COMPARISON AND DESIGN OF SIMPLIFIED GENERAL PERTURBATION MODELS (SGP4) AND CODE FOR NASA JOHNSON SPACE CENTER, ORBITAL DEBRIS PROGRAM OFFICE

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of the Requirements for the Degree

Master of Science in Aerospace Engineering

by

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COMMITTEE MEMBERSHIP

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ABSTRACT

Comparison and Design of Simplified General Perturbation Models (SGP4) and Code for NASA Johnson Space Center, Orbital Debris Program Office

Nicholas Zwiep Miura

This graduate project compares legacy simplified general perturbation model (SGP4) code developed by NASA Johnson Space Center, Orbital Debris Program Office, to a recent public release of SGP4 code by David Vallado. The legacy code is a subroutine in a larger program named PREDICT, which is used to predict the location of orbital debris in GEO. Direct comparison of the codes showed that the new code yields better results for GEO objects, which are more accurate by orders of magnitude (error in meters rather than kilometers). The public release of SGP4 also provides effective results for LEO and MEO objects on a short time scale. The public release code was debugged and modified to provide instant functionality to the Orbital Debris Program Office. Code is provided in an appendix to this paper along with an accompanying CD. A User's Guide is presented in Chapter 7.

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LIST OF ACRONYMS

NASA National Aeronautics and Space Administration

SGP(4) Simplified General Perturbation

PREDICT NASA code that predicts position of orbital debris
SPACETRACK Government program initiated to track satellites
AIAA American Institute of Aeronautics and Astronautics

JSC Johnson Space Center

SATRAK Satellite Tracking (Government owned propagator)

GSFC Goddard Space Flight Center
JPL Jet Propulsion Laboratory

FORTRAN Formula Translating (IBM developed language)

COES Classical Orbital Element Set

TLE Two Line Element

WGS World Geodetic System

LES Lincoln Experimental Satellite
GEO Geosynchronous Earth Orbit

LEO Low Earth Orbit

MEO Medium Earth Orbit

ISS International Space Station

min minutes m meters

cm centimeters
mm millimeters
sec seconds
km kilometers

Polysat Cube satellites developed by Cal Poly

CP3 Polysat 3
CP4 Polysat 4
FOV Field of view

1. Introduction

This project has been completed for NASA Johnson Space Center, Orbital Debris

Program Office, with the purpose of comparing the effectiveness of legacy SGP4

propagation code and a recent public release of SGP4 propagation code. Furthermore,

new SGP4 propagation code based on the public release was developed and delivered.

The Orbital Debris Program Offices uses a program called PREDICT to predict the motion of deep space debris. The program was developed in-house and uses a simplified version of Simplified General Perturbation Theory to propagate the position and velocity of a spacecraft forward in time based on classic orbital elements. In 2004, there was a public release of Simplified General Perturbation 4 (SGP4) code, based on the same theory. The Orbital Debris Program Office wanted to know how their legacy code compared with the public release.

This project compares the two codes to truth data provided by the Orbital Debris Program Office, and shows the accuracy of the delivered results. Additionally, the public release code was debugged and provided for use and the public release code was modified to run within the PREDICT program.

This paper describes the problems involved in this undertaking, as well as the procedure taken to solve these problems. Next, a detailed display of the results plus analysis and conclusions is given. Finally, a User's Guide is provided to help navigate using the public release code and to help update the legacy code.

2. Space Propagation Models

Space propagation models use current state information of a satellite to predict a future state of the satellite. As a simplistic example, imagine a car driving down a highway. If we know the location and the speed of the car now, we can make an accurate prediction of where the car will be in an hour. Similarly for satellites, if we know the position and the velocity now, we can make a reasonable guess where the satellite will be in the future. The satellite however, encounters disturbances, or perturbations, along its path that complicates its motion. These perturbations are caused by the Earth's shape (spherical harmonics), drag, radiation, and effects from other bodies (the sun and moon generally). In our car example, imagine driving up and down hills, over different terrain, with changing speed limits. It makes our simplified example much harder.

Space propagation models are used primarily by agencies that track orbiting objects. The ultimate goal is to know where everything in space is at all times. Propagation models are needed because there are simply too few telescopes to watch everything in the sky at all times. Thus, you find the position and velocity of an object once, and then use the propagator to tell you where it is in the future if you ever need to locate it again. The first attempt to use these models came in 1959, by the National Space Surveillance Control Center. This was a highly simplified model that failed when trying to propagate position more than a week. In 1960, the initial model was replaced by the 'Simplified General Perturbations (SGP)' modelⁱⁱ. Though this model provided accurate results, by 1969 there were too many satellite in orbit to maintain a catalog based on SGP. SGP4, published in SPACETRACK Report #3, was created based on a simplified version of

SGP. By 1979, it was the sole model for catalog maintenance. SGP4 failed when objects were in 'deep space' orbits, so in 1977, an extension to the program was developed to track object with an altitude over 255 km – the height where solar/lunar perturbations have a larger effect than atmospheric drag.

After SPACETRAK Report #3 was released, the 'official' code maintained by the U.S. government became an export controlled black-box. Changes to the code to reflect new programming techniques, better computing power, and more accurate constants were never made public; instead, were made available to select agencies through an executable program called SATRAK.

Agencies not privy to SATRAK, or who needed customized code for their project (including NASA) were forced to manipulate the original code found in SPACETRAK Report #3 themselves, leading to a wide variety of propagation codes based on the same model. David Valladoⁱⁱⁱ, working through the Center for Space, released an AIAA paper in 2004, which attempted to reconcile the many codes into one standardized code. This new code was made available to the public through celestrak.com.

3. SGP4 for Johnson Space Center

Johnson Space Center (JSC) – Orbital Debris Program Office was one project that required the use of a Space Propagation Model. Though they had access to an executable version of SATRAK, customized code was needed to run within a larger program called PREDICT. A telescope observes a debris object and takes right ascension and declination data over a short period of time. The PREDICT code then takes this data and formulates a classical orbital element set (COES) that is fed into the space propagation model. The model can then estimate where the object will be in the near future so it can be found again and analyzed further.

Dr. Mark Matney of the Orbital Debris Program Office was the original architect of the PREDICT space propagation model in 1998. Dr. Matney based his work on the original SPACETRACK Report #3 as well as a publicly released SGP4 model from Goddard Space Flight Center (1997). Though Dr. Matney's program outputs workable results, the Orbital Debris Program Office was intrigued by Vallado's standardized code and wanted to explore the differences between codes. Furthermore, Dr. Matney's code did not accurately predict the location of the objects in some instances, though specific cases were not supplied for analysis.

This project, made possible through a research contract between JSC and Cal Poly San Luis Obispo, explores the differences in the two propagation codes. The scope of the project is as follows:

• Quantify the errors in the two codes based on 'truth' data.

- Deliver new PREDICT code integrated with updated SGP4 module.
- Deliver SGP4 propagator in MATLAB.

All analysis and deliverables are contained within this final report, as well as user guides for attached software code.

4. Procedure

This project was divided into three distinct stages.

- 1. Understanding the software.
- 2. Analyzing differences.
- 3. Programming deliverables.

4.1 Understanding the Software

The starting point of this phase was David Vallado's (et al) AIAA paper, <u>Revisiting Spacetrack Report #3</u>^{iv}. In this paper, updates to the code are described in detail, and a link to the code in both FORTRAN and MATLAB is available at:

http://CelesTrak.com/software/vallado-sw.asp

In Vallado's analysis, results from the new code are compared against selected results from existing public code (i.e. GSFC, JPL, T.S. Kelso's FORTRAN). Though there are obvious differences, there is no proof that the new code is closer or further from the truth. What the paper does do is analyze each difference and explain why the results differ. These differences are commented as 'fixes' within the code and are cataloged in detail in Vallado's paper.

The code that Vallado created first prompts users for an operations mode. This selects how the code calculates epoch of the moon and sun for lunar/solar perturbations. Next,

the code prompts users to input the type of run. Of the three choices, the first is catalog mode, where the code displays position, velocity, and COES for 12 hours forwards and backwards in time with a step of 20 minutes. Second is verification mode, designed to test the output of a given two line element (TLE) set and verify the results against a given file. Third is manual mode, where users define the propagation parameters. If manual mode is chosen, the user gets to determine if they want to use minutes from epoch, epoch, or day of year approach to define the window of time to analyze. The code itself converts epoch and day of year inputs to minutes from epoch when calling the propagator. Next, the user chooses which Earth constants to use, WGS-72 or WGS-84 referring to the World Geodetic System constants and when they became standard. Though the WGS-84 numbers are the current standard (through 2010), WGS-72 values are used in most SGP4 propagators because the 1984 values were not available at the time of creation. Though it is unknown what values SATRAK uses, analysis shows that WGS-72 numbers give more accurate results than WGS-84. Finally, the code calls for TLE inputs. The TLE file must be in the same directory as the main program, and the format of the TLE must be congruent with standards. Multiple TLEs may be in the same file with non TLE lines marked with a '#' symbol. The program then opens or creates an OUT file and prints the data. The OUT file has a header line containing the satellite number. Following lines are in the following form: a column for minutes from epoch, three columns of position data, three columns of velocity data, and four columns of date and time, delineated by a space.

4.1.1 MATLAB Code

The MATLAB code provided by Vallado contained some bugs. There were two major issues encountered. The first involved calling an undefined subroutine. The code tried to call a subroutine called 'days2mdhms', but the subroutine needed was named 'days2mdh'. This subroutine had inputs of the year and day of year, and output the month, day, hour, minute, and second. After this bugged was corrected, another problem was encountered with time. The propagator uses minutes since epoch as an input, but instead was receiving days since epoch. This bug was corrected by dividing critical numbers by 1440 (minutes in a day), and reformatting the output to reflect changes. This code was tested in this debugged form and will be delivered to NASA in its current form to be used as an alternative to SATRAK.

4.1.2 FORTRAN Code

Before working on the FORTRAN code a compiler was needed. The chosen compiler was a free compiler available online called Silverfrost.

(http://download.cnet.com/Silverfrost-FTN95/3000-2069 4-10491439.html)

The code is written in fixed-format FORTRAN 90 language. Additionally, the complete code was spread amongst 25 small .FOR files and 4 common files upon initial download. Based on my understanding of the MATLAB code, I was able to collect all the relevant subroutines and create a new .FOR file that provided the same functionality as the MATLAB code. The code itself did not have any major bugs like the MATLAB code; however, significant time was spent formatting the combined code to fit the fixed-format FORTRAN standards.

4.1.3 PREDICT Code

JSC Orbital Debris Program Office provided their PREDICT code written in FORTRAN. The code is written as a series of subroutines. Though the focus was directed towards the SGP4 subroutine, the interactions with other parts of the program also needed to be taken into account. The SGP4 subroutine is called with a COES input and a time from epoch (when the COES was taken). The output is a single position and velocity vector based on the time from epoch. The code itself is relatively simplistic. It only takes into account J2 perturbations (the first mode of Earth's spherical harmonics), and then uses Kepler's solution with Newton's method to solve for position and velocity.

4.2 Analyzing Differences

JSC had three distinct test cases to analyze that were all debris in geosynchronous orbits. For these cases (8832, 25000, and 30000), JSC sent a TLE and SATRAK truth data. The truth data consists of a data set with 60 minute intervals between predictions and a data set with 360 minute intervals between predictions – but with a larger time range. These objects are of particular importance to the Orbital Debris. All three objects are pieces of debris from a Titan 3C transtage launch vehicle. The pieces themselves are not exactly in GEO, but are relatively close in a subGEO orbit. Below describes the break-up of objects 25000 and 30000.

The breakup of the Titan 3C-4 transtage occurred on 21February 1992 at an altitude of ~35,600 km, inclination (INC) of 11.9 degrees and a right ascension (RA) of 21.8 hours. The operator of the GEODSS sensor on

Maui, Hawaii witnessed approximately 20 pieces in the breakup, but none were tracked at the time. Subsequent to the breakup, the U. S. Space Surveillance Network (SSN) identified three pieces of the debris and assigned them to the catalogue as SSN25000, SSN25001 and SSN30000.

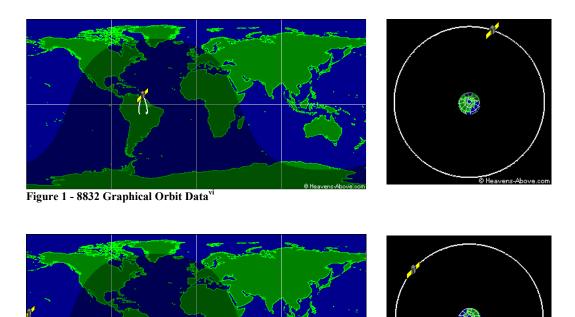


Figure 2 - 25000 Graphical Orbit Datavi

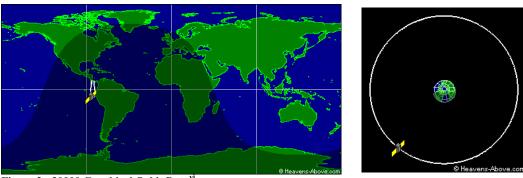


Figure 3 - 30000 Graphical Orbit Datavi

Next, to further the analysis, TLEs were gathered from Celestrak for non-geosynchronous orbits. Then, the same TLEs were again gathered on a later date. Analysis then compared where the object is based on the latter TLE, with where the object was predicted to be based on the former TLE. Analysis was completed on both the MATLAB and FORTRAN codes, as well as JSC's code.

4.2.1 Objects 8832, 25000, and 30000

First, each TLE was processed through the MATLAB and FORTRAN versions of Vallado's code and compared side by side with the SATRAK data. The MATLAB and FORTRAN codes can be programmed to have the same output as the SATRAK data. Results were first compared by hand, and then further analyzed using Excel. Error between SATRAK and Vallado's code is quantified as the absolute magnitudes of difference in the position and velocity vectors.

4.2.1.1 Code Modification

Since the ultimate goal of the project is to replace the current SGP4 propagator in JSC's code, the next step was to modify the FORTRAN code to act as a subroutine, instead of a

stand-alone executable, with the same input and output variables as the original PREDICT model. Because the PREDICT model called for less functionality than the new code provided, much of the work done came in deleting unnecessary lines of code. New lines of code were added to remove any prompts given to the user; however, because the new code can accept LEO objects, a B* prompt was added to the process. There are comments within the new code with instructions on how to remove this if the user does not find it necessary.

Next, a simple program was created in which the user defines the COES from the TLE and gives a 'minutes from epoch' value. The program then calls the propagator and prints the expected position and velocity on the screen.

At this point, the inputs and outputs to the FORTRAN propagator and the PREDICT propagator were the same, and it was trivial to copy and paste the SGP4 subroutine from PREDICT into a new program called by the same simple program from above.

4.2.1.2 SATRAK Comparison

The objects given by JSC were also run through the modified program. However, since the new programs were restrictive in their input and output, only select data points were analyzed until trends were discovered.

4.2.2 Other Objects

To complete the analysis, truth data for LEO and MEO objects were found on SATRAK.

These objects were:

- Polysat CP3 (31129)
- Polysat CP4 (31132)
- Iridium 33 Debris (33771)
- LES 9 (08747)
- ISS Tool bag (33442)

The analysis was completed using the modified code because of the single input and output. Error from truth data is again expressed as the absolute magnitude differences in position and velocity vectors.

4.3 Programming Deliverables

The final step of the procedure was installing the new code into the original PREDICT code. The majority of the leg work had already been completed in the analysis phase, though there were still some issues to be resolved. First, some of the variable and subroutine names needed to be altered to work with the PREDICT code. Specifically, both the PREDICT code and the new code have a subroutine labeled 'JDAY'.

Next, through analysis, it was found that the replacement code needs an extra bit of data to work properly. The PREDICT code only looks at J2 effects, which use a statistical

average of spherical harmonic forces to determine perturbations. This makes epoch data irrelevant because it doesn't matter when you start your calculation, it only matters how long the J2 force has been acting on the object. Thus, when the propagator is called in the PREDICT model, it only inputs time from epoch and not the epoch itself. In contrast, the new code not only takes into account J2 effects, but also looks at forces on the object from lunar and solar influences. These effects are heavily dependent on epoch data because the location of the moon and sun need to be known to correctly calculate the direction and magnitude of the forces.

Epoch data for objects analyzed by the PREDICT code is determined in another subroutine, which is a predecessor for the SGP4 subroutine. To solve the problem, a global common file was created to pass the necessary data to the new SGP4 routine. Details of this can be found in comments in the code and in the User's Guide portion of this report.

Complete testing of the PREDICT code with the modified SGP4 code will be completed by the Orbital Debris Program Office.

5. Results

Sections 5.2 through 5.5 graphically display results that represent the magnitude of the difference in position and velocity vectors between the output of various SGP propagators and SATRAK output over the time span defined by given SATRAK truth data. The given time span ranges from ~15 days to ~27 days. Each section highlights results from the four main codes that were generated from this project. Furthermore, analysis of the results are presented after each section.

Sections 5.6 through 5.8 show the results of the four objects not in GEO orbit, with truth data taken from the TLE data provided by Celestrak.

Section 5.9 links the results to real-world application.

5.1 Data Points

All results were generated from TLEs for the following objects:

- 8832 (Titan debris)
- 25000 (Titan debris)
- 30000 (Titan debris)
- 31129 (Poly Sat 3)
- 31132 (Poly Sat 4)
- 33771 (Iridium 33 debris)
- 33442 (ISS tool bag debris)

5.2 MATLAB vs. SATRAK

Results are displayed for both scenarios created in SATRAK (60 and 360 minute intervals).

5.2.1 Object 8832

The maximum position error calculated between SATRAK and the MATLAB results is 32 cm and the maximum velocity error calculated is 0.023 mm/sec.

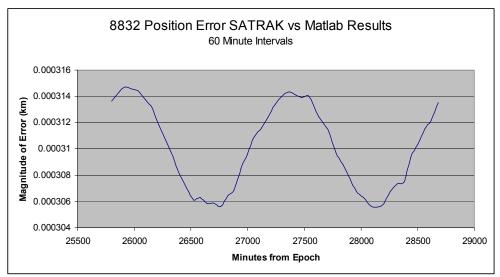


Figure 4 - 8832 Position Error MATLAB (60 min intervals)

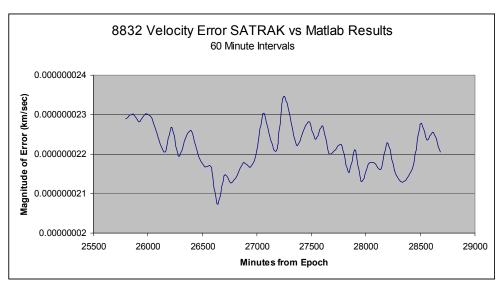


Figure 5 - 8832 Velocity Error MATLAB (60 min intervals)

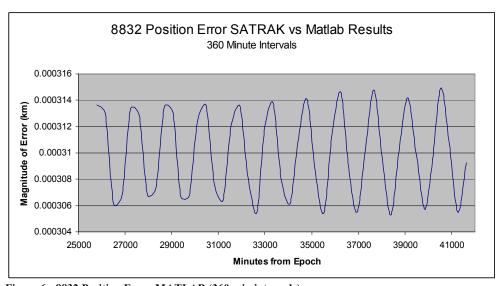


Figure 6 - 8832 Position Error MATLAB (360 min intervals)

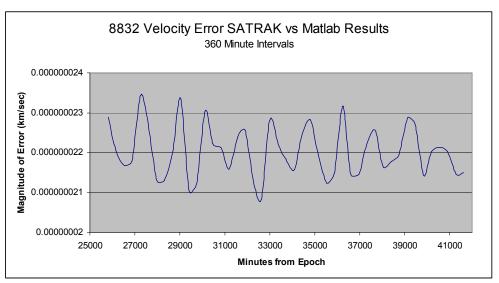


Figure 7 - 8832 Velocity Error MATLAB (360 min intervals)

5.2.2 Object 25000

The maximum position error calculated between the SATRAK and MATLAB results is 1.5m and the maximum velocity error calculated is .11 mm/sec.

