中国股市条件波动率模型的估计与检验

蒋志强 *、储丽娅 #

* zqjiang.ecust@qq.com
#cherry_8621@163.com

条件波动率和收益率模型

收益率模型

$$\begin{cases} r_t = \underline{\mu_t} + \epsilon_t \\ \epsilon_t = \underline{\sigma_t z_t}, \end{cases}$$

条件波动率和收益率模型

收益率模型 $\begin{cases} r_t = \mu_t + \epsilon_t \\ \epsilon_t \equiv \sigma_t z_t, \end{cases}$

μ_t 的设定：

- **常数**
- **ARMA过程**

$$r_t = \phi_0 + \sum_{p=1}^{P} \phi_p r_{t-p} + \sum_{q=1}^{Q} \theta_q \epsilon_{t-q} + \epsilon_t$$

有： $\mathrm{Var}[r_t | \mathbf{I}_{t-1}] = \sigma_t^2$

条件波动率和收益率模型

收益率模型

$$\begin{cases} r_t = \mu_t + \epsilon_t \\ \epsilon_t \equiv \sigma_t z_t, \end{cases}$$

σ_t^2的设定：

- **ARCH**
- **GARCH**
- **EGARCH**
- **ARCH-M**
- **……**

条件波动率和收益率模型

收益率模型
$$\begin{cases} r_t = \mu_t + \epsilon_t \\ \epsilon_t \equiv \sigma_t z_t, \end{cases}$$

Z_t的设定：

- **标准正态分布**
- **标准t分布**
- **标准的广义误差分布**
- **混合正态分布**
- ……

ARCH

- **ARCH(1)**

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2, \quad \alpha_0 > 0, 0 < \alpha_1 < 1.$$

- **ARCH(q)**

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \cdots + \alpha_q \epsilon_{t-q}^2 \equiv \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2$$

✓ **保证 $\sigma_t^2 > 0$，要求 $\alpha_i > 0$ $(i = 0, 1, \ldots, q)$**

✓ **保证平稳，要求** $\sum_{i=1}^{q} \alpha_i < 1$

GARCH

- GARCH(p, q)

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2$$

✓ 保证 $\sigma_t^2 > 0$，要求 $\alpha_i \ \beta_i > 0 \ (i = 0,1,\ldots,q)$

✓ 保证平稳，要求

$$\sum_{i=1}^{p} \beta_i + \sum_{i=1}^{q} \alpha_i < 1$$

EGARCH

- EGARCH(p, q)

$$\log\sigma_t^2=\omega+\sum_{i=1}^{p}\beta_i\log\sigma_{t-i}^2+\sum_{i=1}^{q}\alpha_i(|z_{t-i}|-E|z_{t-i}|)+\sum_{i=1}^{q}\theta_i z_{t-i}$$

- ✓ **杠杆效应** Leverage effect: $\theta_i<0$

Z_t设定

- **标准正态分布**

$$f(z_t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z_t^2}{2}}$$

- **标准 t 分布**

$$g(x) = \frac{\Gamma((v+1)/2)}{\sqrt{v\pi}\,\Gamma(v/2)} \left(1 + \frac{x^2}{v}\right)^{-(v+1)/2},$$

$$f(z_t) = \sigma g(\sigma z_t), \quad \sigma \equiv \sqrt{v/(v-2)}.$$

$$v(v > 2)$$

实验流程

1. 数据描述性统计
2. 检验ARCH效应
3. ARCH(1)和GARCH(1,1)模型参数估计和条件异方差检验
4. EGARCH(1,1)模型参数估计、杠杆效应检验和条件异方差检验

描述性统计

统计量	MATLAB函数
交易日天数	length
均值	mean
标准差	std
偏度	skewness
峰度	kurtosis
最大值	max
最小值	min
自相关系数	autocorr

