Designing Media for Volume Rendering

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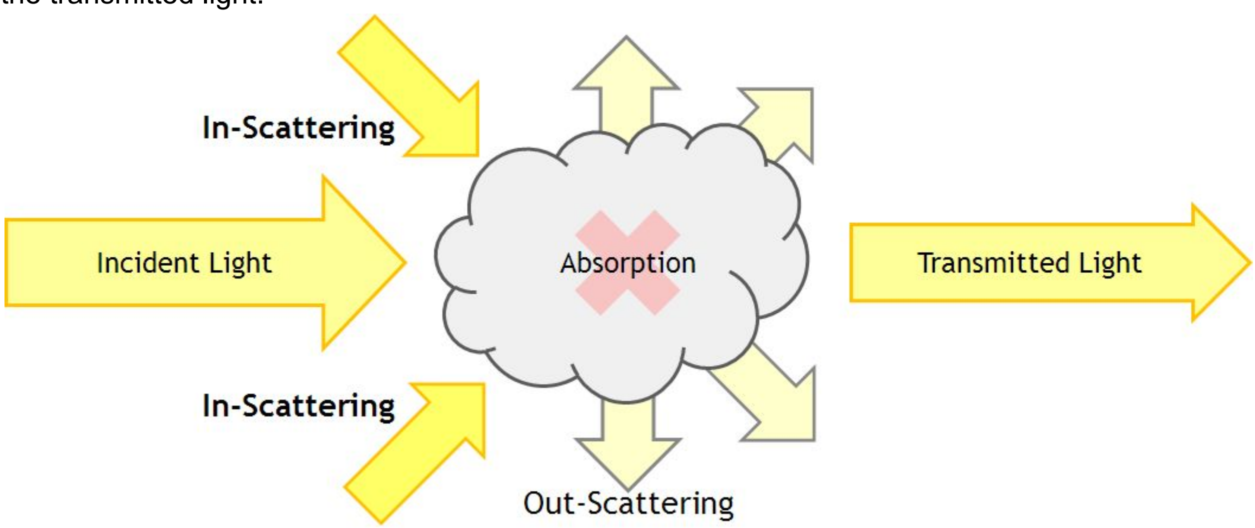
There are two components to the visual effect produced by a volumetric lighting system: the light (source geometry, visibility/occlusion, spectral intensity, etc.) and the scattering medium (spectral phase function, absorption, density, etc.). While the effect of different lighting parameters is very well studied and fairly intuitive, the medium itself can be a little bit harder to explain or parameterize in a way that gives predictable results with a minimum of fiddling on the part of the artists.

Background

As light travels through a medium it is being continuously scattered and absorbed by the mediums constituent particles, at a rate and in a manner derived from the density and specific characteristics of the medium's constituent components. This is modeled as a line of differential scattering events along the light's path. At each event, some of the incident intensity may be absorbed by the medium or scattered such that it no longer travels towards the viewer. The remaining light is unaffected, and transmitted through the differential area along its original direction. In addition, some of the energy from light which also intersected that point in space on a path that might not intersect the viewer can be scattered such that it will, and so is added to the transmitted light.



All of this absorption and scattering is the result of the light probabilistically (in some cases down to the quantum level) with the various particles that make up the atmosphere the medium is modeling. Each of these particles has their own independent absorption and scattering properties. If we assume that the medium itself is generally sparse (such that the different particle types can mix without displacing or interfering with one another) then we can treat the effects of the volume as a whole as the sum of the properties of the individual constituent particle species in isolation (according to their densities).

Light can scatter for a variety of reasons: diffuse scattering from lambertian particles, specular or prismatic reflection from ice crystals, down to wave-like effects for particles that tend to be smaller than the wavelength of visible light. The smallest particles (oxygen, nitrogen, etc.) scatter via a mechanism known as *Rayleigh scattering* which tends to scatter light towards the direction of travel (*forward scattering*) and back towards the source (*back scattering*) rather than orthogonal to that direction. Since Rayleigh scattering is wavelength-dependent, it also tends to scatter shorter (blue) wavelengths more, while affecting longer (red) wavelengths less. Meanwhile, larger particles like water vapor, large molecules, and fine dust scatter via a mechanism known as *Mie scattering*. Mie scattering is generally achromatic, and favors strong but tight forward scattering (possibly with an additional isotropic component scattering in every direction). Rayleigh scattering is why the sky is blue and the sunset is red. Mie scattering is why the sun flares out on a hazy day.

Density and Optical Depth

When we talk about particles in a medium, it's intuitive to think in terms of density (particles per unit volume); however, density itself is not ever directly used. Instead it is often more useful to talk about components in terms of *optical depth* or *optical thickness*, which combines both the number of particles in a given volume and how likely those particles are to interact with the light in a particular way.