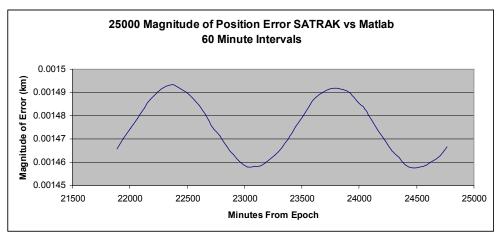


Figure 8 - 25000 Position Error MATLAB (60 min intervals)

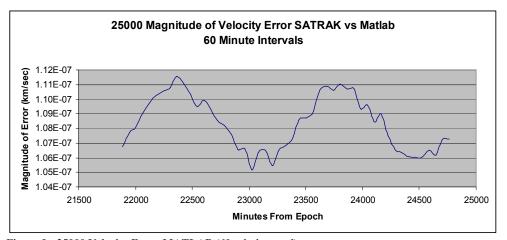


Figure 9 - 25000 Velocity Error MATLAB (60 min interval)

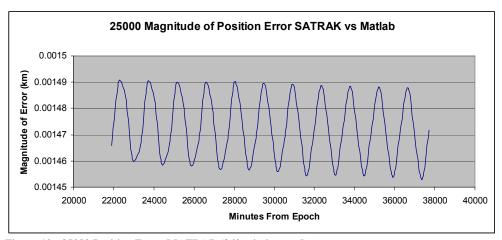


Figure 10 - 25000 Position Error MATLAB (360 min interval)

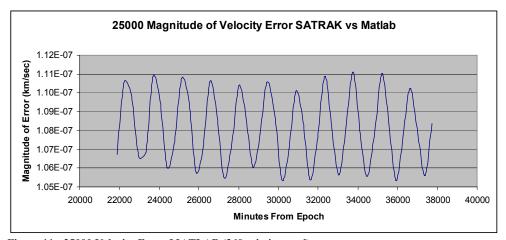


Figure 11 - 25000 Velocity Error MATLAB (360 min interval)

5.2.3 Object 30000

The maximum position error calculated is $51~\mathrm{cm}$ and the maximum velocity error calculated is $0.038~\mathrm{mm/sec}$.

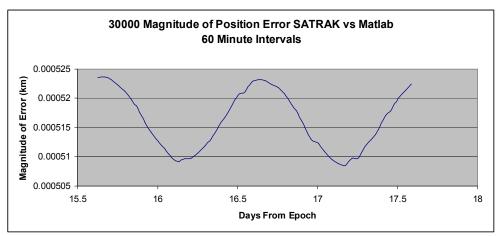


Figure 12 - 30000 Position Error MATLAB (60 min interval)

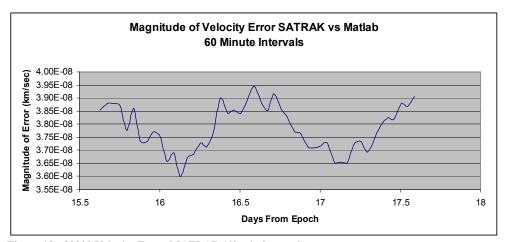


Figure 13 - 30000 Velocity Error MATLAB (60 min interval)

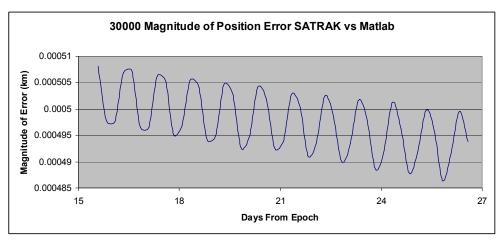


Figure 14 - 30000 Position Error MATLAB (360 min intervals)

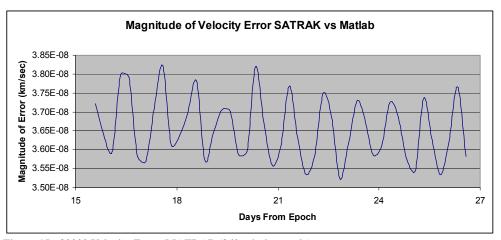


Figure 15 - 30000 Velocity Error MATLAB (360 min intervals)

5.2.4 Analysis - MATLAB

In a broad sense, the error between SATRAK and the MATLAB outputs are insignificant. For practical uses of this data, JSC is looking at units of kilometers, which is orders of magnitudes larger than the error given by MATLAB (1.5 m maximum). However, in each case, there is an obvious periodic variation which either corresponds to the orbital period of the satellite or rotation of the Earth. This can be explained by additional types of perturbations that are modeled in SATRAK that are ignored in Vallado's code. The hypothesis is that SATRAK encompasses some form of triaxilary effects; though there is no way to test this hypothesis.

Next, the difference between the 60 minute intervals and 360 minute intervals graphs was examined_to check for contradictory data (see figure 16) by overlaying data for object 8832). As expected, the two graphs show the same long periodic trend; however, the 60 minute interval captures some short-term perturbations as well. The order of magnitude difference between the two intervals is less than the initial error and can safely be

ignored. Thus, there is no contradictory data. Similar analysis was performed on all subsequent objects and codes with similar results and therefore, are not shown.

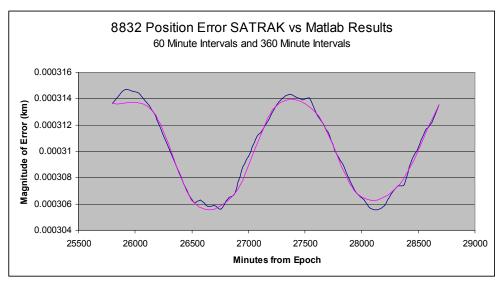


Figure 16 - Comparison of 60 and 360 minute intervals

5.3 FORTRAN vs. SATRAK

Results are displayed for both scenarios created in SATRAK (60 and 360 minute intervals).

5.3.1 Object 8832

The maximum position error calculated is 32 cm and the maximum velocity error calculated is 0.028 mm/sec.

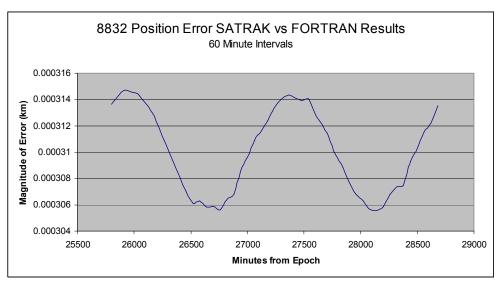


Figure 17 - 8832 Position Error FORTRAN (60 min intervals)

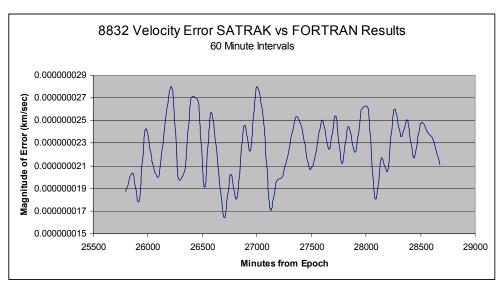


Figure 18 - 8832 Velocity Error FORTRAN (60 min intervals)

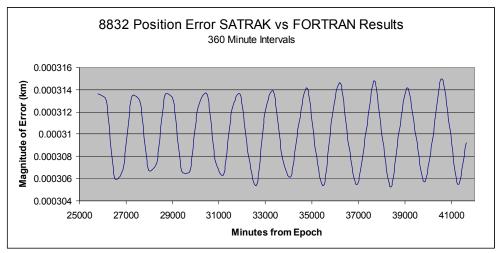


Figure 19 – 8832 Position Error FORTRAN (360 min intervals)

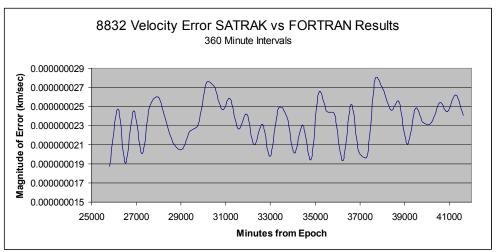


Figure 20 – 8832 Velocity Error FORTRAN (360 min intervals)

5.3.2 Object 25000

The maximum position error calculated is 1.5 m and the maximum velocity error calculated is 0.11 mm/sec

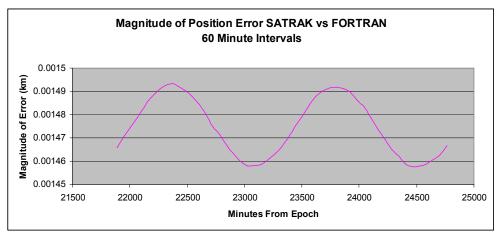


Figure 21 - 25000 Position Error FORTRAN (60 min intervals)

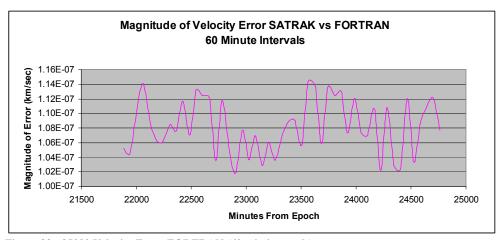


Figure 22 - 25000 Velocity Error FORTRAN (60 min intervals)

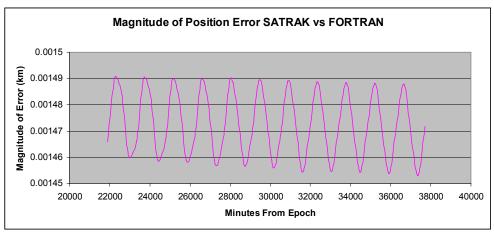


Figure 23 - 25000 Position Error FORTRAN (360 min intervals)

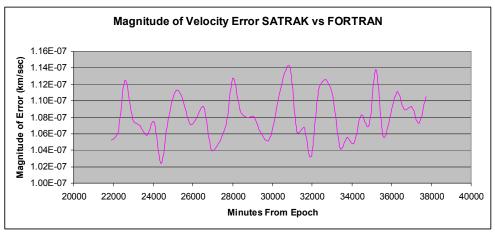


Figure 24 - 25000 Velocity Error FORTRAN (60 min intervals)

5.3.3 Object 30000

The maximum position error between the FORTRAN and SATRAK results is 53 cm and the maximum velocity error is 4.4 mm/sec.

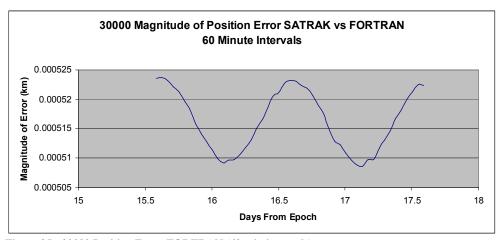


Figure 25 - 30000 Position Error FORTRAN (60 min intervals)

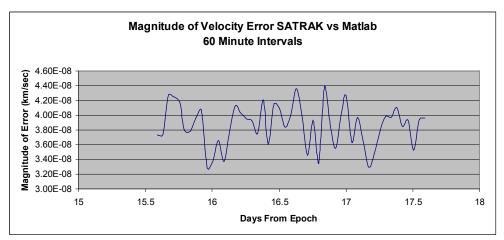


Figure 26 - 30000 Velocity Error FORTRAN (60 min intervals)

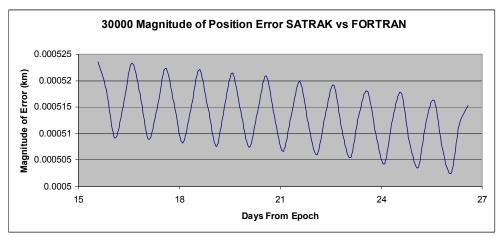


Figure 27 - 30000 Position Error FORTRAN (360 min intervals)

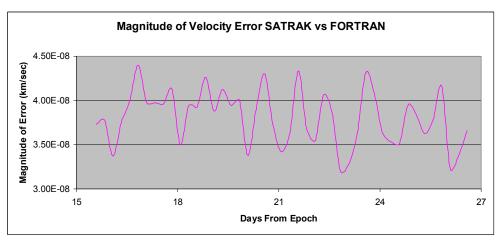


Figure 28 - 30000 Velocity Error FORTRAN (360 min intervals)

5.3.4 Analysis - FORTRAN

The FORTRAN code outputs very similar data to the MATLAB code in terms of position (see below). From visual inspection the two sets of data are indistinguishable.

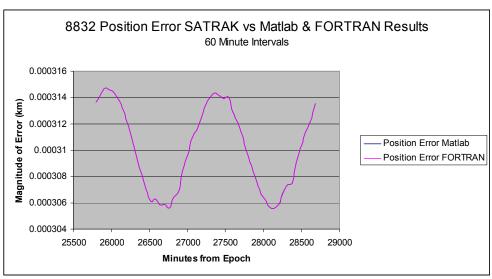


Figure 29 - Comparison of MATLAB and FORTRAN results (8832 position)

There is slightly more difference in the velocity output, see figure 26 (object 8832). The cause of the variation in the output of the FORTRAN code is unknown, but consistent with results found with object 25000. An initial hypothesis was a difference in the precision of the numbers carried through the FORTRAN calculations versus the MATLAB calculations; however, further results from the next section show otherwise. Though the fluctuations look significant, please remember that the graph is displaying differences of .004 mm/sec between the output of the data which is describing the motion of an object traveling roughly 3 km/sec.

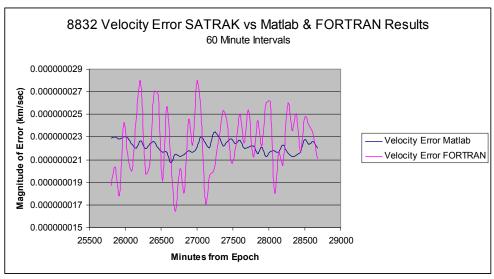


Figure 30 - Comparison of MATLAB and FORTRAN (8832 Velocity)

5.4 Modified Code vs. SATRAK

Results are displayed for both scenarios created in SATRAK (60 and 360 minute intervals). The modified code (labeled Vallado in the graphs) is designed to simulate the inputs and outputs of NASA JSC's existing code. Due to increased complexity, epoch data is also included as an input to the system. This leads to further complications when installing the upgrade into the PREDICT code, which is discussed in a later section.

5.4.1 Object 8832

The maximum position error calculated is 32 cm and the maximum velocity error calculated is 0.024 mm/sec.

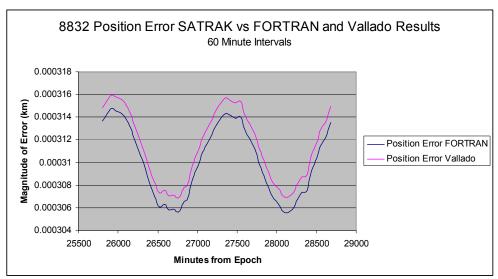


Figure 31 - 8832 Position Error Vallado (60 min intervals)

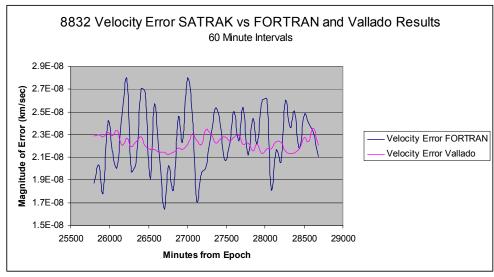


Figure 32 - 8832 Velocity Error Vallado (60 min intervals)

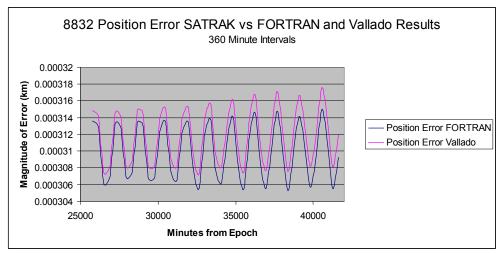


Figure 33 – 8832 Position Error Vallado (360 min intervals)

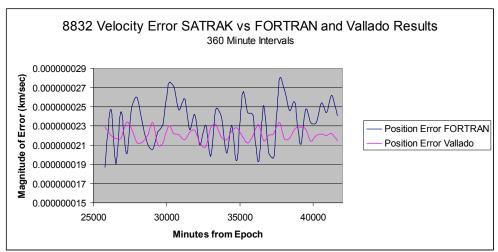


Figure 34 - 8832 Velocity Error Vallado (360 min intervals)

5.4.2 Object 25000

The maximum position error calculated is 1.5 m and the maximum velocity error calculated is .11 mm/sec.

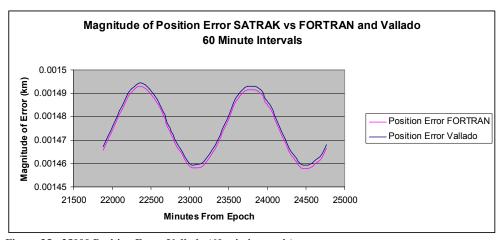


Figure 35 - 25000 Position Error Vallado (60 min intervals)

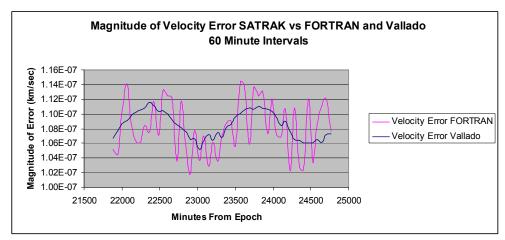


Figure 36 - 25000 Velocity Error Vallado (60 min intervals)

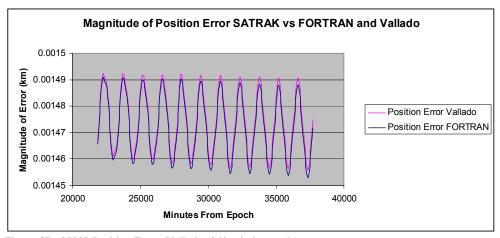


Figure 37 - 25000 Position Error Vallado (360 min intervals)

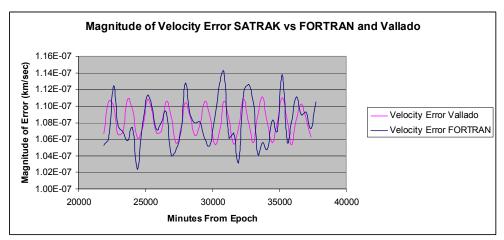


Figure 38 - 25000 Velocity Error Vallado (360 min intervals)

5.4.3 Object 30000

The maximum position error calculated is 6.3 m and the maximum velocity error calculated is 0.47 mm/sec.

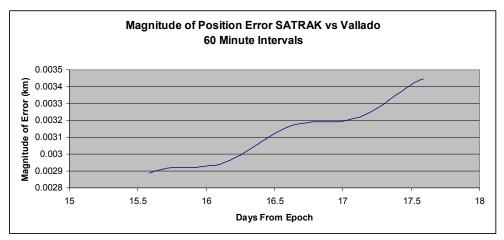


Figure 39 - 30000 Position Error Vallado (60 min intervals)

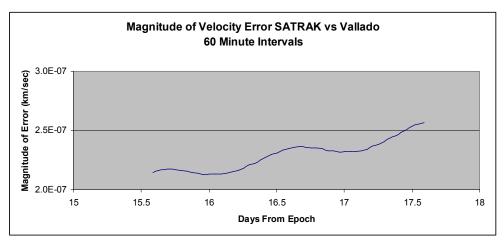


Figure 40 - 30000 Velocity Error Vallado (60 min intervals)

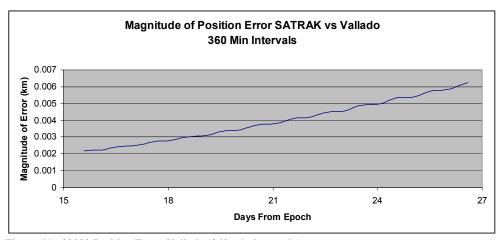


Figure 41 - 30000 Position Error Vallado (360 min intervals)

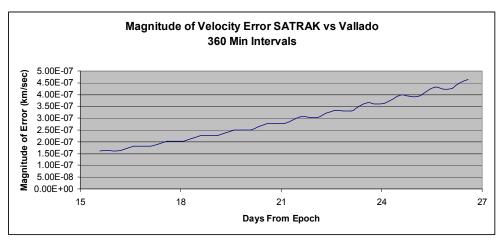


Figure 42 - 30000 Velocity Error Vallado (360 min intervals)

5.4.4 Analysis - Modified Code

Since the Vallado code is a stripped down version of the FORTRAN code, it is expected that the results come out equal or close to equal. Looking at the resultant graphs for object 8832 and object 25000 show some noteworthy trends. First, that the position error

for the full FORTRAN code provides results slightly closer to truth. In the two cases, FORTRAN code is consistently fractions of a millimeter better. Why this happens is unknown. Second, the velocity error is more similar to the MATLAB error than the FORTRAN error. The reason for this is also unknown, but can be seen in figure 39.

