1) $\frac{dy}{dt} = \frac{t}{y}$; $y(0) = 1$

$t\,dt = y\,dy$

$\int t\,dt = \int y\,dy$

$\frac{t^2}{2} = \frac{y^2}{2}$

$y^2 = \left(\frac{t^2}{2}\right)2 + C$

$y^2 = t^2 + C$

$y = \sqrt{t^2 + C}$

Se sustituyen las condiciones iniciales para $y(0) = 1$

$y = \sqrt{t^2 + C}$

$y = \sqrt{1^2 + C}$

$\boxed{y = \sqrt{t^2 + 1}}$

$y = t^2 + 1$

$t = 2$

$y = 2^2 + 1$

$y = 4 + 1$

$y = 5$

$\alpha = 1 \quad \gamma = 1$

2) $\frac{dy}{dt} + \alpha y = \gamma y^2 \; ; \; y(0) = 10 \qquad 2y - 2y^2 =$

$\frac{dy}{dx} = y - y^2 \Rightarrow \int \frac{dy}{y - y^2} = \int dx$

Fracciones parciales

$\frac{1}{y - y^2} = \frac{1}{y(1-y)} = \frac{A}{y} + \frac{B}{1-y}$

$A = \frac{1}{1-y}\Big|_{y=0} \qquad B = \frac{1}{y}\Big|_{y=1}$

$A = \frac{1}{1-0} = \frac{1}{1} = 1 \qquad B = \frac{1}{1} = 1$

$\int \frac{1}{y - y^2} dy = \int \left[\frac{1}{y} + \frac{1}{1-y}\right] dy$

$e^{(\ln \left|\frac{y}{y-1}\right| = x + C)} \qquad \ln\left(\frac{A}{B}\right) = \ln(A) - \ln(B)$

$\left|\frac{y}{y-1}\right| = e^{x+C}$

Sustituyendo las condiciones iniciales

$\frac{10}{10-1} = e^{0+C}$

$\ln\left(\frac{10}{9}\right) = \ln(e^{0+C})$

$1.111 = 10 + C$

$C = 1.111 - 10$

$C = -8.89$

$\frac{1}{y-1} = e^{x - 8.89}$

$\frac{0.5}{0.5 - 1} = e^{0.5 - 8.89}$

$0.5 = -9.24$

3) $\frac{dy}{dt} = e^{2y}\,sen\,t \;;\; y(0)=0$

$\frac{1}{dy} = \frac{1}{e^{2y}\,sen\,t}$

$e^{-2y}\,dy = sen\,t\,dt$

$\int e^{-2y}\,dy = \int sen\,t\,dt$

$u = -2y$ $\qquad \int e^{u}\frac{du}{-2} = \int sen\,t\,dt$

$du = -2dy$

$-\frac{1}{2}\int e^{u}\,du = \int sen\,t\,dt$

$-\frac{1}{2}(e^{u}+c) = -\cos x + c$

$-\frac{1}{2}e^{-2y} = -\cos x + c$

$e^{-2y} = -2(-\cos x + c)$

$e^{-2y} = 2\cos x + c$

$\ln e^{-2y} = \ln(2\cos x + c)$

$-2y = \ln(2\cos x + c)$

$y = \frac{\ln(2\cos x + c)}{-2}$

$0 = \frac{\ln(2\cos(0) + c)}{-2}$

$0 = \ln(2 + c)$

$e^{0} = e^{\ln(2+c)}$

$1 = 2 + c$

$c = 1 - 2$

$c = 1$

$$y = \frac{\ln(2\cos x - 1)}{-2}$$

$y = 2$

$y = 0$

$y = \frac{\ln(2\cos 2 - 1)}{-2}$

$\therefore y = 0{,}153$

4) $\frac{dy}{dx} = \frac{e^x}{2y}$; $y(0) = 1$

$2y\,dy = e^x\,dx$

$\int 2y\,dy = \int e^x\,dx$

$2\int y\,dy = \int e^x\,dx$

$2\,\frac{y^{1+1}}{1+1} = \begin{matrix} u = x \\ dv = dx \end{matrix}$

$\not{2}\,\frac{y^2}{\not{2}} = \int e^u\,du$

$y^2 + C = e^x + C$

$y^2 = e^x + C$

$y = \sqrt{e^x + C} \Rightarrow y = \sqrt{e^x}$

$1 = \sqrt{e^x + C} \rightarrow 1 = \sqrt{e^0 + C}$

$1^2 = (\sqrt{e^x + C})^2 \qquad 1 = \sqrt{1 + C}$

$1 = e^x + C \qquad 1 = 1 + C$

$1 = 1 + C$

$C = 1 - 1$

$\underline{C = 0}$

$y = \sqrt{e^{0.5646}}$

$\underline{y = 1.2388 \approx 1.24}$