We can understand optical thickness more intuitively by thinking about it in relation to *transmittance*, which is the proportion of light traveling through a volume that reaches its original destination (or in other words is transmitted without interference). Transmittance is defined as follows:

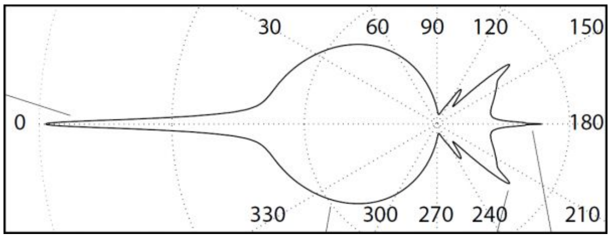
$$T(l) = e^{-\tau l}$$

Where T is the transmittance (from 0 to 1), l is the length of the path the light is traveling through the volume, and τ is the optical thickness of the medium within the volume. From this perspective we can understand that the optical thickness correlated with the transmittance -- at $\tau = 0$ the volume is perfectly transmissive, but it slowly becomes opaque as $\tau \to \infty$. Since the optical thickness is multiplied by a distance to get this scale factor, the units are m^{-1}.

Scattering and the Phase Function

The *phase function* describes how energy intersecting a volume is scattered (when scattering occurs). It is essentially the volumetric equivalent of a BRDF: it defines over the sphere the proportion of energy being scattered in any given direction based on its relation to the incident direction. Though it defines a sphere, in practice the phase function is rotationally invariant, and so can be defined as a function simply of the angle between the incident direction the outgoing direction in question (or more often, the cosine of that angle).

Phase functions incorporate a variety of different phenomenon, from the diffuse scattering of dust particles to the specific angular reflections of atmospheric ice crystals and the wave-like interactions around molecules that are smaller than the light's actual wavelength itself. As such they can be arbitrarily complex; however, in practice there are a few general-purpose phase functions which can be manipulated and composited together to acceptably cover a wide variety of common phenomenon.



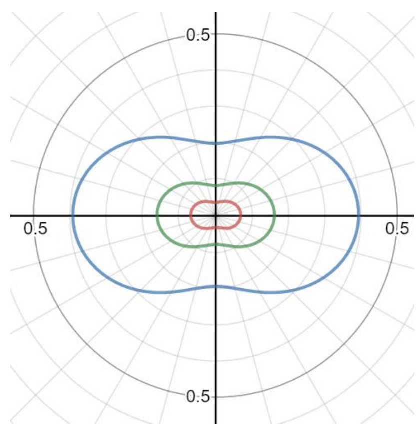
Isotropic Scattering

The simplest of all phase functions is the isotropic function. This is the equivalent of a Lambertian BRDF, which is uniform in all directions. Isotropic scattering is most appropriate for dust or very thick fog.

Rayleigh Scattering

Rayleigh scattering models interactions of light with particles smaller than 1/4th of the light's wavelength. This scattering is described by the function:

$$\rho_R(\theta) = \frac{3}{16\pi}(1 + \cos^2\theta)$$

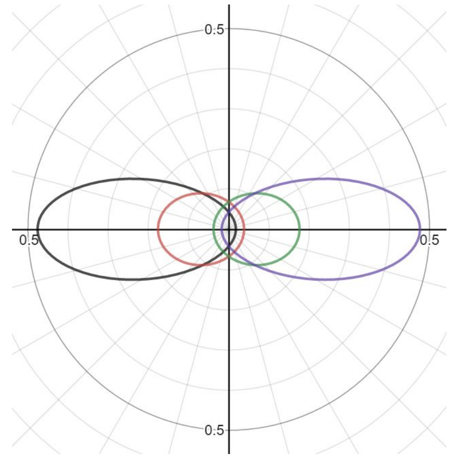


Since Rayleigh scattering is wavelength-dependent, it has an inherent chromatic component. This can be approximated by scaling the optical thickness for the different color components appropriately for the primaries in that color space. For example, the Rayleigh scattering from Earth's atmosphere at sea level can be described by using an optical depth of $[5.8 \times 10^-6, 1.36 \times 10^-5, 3.31 \times 10^-5]$.

Henyey-Greenstein Scattering

The Henyey-Greenstein scattering function ("HG scattering" for short) is a generalized scattering formula that accounts for a variable amount of forward or back scattering. This is done by adding an "eccentricity" parameter g which is in the range $(-1, 1)$. If you consider the phase function to be a weighted distribution over all possible angles relative to the incident direction, you can think of g as the mean cosine value of those angles as weighted by that function. The HG function is as follows:

$$\rho_{HG}(\theta, g) = \frac{1}{4\pi}\frac{1-g^2}{(1+g^2-2g\cos\theta)^{3/2}}$$



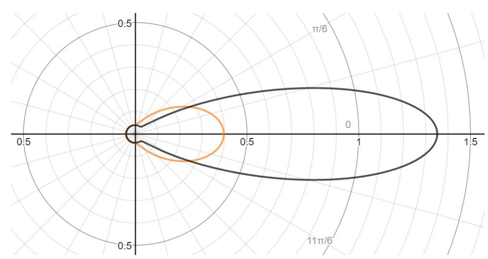
At $g = 0$ the function matches an isotropic distribution (i.e. equal scattering in all directions). As $g \to \pm 1$ the distribution is biased more towards back or forward scattering depending on the sign. Thus the HG function can be used to approximate a variety of phase functions with a single lobe. In fact, even very complex phase functions could be approximated by adding together weighted HG functions with different eccentricity terms.