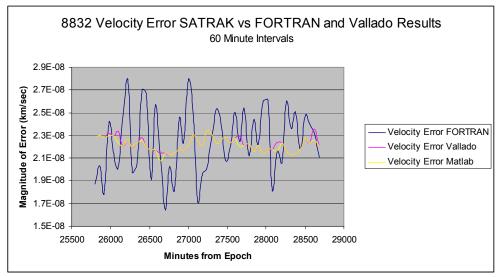


Figure 43 - Comparison of Vallado to MATLAB (8832 Velocity Error)

Finally, the results of object 30000 do not match the pattern initiated by the first two objects. The error involved is much greater than other runs and though has periodic tendencies, adheres more towards linear growth. Even though the error is larger than other runs, it is still orders of magnitude less than the operating threshold.

5.5 JSC Code vs. SATRAK

The JSC SGP4 subroutine code was taken directly from October 2008 version of the PREDICT code. In these tests, it is called by the same program that calls the modified code. In the resultant graphs, the JSC code is labeled as Matney, after Dr. Matney who wrote the original code.

5.5.1 Object 8832

The maximum position error calculated is 356.1 km and the maximum velocity error calculated is 25.4 m/sec.

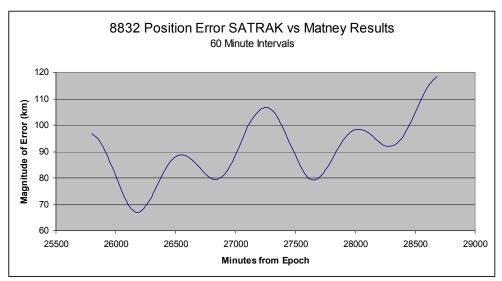


Figure 44 - 8832 Position Error Matney (60 min intervals)

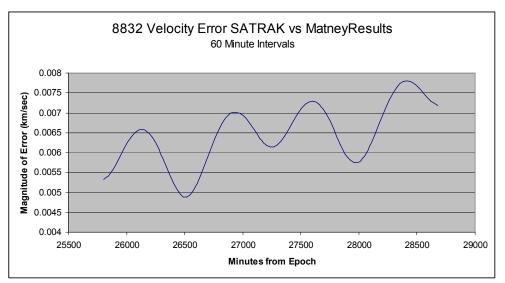


Figure 45 - 8832 Velocity Error Matney (60 min intervals)

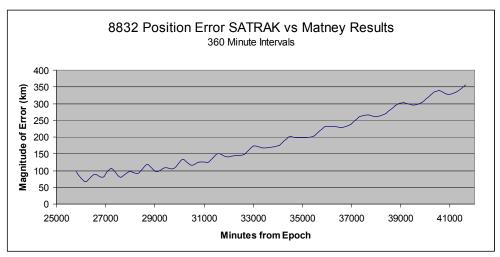


Figure 46 - 8832 Position Error Matney (360 min intervals)

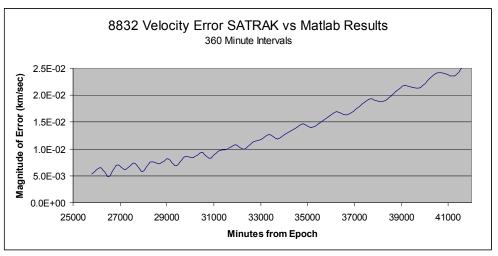


Figure 47 - 8832 Velocity Error Matney (360 min intervals)

5.5.2 Object 25000

The maximum position error calculated is 137.8 km and the maximum velocity error calculated is 10 m/sec.

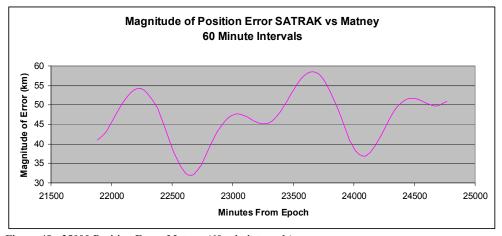


Figure 48 - 25000 Position Error Matney (60 min intervals)

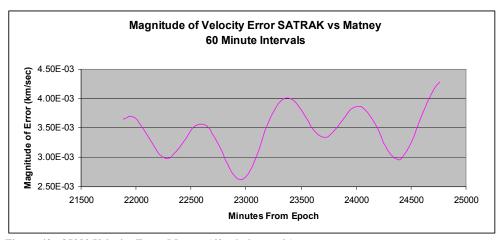


Figure 49 - 25000 Velocity Error Matney (60 min intervals)

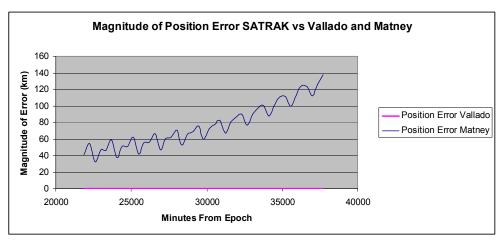


Figure 50 - 25000 Position Error Matney (360 min intervals)

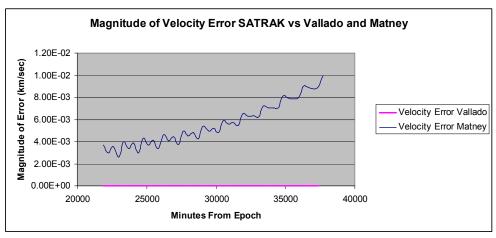


Figure 51 - 25000 Velocity Error Matney (360 min intervals)

5.5.3 Object 30000

The maximum position error calculated is 61.2 km and the maximum velocity error calculated is 4.4 m/sec.

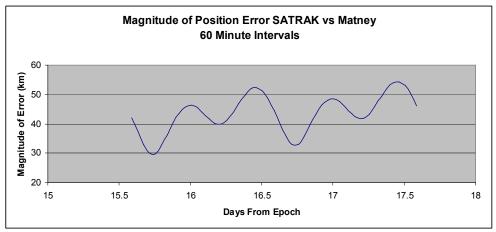


Figure 52 - 30000 Position Error Matney (60 min intervals)

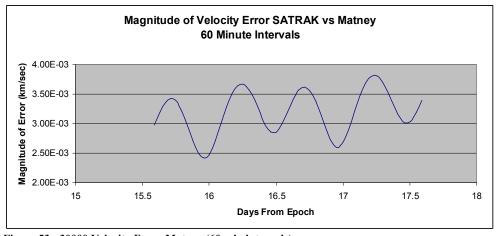


Figure 53 - 30000 Velocity Error Matney (60 min intervals)

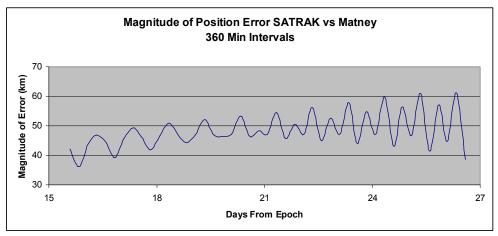


Figure 54 - 30000 Position Error Matney (360 min intervals)

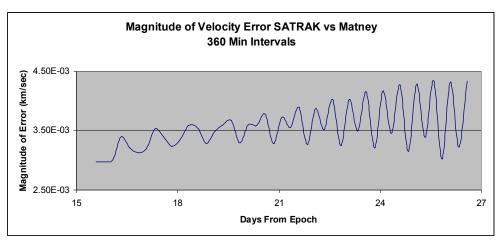


Figure 55 - 30000 Velocity Error Matney (360 min intervals)

5.5.4 Analysis - JSC Code

From visual inspection and direct comparison with the three previous codes, it is easy to see that the JSC code does not provide the same level of accuracy in the output. It is suspected that there are two main reasons for this. First, it is believed that there is a fundamental flaw in the programming of Kepler's solution. This is skewing output results even before any perturbations are taken into account. Second, the JSC code does

not account for any third body interactions. This causes the periodic variations seen in the graphs, as sometimes the third body effects help the simulation while at other times it hurts the simulation.

Furthermore, the scope of the error is of similar magnitude to the threshold, and thus cannot be ignored like the previous three cases. This will be discussed further in Section 5.9 Operational Issues.

5.6 Polysat

Polysats are small satellites designed by Cal Poly under the Cube Sat project.. CP3 and CP4 were launched into 640 km sun-synchronous orbits on April 17, 2007. For the Polysats, there were two sets of truth data. First, there was SATRAK data of the CP3 available. Second, there was TLE data provided by Celestrak. Using archived and current TLE data, it is possible to see how far away the prediction is from official data.

For this analysis, only the modified code was compared to truth data. This is to check the operation potential of the new software beyond GEO objects. The original code was not tested because it was not designed for LEO or MEO objects.

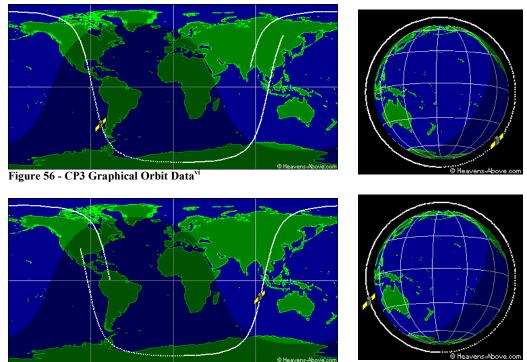


Figure 57 - CP4 Graphical Orbit Datavi

5.6.1 CP3

Below are graphs showing the magnitude of the vector error in both position and velocity of the modified code compared to SATRAK truth data.

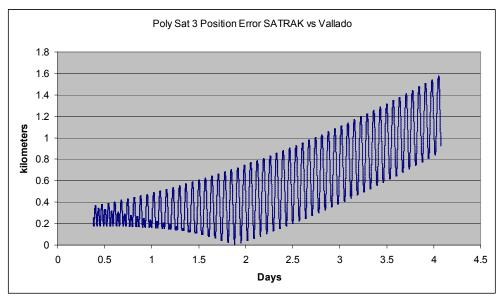


Figure 58 - CP3 Position Error Vallado (10 sec intervals)

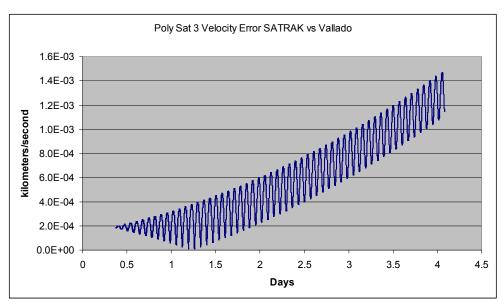


Figure 59 - CP3 Velocity Error Vallado (10 sec intervals)

These graphs show two trends. First, the variation in error is a function of the objects period and not of the Earth's rotation. The periodic motion of both the position and velocity corresponds to the mean motion of the spacecraft. Second, the slope of the error

trends is also proportional to the mean motion of the object. In other words, the faster the object is moving, the faster error will be a significant factor.

Beyond SATRAK, there is also TLE data that can be used as truth data. TLE's were pulled from Celestrak at four different times and are shown in the table below.

Table 1 - TLE readings for CP3

Table 1 - TLE readil	
Date of Reading	TLE
4/22/09	1 31129U 07012N 09112.08372896 .00000092 00000-0 31739-4 0 8719
	2 31129 97.9954 167.8345 0103695 98.8601 262.4373 14.52123890106741
5/1/09	1 31129U 07012N 09121.11032672 .00000119 00000-0 37832-4 0 8897
	2 31129 97.9938 176.4337 0103587 70.9779 290.2602 14.52126376108059
5/6/09	1 31129U 07012N 09126.14038987 .00000017 00000-0 14486-4 0 9001
	2 31129 97.9934 181.2249 0103743 55.7944 305.3039 14.52126523108786
5/23/09	1 31129U 07012N 09143.15980035 .00000024 00000-0 16378-4 0 9438
	2 31129 97.9925 197.4331 0102509 3.5446 356.6451 14.52130919111251

Table 2 - CP3 TLE Results

Days since 4/22 TLE	Error in Position (km)	Error in Velocity (km/sec)
9.03	0.638	0.008662
14.06	2.802	0.002583
31.08	9.197	0.000703

These results are significantly better than expected given the SATRAK analysis. This shows that SATRAK data and TLE data do not necessarily match. Based on this, the modified code may give more accurate results than SATRAK. If the new code contains

more modeled perturbations than SATRAK, this would also explain the periodic variations seen in most of the preceding figures.

5.6.2 CP4

Table 3 - CP4 TLE Data

Table 3 - CI 4 ILE L	7414
Date of Reading	TLE
4/22/09	1 31132U 07012Q 09112.17610138 .00000080 00000-0 27710-4 0 5883
	2 31132 97.9989 171.5508 0086862 87.9115 273.2019 14.55199299106882
5/1/09	1 31132U 07012Q 09120.15222967 .00000158 00000-0 45009-4 0 5943
	2 31132 97.9986 179.1896 0086768 63.3890 297.6162 14.55201137108040
5/23/09	1 31132U 07012Q 09143.1865555600000231 00000-0 -40476-4 0 6101
	2 31132 097.9973 201.2422 0085319 352.7051 007.2899 14.55204047111390

Table 4 - CP4 Results

Error in Position (km)	Error in Velocity (km/sec)
0.5924	0.000514
12.253	0.0115
	0.5924

The CP4 results are similar to the CP3 results. The error in both position and velocity are of similar magnitude after similar amounts of time. From this, a general rate of error propagation can be assessed.

5.7 ISS Tool bag

The ISS Tool bag was a piece of debris dropped by astronauts working on the outside of the International Space Station. The ISS is in orbit at approximately 350 km altitude. The ISS Tool bag is currently in orbit at approximately 300 km altitude due to drag perturbations and lack of station keeping.



Figure 60 - ISS Tool_bag Graphical Orbit Datavi

The following TLE's were downloaded from Celestrak.

Table 5 - ISS Tool bag TLE Data

1 abic 3 - 135 1001 ba	is the bata
Date of Reading	TLE
5/1/09	1 33442U 98067BL 09120.27106401 .00062601 00000-0 24248-3 0 2271
	2 33442 51.6371 184.2223 0003583 339.3988 20.6682 15.86587612 25511
5/6/09	1 33442U 98067BL 09125.49770747 .00067504 00000-0 25238-3 0 2356
	2 33442 51.6390 156.8544 0001434 341.7735 18.3083 15.87287894 26344
5/23/09	1 33442U 98067BL 09143.17121749 .00091180 14199-4 29936-3 0 2571
	2 33442 51.6360 64.0961 0002463 89.7331 270.1497 15.90545869 29150

Table 6 - ISS Tool bag Results

Table 0 - 155 Tool dag Kesults		
Days since 4/22 TLE	Error in Position (km)	Error in Velocity (km/sec)
5.23	33.645	0.0388
22.90	3041.2	3.519

The SGP4 propagator breaks down quickly when dealing with extremely low orbits. After 5 days, the error in position has already become significant and after 23 days, the projection is useless. The error in the projection is caused by an ever changing B* term in the TLE, or the term used to approximate drag. The force of drag is changing constantly due to the object falling further and further into the atmosphere. Propagation for LEO objects should be limited to short periods of time using the new code.

5.8 Iridium Debris

Iridium Debris is a piece of the Iridium 33 satellite that broke up in LEO. The altitude of the debris is approximately 775 km, putting it in a similar orbit to the Polysats.

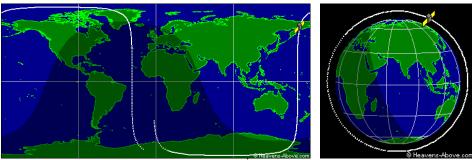


Figure 61 - Iridium Debris Graphical Orbit Data vi

Table 7 - Iridium Debris TLE Data

Table 7 - Illulum Deblis TLE Data		
Date of Reading	TLE	
4/22/09	1 33771U 97051J 09111.44856353 .00002894 00000-0 10166-2 0 511	
	2 33771 86.4092 92.3272 0016638 106.1355 254.1660 14.34588702 9984	
5/1/09	1 33771U 97051J 09120.30665597 .00004031 00000-0 14163-2 0 587	
	2 33771 86.4081 88.6364 0017979 83.2643 277.0615 14.34660268 11251	
5/23/09	1 33771U 97051J 09142.83354878 .00003334 00000-0 11665-2 0 776	
	2 33771 086.4045 079.2409 0018814 026.0516 334.1596 14.34826651 14481	

Table 8 - Iridium Results

Days since 4/22 TLE	Error in Position (km)	Error in Velocity (km/sec)
8.59	41.315	0.0444
31.39	470.04	0.4969

5.9 Operational Issues

There are two main issues to consider when investigating whether or not to switch to the new system. First, how will the improvements in the code affect the program effectiveness? Second, how will the additions to the code affect the speed of operation?

5.9.1 Program Effectiveness

Working with the assumption that the original JSC code provides 'good enough' answers, the following estimates how long the JSC code could successfully track the satellite. If this range of time falls within the scope of program operations, then JSC would not be able to differentiate between the old and new SGP4 versions.

To calculate this estimation, there were some simplifying assumption made. First, NASA uses a telescope with a 2 degree square FOV to initially find the object. It then uses the PREDICT software to try to find the same object with a 0.2 degree square FOV telescope. For this estimation, a 0.2 degree round FOV was assumed. This means that

the threshold for the angle away from the position truth vector is half of this, or 0.1 degrees. Second, the angle from the center of the Earth was calculated rather than from some point on the surface of the Earth. This assumption has both the ability to help or harm the prediction based on the vector error and where the telescope actually is, but should give a reasonable answer. Third, since SATRAK data was not available for all the points needed, the MATLAB data was used as truth. Earlier tests showed that MATLAB provided accurate outputs consistently.

Below are graphs of the three objects defined by NASA JSC showing the angle at which the object is from where the old SGP4 program tells the telescope to point. A threshold line of 0.1 degrees is also drawn for a point of comparison.

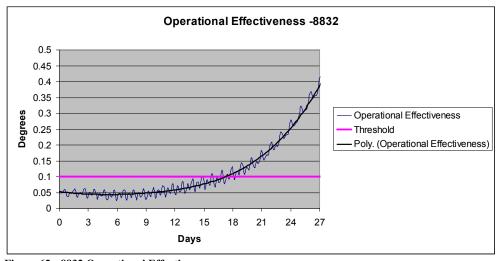


Figure 62 - 8832 Operational Effectiveness

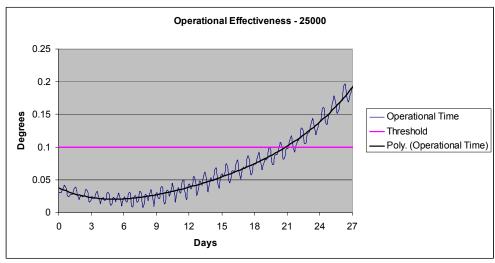


Figure 63 - 25000 Operational Effectiveness

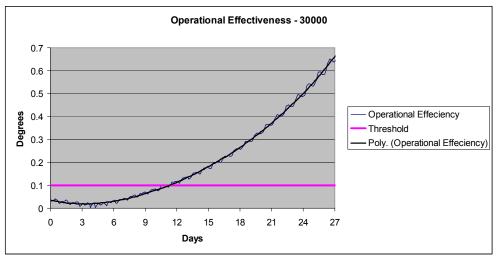


Figure 64 - 30000 Operational Effectiveness

All three cases show that the existing SGP4 propagator breaks down and is no longer effective after a matter of days. From these limited examples, it takes 12 days for the first object to leave the FOV. Furthermore, these examples show that the propagator is most effective between three and nine days. What is troubling about these graphs,

mentioned briefly earlier, is the fact that at time zero, the telescope will not be centered on the target. Again, it is believed this to be caused by a flaw in programming Kepler's solution. The above graphs can be compared to the following:

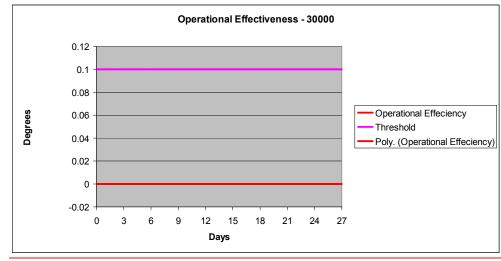


Figure 65 - 30000 Operational Effectiveness (new code)

The error with the new code is essentially zero throughout the entirety of the simulation and is difficult to read in the above chart. Object 30000 offered the poorest results and thus was chosen as a point of comparison. The new code offers significant improvement over the old code and will stay within threshold limits indefinitely. Additionally, even in short term situations, the new code offers superior accuracy.