检验收益率的ARCH效应

ARCH效应检验（条件异方差？）

Engle(1982)'s ARCH test

- Denote $\hat{\epsilon}_t$ as the demeaned return series, and run regression:

$$\hat{\epsilon}_t^2 = \alpha_0 + \alpha_1 \hat{\epsilon}_{t-1}^2 + \alpha_2 \hat{\epsilon}_{t-2}^2 + \cdots + +\alpha_m \hat{\epsilon}_{t-m}^2 + u_t.$$

- Record the R^2 of the regression, then under the null hypothesis of no ARCH effect

$$T \cdot R^2 \overset{a}{\sim} \chi^2(m).$$

检验ARCH效应

```
h = archtest(res)
h = archtest(res,Name,Value)
[h,pValue] = archtest(___)
[h,pValue,stat,cValue] = archtest(___)
```

Example:

```
ret_demeaned = ret - mean(ret);
[h, pValue, stat] = archtest(ret_demeaned, 'Lags', 2);
```

模型参数原理

1. 推导收益率条件概率分布
2. 似然函数
3. 极大化似然函数估计模型参数

ARCH(1)参数估计

- ARCH(1) model:

$$\begin{cases} r_t = \mu + \epsilon_t \\ \epsilon_t = \sigma_t z_t, \quad z_t \sim \mathcal{N}(0,1) \\ \sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2. \end{cases}$$

- Conditional density:

$$f(r_t | r_{t-1}) = \frac{1}{\sqrt{2\pi}\sigma_t} \exp\left[-\frac{(r_t - \mu)^2}{2\sigma_t^2} \right].$$

- Log likelihood function:

$$L(\theta) \equiv \ln f(r_T, \cdots, r_t | r_0) = \sum_{t=1}^{T} \ln f(r_t | r_{t-1})$$

$$= -\frac{T}{2} \ln(2\pi) - \sum_{t=1}^{T} \ln \sigma_t - \frac{1}{2} \sum_{t=1}^{T} \frac{(r_t - \mu)^2}{\sigma_t^2}$$

GARCH(1, 1)参数估计

- GARCH(1,1) model:

$$\begin{cases} r_t = \mu + \epsilon_t \\ \epsilon_t = \sigma_t z_t, \quad z_t \sim \mathcal{N}(0,1) \\ \sigma_t^2 = \omega + \beta_1 \sigma_{t-1}^2 + \alpha_1 \epsilon_{t-1}^2, \end{cases}$$

- Log likelihood function:

$$L(\boldsymbol{\theta}) \equiv -\frac{T}{2}\ln(2\pi) - \sum_{t=1}^{T} \ln \sigma_t - \frac{1}{2}\sum_{t=1}^{T} \frac{(r_t - \mu)^2}{\sigma_t^2},$$

where $\sigma_t^2 = \omega + \beta_1 \sigma_{t-1}^2 + \alpha_1 \epsilon_{t-1}^2$.

- Initial value σ_0^2: Bollerslev proposes using sample variance:

$$\hat{\sigma}^2 \equiv \frac{1}{T}\sum_{t=1}^{T}(r_t - \mu)^2$$

EARCH(1, 1)参数估计

- EGARCH(1,1) model:

$$\begin{cases} r_t = \mu + \epsilon_t \\ \epsilon_t = \sigma_t z_t, \quad z_t \sim \mathcal{N}(0,1) \\ \log \sigma_t^2 = \omega + \beta_1 \log \sigma_{t-1}^2 + \alpha_1(|z_{t-1}| - E|z_{t-1}|) + \theta z_{t-1}. \end{cases}$$

- Log likelihood function:

$$L(\boldsymbol{\theta}) \equiv -\frac{T}{2}\ln(2\pi) - \sum_{t=1}^{T} \ln \sigma_t - \frac{1}{2}\sum_{t=1}^{T} \frac{(r_t - \mu)^2}{\sigma_t^2},$$

where

$$\sigma_t^2 = \exp\left[\omega + \beta_1 \log \sigma_{t-1}^2 + \alpha_1(|z_{t-1}| - E|z_{t-1}|) + \theta z_{t-1}\right].$$

