Assignment 1: Just Logic
15-316 Software Foundations of Security and Privacy

Total Points: 50

1. **Propositional soundness (15 points).** Use the semantics of propositional logic to prove that the $\neg$L rule is sound by showing that the validity of the premises imply the validity of the conclusion.

$$(\neg\mathrm{L}) \quad \frac{\Gamma \vdash P, \Delta}{\Gamma, \neg P \vdash \Delta}$$

Hint: use the proof for $\wedge R$ given in Lecture 2 as a guide to structure your argument.

Solution.

2. **Exclusionary rules (15 points)**

Exclusive-or is a logical operation that is true exactly when its arguments take different values. Extending the semantics of propositional logic to incorporate this connective, we define:

$$I \models P \oplus Q \text{ iff either } I \models P \text{ or } I \models Q\text{, but not both}$$

First, write left and write inference rules for exclusive-or. You do not need to prove that they are sound, but you should explain the reasoning that led you to the premises for each rule.

$$(\oplus\text{L})\ \frac{\cdots}{\Gamma, P \oplus Q \vdash \Delta} \qquad (\oplus\text{R})\ \frac{\cdots}{\Gamma \vdash P \oplus Q, \Delta}$$

Then, check your work by using the rules to give a sequent calculus proof that the formula below is valid. You should assume that $\oplus$ takes precedence over implication:

$$q \oplus (p \oplus q) \to p$$

Solution.

3. **Derived Resolution (15 points)** The *resolution rule* R is a very powerful tool for propositional inference that serves as the workhorse for many automated solvers.

$$(\mathrm{R}) \quad \frac{\Gamma \vdash P, \Delta_1 \quad \Gamma \vdash \neg P, \Delta_2}{\Gamma \vdash \Delta_1, \Delta_2}$$

Show that R is a derived rule. In addition to the inference rules presented in lecture, you may need to use one or both of the weakening rules WL,WR shown below.

$$(\mathrm{WL}) \quad \frac{\Gamma \vdash \Delta}{\Gamma, P \vdash \Delta} \quad (\mathrm{WR}) \quad \frac{\Gamma \vdash \Delta}{\Gamma \vdash P, \Delta}$$

Solution.