Below is the operational effectiveness of the new code in relation to Polysat 3 (CP3):

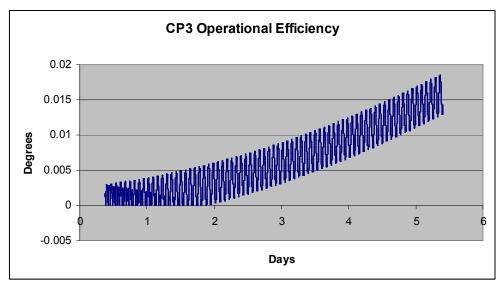


Figure 66 - CP3 Operational Effectiveness

Though the values only extend to roughly 5 days, extrapolating the data using a quadratic suggests that the modified code will be able to keep the object in the FOV for roughly 15 days. Using the TLE truth data points to a <u>different model</u>. Using data between 4/22/09 and 5/23/2009 which is roughly 31 days, shows that the object will be about .071 degrees away from the center of the FOV – still within threshold limits. This shows that the growth in error is linear instead of polynomial, suggesting that the modified code will be able to keep the object in the FOV for roughly 40 days.

5.9.2 Operational Speed

Comparing lines of code, the new program contains 4475 lines with comments and blank lines, while the existing code only contains 2510 lines. No noticeable difference was encountered when running the two programs.

6. Conclusion

The preceding results lead to the following conclusions:

In reference to the original question of whether or not NASA JSC should change their system to incorporate the new Vallado code, the answer is yes. This should be done for two distinct reasons. First, the modified code is more accurate by orders of magnitude over a much longer period of time. Second, the old code provides flawed answers off by kilometers even at time zero. Though the old code provides 'good enough' answers for periods up to about one week, this assumes that the input data is accurate. In reality, the nature of the input data may contain significant errors as well. This is because the code uses right ascension and declination data separated by brief periods of time (less than 5 minutes), thus the object being followed does not move far and the right ascension and declination change only slightly. Calculating COES from this data is limited to the accuracy and the significant figures provided by the telescope data. In such, the 'good enough' answers provided by the old code may not be 'good enough' when compounded with errors in telescope readings.

From the analysis, it is apparent that Vallado's code does not exactly match the output of SATRAK, though it is close. The periodic nature of the error between the two codes, which mimics the period of the orbiting object around the Earth, implies that there are perturbations or constants that differ between the two programs. For this analysis, SATRAK data is taken as truth; however, the error in the SATRAK program is unknown. It is possible that Vallado better predicts the position and velocity of orbiting objects. For the purposes of this project, the two outputs are statistically identical.

If using the Vallado code to replace SATRAK, the MATLAB version of the code is more reliable than the FORTRAN version. Even though the two programs yield indistinguishable results, the MATLAB version of the code is more intuitive and easier to follow. Furthermore, FORTRAN proved more difficult to use as the program balked at times for unknown reasons. Both programs have been provided for use on the attached CD with a User's Guide in section 7.

Finally, the modified code has limitations based on the altitude of the satellite. For objects in deep space orbits such as GEO, the code accurately predicts (to the same level as SATRAK) the position and the velocity of the object within threshold limits indefinitely. In MEO, the effectiveness drops to roughly two weeks of predictions within the threshold limits. In LEO, the effectiveness drops to about one day of predictions within the threshold limits. Thus, the new code will work for all orbiting objects; however, it is important to keep in mind the operational timeline of activities.

Furthermore, for LEO and MEO objects, the B* term needs to be added to the system, or else the accuracy of the results may decrease. Improvements to the PREDICT code must be made to accommodate this before use.

7. User's Guide

This section provides instructions on how to install and run the three different codes discussed in this paper. Sections 7.1 and 7.2 describe the SATRAK replacement codes written in MATLAB and FORTRAN. Section 7.3 describes how to incorporated the modified code into the existing NASA JSC PREDICT code as well as the various options the user has within the code itself.

7.1 MATLAB

Note that the output file will be over-written each time the program is run unless the filename is changed.

The MATLAB code mimics the SATRAK code and outputs very similar results. Work was done using MATLAB 7.0.

7.1.1 Installation

On the attached CD, there is a folder named SGP4 MATLAB. Copy and paste this folder to MATLAB's current directory. The folder contains the following files:

• testmat.m Driver script for testing and example usage

• sgp4.m Main sgp4 routine

sgp4init.m
 Initialization routine for sgp4

• initl.m Initialization for sgp4

• dsinit.m Deep space initialization

• dspace.m Deep space perturbations

• dpper.m Deep Space periodics

• dscom.m Deep Space common variables

• twoline2rv.m TLE conversion routine

• angl.m Find the angle between two vectors

• constmath.m set mathematical constants

• days2mdh.m convert days to month day hour minute second

• getgravc.m Get the gravity constants

• gstime.m Find Greenwich sidereal time

• invjday.m Inverse Julian Date

• jday.m Find the Julian Date

• mag.m Magnitude of a vector

• newtonnnu.m Kepler's iteration given eccentricity and true anomaly

• rv2coe.m Convert position and velocity vectors to classical orbital

elements

• SGP4-VER.TLE Verification file

Sgp4 CodeReadme.pdf

7.1.2 Verification Run

1. Run testmat.m

2. Enter opsmode 'i' for improved method

3. Enter typerun 'v' for verification mode

- 4. Enter constant '72' corresponding to WGS-72 standards
- 5. Enter file name 'SGP4-VER.TLE' see appendix 1 for file information
- 6. The program should output a file called 'tmatver.out' can be compared to the results found in appendix 2 for verification that the program is running properly.

7.1.3 TLE inputs

The program does not discriminate against file types when opening TLEs, though it has only been tested with .TXT and .TLE file extensions. Make sure that the input TLE does have proper the proper format. A simple way to load a TLE is to open a new script in MATLAB and copy and paste the desired TLE, then save the script with a .TLE file extension.

The program is capable of handling multiple TLEs in a single run and will output the file with a header representing the satellite number before each set of data. When formatting a multiple TLE input, use a '#' symbol to begin each line that does not contain TLE data. This tells the computer to skip the line and move to the next line.

7.1.4 Catalog Mode

Catalog mode takes a TLE input and outputs data for -24 hours to +24 hours with 20 minute intervals. This is useful for short term propagations of many objects (i.e. multiple TLEs).

1. Run 'testmat.m'

- 2. Enter 'a' for afspc or 'i' for improved method
- 3. For type of run, enter 'c' for catalog mode
- 4. Enter TLE data file must be completely written with the file extension name
- 5. A new file should be created with the name 'tmatall.out' with the catalog data.

7.1.5 Manual Mode

Manual mode gives the user the opportunity to define when to start and stop calculating and at what time intervals. To do this, the user is given three options to define time:

- Minutes from epoch
- Epoch
- Day of Year

The minutes from epoch approach assumes that the epoch is defined by the time stated within the TLE. Simply input (in minutes) how much time you want to elapse before the first data point, and how much time you want to elapse before the last data point. The epoch approach asks for a start date and time and a stop date and time. Finally, the day of year approach asks for a year and a day of the year. The day of the year can be in decimal form to indicate a fraction of a day. For all three approaches, the reference time frame is UTC. Furthermore, whatever mode chosen will prompt for the time step that will be used (in minutes).

Note, when working with multiple TLEs in manual mode, the user will be prompted for start and stop points and time step for each individual TLE.

- 1. run 'testmat.m'
- 2. Enter 'a' for afspc or 'i' for improved mehod
- 3. For type of run, enter 'm' for manual operation
- 4. Enter how the start and stop points are determined
 - a. m = minutes from epoch
 - b. e = epoch (year, month, day, hour, min, sec)
 - c. d = day of year (year, day)
- 5. Enter constant choice (72 gives answers closest to truth)
- 6. Enter TLE filename
- 7. Follow prompts on screen to input start and stop points and time step
- 8. Program sends output to file named 'tmat.out'

7.1.6 Understanding the Output File

The output comes without column headers, so it is important to know what the values represent. The first line is a header line with information about the satellite number. When working with multiple TLEs, data from each TLE is separated by a similar header with the satellite number. For each satellite, there are 7-13 columns of data (columns 8-13 are date and time data which is not calculated for catalog mode):

Column 1 Minutes from Epoch

Column 2 X-Position

Column 3 Y-Position

Column 4 Z-Position

Column 5 X-Velocity

Column 6 Y-Velocity

Column 7 Z-Velocity

Column 8 Year

Column 9 Month

Column 10 Day

Column 11 Hour

Column 12 Minute

Column 13 Second

7.2 FORTRAN (Vallado Code)

The main FORTRAN program has many similarities with the MATLAB program and may reference certain sections.

^{**}Note, the position vector is based in ECI coordinate frame.**

^{**}Note that the output file will be over-written each time the program is run unless the filename is changed.**

7.2.1 Installation

On the attached CD, there is a folder labeled SGP4 FORTRAN. Copy and paste the folder to a working directory. The files contained in the folder are:

ASTMATH.CMN Common file with mathematical constants

SGP4.CMN Common file with constants applicable to SGP4 model

TESTFOR.FOR Fixed Format FORTRAN file

TESTFOR.EXE Executable file of TESTFOR.FOR

Validation.OUT Data to validate correct installation

SGP4-VER.TLE Validation TLE Data

Either use the executable (compiled and built on Silverfrost), or compile and build the FORTRAN file using another compiler. Correct installation should result in the program running in a command prompt.

7.2.2 Verification Run

** FORTRAN is case sensitive, please keep this in mind when entering text. **

- 1. Run TESTFOR.EXE
- 2. Select opsmode 'i'
- 3. Select run type 'v'
- 4. Input filename 'SGP4-VER.TLE'
- 5. Program will output a file named 'tfor.out' to the working directory

6. Compare this output with data from the file labeled 'Verification.OUT'

7.2.3 TLE Input

This function is identical to MATLAB. See section 7.1.3.

7.2.4 Catalog Mode

Catalog mode takes a TLE input and outputs data for -24 hours to +24 hours with 10 minute intervals. This is useful for short term propagations of many objects (i.e. multiple TLEs).

- 1. Run 'TESTFOR.EXE This should bring up a command prompt
- 2. Enter 'a' for afspc or 'i' for improved method
- 3. For type of run, enter 'c' for catalog mode
- 4. Enter TLE filename– file must be completely written with the file extension name
- 5. A new file should be created with the name 'tfor.out' with the catalog data.

7.2.5 Manual Mode

Manual mode gives the user the opportunity to define when to start and stop calculating and at what time intervals. To do this, the user is given three options to define time:

- Minutes from epoch
- Epoch
- Day of Year

The minutes from epoch approach assumes that the epoch is defined by the time stated within the TLE. Simply input (in minutes) how much time you want to elapse before the first data point, and how much time you want to elapse before the last data point. The epoch approach asks for a start date and time and a stop date and time. Finally, the day of year approach asks for a year and a day of the year. The day of the year can be in decimal form to indicate a fraction of a day. For all three approaches, the reference time frame is UTC. Furthermore, whatever mode chosen will prompt for the time step that will be used (in minutes).

Note, when working with multiple TLEs in manual mode, the user will be prompted for start and stop points and time step for each individual TLE.

- 9. run 'TESTFOR.EXE' a new command prompt should appear
- 10. Enter 'a' for afspc or 'i' for improved method
- 11. For type of run, enter 'm' for manual operation
- 12. Enter how the start and stop points are determined (Case sensitive)
 - a. M = minutes from epoch
 - b. E = epoch (year, month, day, hour, min, sec)
 - c. D = day of year (year, day)
- 13. Enter constant choice (72 gives answers closest to truth)
- 14. Enter TLE filename
- 15. Follow prompts on screen to input start and stop points and time step

16. Program sends output to file named 'tfor.out'

7.2.6 Understanding the Output File

The output file created is in the same format at the MATLAB output. Please see section 7.1.6 for more details.

7.3 Modified FORTRAN Code - For JSC

The modified FORTRAN code is designed to be compatible with the current PREDICT program; however, epoch data needs to be passed to the new program. This section does not describe how to run the PREDICT code. Rather, it explains how to integrate the modified code with the existing code.

7.3.1 Installation

On the attached CD in the folder 'MODIFIED CODE', there is a file called:

SGP4_Classic Subroutine.txt

Open this file and copy all of the contents. Next in the PREDICT source code, locate the SGP4_Classic subroutine (line 853 – line 1116 in Feb. version). Delete the entire subroutine and replace it with the text copied above.

To solve the epoch data problem, a global file called /global2/ is created in the surveychasenew subroutine. In the line following the definition of /global1/ (or line 4314 after addition of the new code) add the following line (□ represents a space):

□□□□□COMMON /global2/ DATE, UT

```
cccccccccccccccccccccccccccccc
C SUBROUTINE SURVEYCHASENEW
cccccccccccccccccccccccccc
     esb 3/21/09 changed the a58 formats back to a80 to match the original data formats
      for xline, cline, testline
        this will allow additional data to be in the original data files.
     Last Change: 16 April 2008
     KJA 16 April, made the propagation to start at 2200 UTday -1 and for 24 hours
     KJA 13 Mar 2007
     Changed the prop time to be rounded up to the minute, changed inputs
     KJA 1 Mar 2007
     New formats to RA,DEC, and MAG
     This program will be used to build the prediction files for the
     survey and chance program
     Inputs need to be one set of observations with the
     YYYYDOY.obj# line at the top and the observations following it
     SUBROUTINE SURVEYCHASENEW
                                tele, FILEIN
        mmON /global1/ PROPNAm-
     COMMON /global2/ DATE, UT
      INTEGER YYYY, JDOY, NDET, MONTE, i
      INTEGER TIMEhrs, TIMEmins, JDET, DAY, HOUR
     DOUBLE PRECISION TIME, MINS, HOURb4mid, newUTb4mid, newUT
     DOUBLE PRECISION HOURnew, MINsnew, UT
     CHARACTER*20 FILEOUT, FILETEMP, FILEOUT2
     CHARACTER*80 XLINE, CLINE, TESTLINE
      CHARACTER*12 XXX
      CHARACTER*34 SHORT
      CHARACTER*10 DATE
     CHARACTER*8 BEGDATE, CHECKHOUR
     CHARACTER*51 CHARSTRING2
      CHARACTER*52 CHARSTRING1
     CHARACTER*20 PROPNAME
      CHARACTER*2 TELE
```

Figure 67 - Code Addition to SURVEYCHASENEW

Next, in the same directory on the CD, there are two common files entitled:

- SGP4.CMN
- ASTMATH.CMN

Copy and paste these files into the directory with the PREDICT source code, as the new

SGP4 propagator references these files.

Now, the updated PREDICT code will be ready to be compiled.

7.3.2 Code Options

The Vallado code contains various options that have been pre-programmed for the Orbital Debris Program Office use; however, these can changed if necessary.

7.3.2.1 Constant Values

The program is set to use World Geodetic System (WGS) 72 parameters for Earth defined constants. These parameters are legacy values from the first code release of SGP4, and best match SATRAK truth data. The Vallado code is programmed to allow an update to the WGS 84 parameters, which are considered up-to-date until 2010^{vii}. To make this switch, find where 'whichconst' is defined in the SGP4_Classic subroutine (~line 925) and change the value from 72 to 84.

```
Real*8 ro(3),vo(3)
      ----- Locals -----
REAL*8 J2, TwoPi, Rad, mu, RadiusEarthKm, xke,
      de2ra, xpdotp, T, sec, JD, pi, j3, j4, j3oj2, tumin
INTEGER EYr, EDay, EMon, EHr, EMin
INCLUDE 'SGP4.CMN'
COMMON /global2/ Date, UT
----- Implementation
Opsmode = 'i'
typerun = 'm'
typeinput = 'M'
                   4.0D0 * datan(1.0D0) ! 3.14159265358979D0
                 2.0D0 * pi ! 6.28318530717959D0
180.0D0 / pi ! 57.29577951308230D0
TwoPi
Rad
                  pi / 180.0D0 ! 0.01745329251994330D0
DE2RA
xpdotp
              = 1440.0 / (2.0 *pi) ! 229.1831180523293D0
```

Figure 68 - Changing WGS values (1)

If manual changes are necessary to the constants, find the subroutine 'getgravconst' (~line 1012). The constants for specific called cases are listed in various IF statements.

The user can define radius of Earth, J2, J3, and J4 harmonics for either the WGS 72 or WGS 84 system.

```
j3oj2 <mark>=</mark>
              j3 / j2
  ENDIF
if (whichconst.eq.72) THEN
    ! ----- wgs-72 constants -
           = 398600.8D0
                                    ! in km3 / s2
   mu
                                   ! km
   radiusearthkm = 6378.135DO
           = 60.0D0 / dsqrt(radiusearthkm**3/mu)
    xke
    tumin = 1.000 / xke
               0.001082616D0
    jЗ
              -0.00000253881D0
             -0.00000165597D0
    j4
    j3oj2 =
              j3 / j2
  ENDIF
   (whichconst.eq.84) THEN
                   wgs-84 constant
                                      in km3 /
           = 398600.5D0
    radiusearthkm = 6378.137DO
           = 60.0D0 / dsqrt(radiusearthkm**3/mu)
           = 1.0D0 / xke
               0.00108262998905D0
              -0.00000253215306D0
    13
              -0.00000161098761D0
              j3 / j2
```

Figure 69 - Changing WGS values (2)

7.3.2.2 B*

The B* term is used to estimate the drag force felt by the object in orbit. For the Orbital Debris Program Office, this value has been set to zero since B* is not calculated by the PREDICT program and the objects in GEO do not experience significant drag forces. There are two ways to change the default value of B*. First, the user can define it as something other than zero inside the program. To do this, find where B* is defined in the TwoLine2RVSGP4 subroutine (~line 1157). Change the 0.0d0 value to desired B* value. Second, the user can comment the above line out completely and uncomment the two lines above. Doing this will prompt the user to input a B* term. This may cause some problems in running the PREDICT code since it will prompt the user during every

iteration of the SGP4 program. To solve this, the user can comment out all three lines and define the B* term in the main program. The term can then be accessed by the subroutine either by a global variable or by having the B* variable be called by the subroutine itself.

```
Code = 0
        ----- CONVERT TO INTERNAL UNITS ------
* ---- RADIANS, DISTANCE IN EARTH RADII, AND VELOCITY IN ER/KEMIN) ----
       NDot = MMDOTO2 / (XPDOTP*1440)
       NDDot = MMDDOTO6 / (XPDOTP*1440*1440)
c added the possibility to put in a Bstar term. If this is unnecessary
c comment out the next two lines and uncomment the third line.
        !Write(*,*) 'Input Bstar term: '
        !read(*,*) Bstar
        Bstar = 0.0d0
c Changed initial element set to match NASA input
            = MMO / XPDOTP
       No
              = (No*TUMin) ** (-2.0D0/3.0D0)
       Inclo = INCDEGO * Deg2Rad
       nodeo = RAANDEGO * Deg2Rad
       Argpo = APDEGO * Deg2Rad
Mo = MADEGO* Deg2Rad
        IF (DABS(ECCO-1.0DO) .gt. 0.000001DO) THEN
           Altp= (a*(1.0D0-ECC0))-1.0D0
           Alta= (a*(1.0D0+ECC0))-1.0D0
```

Figure 70 - Changing the B* term

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APPENDIX A. SGP4 Code

All working codes are included in the accompanying CD. The FORTRAN code designed to replace the legacy SGP4 propagator within PREDICT is displayed below for reference.

