduction in marks. Absence of specifically requested tables and plots will also receive a deduction. Submission in multiple pdf/image files will result in a fail or request to resubmit or defer.
- Computation can be coded in Python/other languages, or done in Excel because this is our first exam. Arrange all computational workings (code files) in a zip file and name it E1 YOURNAME CODE_zip

Upload and deadline questions should be directed to CQFProgram@fitchlearning.com and clarifying questions to Richard.Diamond@fitchlearning.com. Tutor is unable to re-explain calculation or confirm correct numerical answers. Please make a good use of lecture material.

Please follow the instructions on file naming. It will facilitate the faster turnaround of exam scripts.

Marking Scheme: Q1 16% **Q2** 18% **Q3** 20% **Q4** 10% **Q5** 16% **Q6** 20%. Total is 100%.

Optimal Portfolio Allocation

An investment universe of the following risky assets with a dependence structure (correlation) is given:

Asset	$\boldsymbol{\mu}$	$\boldsymbol{\sigma}$	$\boldsymbol{w}$
A	0.02	0.05	w_1
B	0.07	0.12	w_2
C	0.15	0.17	w_3
D	0.20	0.25	w_4

$$\boldsymbol{R} = \begin{pmatrix} 1 & 0.3 & 0.3 & 0.3 \\ 0.3 & 1 & 0.6 & 0.6 \\ 0.3 & 0.6 & 1 & 0.6 \\ 0.3 & 0.6 & 0.6 & 1 \end{pmatrix}$$

Question 1. Consider minimum variance portfolio with a target return m.

$$\underset{\boldsymbol{w}}{\operatorname{argmin}} \frac{1}{2}\boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w} \qquad \text{s.t. } \boldsymbol{w}'\mathbf{1} = 1, \qquad \mu_\Pi = \boldsymbol{w}'\boldsymbol{\mu} = m$$

- Formulate the Lagrangian and give its partial derivatives.
- Write down the analytical solution for optimal allocations $\boldsymbol{w}^*$ (derivation not required).
- Inverse optimisation: generate above 700 random allocation sets (vectors) 4×1, those will not be optimal allocations.

  Standardise each set to satisfy $w'\mathbf{1} = 1$, in fact you can generate 3 allocations and compute the 4th.

  For each vector of allocations compute $\mu_\Pi = \boldsymbol{w}'\boldsymbol{\mu}$ and $\sigma_\Pi = \sqrt{\boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w}}$.

  Plot the cloud of points of μ_Π vertically on σ_Π horizontally. Explain this plot.

Hint: Since we treat this as an inverse optimisation, there is no computation of $\boldsymbol{w}^*$ from the ready formula.

Question 2. Consider optimisation for a tangency portfolio (maximum Sharpe Ratio).

- Formulate optimisation expression.
- Formulate Lagrangian function and give its partial derivatives only.
- For the range of tangency portfolios given by $r_f = 50bps, 100bps, 150bps, 175bps$ optimal compute allocations (ready formula) and σ_Π. Present results in a table.

  Plot the efficient frontier in the presence of a risk-free asset for $r_f = 100bps, 175bps$.

Products and Market Risk

Question 3. Implement the multi-step binomial method as described in Binomial Method lecture with the following variables and parameters: stock $S = 100$, interest rate $r = 0.05$ (continuously compounded) for a call option with strike $E = 100$, and maturity $T = 1$.

- Use any suitable parametrisation for up and down moves uS, vS.
- Compute the option value for a range of volatilities $[0.05, \ldots, 0.80]$ and plot the result. Set trees to have a minimum four time steps.
- Now, compute and plot the value of one option, $\sigma_{imp} = 0.2$ as you increase the number of time steps $NTS = 4, 5, \ldots, 50$.

Hint: This is a computational problem, best coded in Python. Plots must be presented.

Question 4. Use the ready formula for Expected Shortfall in order to compute the standardised value of Expected Shortfall for $N(0, 1)$.

- Compute for the following range of percentiles [99.95; 99.75; 99.5; 99.25; 99; 98.5; 98; 97.5]
  Provide a table.
- The formula to use, and $1 - c$ refers to $1 - 99.95$ and so on,

$$\mathrm{ES}_c(X) \quad = \quad \mu - \sigma \, \frac{\phi(\Phi^{-1}(1-c))}{1-c}.$$

Hint: For the derivation and explanation of the formula please refer to VaR and ES lecture solutions.

Question 5. For this questions, use S&P500 index data provided in order to implement backtesting for 99%/10day Value at Risk and report the following:

(a) The count and percentage of VaR breaches.

(b) The count and percentage of consecutive VaR breaches. Example: $1, 1, 1$ indicates two consecutive occurrences.

(c) Provide a plot which identifies the breaches.

$$\text{VaR}_{10D,t} = \text{Factor} \times \sigma_t \times \sqrt{10}$$

- Compute the rolling standard deviation σ_t from 21 daily returns.
- Timescale of the standard deviation is 'daily' regardless of how many returns are in the sample. We project from 1-day to 10-day using the additivity of variance $\sigma_{10D} = \sqrt{\sigma_t \times 10}$.
- VaR is fixed at time t and compared to the return realised from t to $t + 10$. A breach occurs when that forward realised 10-day return $\ln(S_{t+10}/S_t)$ is below the VaR_t quantity.

$$r_{10D,t+10} < \text{VaR}_{10D,t} \qquad \text{means breach, given both numbers are negative.}$$

Question 6. Re-implement backtesting using assumptions above (as necessary) and the EWMA variance forecast equation below. This is also known as RiskMetrics approach.

The tutor will not reconfirm how to compute this model, but you can use the variance of your computed log-returns for the entire dataset to initialise the scheme.

$$\sigma^2_{t+1\,|\,t} = \lambda\,\sigma^2_{t\,|\,t-1} + (1-\lambda)\,r_t^2$$

with $\lambda = 0.72$ value set to minimise out of sample forecasting error, and r_t refers to a return.

Provide the same deliverables (a), (b), and (c) as in the previous Question.

END OF EXAM