2019 Asia and Pacific Mathematical Contest in Modeling

Problem A

Melting Representation Model of Silicon Dioxide Analyzed Based on Image

The main component of iron tailings is silicon dioxide, while the silicon dioxide is the hardest part to melt among the iron tailing components. Therefore, the melting behavior of iron tailing can be represented by the melting behavior of silicon dioxide. However, the temperature of high-temperature molten tank is over 1,500℃, in which the service life of routine detection equipment under the environment is very short. To solve the problem, relevant research group firstly adopted a kind of rifted CCD video shooting system with amplification effect for the first time at home and abroad. In the non-contact way, the group has obtained the dynamic visual data of silicon dioxide in the high-temperature molten pool (sequence image under time sequence), and has observed the real-time melting rate of silicon dioxide in the time sequence through video analysis, which provided guidance for the tailing addition and heat compensation in the process of slag cotton preparation, thus indirectly improved the direct fiber forming technology of blast furnace slag.

Time sequential image of silicon dioxide in high-temperature molten pool during the melting process (see Attachment, 114 images in total; the serial number of document name is the time sequence, collecting one image every other 1s). See the figure below for the situation of adopted experimental equipment:

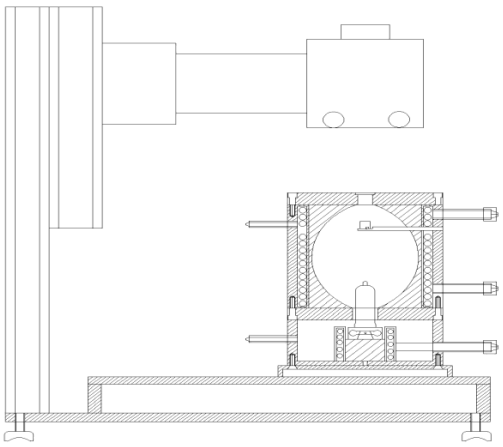

Figure 1 Drawing of Refitted CCD Equipment with Amplification Effect

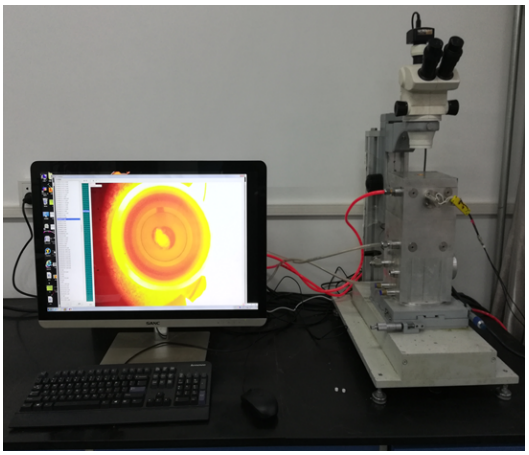

Figure 2 Diagram of Physical Equipment Object

To reveal the dissolution behavior of iron tailings in blast furnace slag, the main component SiO_2 of iron tailings is adopted for research, researching the melting process of SiO_2 particles at high temperature, so as to represent the melting of iron tailings. Materials to be prepared for the test are: pure SiO_2 particles for analysis (see Figure 3) and corundum crucible with diameter of 8mm (see Figure 4) respectively:

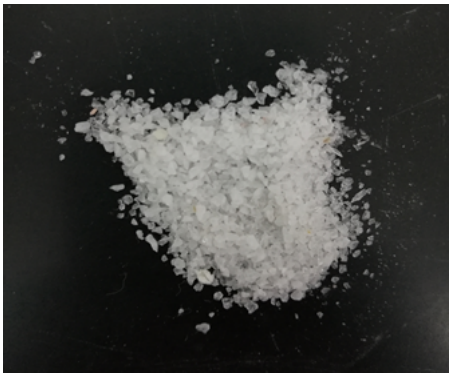

Figure 3 Pure SiO_2 Particles for Analysis

Figure 4 Corundum Crucible with Diameter of 8mm

The information shown in the digital image of Attachment includes:

(1) The fixed position of crucible in the image;

(2) The position of SiO2 particles which are not molten may change along with time sequence.

It is required to conduct the following analysis on the image in the Attachment

1. The position of silicon dioxide particles in the high-temperature molten pool during the melting process is changing constantly, and the first step to analyze the melting behavior of silicon dioxide is to track the target. Please establish a mathematical model to track the centroid position of silicon dioxide particles during the melting process, and present the motion trail of centroid of silicon dioxide.

2. Establish indexes which represent the edge outline characteristics of silicon dioxide during the melting process of silicon dioxide (such as shape, perimeter, area, generalized radius, etc.); the participants can select by themselves, as long as they can represent the melting process of silicon dioxide.

3. For the image given in the attachment is 2D, while the key parameter which represents the melting rate of silicon dioxide is mass, and the mass is in direct proportion to the 3D volume, estimate the actual melting rate of silicon dioxide based on edge outline characteristic indexes of silicon dioxide in the No. 2 question.