A.1 Modified FORTRAN Code (Vallado)

This code compiles and runs on LAHEY express 7.2. Output files are generated; however, when testing the code with PREDICT_Feb09, there were issues with rewriting a file at the end of the program. There is no evidence that this is linked with the new SGP4 propagator.

```
Subroutine SGP Classic
* code modified by Nicholas Miura
* Cal Poly - San Luis Obispo
* nicholas.miura@gmail.com
* subroutine should interface with existing NASA predictor code
* DTDAYS = Time since final observation in Days
* MMO = Mean motion at final observation
* ECCO = Eccentricity at final observation
* INCDEGO = Inclination at final observation
* APDEGO = Argument of Perigee (Degrees)at observation
* RAANDEGO = Right Ascension of Ascending Node (Degrees)at observation
* MADEGO = Mean Anomaly (Degrees) at observation
* MMDOTO2 = First time derivative
* MMDDOTO6 = Second time derivative
* Outputs
* POS = Position of Spacecraft
* VEL = Velocity of Spacecraft
* UP = Is spacecraft still in orbit?
* Called Subroutines
```

* getgravconst

Subroutine SGP_Classic(DTDAYS,MM0,ECC0, & INCDEG0,APDEG0,RAANDEG0,MADEG0, & MMDOTO2,MMDDOTO6,POS,VEL,UP)

IMPLICIT NONE

c these declarations are copied from NASA Predictor code c and define the input and output of the subroutine

REAL*8 DTDAYS,MM0,ECC0
REAL*8 INCDEG0,APDEG0,RAANDEG0,MADEG0
REAL*8 MMDOTO2,MMDDOTO6,EHour, ESec
Real*8 JDSatEpoch2
REAL*8 POS(1:3),VEL(1:3)
LOGICAL UP
DOUBLE PRECISION UT

c these declarations are used in the code

typerun = 'm' typeinput = 'M'

```
which const = 72
```

```
pi = 4.0D0 * datan(1.0D0) ! 3.14159265358979D0

TwoPi = 2.0D0 * pi ! 6.28318530717959D0

Rad = 180.0D0 / pi ! 57.29577951308230D0

DE2RA = pi / 180.0D0 ! 0.01745329251994330D0

xpdotp = 1440.0 / (2.0 *pi) ! 229.1831180523293D0
```

c sgp4fix identify constants and allow alternate values

CALL getgravconst(which const, tumin, mu, radiusearthkm, xke, & j2, j3, j4, j3oj2)

a j2, j3, j4, j30j2)

c Read Date and Time

- c Open a temporary file containing initialized orbital data
- c Called by other subroutines
- c OPEN(115,FILE = 'Sgp4Rec.bak', ACCESS = 'DIRECT', c & FORM = 'UNFORMATTED', RECL = 1100, STATUS = 'REPLACE')

NumSats = 1

- c bring orbital elements to the new two line element converter trick the system CALL TwoLine2RVSGP4 (NumSats,typerun,typeinput,whichconst,
 - & Code, DTDAYS,MM0,ECC0,
 - & INCDEG0,APDEG0,RAANDEG0,MADEG0, & MMDOTO2,MMDDOTO6, JDSatEpoch2, Error)
- c Call the SGP4 propagator based on Time in minutes

```
T = DTDAYS*1440
CALL SGP4 ( whichconst, T, Ro, Vo, Error )
If (Error .eq. 1)Then
Up = .FALSE.
```

ENDIF

```
c the next WRITE line displays the result on the screen
c comment this out if you don't want to test the results
c
     WRITE(*,800) T, ro(1),ro(2),ro(3),vo(1),vo(2),vo(3)
c 800 FORMAT(F17.8,3F17.8,3(1X,F14.9))
   Pos = Ro
   Vel = Vo
    CLOSE(115)
c
    STOP
   END
                 function getgravconst
  this function gets constants for the propagator. note that mu is identified to
   facilitiate comparisons with newer models.
  author
             : david vallado
                                      719-573-2600 21 jul 2006
  inputs
   which const - which set of constants to use 721, 72, 84
  outputs
   tumin
            - minutes in one time unit
            - earth gravitational parameter
   radiusearthkm - radius of the earth in km
            - reciprocal of tumin
   j2, j3, j4 - un-normalized zonal harmonic values
          - j3 divided by j2
   j3oj2
   norad spacetrack report #3
   vallado, crawford, hujsak, kelso 2006
    SUBROUTINE getgravconst (whichconst, tumin, mu,
            radiusearthkm, xke, j2, j3, j4, j3oj2)
    IMPLICIT NONE
```

```
REAL*8 radiusearthkm, xke, j2, j3, j4, j3oj2, mu, tumin INTEGER whichconst
```

```
if (whichconst.eq.721) THEN
  ! -- wgs-72 low precision str#3 constants --
  radiusearthkm = 6378.135D0 ! km
  xke = 0.0743669161D0
  mu = 398600.79964D0
                             ! in km3 / s2
  tumin = 1.0D0 / xke
  i2 = 0.001082616D0
  j3 = -0.00000253881D0
  j4 = -0.00000165597D0
  j30j2 = j3 / j2
 ENDIF
if (whichconst.eq.72) THEN
  ! ----- wgs-72 constants -----
  mu = 398600.8D0
                         ! in km3 / s2
  radiusearthkm = 6378.135D0 ! km
  xke = 60.0D0 / dsqrt(radiusearthkm**3/mu)
  tumin = 1.0D0 / xke
     = 0.001082616D0
  j2
 j3 = -0.00000253881D0
 j4 = -0.00000165597D0
  j30j2 = j3 / j2
 ENDIF
if (whichconst.eq.84) THEN
  ! ----- wgs-84 constants -----
  mu = 398600.5D0
                         ! in km3 / s2
  radiusearthkm = 6378.137D0 ! km
  xke = 60.0D0 / dsqrt(radiusearthkm**3/mu)
  tumin = 1.0D0 / xke
  j2 = 0.00108262998905D0
  i3 = -0.00000253215306D0
  j4 = -0.00000161098761D0
  j30j2 = j3 / j2
 ENDIF
RETURN
END! SUBROUTINE getgravconst
           SUBROUTINE TWOLINE2RVSGP4
```

* this function converts the two line element set character string data to

^{*} variables and initializes the sgp4 variables. several intermediate variables

```
and quantities are determined. note that the result is a "structure" so multiple
   satellites can be processed simultaneously without having to reinitialize, the
   verification mode is an important option that permits quick checks of any
   changes to the underlying technical theory, this option works using a
   modified TLE file in which the start, stop, and delta time values are
   included at the end of the second line of data. this only works with the
   verification mode. the catalog mode simply propagates from -1440 to 1440 min
   from epoch and is useful when performing entire catalog runs.
             : david vallado
                                      719-573-2600 1 mar 2001
  author
  inputs
   Numsats - Number of satellites processed. It also becomes the record
           number for each satellite
                                     verification 'V', catalog 'C',
   typerun - type of run
   typeinput - type of manual input
                                          mfe 'M', epoch 'E', dayofyr 'D'
   whichconst - which set of constants to use 72, 84
   opsmode - type of manual input
                                          afspc 'a', imporved 'i'
  outputs
             - EOF indicator. Code = 999 when EOF reached
   Code
                                          min from epoch
   startmfe - starttime of simulation,
   stopmfe - stoptime of simulation,
                                           min from epoch
   deltamin - time step
                                     min
  coupling
   days2mdhms - conversion of days to month, day, hour, minute, second
            - convert day month year hour minute second into Julian date
   sgp4init - initialize the sgp4 variables
  Files
   Unit 10 - test.elm
                           input 2-line element set file
   Unit 11

    test.bak

                           output file
                             temporary file of record for 2 line element sets
   Unit 15
            - sgp4rec.bak
* references :
  norad spacetrack report #3
   vallado, crawford, hujsak, kelso 2006
   SUBROUTINE TwoLine2RVSGP4 (NumSats, Typerun, typeinput,
                     whichconst, Code, DTDAYS, MM0, ECC0,
   &
```

& whichconst, Code, DTDAYS,MM0,ECC0,
 & INCDEG0,APDEG0,RAANDEG0,MADEG0,
 & MMDOTO2,MMDDOTO6, JDSatEpoch1, Error)

IMPLICIT NONE Character Typerun, typeinput Integer Code, NumSats, whichconst REAL*8 startmfe, stopmfe, deltamin

REAL*8 J2, mu, RadiusEarthKm,VKmPerSec, xke, tumin
REAL*8 BC,EPDay, sec, xpdotp, j3, j4, j3oj2
REAL*8 startsec, stopsec, startdayofyr, stopdayofyr, jdstart,
& jdstop, JDSatEpoch1
INTEGER startyear, stopyear, startmon, stopmon, startday,
& stopday, starthr, stophr, startmin, stopmin
INTEGER Yr, Mon, Day, Hr, Minute, ICrdno, nexp, bexp, error
CHARACTER Show
Character*130 LongStr1,LongStr2
REAL*8 DTDAYS,MM0,ECC0
REAL*8 INCDEG0,APDEG0,RAANDEG0,MADEG0
REAL*8 MMDOTO2,MMDDOTO6
COMMON /DebugHelp/ Help
CHARACTER Help
INCLUDE 'SGP4.CMN'
INCLUDE 'ASTMATH.CMN'
! Implementation
Show = $'N'$
xpdotp = 1440.0D0 / (2.0D0 * pi) ! 229.1831180523293
CALL getgravconst(whichconst, tumin, mu, radiusearthkm, xke,
& j2, j3, j4, j3oj2);
VKmPerSec = RadiusEarthKm * xke / 60.0D0
READ THE FIRST LINE OF ELEMENT SET
Code = 0
CONVERT TO INTERNAL UNITS
RADIANS, DISTANCE IN EARTH RADII, AND VELOCITY IN ER/KEMIN)
ND-4 = MMDOTO2 / (VBDOTD*1446)
NDot = MMDOTO2 / (XPDOTP*1440) NDDot = MMDDOTO6 / (XPDOTP*1440*1440)
NDD00 = MMDD0100/(ACD01C.1440)

c added the possibility to put in a Bstar term. If this is unnecessary c comment out the next two lines and uncomment the third line.

```
!Write(*,*) 'Input Bstar term: '
    !read(*,*) Bstar
    Bstar = 0.0d0
c Changed initial element set to match NASA input
    No = MM0 / XPDOTP
    a = (No*TUMin)**(-2.0D0/3.0D0)
    Inclo = INCDEG0 * Deg2Rad
    nodeo = RAANDEG0 * Deg2Rad
    Argpo = APDEG0 * Deg2Rad
    Mo = MADEG0* Deg2Rad
    IF (DABS(ECC0-1.0D0) .gt. 0.000001D0) THEN
      Altp= (a*(1.0D0-ECC0))-1.0D0
      Alta=(a*(1.0D0+ECC0))-1.0D0
     ELSE
      Alta= 999999.9D0
      Altp= 2.0D0* (4.0D0/(No*No)**(1.0D0/3.0D0))
     ENDIF
    ! ---- Ballistic Coefficient ----
    IF (DABS(BStar) .gt. 0.00000001D0) THEN
      BC= 1.0D0/(12.741621D0*BStar)
     ELSE
      BC= 1.1111111111111100
     ENDIF
    ! find sgp4epoch time of element set
    ! remember that sgp4 uses units of days from 0 jan 1950 (sgp4epoch)
    ! and minutes from the epoch (time)
* ----- MAKE INITIAL PREDICTION AT EPOCH -----
   !2433281.5 - 2400000.5 = 33281.0, thus time from 1950
   CALL SGP4Init( which const,
     SatNum,BStar, ECC0, JDSatEpoch1-2433281.5D0,
                                                                                 Formatted: German
                                                                                  (Germany)
            Argpo, Inclo, Mo, No, nodeo, Error)
    IF(Error .GT. 0) THEN
      WRITE(*,*) '# *** SGP4 Model Error ***',Error
     ENDIF
```

```
write tle output details
c
    INCLUDE 'debug8.for'
    ! ---- Fix to indicate end-of-file
    GOTO 1000
 999 Code = 999
1000 CONTINUE
    RETURN
    END! SUBROUTINE TwoLine2RVSGP4
                  SUBROUTINE SGP4INIT
  This subroutine initializes variables for SGP4.
  author
             : david vallado
                                     719-573-2600 28 jun 2005
  inputs
           - satellite number
   satn
   bstar
            - sgp4 type drag coefficient
                                              kg/m2er
   ecco
            - eccentricity
            - epoch time in days from jan 0, 1950. 0 hr
   epoch
            - argument of perigee (output if ds)
   argpo
            - inclination
   inclo
            - mean anomaly (output if ds)
   mo
           - mean motion
   no
   nodeo
            - right ascension of ascending node
  outputs
            - common block values for subsequent calls
   return code - non-zero on error.
            1 - mean elements, ecc \ge 1.0 or ecc < -0.001 or a < 0.95 er
            2 - mean motion less than 0.0
            3 - pert elements, ecc < 0.0 or ecc > 1.0
            4 - semi-latus rectum < 0.0
            5 - epoch elements are sub-orbital
            6 - satellite has decayed
  coupling
   getgravconst-
   initl
   dscom
   dpper
```

```
dsinit -
  references:
  hoots, roehrich, norad spacetrack report #3 1980
 hoots, norad spacetrack report #6 1986
  hoots, schumacher and glover 2004
  vallado, crawford, hujsak, kelso 2006
  SUBROUTINE SGP4Init (whichconst,
               Satn, xBStar, xEcco, Epoch, xArgpo,
  &
  &
               xInclo, xMo, xNo, xnodeo, Error)
    IMPLICIT NONE
    INTEGER Satn, error, which const
    REAL*8 xBStar, xEcco, Epoch, xArgpo, xInclo, xMo, xNo, xnodeo
    REAL*8 T, r(3), v(3)
    INCLUDE 'SGP4.CMN'
    !COMMON /DebugHelp/ Help! removed by NM 4/16
    CHARACTER Help
* ------ Local Variables -----
    REAL*8 Ao,ainv,con42,cosio,sinio,cosio2,Eccsq,omeosq,
         posq,rp,rteosq, CNODM, SNODM, COSIM, SINIM, COSOMM,
  &
  &
         SINOMM, Cc1sq,
  &
         Cc2 , Cc3 , Coef , Coef1 , Cosio4, DAY , Dndt ,
  &
         Eccm , EMSQ , Eeta , Etasq , GAM , Argpm , nodem,
         Inclm, Mm, Xn, Perige, Pinvsq, Psisq, Qzms24,
  &
         RTEMSQ, S1 , S2 , S3 , S4 , S5 , S6 ,
  &
         S7 , SFour , SS1 , SS2 , SS3 , SS4 , SS5
  &
  &
         SS6 , SS7 , SZ1 , SZ2 , SZ3 , SZ11 , SZ12
         SZ13 , SZ21 , SZ22 , SZ23 , SZ31 , SZ32 , SZ33 , Tc , Temp , Temp1 , Temp2 , Temp3 , Tsi , XPIDOT ,
  &
  &
         Xhdot1, Z1 , Z2 , Z3 , Z11 , Z12 , Z13 ,
  &
         Z21 , Z22 , Z23 , Z31 , Z32 , Z33
    REAL*8 qzms2t, SS, mu, RadiusEarthKm, J2, j3oJ2,J4,X2o3,
         temp4, j3, xke, tumin
    INCLUDE 'ASTMATH.CMN'
* ------ INITIALIZATION ------
    method = 'n'
    clear sgp4 flag
    Error = 0
```

```
c
    variables, but the user would need to set the common values first. we
    include the additional assignment in case twoline2rv is not used.
    bstar = xbstar
    ecco = xecco
    argpo = xargpo
    inclo = xinclo
    mo = xmo
    no
        = xno
    nodeo = xnodeo
    ! sgp4fix identify constants and allow alternate values
    CALL getgravconst( which const, tumin, mu, radiusearthkm, xke,
       j2, j3, j4, j3oj2)
    SS = 78.0D0/RadiusEarthKm + 1.0D0
    QZMS2T = ((120.0D0-78.0D0)/RadiusEarthKm) ** 4
    X2o3 = 2.0D0 / 3.0D0
   sgp4fix divisor for divide by zero check on inclination
   the old check used 1.0D0 + \cos(pi-1.0D-9), but then compared it to
   1.5D-12, so the threshold was changed to 1.5D-12 for consistency
    temp4 = 1.5D-12
                                                                                       Formatted: French
                                                                                        (France)
    Init = 'y'
    T = 0.0D0
    CALL INITL( Satn, which const, Ecco, EPOCH, Inclo, No,
      Method, AINV, AO, CON41, CON42, COSIO, COSIO2,
      Eccsq, OMEOSQ, POSQ, rp, RTEOSQ, SINIO,
  &
  & GSTo, Opsmode)
    IF(rp.lt. 1.0D0) THEN
      Error = 5
     ENDIF
    IF(OMEOSQ .ge. 0.0D0 .OR. No .ge. 0.0D0) THEN
      ISIMP = 0
      IF (rp.lt. (220.0D0/RadiusEarthKm+1.0D0)) THEN
        ISIMP = 1
                                                                                       Formatted: French
                                                                                        (France)
       ENDIF
      SFour = SS
      QZMS24 = QZMS2T
```

sgp4fix - note the following variables are also passed directly via sgp4 common.

it is possible to streamline the sgp4init call by deleting the "x"

c

c

```
PERIGE = (rp-1.0D0)*RadiusEarthKm
* ----- For perigees below 156 km, S and Qoms2t are altered ----
     IF(PERIGE .lt. 156.0D0) THEN
       SFour = PERIGE-78.0D0
       IF(PERIGE .le. 98.0D0) THEN
         SFour = 20.0D0
        ENDIF
       QZMS24 = ((120.0D0-SFour)/RadiusEarthKm)**4
       SFour = SFour/RadiusEarthKm + 1.0D0
      ENDIF
     PINVSQ = 1.0D0/POSQ
     TSI = 1.0D0/(AO-SFour)
     ETA = AO*Ecco*TSI
                                                                             Formatted: French
     ETASO = ETA*ETA
                                                                             (France)
     EETA = Ecco*ETA
     PSISQ = DABS(1.0D0-ETASQ)
     COEF = QZMS24*TSI**4
     COEF1 = COEF/PSISQ**3.5D0
     CC2 = COEF1*No* (AO* (1.0D0+1.5D0*ETASQ+EETA*)
  &
           (4.0D0+ETASO))+0.375D0*
        J2*TSI/PSISQ*CON41*(8.0D0+3.0D0*ETASQ*(8.0D0+ETASQ)))
  &
     CC1 = BSTAR*CC2
     CC3 = 0.0D0
     IF(Ecco .GT. 1.0D-4) THEN
       CC3 = -2.0D0*COEF*TSI*J3OJ2*No*SINIO/Ecco
      ENDIF
     X1MTH2 = 1.0D0-COSIO2
     CC4 = 2.0D0*No*COEF1*AO*OMEOSQ*(ETA*(2.0D0+0.5D0*ETASQ)
  &
          +Ecco*(0.5D0 + 2.0D0*ETASQ) - J2*TSI/(AO*PSISQ)*
                                                                            Formatted: French
                                                                             (France)
          (-3.0D0*CON41*(1.0D0-2.0D0*
  &
       EETA+ETASQ*(1.5D0-0.5D0*EETA))+0.75D0*X1MTH2*(2.0D0*ETASQ
  &
       -EETA*(1.0D0+ETASQ))*DCOS(2.0D0*Argpo)))
     CC5 = 2.0D0*COEF1*AO*OMEOSQ*(1.0D0 + 2.75D0*)
  &
           (ETASQ + EETA) + EETA*ETASQ)
     COSIO4 = COSIO2*COSIO2
     TEMP1 = 1.5D0*J2*PINVSQ*No
```

MDot = No + 0.5D0*TEMP1*RTEOSO*CON41 + 0.0625D0*TEMP2*

RTEOSQ*(13.0D0 - 78.0D0*COSIO2 + 137.0D0*COSIO4)

395.0D0*COSIO4)+TEMP3*(3.0D0-36.0D0*COSIO2+49.0D0*COSIO4)