Mie-Murky and Mie-Hazy

Using a Henyey-Greenstein term is one way of getting the strong forward scattering terms seen in atmospheric rendering; however, while HG is a general term useful for a lot of different approximations, other models exist which are designed to explicitly model this atmospheric behavior and so are much simpler. One such approximation comes in the form of two functions (one for “Hazy” atmospheres, and one for denser “Murky” ones):

$$\rho_{MH}(\theta) = \frac{1}{4\pi}\left(\frac{1}{2} + \frac{9}{2}\left(\frac{1+\cos\theta}{2}\right)^8\right)$$

$$\rho_{MH}(\theta) = \frac{1}{4\pi}\left(\frac{1}{2} + \frac{33}{2}\left(\frac{1+\cos\theta}{2}\right)^3 2\right)$$



These functions effectively combine an isotropic distribution and a forward-scattering lobe into one statement. These could also be modeled using a combination of isotropic and HG terms, but if back scattering or fine-grained control over the eccentricity of the forward scattering lobe are not required, this can be a more straightforward approach.

Practical Implementation

Background Atmosphere

For most engines trying to approximate a normal “Earth-like” environment, you will want to use a constant Rayleigh term (unless you are dramatically changing in atmospheric altitude) with RGB optical thicknesses tuned to the engine’s unit scale. As mentioned before, the “real world” value should be something like $[5.8 \times 10^-6, 1.36 \times 10^-5, 3.31 \times 10^-5]$; however, this is generally very subtle on the for the scale at which most games work. Instead, it’s recommended that you start with a single Rayleigh term at this value (which assumes a “1 world-unit = 1m” ratio) and then scale these values up or down until you come up with something that looks good for a “clear day” atmosphere.

Once you have a baseline for your Rayleigh scattering, it should be fixed as a constant (unless, again, the base atmosphere changes -- by climbing from sea-level to the top of a mountain, for example). Technically it may be correct to also add a small Absorption component (accounting for Ozone completely absorbing light) but this may or may not be necessary depending on the scale of the scenes involved.

Varying Conditions

Once your constant components are accounted for, you can incorporate variable components like haze, smoke, and fog. You can use any combination of Isotropic, Henyey-Greenstein, or Mie-Hazy/Mie-Murky terms to do this.

Simple Approach

Use an Isotropic term to incorporate things like dust and smoke in the atmosphere. Use both Mie-Murky and Mie-Hazy terms to account for things like fog, by blending between them according to the fog thickness. For example, use a density parameter w and scale-factors k_{MH} and K_{MM} for Mie-Hazy and Mie-Murky (respectively) as follows:

$$k_{MH}(w) = 1 - |1 - 2w|$$
$$k_{MM}(w) = max(0, 2w - 1)$$

This will cause w to control a smooth blending between no Mie effect (at $w = 0$) to a Mie-Hazy effect (at $w = 0.5$) to a fully Mie-Murky effect (at $w = 1$). Expose the parameter w to the artists, along with an optical thickness parameter to control the density of the atmosphere being created.

When blending between atmospheric conditions, simply interpolate between the w and optical thickness settings to produce the desired effect. You may optionally expose the color component of the varying conditions (i.e. setting the R, G, and B values separately) for more versatility.

Three-Parameter Scattering

With this approach two HG terms are used. The artists see three parameters (r, g_1, and g_2) and a single set of optical thickness values. The value g_1 controls the eccentricity of the first HG term. The value g_2 controls the eccentricity of the second HG term. Finally, the value r controls the ratio of the optical thickness that each term represents (where $r = 0$ would mean it's all applied to the first HG term, $r = 1$ meaning it's all applied to the second term, and $r = 0.5$ meaning it's split evenly between the two).

This approach has the advantage of giving artists explicit control over the sharpness of the eccentricity, and allowing them to combine things like a strong forward-scattering lobe and a subtle back-scattering component. This frees them up to achieve potentially more exacting effects (at the cost of having to fiddle slightly more with the system to get the desired results). The ratio term r could be eliminated completely and replaced with specifying the optical thicknesses for the two HG terms explicitly, but this may not be particularly useful.

Much like with the previous method, different media can be blended together simply by blending the individual high-level components. And as above, an additional isotropic term may be included to handle dense media like dust or smoke.