- Initial value σ_0^2:

$$\hat{\sigma}^2 \equiv \frac{1}{T}\sum_{t=1}^{T}(r_t - \mu)^2$$

MATLAB估计模型参数

```
fit_to_garch = garch('Offset', NaN,
'GARCH', NaN, 'ARCH', NaN,
'GARCHLags', 1, 'ARCHLags', 1);

[EstPars_GARCH, EstParsCov_GARCH] =
estimate(fit_to_garch, ret)
```

自学matlab garch函数和egarch函数

https://www.mathworks.com/help/econ/specify-garch-models-using-garch.html

https://www.mathworks.com/help/econ/specify-egarch-models-using-egarch.html

MATLAB估计模型参数

```
GARCH(1,1) Conditional Variance Model with Offset (Gaussian Distribution):

                Value        StandardError    TStatistic      PValue
              __________    _____________    __________    __________

Constant      5.0872e-06     6.7316e-07        7.5571      4.1215e-14
GARCH{1}         0.83909      0.0068551         122.4               0
ARCH{1}          0.15865      0.0081759        19.404       7.106e-84
Offset        0.00019495      0.0001412        1.3807         0.16738


EstPars_GARCH =

  garch - 属性:

     Description: "GARCH(1,1) Conditional Variance Model with Offset (Gaussian Distribution)"
    Distribution: Name = "Gaussian"
               P: 1
               Q: 1
        Constant: 5.08717e-06
           GARCH: {0.839088} at lag [1]
            ARCH: {0.158646} at lag [1]
          Offset: 0.000194952

EstParsCov_GARCH =

   1.0e-04 *

    0.0000   -0.0000    0.0000    0.0000
   -0.0000    0.4699   -0.4724    0.0013
    0.0000   -0.4724    0.6684   -0.0013
    0.0000    0.0013   -0.0013    0.0002
```

MATLAB估计模型参数

$$\begin{cases} r_t = \mu + \epsilon_t \\ \epsilon_t = \sigma_t z_t, \quad z_t \sim \mathcal{N}(0,1) \\ \sigma_t^2 = \omega + \beta_1 \sigma_{t-1}^2 + \alpha_1 \epsilon_{t-1}^2, \end{cases}$$

GARCH(1,1) Conditional Variance Model with Offset (Gaussian Distribution):

	Value	StandardError	TStatistic	PValue
Constant	5.0872e-06	6.7316e-07	7.5571	4.1215e-14
GARCH{1}	0.83909	0.0068551	122.4	0
ARCH{1}	0.15865	0.0081759	19.404	7.106e-84
Offset	0.00019495	0.0001412	1.3807	0.16738

检验冲击 Z_t 的ARCH效应

$$\begin{cases} r_t = \mu + \epsilon_t \\ \epsilon_t = \sigma_t z_t, \quad z_t \sim \mathcal{N}(0,1) \\ \sigma_t^2 = \omega + \beta_1 \sigma_{t-1}^2 + \alpha_1 \epsilon_{t-1}^2, \end{cases}$$

1. **利用估计参数计算ε_t**
2. **设σ_0=0，计算σ_t**
3. **计算冲击$Z_t = \varepsilon_t / \sigma_t$**
4. **利用archtest检验Z_t**

GARCH

基于GARCH(1, 1)的收益率模拟

1.设置模型参数 [garch]

2.利用标准正态分布产生随机数 [simulate]

```
simulate_garch = garch('Offset', 0.000245, 'GARCH', 0.8869,
'ARCH', 0.1010, 'Constant', 5.18*10^-6, 'GARCHLags', 1,
'ARCHLags', 1);
[vlt_sim_garch, ret_sim_garch] = simulate(simulate_garch, 1000);
```

文献阅读

- 2-2004AER Risk and volatility econometric models and financial practice.pdf
- 2-波动率预测_GARCH模型与隐含波动率_郑振龙.pdf
- 2-基于GARCH模型的波动率与隐含波动率实证分析_以上证50ETF期权为例_杨晓辉.pdf
- 2-中国股票市场的波动率预测模型及其SPA检验_魏宇.pdf