TEMP2 = 0.5D0*TEMP1*J2*PINVSQ

(7.0D0 - 114.0D0*COSIO2 +

XHDOT1 = -TEMP1*COSIO

&

&

TEMP3 = -0.46875D0*J4*PINVSO*PINVSO*No

ArgpDot = -0.5D0*TEMP1*CON42 + 0.0625D0*TEMP2*

```
nodeDot = XHDOT1 + (0.5D0*TEMP2*(4.0D0-19.0D0*COSIO2) + (0.5D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D0*TEMP2*(4.0D0-19.0D
      &
                               2.0D0*TEMP3*(3.0D0 - 7.0D0*COSIO2))*COSIO
               XPIDOT = ArgpDot+nodeDot
               OMGCOF = BSTAR*CC3*DCOS(Argpo)
               XMCOF = 0.0D0
               IF(Ecco .GT. 1.0D-4) THEN
                    XMCOF = -X2O3*COEF*BSTAR/EETA
               XNODCF = 3.5D0*OMEOSQ*XHDOT1*CC1
               T2COF = 1.5D0*CC1
c
                sgp4fix for divide by zero with xinco = 180 deg
               if (dabs(cosio+1.0).gt. 1.5d-12) THEN
                    XLCOF = -0.25D0*J3OJ2*SINIO*
      &
                                  (3.0D0+5.0D0*COSIO)/(1.0D0+COSIO)
                  else
                    XLCOF = -0.25D0*J3OJ2*SINIO*
      &
                                  (3.0D0+5.0D0*COSIO)/temp4
                 ENDIF
               AYCOF = -0.5D0*J3OJ2*SINIO
               DELMO = (1.0D0+ETA*DCOS(Mo))**3
                                                                                                                                                                                                     Formatted: German
                                                                                                                                                                                                      (Germany)
               SINMAO = DSIN(Mo)
               X7THM1 = 7.0D0*COSIO2-1.0D0
 * ------ Deep Space Initialization ------
               IF ((TWOPI/No) .ge. 225.0D0) THEN
                    METHOD = 'd'
                    ISIMP = 1
                    TC = 0.0D0
                    Inclm = Inclo
                    CALL DSCOM( EPOCH , Ecco , Argpo , Tc , Inclo ,
      &
                                 nodeo, No
      &
                                 SNODM, CNODM, SINIM, COSIM, SINOMM, COSOMM,
                                 DAY, E3, Ee2, Eccm, EMSQ, GAM,
      &
                                 Peo , Pgho , Pho , PInco , Plo
      &
                                 RTemSq, Se2 , Se3 , Sgh2 , Sgh3 , Sgh4 ,
      &
                                                                                                                                                                                                    Formatted: French
                                                                                                                                                                                                      (France)
                                 Sh2 , Sh3 , Si2 , Si3 , Sl2 , Sl3 ,
      &
                                 S14 , S1 , S2 , S3 , S4 , S5 ,
S6 , S7 , SS1 , SS2 , SS3 , SS4
      &
      &
                                 SS5 , SS6 , SS7 , SZ1 , SZ2 , SZ3
      &
      &
                                 SZ11 , SZ12 , SZ13 , SZ21 , SZ22 , SZ23
                                                                                                                                                                                                     Formatted: German
                                                                                                                                                                                                      (Germany)
                                 SZ31 , SZ32 , SZ33 , Xgh2 , Xgh3 , Xgh4 ,
      &
                                 Xh2 , Xh3 , Xi2 , Xi3 , Xl2 , Xl3 ,
      &
                                 X14 , Xn , Z1 , Z2 , Z3 , Z11
      &
      &
                                 Z12 , Z13 , Z21 , Z22 , Z23 , Z31 ,
                                 Z32 , Z33 , Zmol , Zmos )
      &
```

```
CALL DPPER(e3, ee2, peo, pgho, pho, pinco,
             plo , se2 , se3 , sgh2 , sgh3 , sgh4 ,
  &
                                                                                  Formatted: French
                                                                                  (France)
  &
             sh2 , sh3 , si2 , si3 , sl2 , sl3 ,
             sl4 , T , xgh2 , xgh3 , xgh4 , xh2 ,
  &
             xh3 , xi2 , xi3 , xl2 , xl3 , xl4 ,
  &
  &
             zmol, zmos, Inclm, init,
  &
             Ecco, Inclo, nodeo, Argpo, Mo, Opsmode)
        Argpm = 0.0D0! add for DS to work initial
        nodem = 0.0D0
        Mm = 0.0D0
        CALL DSINIT( which const,
  &
              Cosim ,Emsq, Argpo, S1 , S2 , S3
  &
              S4 , S5 , Sinim , Ss1 , Ss2 , Ss3
  &
              Ss4 , Ss5 , Sz1 , Sz3 , Sz11 , Sz13 ,
  &
              Sz21 , Sz23 , Sz31 , Sz33 , T , Tc
                                                                                  Formatted: German
                                                                                  (Germany)
              GSTo, Mo, MDot, No, nodeo, nodeDot,
  &
  &
              XPIDOT, Z1 , Z3 , Z11 , Z13 , Z21 ,
              Z23 , Z31 , Z33 , ecco , eccsq,
  &
  &
              Eccm, Argpm, Inclm, Mm, Xn, nodem,
              IREZ, Atime, D2201, D2211, D3210, D3222
  &
              D4410, D4422, D5220, D5232, D5421, D5433
                                                                                  Formatted: German
  &
              Dedt, Didt, DMDT, DNDT, DNODT, DOMDT,
                                                                                  (Germany)
              Del1, Del2, Del3, Xfact, Xlamo, Xli,
  &
                                                                                  Formatted: French
                                                                                  (France)
              Xni)
  &
      ENDIF
* ----- Set variables if not deep space or rp < 220 -----
      IF (ISIMP .ne. 1) THEN
        CC1SQ = CC1*CC1
        D2 = 4.0D0*AO*TSI*CC1SQ
        TEMP = D2*TSI*CC1 / 3.0D0
        D3 = (17.0D0*AO + SFour) * TEMP
        D4 = 0.5D0*TEMP*AO*TSI*
             (221.0D0*AO + 31.0D0*SFour)*CC1
                                                                                  Formatted: French
                                                                                  (France)
        T3COF = D2 + 2.0D0*CC1SQ
        T4COF = 0.25D0*(3.0D0*D3+CC1*(12.0D0*D2+10.0D0*CC1SQ))
        T5COF = 0.2D0*(3.0D0*D4 + 12.0D0*CC1*D3 + 6.0D0*D2*D2 +
  &
             15.0D0*CC1SQ*(2.0D0*D2 + CC1SQ))
       ENDIF
     ENDIF! ----- if nodeo and No are gtr 0
   init = 'n'
```

CALL SGP4(whichconst, 0.0D0, r, v, error)

RETURN

END! end sgp4init

^k ------

SUBROUTINE SGP4

this procedure is the sgp4 prediction model from space command. this is an updated and combined version of sgp4 and sdp4, which were originally published separately in spacetrack report #3. this version follows the methodology from the aiaa paper (2006) describing the history and development of the code.

author : david vallado 719-573-2600 28 jun 2005

inputs

satrec - initialised structure from sgp4init() call.

tsince - time eince epoch (minutes)

outputs

r - position vector km v - velocity km/sec

return code - non-zero on error.

- 1 mean elements, ecc >= 1.0 or ecc < -0.001 or a < 0.95 er
- 2 mean motion less than 0.0
- 3 pert elements, ecc < 0.0 or ecc > 1.0
- 4 semi-latus rectum < 0.0
- 5 epoch elements are sub-orbital
- 6 satellite has decayed

coupling

getgravconst-

dpper

dpspace

*

* references :

- hoots, roehrich, norad spacetrack report #3 1980
- * hoots, norad spacetrack report #6 1986
- * hoots, schumacher and glover 2004
- * vallado, crawford, hujsak, kelso 2006

```
SUBROUTINE SGP4 (whichconst, T, r, v, Error)
    IMPLICIT NONE
    INTEGER Error, which const
    REAL*8 T, r(3), v(3)
    INCLUDE 'SGP4.CMN'
* ------ Local Variables -----
    REAL*8 AM , Axnl , Aynl , Betal , COSIM , Cnod ,
        Cos2u, Coseo1, Cosi, Cosip, Cosisq, Cossu, Cosu,
  &
  &
        Delm, Delomg, Eccm, EMSQ, Ecose, El2, Eo1,
  &
        Eccp, Esine, Argpm, Argpp, Omgadf, Pl,
        Rdotl, Rl , Rvdot, Rvdotl, SINIM,
  &
        Sin2u, Sineo1, Sini, Sinip, Sinsu, Sinu,
  &
                                                                               Formatted: French
                                                                                (France)
        Snod, Su, T2, T3, T4, Tem5, Temp,
  &
        Temp1, Temp2, Tempa, Tempe, Templ, U, Ux,
  &
        Uy , Uz , Vx , Vy , Vz , Inclm , Mm ,
  &
        XN , nodem, Xinc, Xincp, Xl , Xlm , Mp ,
  &
  &
        Xmdf, Xmx, Xmy, Xnoddf, Xnode, nodep,
  &
        Tc , Dndt
    REAL*8 X2O3, J2,J3,XKE,J3OJ2, mr,mv,
        mu, RadiusEarthkm, VKmPerSec, temp4, tumin, j4
                                                                               Formatted: German
        INTEGER iter
    CHARACTER Help
    INCLUDE 'ASTMATH.CMN'
* ------ WGS-72 EARTH CONSTANTS ------
* ----- SET MATHEMATICAL CONSTANTS ------
  X2O3 = 2.0D0/3.0D0
 Keep compiler ok for warnings on uninitialized variables
   mr = 0.0D0
   Coseo1 = 1.0D0
   Sineo1 = 0.0D0
  ! sgp4fix identify constants and allow alternate values
    CALL getgravconst( which const, tumin, mu, radiusearthkm, xke,
       j2, j3, j4, j3oj2)
c sgp4fix divisor for divide by zero check on inclination
c the old check used 1.0D0 + cos(pi-1.0D-9), but then compared it to
c 1.5D-12, so the threshold was changed to 1.5D-12 for consistency
```

temp4 = 1.5D-12

```
VKmPerSec = RadiusEarthKm * xke/60.0D0
* ------ CLEAR SGP4 ERROR FLAG ------
  Error = 0
* ------ UPDATE FOR SECULAR GRAVITY AND ATMOSPHERIC DRAG ------
  XMDF = Mo + MDot*T
  OMGADF = Argpo + ArgpDot*T
  XNODDF = nodeo + nodeDot*T
  Argpm = OMGADF
  Mm = XMDF
  T2 = T*T
  nodem = XNODDF + XNODCF*T2
                                                                        Formatted: French
  TEMPA = 1.0D0 - CC1*T
                                                                         (France)
  TEMPE = BSTAR*CC4*T
  TEMPL = T2COF*T2
  IF (ISIMP .ne. 1) THEN
    DELOMG = OMGCOF*T
    DELM = XMCOF*((1.0D0+ETA*DCOS(XMDF))**3-DELMO)
    TEMP = DELOMG + DELM
    Mm = XMDF + TEMP
    Argpm = OMGADF - TEMP
    T3 = T2*T
    T4 = T3*T
    TEMPA = TEMPA - D2*T2 - D3*T3 - D4*T4
    TEMPE = TEMPE + BSTAR*CC5*(DSIN(Mm) - SINMAO)
    TEMPL = TEMPL + T3COF*T3 + T4*(T4COF + T*T5COF)
                                                                        Formatted: French
   ENDIF
  XN = No
  Eccm = Ecco
  Inclm = Inclo
  IF(METHOD .EQ. 'd') THEN
    TC = T
    CALL DSPACE(IRez, D2201, D2211, D3210, D3222, D4410,
           D4422, D5220, D5232, D5421, D5433, Dedt,
                                                                        Formatted: German
                                                                         (Germany)
           Del1, Del2, Del3, Didt, Dmdt, Dnodt,
  &
           Domdt, Argpo, ArgpDot, T, TC, GSTo,
  &
  &
           Xfact, Xlamo, No,
           Atime, Eccm, Argpm, Inclm, Xli, Mm,
  &
  &
           XNi , nodem, Dndt , XN )
   ENDIF
  mean motion less than 0.0
  IF(XN .LE. 0.0D0) THEN
    Error = 2
```

```
ENDIF
   AM = (XKE/XN)**X2O3*TEMPA**2
   XN = XKE/AM**1.5D0
   Eccm = Eccm-TEMPE
c fix tolerance for error recognition
   IF (Eccm .GE. 1.0D0 .or. Eccm.lt.-0.001D0 .or. AM .lt. 0.95) THEN
       write(6,*) '# Error 1, Eccm = ', Eccm, 'AM = ', AM
c
     Error = 1
    ENDIF
c sgp4fix change test condition for eccentricity
   IF (Eccm .lt. 1.0D-6) Eccm = 1.0D-6
   Mm = Mm + No*TEMPL
   XLM = Mm + Argpm + nodem
   EMSQ = Eccm*Eccm
   TEMP = 1.0D0 - EMSQ
   nodem = DMOD(nodem, TwoPi)
   Argpm = DMOD(Argpm,TwoPi)
   XLM = DMOD(XLM,TwoPi)
   Mm = DMOD(XLM - Argpm - nodem, TwoPi)
* ----- COMPUTE EXTRA MEAN QUANTITIES ------
   SINIM = DSIN(Inclm)
   COSIM = DCOS(Inclm)
* ------ ADD LUNAR-SOLAR PERIODICS -----
   Eccp = Eccm
   XINCP = Inclm
   Argpp = Argpm
   nodep = nodem
   Mp = Mm
   SINIP = SINIM
   COSIP = COSIM
   IF(METHOD .EQ. 'd') THEN
     CALL DPPER(e3, ee2, peo, pgho, pho, pinco,
           plo , se2 , se3 , sgh2 , sgh3 , sgh4 ,
                                                                             Formatted: French
                                                                             (France)
           sh2 , sh3 , si2 , si3 , sl2 , sl3 ,
  &
  &
           sl4 , T , xgh2 , xgh3 , xgh4 , xh2 ,
           xh3 , xi2 , xi3 , xl2 , xl3 , xl4 ,
  &
  &
           zmol, zmos, Inclo, 'n',
           Eccp , XIncp , nodep, Argpp, Mp, Opsmode )
     IF(XINCP .lt. 0.0D0) THEN
      XINCP = -XINCP
      nodep = nodep + PI
       Argpp = Argpp - PI
```

```
ENDIF
    IF(Eccp .lt. 0.0D0 .OR. Eccp .GT. 1.0D0) THEN
      Error = 3
     ENDIF
    ENDIF
* ------ LONG PERIOD PERIODICS -----
  IF(METHOD .EQ. 'd') THEN
    SINIP = DSIN(XINCP)
    COSIP = DCOS(XINCP)
    AYCOF = -0.5D0*J3OJ2*SINIP
     sgp4fix for divide by zero with xincp = 180 deg
     if (dabs(cosip+1.0).gt. 1.5d-12) THEN
      XLCOF = -0.25D0*J3OJ2*SINIP*
            (3.0D0+5.0D0*COSIP)/(1.0D0+COSIP)
  &
      else
      XLCOF = -0.25D0*J3OJ2*SINIP*
  &
            (3.0D0+5.0D0*COSIP)/temp4
     ENDIF
   ENDIF
   AXNL = Eccp*DCOS(Argpp)
  TEMP = 1.0D0 / (AM*(1.0D0-Eccp*Eccp))
  AYNL = Eccp*DSIN(Argpp) + TEMP*AYCOF
  XL = Mp + Argpp + nodep + TEMP*XLCOF*AXNL
* ----- SOLVE KEPLER'S EQUATION -----
  U = DMOD(XL-nodep,TwoPi)
  EO1 = U
  ITER=0
c sgp4fix for kepler iteration
c the following iteration needs better limits on corrections
  Temp = 9999.9D0
  DO WHILE ((Temp.ge.1.0D-12).and.(ITER.lt.10))
    ITER=ITER+1
    SINEO1= DSIN(EO1)
    COSEO1= DCOS(EO1)
    TEM5 = 1.0D0 - COSEO1*AXNL - SINEO1*AYNL
    TEM5 = (U - AYNL*COSEO1 + AXNL*SINEO1 - EO1) / TEM5
    Temp = DABS(Tem5)
    IF(Temp.gt.1.0D0) Tem5=Tem5/Temp! Stop excessive correction
    EO1 = EO1 + TEM5
    ENDDO
```

* ----- SHORT PERIOD PRELIMINARY QUANTITIES -----

```
ECOSE = AXNL*COSEO1+AYNL*SINEO1
  ESINE = AXNL*SINEO1-AYNL*COSEO1
  EL2 = AXNL*AXNL+AYNL*AYNL
  PL = AM*(1.0D0-EL2)
  semi-latus rectum < 0.0
  IF (PL.lt. 0.0D0) THEN
    Error = 4
   ELSE
    RL = AM*(1.0D0-ECOSE)
    RDOTL = DSQRT(AM)*ESINE/RL
    RVDOTL= DSQRT(PL)/RL
    BETAL = DSQRT(1.0D0-EL2)
    TEMP = ESINE/(1.0D0+BETAL)
    SINU = AM/RL*(SINEO1-AYNL-AXNL*TEMP)
    COSU = AM/RL*(COSEO1-AXNL+AYNL*TEMP)
    SU = DATAN2(SINU,COSU)
                                                                        Formatted: French
                                                                        (France)
    SIN2U = (COSU + COSU) * SINU
    COS2U = 1.0D0-2.0D0*SINU*SINU
    TEMP = 1.0D0/PL
                                                                        Formatted: French
    TEMP1 = 0.5D0*J2*TEMP
    TEMP2 = TEMP1*TEMP
* ------ UPDATE FOR SHORT PERIOD PERIODICS ------
    IF(METHOD .EQ. 'd') THEN
      COSISQ = COSIP*COSIP
      CON41 = 3.0D0*COSISQ - 1.0D0
      X1MTH2 = 1.0D0 - COSISQ
      X7THM1 = 7.0D0*COSISQ - 1.0D0
     ENDIF
    mr = RL*(1.0D0 - 1.5D0*TEMP2*BETAL*CON41) +
        0.5D0*TEMP1*X1MTH2*COS2U
    SU = SU - 0.25D0*TEMP2*X7THM1*SIN2U
                                                                        Formatted: French
                                                                        (France)
    XNODE= nodep + 1.5D0*TEMP2*COSIP*SIN2U
    XINC = XINCP + 1.5D0*TEMP2*COSIP*SINIP*COS2U
    mv = RDOTL - XN*TEMP1*X1MTH2*SIN2U / XKE
    RVDOT= RVDOTL + XN*TEMP1* (X1MTH2*COS2U+1.5D0*CON41) / XKE
    ------ ORIENTATION VECTORS ------
                                                                        Formatted: French
                                                                        (France)
    SINSU= DSIN(SU)
    COSSU= DCOS(SU)
    SNOD = DSIN(XNODE)
    CNOD = DCOS(XNODE)
    SINI = DSIN(XINC)
    COSI = DCOS(XINC)
    XMX = -SNOD*COSI
```

```
XMY = CNOD*COSI
     UX = XMX*SINSU + CNOD*COSSU
     UY = XMY*SINSU + SNOD*COSSU
     UZ = SINI*SINSU
     VX = XMX*COSSU - CNOD*SINSU
     VY = XMY*COSSU - SNOD*SINSU
     VZ = SINI*COSSU
* ----- POSITION AND VELOCITY -----
     r(1) = mr*UX * RadiusEarthkm
     r(2) = mr*UY * RadiusEarthkm
     r(3) = mr*UZ * RadiusEarthkm
     v(1) = (mv*UX + RVDOT*VX) * VKmPerSec
     v(2) = (mv*UY + RVDOT*VY) * VKmPerSec
     v(3) = (mv*UZ + RVDOT*VZ) * VKmPerSec
    ENDIF
* ----- ERROR PROCESSING -----
c sgp4fix for decaying satellites
   if (mr.lt. 1.0D0) THEN
    error = 6
    ENDIF
  RETURN
  END! end sgp4
              SUBROUTINE INITL
  this subroutine initializes the spg4 propagator. all the initialization is
  consolidated here instead of having multiple loops inside other routines.
           : david vallado
                                719-573-2600 28 jun 2005
  author
  inputs
          - eccentricity
                                   0.0 - 1.0
  ecco
  epoch
           - epoch time in days from jan 0, 1950. 0 hr
          - inclination of satellite
  inclo
          - mean motion of satellite
  no
          - satellite number
  satn
  outputs
  ainv
          - 1.0 / a
          - semi major axis
  ao
```

```
con41
            -1.0 - 5.0 \cos(i)
   con42
           - cosine of inclination
   cosio
   cosio2
           - cosio squared
           - eccentricity squared
   eccsq
           - flag for deep space
                                         'd', 'n'
   method
            - 1.0 - ecco * ecco
   omeosq
           - semi-parameter squared
   posq
          - radius of perigee
   rp
           - square root of (1.0 - ecco*ecco)
   rteosq
   sinio
           - sine of inclination
   gsto
           - gst at time of observation
                                           rad
          - mean motion of satellite
   no
  coupling
   getgravconst-
 references:
  hoots, roehrich, norad spacetrack report #3 1980
  hoots, norad spacetrack report #6 1986
   hoots, schumacher and glover 2004
   vallado, crawford, hujsak, kelso 2006
   SUBROUTINE INITL(Satn, whichconst, Ecco, EPOCH, Inclo, No,
         Method, AINV, AO, CON41, CON42, COSIO, COSIO2,
  &
  &
         Eccsq, OMEOSQ, POSQ, rp, , RTEOSQ, SINIO,
  &
         GSTo, operationmode)
    IMPLICIT NONE
    CHARACTER Method, operationmode
    INTEGER Satn, which const
    REAL*8 Ecco , EPOCH , Inclo , No ,
         AINV, AO, CON41, CON42, COSIO, COSIO2,
  &
  &
         Eccsq, OMEOSQ, POSQ, rp, , RTEOSQ, SINIO, GSTo
    COMMON /DebugHelp/ Help
    CHARACTER Help
* ------ Local Variables -----
    sgp4fix use old way of finding gst
    Integer ids70
    REAL*8 ts70, ds70, tfrac, c1, thgr70, fk5r, c1p2p, thgr, thgro
    REAL*8 RadPerDay, Temp, TUT1
    REAL*8 ak, d1, del, adel, po
```

```
* ------ WGS-72 EARTH CONSTANTS ------
   X2o3 = 2.0D0/3.0D0
   ! sgp4fix identify constants and allow alternate values
   CALL getgravconst( which const, tumin, mu, radiusearthkm, xke,
      j2, j3, j4, j3oj2)
  &
* ------ CALCULATE AUXILLARY EPOCH QUANTITIES ------
   Eccsq = Ecco*Ecco
   OMEOSQ = 1.0D0 - Eccsq
   RTEOSQ = DSQRT(OMEOSQ)
   COSIO = DCOS(Inclo)
   COSIO2 = COSIO*COSIO
* ------ UN-KOZAI THE MEAN MOTION ------
   AK = (XKE/N_0)**X2O3
   D1 = 0.75D0*J2*(3.0D0*COSIO2-1.0D0)/(RTEOSQ*OMEOSQ)
   DEL = D1/(AK*AK)
                                                                            Formatted: German
                                                                             (Germany)
   ADEL = AK * (1.0D0 - DEL*DEL - DEL*
            (1.0D0/3.0D0 + 134.0D0*DEL*DEL / 81.0D0))
   DEL = D1/(ADEL*ADEL)
   No = No/(1.0D0 + DEL)
   AO = (XKE/N_0)**X2O3
   SINIO= DSIN(Inclo)
   PO = AO*OMEOSQ
   CON42= 1.0D0-5.0D0*COSIO2
   CON41= -CON42-COSIO2-COSIO2
   AINV = 1.0D0/AO
                                                                            Formatted: French
                                                                             (France)
   POSQ = PO*PO
   rp = AO*(1.0D0-Ecco)
   METHOD = 'n'
* ----- CALCULATE GREENWICH LOCATION AT EPOCH -----
    sgp4fix modern approach to finding sidereal time
   IF (operationmode .ne. 'a') THEN
     RadPerDay = twopi * 1.002737909350795D0 !6.30038809866574D0
     Temp = Epoch + 2433281.5D0
                                                                            Formatted: German
     TUT1 = (DINT(Temp-0.5D0) + 0.5D0 - 2451545.0D0) / 36525.0D0
     Gsto= 1.75336855923327D0 + 628.331970688841D0*TUT1
         + 6.77071394490334D-06*TUT1*TUT1
                                                                            Formatted: French
                                                                             (France)
          - 4.50876723431868D-10*TUT1*TUT1*TUT1
```

