CQF Exercise 2.1 Portfolio Theory

1. **The Global Minimum Variance Portfolio**

   - Express the variance of the portfolio returns, $\sigma_\Pi^2(w_A)$ as a sole function of the weight w_A invested in asset A (**Hint:** use the budget equation $w_B + w_A = 1$).

   - Minimize the function $\sigma_\Pi^2(w_A)$ to find the asset allocation of the global minimum variance portfolio.

   - Under what conditions on σ_A, σ_B and/or ρ_{AB} does this result hold?

2. **The Tangency Portfolio**

   - Find a functional form $S(w_A^t, w_B^t)$ for the slope of the tangency line.

   - Express the slope of the tangency line, as a sole function $S(w_A^t)$ of the weight w_A^t invested in asset A (**Hint:** use the budget equation $w_B^t + w_A^t = 1$).

   - Find the tangency portfolio's asset allocation. (**Hint:** rather than maximizing $S(w_A^t)$, consider maximizing $S^2(w_A^t)$)

3. (Optional) What are the economic significance of α and β in Sharpe's Market Model and how are they measured or estimated in practice?