+ RadPerDay*(Temp-0.5D0-DINT(Temp-0.5D0))

REAL*8 X2o3, J2, XKE, tumin, mu, radiusearthkm, j3, j4, j3oj2

INCLUDE 'ASTMATH.CMN'

&

ELSE

```
! sgp4fix use old way of finding gst
    ! count integer number of days from 0 jan 1970
    TS70 = EPOCH-7305.0D0
   IDS70 = TS70 + 1.0D-8
   TFRAC = TS70-IDS70
    ! find greenwich location at epoch
   C1 = 1.72027916940703639D-2
    THGR70= 1.7321343856509374D0
    FK5R = 5.07551419432269442D-15
   C1P2P = C1+TWOPI
   gsto = THGR70+C1*IDS70+C1P2P*TFRAC+TS70*TS70*FK5R
  ENDIF
  ! ----- Check quadrants -----
  Gsto = DMOD( Gsto, TwoPi )
  IF (Gsto.lt. 0.0D0) THEN
    Gsto=Gsto + TwoPi
   ENDIF
RETURN
END! end initl
             SUBROUTINE DPPER
This Subroutine provides deep space long period periodic contributions
to the mean elements. by design, these periodics are zero at epoch.
this used to be dscom which included initialization, but it's really a
recurring function.
author
         : david vallado
                                719-573-2600 28 jun 2005
inputs
e3
ee2
peo
pgho
pho
                                                                                   Formatted: French
pinco
plo
se2, se3, Sgh2, Sgh3, Sgh4, Sh2, Sh3, Si2, Si3, Sl2, Sl3, Sl4 -
                                                                                   Formatted: German
                                                                                    (Germany)
xh2, xh3, xi2, xi3, xl2, xl3, xl4 -
zmol
zmos
```

```
- eccentricity
                                    0.0 - 1.0
 ep
         - inclination - needed for lyddane modification
 inclo
 nodep
          - right ascension of ascending node
 argpp
          - argument of perigee
         - mean anomaly
 mp
outputs
                                    0.0 - 1.0
         - eccentricity
 ep
 inclp
         - inclination
          - right ascension of ascending node
 nodep
          - argument of perigee
 argpp
 mp
         - mean anomaly
coupling
 none.
references:
 hoots, roehrich, norad spacetrack report #3 1980
 hoots, norad spacetrack report #6 1986
 hoots, schumacher and glover 2004
 vallado, crawford, hujsak, kelso 2006
 SUBROUTINE\ DPPER(\ e3\quad,\ ee2\quad,\ peo\quad,\ pgho\ \ ,\ pho\quad,\ pinco\ ,
            plo , se2 , se3 , sgh2 , sgh3 , sgh4 ,
                                                                                       Formatted: French
            sh2 , sh3 , si2 , si3 , sl2 , sl3
                                                                                       (France)
&
            sl4 \ , T \ , xgh2 \ , xgh3 \ , xgh4 \ , xh2 \ ,
&
&
            xh3 , xi2 , xi3 , xl2 , xl3 , xl4 ,
&
            zmol, zmos, inclo, init,
            Eccp , Inclp , nodep, Argpp , Mp,
&
            operationmode)
&
  IMPLICIT NONE
  CHARACTER Init, operationmode
  REAL*8 e3 , ee2 , peo , pgho , pho , pinco , plo ,
        se2, se3, sgh2, sgh3, sgh4, sh2, sh3,
                                                                                       Formatted: French
                                                                                       (France)
        si2 , si3 , sl2 , sl3 , sl4 , T , xgh2 ,
&
&
        xgh3 , xgh4 , xh2 , xh3 , xi2 , xi3 , xl2 ,
        xl3, xl4, zmol, zmos, inclo,
&
        Eccp , Inclp , nodep, Argpp , Mp
  ----- Local Variables
  REAL*8 alfdp, betdp, cosip, cosop, dalf, dbet, dls,
&
        f2, f3, pe, pgh, ph, pinc, pl,
&
        sel, ses, sghl, sghs, shl, shs, sil,
&
        sinip, sinop, sinzf, sis, sll, sls, xls,
&
        xnoh, zf, zm
```

REAL*8 Zel , Zes , Znl , Zns !COMMON /DebugHelp/ Help CHARACTER Help INCLUDE 'ASTMATH.CMN'

```
------ Constants -----
                                                                            Formatted: German
                                                                             (Germany)
   ZES = 0.01675D0
   ZEL = 0.05490D0
   ZNS = 1.19459D-5
   ZNL = 1.5835218D-4
* ------ CALCULATE TIME VARYING PERIODICS ------
   ZM = ZMOS + ZNS*T
   IF (Init.eq.'y') ZM = ZMOS
   ZF = ZM + 2.0D0*ZES*DSIN(ZM)
   SINZF = DSIN(ZF)
                                                                            Formatted: German
                                                                             (Germany)
   F2 = 0.5D0*SINZF*SINZF - 0.25D0
   F3 = -0.5D0*SINZF*DCOS(ZF)
   SES = SE2*F2 + SE3*F3
                                                                            Formatted: French
   SIS = SI2*F2 + SI3*F3
   SLS = SL2*F2 + SL3*F3 + SL4*SINZF
   SGHS = SGH2*F2 + SGH3*F3 + SGH4*SINZF
   SHS = SH2*F2 + SH3*F3
   ZM = ZMOL + ZNL*T
   IF (Init.eq.'y') ZM = ZMOL
   ZF = ZM + 2.0D0*ZEL*DSIN(ZM)
                                                                            Formatted: German
   SINZF= DSIN(ZF)
   F2 = 0.5D0*SINZF*SINZF - 0.25D0
   F3 = -0.5D0*SINZF*DCOS(ZF)
   SEL = EE2*F2 + E3*F3
   SIL = XI2*F2 + XI3*F3
   SLL = XL2*F2 + XL3*F3 + XL4*SINZF
   SGHL = XGH2*F2 + XGH3*F3 + XGH4*SINZF
   SHL = XH2*F2 + XH3*F3
                                                                            Formatted: French
                                                                             (France)
   PE = SES + SEL
   PINC = SIS + SIL
   PL = SLS + SLL
   PGH = SGHS + SGHL
   PH = SHS + SHL
   IF (Init.eq.'n') THEN
     PE = PE - PEO
     PINC = PINC - PINCO
     PL = PL - PLO
```

```
PGH = PGH - PGHO
      PH = PH - PHO
      Inclp = Inclp + PINC
      Eccp = Eccp + PE
      SINIP = DSIN(Inclp)
      COSIP = DCOS(Inclp)
* ------ APPLY PERIODICS DIRECTLY ------
c sgp4fix for lyddane choice
c strn3 used original inclination - this is technically feasible
c gsfc used perturbed inclination - also technically feasible
c probably best to readjust the 0.2 limit value and limit discontinuity
c = 0.2 \text{ rad} = 11.45916 \text{ deg}
c use next line for original strn3 approach and original inclination
       IF (inclo.ge.0.2D0) THEN
c
  use next line for gsfc version and perturbed inclination
      IF (Inclp.ge.0.2D0) THEN
        PH = PH/SINIP
        PGH = PGH - COSIP*PH
        Argpp = Argpp + PGH
        nodep = nodep + PH
        Mp = Mp + PL
       ELSE
* ----- APPLY PERIODICS WITH LYDDANE MODIFICATION ------
        SINOP = DSIN(nodep)
        COSOP = DCOS(nodep)
        ALFDP = SINIP*SINOP
        BETDP = SINIP*COSOP
        DALF = PH*COSOP + PINC*COSIP*SINOP
        DBET = -PH*SINOP + PINC*COSIP*COSOP
        ALFDP = ALFDP + DALF
        BETDP = BETDP + DBET
        nodep = DMOD(nodep, TwoPi)
        ! sgp4fix for afspc written intrinsic functions
        ! nodep used without a trigonometric function ahead
        IF ((nodep .LT. 0.0D0) .and. (operationmode .eq. 'a'))
  &
             THEN
          nodep = nodep + twopi
         ENDIF
        XLS = Mp + Argpp + COSIP*nodep
        DLS = PL + PGH - PINC*nodep*SINIP
        XLS = XLS + DLS
        XNOH = nodep
```

```
nodep = DATAN2(ALFDP, BETDP)
       ! sgp4fix for afspc written intrinsic functions
       ! nodep used without a trigonometric function ahead
       IF ((nodep .LT. 0.0D0) .and. (operationmode .eq. 'a'))
&
           THEN
         nodep = nodep + twopi
        ENDIF
       IF (DABS(XNOH-nodep) .GT. PI) THEN
         IF(nodep .lt. XNOH) THEN
           nodep = nodep+TWOPI
          ELSE
           nodep = nodep-TWOPI
          ENDIF
        ENDIF
       Mp = Mp + PL
       Argpp= XLS - Mp - COSIP*nodep
     ENDIF
   ENDIF
 RETURN
 END! end dpper
              SUBROUTINE DSCOM
This Subroutine provides deep space common items used by both the secular
 and periodics subroutines. input is provided as shown, this routine
 used to be called dpper, but the functions inside weren't well organized.
          : david vallado
                                  719-573-2600 28 jun 2005
author
inputs
 epoch
         - eccentricity
 ep
          - argument of perigee
 argpp
 tc
 inclp
         - inclination
          - right ascension of ascending node
 nodep
         - mean motion
outputs
```

```
sinim, cosim, sinomm, cosomm, snodm, cnodm
 day
 e3
 ee2
         - eccentricity
 em
 emsq
          - eccentricity squared
 gam
 peo
 pgho
 pho
 pinco
 plo
                                                                                      Formatted: French
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rtemsq
 se2, se3
 sgh2, sgh3, sgh4
 sh2, sh3, si2, si3, sl2, sl3, sl4
 s1, s2, s3, s4, s5, s6, s7
 ss1, ss2, ss3, ss4, ss5, ss6, ss7, sz1, sz2, sz3
 sz11, sz12, sz13, sz21, sz22, sz23, sz31, sz32, sz33
 xgh2, xgh3, xgh4, xh2, xh3, xi2, xi3, xl2, xl3, xl4
 nm - mean motion
                                                                                      Formatted: German
                                                                                      (Germany)
 z1, z2, z3, z11, z12, z13, z21, z22, z23, z31, z32, z33
 zmol
 zmos
coupling
 none.
references:
 hoots, roehrich, norad spacetrack report #3 1980
 hoots, norad spacetrack report #6 1986
hoots, schumacher and glover 2004
 vallado, crawford, hujsak, kelso 2006
 SUBROUTINE DSCOM( EPOCH , Eccp , Argpp , Tc , Inclp , nodep,
&
&
            SNODM, CNODM, SINIM, COSIM, SINOMM, COSOMM,
            DAY, E3, Ee2, Eccm, EMSQ, GAM,
&
&
            Peo , Pgho , Pho , PInco , Plo
            RTemSq, Se2 , Se3 , Sgh2 , Sgh3 , Sgh4
&
                                                                                      Formatted: French
                                                                                      (France)
            Sh2 , Sh3 , Si2 , Si3 , Sl2 , Sl3 ,
&
&
            S14 , S1 , S2 , S3 , S4 , S5
&
            S6 , S7 , SS1 , SS2 , SS3 , SS4
&
            SS5 , SS6 , SS7 , SZ1 , SZ2 , SZ3 ,
```

```
SZ11 , SZ12 , SZ13 , SZ21 , SZ22 , SZ23
                                                                              Formatted: German
                                                                              (Germany)
  &
             SZ31 , SZ32 , SZ33 , Xgh2 , Xgh3 , Xgh4 ,
  &
             Xh2 , Xh3 , Xi2 , Xi3 , Xl2 , Xl3 ,
             X14 , Xn , Z1 , Z2 , Z3 , Z11
  &
             Z12 , Z13 , Z21 , Z22 , Z23 , Z31 ,
  &
  &
             Z32 , Z33 , Zmol , Zmos )
   IMPLICIT NONE
   REAL*8 EPOCH, Eccp, Argpp, Tc, Inclp, nodep, Np
         SNODM, CNODM, SINIM, COSIM, SINOMM, COSOMM, DAY,
  &
        E3 , Ee2 , Eccm , EMSQ , GAM , RTemSq, Se2 ,
  &
        Peo , Pgho , Pho , PInco , Plo ,
  &
        Se3 , Sgh2 , Sgh3 , Sgh4 , Sh2 , Sh3 , Si2 ,
  &
         Si3 , Sl2 , Sl3 , Sl4 , Sl , S2 , S3
         S4 , S5 , S6 , S7 , SS1 , SS2 , SS3
  &
  &
        SS4 , SS5 , SS6 , SS7 , SZ1 , SZ2 , SZ3
  &
        SZ11 , SZ12 , SZ13 , SZ21 , SZ22 , SZ23 , SZ31 ,
  &
        SZ32, SZ33, Xgh2, Xgh3, Xgh4, Xh2, Xh3,
        Xi2 , Xi3 , Xl2 , Xl3 , Xl4 , Xn , Z1
  &
                                                                              Formatted: German
                                                                              (Germany)
  &
        Z2 , Z3 , Z11 , Z12 , Z13 , Z21 , Z22
  &
        Z23 , Z31 , Z32 , Z33 , Zmol , Zmos
        ----- Local Variables -----
   REAL*8 c1ss, c1L, zcosis, zsinis, zsings, zcosgs,
  &
        Zes, zel
   INTEGER LsFlg
   REAL*8 a1 , a2 , a3 , a4 , a5 , a6 , a7 ,
        a8, a9, a10, betasq, cc, ctem, stem,
  &
        x1 , x2 , x3 , x4 , x5 , x6 , x7
  &
  &
        x8 , xnodce, xnoi , zcosg , zcosgl, zcosh , zcoshl,
  &
         zcosi, zcosil, zsing, zsingl, zsinh, zsinhl, zsini,
  &
        zsinil, zx , zy
   CHARACTER Help
   INCLUDE 'ASTMATH.CMN'
* ----- Constants -----
                                                                              Formatted: German
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   ZES = 0.01675D0
   ZEL = 0.05490D0
   C1SS = 2.9864797D-6
   C1L = 4.7968065D-7
   ZSINIS = 0.39785416D0
   ZCOSIS = 0.91744867D0
   ZCOSGS = 0.1945905D0
   ZSINGS = -0.98088458D0
* ------ DEEP SPACE PERIODICS INITIALIZATION ------
```

```
XN = Np
   Eccm = Eccp
   SNODM = DSIN(nodep)
   CNODM = DCOS(nodep)
   SINOMM = DSIN(Argpp)
   COSOMM = DCOS(Argpp)
   SINIM = DSIN(Inclp)
   COSIM = DCOS(Inclp)
   EMSQ = Eccm*Eccm
   BETASQ = 1.0D0-EMSQ
   RTEMSQ = DSQRT(BETASQ)
* ----- INITIALIZE LUNAR SOLAR TERMS -----
   PEO = 0.0D0
   PINCO = 0.0D0
   PLO = 0.0D0
   PGHO = 0.0D0
   PHO = 0.0D0
   DAY = EPOCH + 18261.5D0 + TC/1440.0D0
   XNODCE = DMOD(4.5236020D0 - 9.2422029D-4*DAY,TwoPi)
   STEM = DSIN(XNODCE)
   CTEM = DCOS(XNODCE)
   ZCOSIL = 0.91375164D0 - 0.03568096D0*CTEM
   ZSINIL = DSQRT(1.0D0 - ZCOSIL*ZCOSIL)
   ZSINHL = 0.089683511D0*STEM / ZSINIL
   ZCOSHL = DSQRT(1.0D0 - ZSINHL*ZSINHL)
   GAM = 5.8351514D0 + 0.0019443680D0*DAY
   ZX = 0.39785416D0*STEM/ZSINIL
   ZY = ZCOSHL*CTEM + 0.91744867D0*ZSINHL*STEM
                                                                      Formatted: German
   ZX = DATAN2(ZX,ZY)
                                                                      (Germany)
   ZX = GAM + ZX - XNODCE
   ZCOSGL = DCOS(ZX)
   ZSINGL = DSIN(ZX)
* ------ DO SOLAR TERMS ------
   ZCOSG = ZCOSGS
   ZSING = ZSINGS
   ZCOSI = ZCOSIS
   ZSINI = ZSINIS
   ZCOSH = CNODM
   ZSINH = SNODM
   CC = C1SS
   XNOI = 1.0D0 / XN
   DO LSFlg = 1.2
```

A1 = ZCOSG*ZCOSH + ZSING*ZCOSI*ZSINH

```
A3 = -ZSING*ZCOSH + ZCOSG*ZCOSI*ZSINH
     A7 = -ZCOSG*ZSINH + ZSING*ZCOSI*ZCOSH
     A8 = ZSING*ZSINI
     A9 = ZSING*ZSINH + ZCOSG*ZCOSI*ZCOSH
     A10= ZCOSG*ZSINI
     A2 = COSIM*A7 + SINIM*A8
     A4 = COSIM*A9 + SINIM*A10
     A5 = -SINIM*A7 + COSIM*A8
     A6 = -SINIM*A9 + COSIM*A10
     X1 = A1*COSOMM + A2*SINOMM
     X2 = A3*COSOMM + A4*SINOMM
     X3 = -A1*SINOMM + A2*COSOMM
     X4 = -A3*SINOMM + A4*COSOMM
     X5 = A5*SINOMM
     X6 = A6*SINOMM
     X7 = A5*COSOMM
     X8 = A6*COSOMM
     Z31 = 12.0D0*X1*X1 - 3.0D0*X3*X3
     Z32 = 24.0D0*X1*X2 - 6.0D0*X3*X4
     Z33= 12.0D0*X2*X2 - 3.0D0*X4*X4
     Z1 = 3.0D0* (A1*A1 + A2*A2) + Z31*EMSQ
     Z2 = 6.0D0* (A1*A3 + A2*A4) + Z32*EMSQ
     Z3 = 3.0D0* (A3*A3 + A4*A4) + Z33*EMSQ
     Z11 = -6.0D0*A1*A5 + EMSQ*(-24.0D0*X1*X7-6.0D0*X3*X5)
     Z12 = -6.0D0* (A1*A6 + A3*A5) + EMSQ*
  &
         (-24.0D0*(X2*X7+X1*X8) - 6.0D0*(X3*X6+X4*X5))
     Z13 = -6.0D0*A3*A6 + EMSQ*(-24.0D0*X2*X8 - 6.0D0*X4*X6)
     Z21 = 6.0D0*A2*A5 + EMSQ*(24.0D0*X1*X5-6.0D0*X3*X7)
     Z22= 6.0D0* (A4*A5 + A2*A6) + EMSQ*
        ( 24.0D0*(X2*X5+X1*X6) - 6.0D0*(X4*X7+X3*X8) )
     Z23 = 6.0D0*A4*A6 + EMSQ*(24.0D0*X2*X6 - 6.0D0*X4*X8)
     Z1 = Z1 + Z1 + BETASQ*Z31
     Z2 = Z2 + Z2 + BETASQ*Z32
     Z3 = Z3 + Z3 + BETASQ*Z33
     S3 = CC*XNOI
     S2 = -0.5D0*S3 / RTEMSQ
     S4 = S3*RTEMSQ
     S1 = -15.0D0*Eccm*S4
     S5 = X1*X3 + X2*X4
     S6 = X2*X3 + X1*X4
     S7 = X2*X4 - X1*X3
* ------ DO LUNAR TERMS ------
     IF (LSFLG.eq.1) THEN
```

```
SS1 = S1
       SS2 = S2
       SS3 = S3
       SS4 = S4
       SS5 = S5
       SS6 = S6
       SS7 = S7
       SZ1 = Z1
       SZ2 = Z2
       SZ3 = Z3
                                                                          Formatted: German
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       SZ11 = Z11
       SZ12 = Z12
       SZ13 = Z13
       SZ21 = Z21
       SZ22 = Z22
       SZ23 = Z23
       SZ31 = Z31
       SZ32 = Z32
       SZ33 = Z33
       ZCOSG = ZCOSGL
       ZSING = ZSINGL
       ZCOSI = ZCOSIL
       ZSINI = ZSINIL
       ZCOSH = ZCOSHL*CNODM+ZSINHL*SNODM
       ZSINH = SNODM*ZCOSHL-CNODM*ZSINHL
       CC = C1L
      ENDIF
    ENDDO
   ZMOL = DMOD(4.7199672D0 + 0.22997150D0*DAY-GAM,TwoPi)
   ZMOS = DMOD(6.2565837D0 + 0.017201977D0*DAY,TwoPi)
* ------ DO SOLAR TERMS ------
   SE2 = 2.0D0*SS1*SS6
   SE3 = 2.0D0*SS1*SS7
                                                                          Formatted: French
                                                                          (France)
   SI2 = 2.0D0*SS2*SZ12
   SI3 = 2.0D0*SS2*(SZ13-SZ11)
   SL2 = -2.0D0*SS3*SZ2
   SL3 = -2.0D0*SS3*(SZ3-SZ1)
   SL4 = -2.0D0*SS3*(-21.0D0-9.0D0*EMSQ)*ZES
   SGH2= 2.0D0*SS4*SZ32
   SGH3= 2.0D0*SS4*(SZ33-SZ31)
   SGH4= -18.0D0*SS4*ZES
   SH2 = -2.0D0*SS2*SZ22
   SH3 = -2.0D0*SS2*(SZ23-SZ21)
```

```
* ------ DO LUNAR TERMS ------
    EE2 = 2.0D0*S1*S6
    E3 = 2.0D0*S1*S7
    XI2 = 2.0D0*S2*Z12
    XI3 = 2.0D0*S2*(Z13-Z11)
    XL2 = -2.0D0*S3*Z2
    XL3 = -2.0D0*S3*(Z3-Z1)
    XL4 = -2.0D0*S3*(-21.0D0-9.0D0*EMSQ)*ZEL
    XGH2= 2.0D0*S4*Z32
    XGH3= 2.0D0*S4*(Z33-Z31)
    XGH4= -18.0D0*S4*ZEL
    XH2 = -2.0D0*S2*Z22
    XH3 = -2.0D0*S2*(Z23-Z21)
  RETURN
  END! dscom
               SUBROUTINE DSINIT
 This Subroutine provides Deep Space contributions to Mean Motion Dot due
  to geopotential resonance with half day and one day orbits.
  Inputs
  Cosim, Sinim-
           - Eccentricity squared
  Emsq
  Argpo
           - Argument of Perigee
  S1, S2, S3, S4, S5
  Ss1, Ss2, Ss3, Ss4, Ss5 -
  Sz1, Sz3, Sz11, Sz13, Sz21, Sz23, Sz31, Sz33 -
  T
          - Time
  Tc
  GSTo
            - Greenwich sidereal time
                                            rad
  Mo
           - Mean Anomaly
            - Mean Anomaly dot (rate)
  MDot
  No
          - Mean Motion
           - right ascension of ascending node
  nodeo
  nodeDot - right ascension of ascending node dot (rate)
  XPIDOT
  Z1, Z3, Z11, Z13, Z21, Z23, Z31, Z33 -
           - Eccentricity
  Eccm
   Argpm
            - Argument of perigee
  Inclm
           - Inclination
```

```
Mm
          - Mean Anomaly
 Xn
         - Mean Motion
           - right ascension of ascending node
 nodem
Outputs
          - Eccentricity
 Eccm
 Argpm
           - Argument of perigee
          - Inclination
 Inclm
 Mm
          - Mean Anomaly
         - Mean motion
 Xn
           - right ascension of ascending node
 nodem
         - Resonance flags
                                  0-none, 1-One day, 2-Half day
 IRez
 Atime
                                                                                      Formatted: German
                                                                                       (Germany)
 D2201, D2211, D3210, D3222, D4410, D4422, D5220, D5232, D5421, D5433
 Dedt
 Didt
DMDT
 DNDT
 DNODT
 DOMDT
Del1, Del2, Del3 -
 Ses, Sghl, Sghs, Sgs, Shl, Shs, Sis, Sls
 THETA
                                                                                      Formatted: French
                                                                                       (France)
 Xfact
 Xlamo
 Xli
 Xni
Locals
 ainv2
 aonv
 cosisq
 eoc
 f220, f221, f311, f321, f322, f330, f441, f442, f522, f523, f542, f543
 g200, g201, g211, g300, g310, g322, g410, g422, g520, g521, g532, g533
 sini2
 temp, temp1 -
 Theta
 xno2
Coupling
 getgravconst-
references:
 hoots, roehrich, norad spacetrack report #3 1980
 hoots, norad spacetrack report #6 1986
```

```
hoots, schumacher and glover 2004
vallado, crawford, hujsak, kelso 2006
SUBROUTINE DSINIT( which const,
           Cosim, Emsq, Argpo, S1, S2, S3,
&
&
           S4 , S5 , Sinim , Ss1 , Ss2 , Ss3 ,
&
           Ss4 , Ss5 , Sz1 , Sz3 , Sz11 , Sz13 ,
           Sz21 , Sz23 , Sz31 , Sz33 , T , Tc
&
                                                                             Formatted: German
                                                                              (Germany)
&
           GSTo, Mo, MDot, No, nodeo, nodeDot,
&
           XPIDOT, Z1 , Z3 , Z11 , Z13 , Z21 ,
           Z23 , Z31 , Z33 , Ecco , EccSq ,
&
&
           Eccm, Argpm, Inclm, Mm, Xn, nodem,
&
           IREZ, Atime, D2201, D2211, D3210, D3222
                                                                             Formatted: French
           D4410, D4422, D5220, D5232, D5421, D5433
&
                                                                             Formatted: German
           Dedt, Didt, DMDT, DNDT, DNODT, DOMDT,
&
                                                                             (Germany)
&
           Del1, Del2, Del3, Xfact, Xlamo, Xli,
&
           Xni)
  IMPLICIT NONE
  INTEGER IRez, which const
  REAL*8 Cosim, Emsq, Argpo, S1, S2, S3, S4,
       S5 , Sinim , Ss1 , Ss2 , Ss3 , Ss4 , Ss5
&
       Sz1 , Sz3 , Sz11 , Sz13 , Sz21 , Sz23 , Sz31 ,
&
                , Tc , GSTo , Mo , MDot , No .
&
       Sz33, T
       nodeo, nodeDot, XPIDOT, Z1, Z3, Z11, Z13,
&
&
       Z21 , Z23 , Z31 , Z33 , Eccm , Argpm , Inclm ,
&
       Mm , Xn , nodem , Atime , D2201 , D2211 , D3210
                                                                             Formatted: German
                                                                             (Germany)
&
       D3222, D4410, D4422, D5220, D5232, D5421, D5433.
       Dedt, Didt, DMDT, DNDT, DNODT, DOMDT, Del1,
&
       Del2, Del3, Xfact, Xlamo, Xli, Xni, Ecco,
&
&
       Eccsq
----- Local Variables
 REAL*8 ainv2, aonv, cosisq, eoc, f220, f221, f311,
       f321 , f322 , f330 , f441 , f442 , f522 , f523
&
```

- & f542, f543, g200, g201, g211, g300, g310
- & g322, g410, g422, g520, g521, g532, g533,
- & ses, sgs, sghl, sghs, shs, shl, sis,
- & sini2, sls, temp, temp1, Theta, xno2

REAL*8 Q22 , Q31 , Q33 , ROOT22, ROOT44, ROOT54,

- & RPTim, Root32, Root52, X2o3, XKe, Znl,
- & Zns, Emo, emsqo, tumin, mu, radiusearthkm, j2, j3, j4,
- & j3oj2

CHARACTER Help

INCLUDE 'ASTMATH.CMN'

```
Q22 = 1.7891679D-6
    Q31 = 2.1460748D-6
    Q33 = 2.2123015D-7
    ROOT22 = 1.7891679D-6
    ROOT44 = 7.3636953D-9
    ROOT54 = 2.1765803D-9
    RPTim = 4.37526908801129966D-3! this equates to 7.29211514668855e-5 rad/sec
    Root32 = 3.7393792D-7
    Root52 = 1.1428639D-7
    X2o3 = 2.0D0 / 3.0D0
    ZNL = 1.5835218D-4
    ZNS = 1.19459D-5
    ! sgp4fix identify constants and allow alternate values
    CALL getgravconst( which const, tumin, mu, radiusearthkm, xke,
     j2, j3, j4, j3oj2)
* ------ DEEP SPACE INITIALIZATION ------
    IF ((XN.lt.0.0052359877D0).AND.(XN.GT.0.0034906585D0)) THEN
     IREZ = 1
    ENDIF
    IF ((XN.ge.8.26D-3).AND.(XN.LE.9.24D-3).AND.(Eccm.GE.0.5D0))THEN
     IREZ = 2
    ENDIF
* ------ DO SOLAR TERMS ------
   SES = SS1*ZNS*SS5
                                                                             Formatted: French
                                                                              (France)
   SIS = SS2*ZNS*(SZ11 + SZ13)
   SLS = -ZNS*SS3*(SZ1 + SZ3 - 14.0D0 - 6.0D0*EMSQ)
    SGHS = SS4*ZNS*(SZ31 + SZ33 - 6.0D0)
    SHS = -ZNS*SS2*(SZ21 + SZ23)
    sgp4fix for 180 deg incl
c
    IF ((Inclm.lt.5.2359877D-2).or.(Inclm.gt.pi-5.2359877D-2)) THEN
     SHS = 0.0D0
     ENDIF
    IF (SINIM.ne.0.0D0) THEN
     SHS = SHS/SINIM
    ENDIF
    SGS = SGHS - COSIM*SHS
* ----- DO LUNAR TERMS ------
   DEDT = SES + S1*ZNL*S5
   DIDT = SIS + S2*ZNL*(Z11 + Z13)
```

```
DMDT = SLS - ZNL*S3*(Z1 + Z3 - 14.0D0 - 6.0D0*EMSQ)
   SGHL = S4*ZNL*(Z31 + Z33 - 6.0D0)
   SHL = -ZNL*S2*(Z21 + Z23)
   sgp4fix for 180 deg incl
   IF ((Inclm.lt.5.2359877D-2).or.(Inclm.gt.pi-5.2359877D-2)) THEN
     SHL = 0.0D0
    ENDIF
   DOMDT= SGS+SGHL
   DNODT= SHS
   IF (SINIM .ne. 0.0D0) THEN
     DOMDT = DOMDT-COSIM/SINIM*SHL
     DNODT = DNODT+SHL/SINIM
   ENDIF
* ------ CALCULATE DEEP SPACE RESONANCE EFFECTS ------
   DNDT = 0.0D0
   THETA = DMOD(GSTo + TC*RPTIM,TwoPi)
   Eccm = Eccm + DEDT*T
   emsq = eccm**2
   Inclm = Inclm + DIDT*T
                                                                            Formatted: German
                                                                            (Germany)
   Argpm = Argpm + DOMDT*T
   nodem = nodem + DNODT*T
   Mm = Mm + DMDT*T
 ----- Initialize the resonance terms -----
   IF (IREZ .ne. 0) THEN
     AONV = (XN/XKE)**X2O3
* ----- GEOPOTENTIAL RESONANCE FOR 12 HOUR ORBITS ------
   IF (IREZ .eq. 2) THEN
     COSISQ = COSIM*COSIM
     emo = Eccm
     emsqo = emsq
     Eccm = ecco
     emsq = eccsq
     EOC = Eccm*EMSQ
     G201 = -0.306D0-(Eccm-0.64D0)*0.440D0
     IF (Eccm.le.0.65D0) THEN
       G211 = 3.616D0 - 13.2470D0*Eccm + 16.2900D0*EMSO
       G310 = -19.302D0 + 117.3900D0*Eccm - 228.4190D0*EMSQ +
  &
            156.591D0*EOC
       G322 = -18.9068D0 + 109.7927D0 * Eccm - 214.6334D0 * EMSQ +
  &
            146.5816D0*EOC
```

G410 = -41.122D0 + 242.6940D0*Eccm - 471.0940D0*EMSQ +

```
&
          313.953D0*EOC
     G422 = -146.407D0 + 841.8800D0*Eccm - 1629.014D0*EMSQ +
&
         1083.435D0*EOC
     G520 = -532.114D0 + 3017.977D0*Eccm - 5740.032D0*EMSQ +
         3708.276D0*EOC
&
    ELSE
     G211 = -72.099D0 + 331.819D0*Eccm - 508.738D0*EMSO +
&
          266.724D0*EOC
     G310 = -346.844D0 + 1582.851D0*Eccm - 2415.925D0*EMSQ +
&
          1246.113D0*EOC
     G322 = -342.585D0 + 1554.908D0*Eccm - 2366.899D0*EMSQ +
&
          1215.972D0*EOC
     G410 = -1052.797D0 + 4758.686D0*Eccm - 7193.992D0*EMSO +
&
          3651.957D0*EOC
     G422 = -3581.690D0 + 16178.11D0*Eccm - 24462.77D0*EMSQ +
&
         12422.52D0*EOC
     IF (Eccm.gt.0.715D0) THEN
       G520 = -5149.66D0 + 29936.92D0*Eccm -54087.36D0*EMSQ
&
           + 31324.56D0*EOC
       G520 = 1464.74D0 - 4664.75D0*Eccm + 3763.64D0*EMSQ
      ENDIF
    ENDIF
   IF (Eccm.lt.0.7D0) THEN
     G533 = -919.22770D0 + 4988.6100D0*Eccm-9064.7700D0*EMSQ
&
         + 5542.21D0*EOC
     G521 = -822.71072D0 + 4568.6173D0*Eccm-8491.4146D0*EMSQ
&
         + 5337.524D0*EOC
     G532 = -853.66600D0 + 4690.2500D0*Eccm-8624.7700D0*EMSQ
&
        + 5341.4D0*EOC
    ELSE
     G533 =-37995.780D0 + 161616.52D0*Eccm-229838.20D0*EMSQ+
&
        109377.94D0*EOC
     G521 =-51752.104D0 + 218913.95D0*Eccm-309468.16D0*EMSQ+
&
        146349.42D0*EOC
     G532 = -40023.880D0 + 170470.89D0*Eccm-242699.48D0*EMSQ+
&
        115605.82D0*EOC
    ENDIF
   SINI2 = SINIM*SINIM
   F220 = 0.75D0*(1.0D0+2.0D0*COSIM+COSISQ)
   F221 = 1.5D0*SINI2
   F321 = 1.875D0*SINIM * (1.0D0-2.0D0*COSIM-3.0D0*COSISQ)
   F322 = -1.875D0*SINIM * (1.0D0+2.0D0*COSIM-3.0D0*COSISQ)
   F441 = 35.0D0*SINI2*F220
                                                                            Formatted: French
                                                                             (France)
   F442 = 39.3750D0*SINI2*SINI2
   F522 = 9.84375D0*SINIM * (SINI2* (1.0D0-2.0D0*COSIM-
```

```
&
          5.0D0*COSISO)+0.33333333D0 * (-2.0D0+4.0D0*COSIM+
  &
          6.0D0*COSISO))
     F523 = SINIM * (4.92187512D0*SINI2 * (-2.0D0-4.0D0*COSIM+
          10.0D0*COSISQ) + 6.56250012D0*
  &
          (1.0D0+2.0D0*COSIM-3.0D0*COSISQ))
  &
     F542 = 29.53125D0*SINIM*(2.0D0-8.0D0*COSIM+COSISQ*)
  &
          (-12.0D0+8.0D0*COSIM+10.0D0*COSISO))
     F543 = 29.53125D0*SINIM * (-2.0D0-8.0D0*COSIM+COSISO*
          (12.0D0+8.0D0*COSIM-10.0D0*COSISQ))
  &
     XNO2 = XN * XN
     AINV2 = AONV * AONV
     TEMP1 = 3.0D0*XNO2*AINV2
     TEMP = TEMP1*ROOT22
     D2201 = TEMP*F220*G201
     D2211 = TEMP*F221*G211
     TEMP1 = TEMP1*AONV
     TEMP = TEMP1*ROOT32
     D3210 = TEMP*F321*G310
     D3222 = TEMP*F322*G322
     TEMP1 = TEMP1*AONV
     TEMP = 2.0D0*TEMP1*ROOT44
     D4410 = TEMP*F441*G410
     D4422 = TEMP*F442*G422
     TEMP1 = TEMP1*AONV
     TEMP = TEMP1*ROOT52
     D5220 = TEMP*F522*G520
     D5232 = TEMP*F523*G532
     TEMP = 2.0D0*TEMP1*ROOT54
     D5421 = TEMP*F542*G521
     D5433 = TEMP*F543*G533
     XLAMO = DMOD(Mo+nodeo+nodeo-THETA-THETA,TwoPi)
     XFACT = MDot + DMDT + 2.0D0 * (nodeDot+DNODT-RPTIM) - No
     Eccm = emo
     emsq = emsqo
    ENDIF
   IF (Irez .eq. 1) THEN
* ----- SYNCHRONOUS RESONANCE TERMS -----
     G200 = 1.0D0 + EMSO * (-2.5D0 + 0.8125D0 * EMSO)
     G310 = 1.0D0 + 2.0D0*EMSQ
     G300 = 1.0D0 + EMSQ * (-6.0D0 + 6.60937D0 * EMSQ)
     F220 = 0.75D0 * (1.0D0 + COSIM) * (1.0D0 + COSIM)
     F311 = 0.9375D0*SINIM*SINIM*
          (1.0D0+3.0D0*COSIM) - 0.75D0*(1.0D0+COSIM)
  &
```

```
F330 = 1.0D0 + COSIM
      F330 = 1.875D0*F330*F330*F330
      DEL1 = 3.0D0*XN*XN*AONV*AONV
      DEL2 = 2.0D0*DEL1*F220*G200*Q22
                                                                                  Formatted: German
                                                                                   (Germany)
      DEL3 = 3.0D0*DEL1*F330*G300*Q33*AONV
      DEL1 = DEL1*F311*G310*Q31*AONV
      XLAMO = DMOD(Mo+nodeo+Argpo-THETA,TwoPi)
      XFACT = MDot + XPIDOT - RPTIM + DMDT + DOMDT + DNODT - No
     ENDIF
* ----- FOR SGP4, INITIALIZE THE INTEGRATOR -----
    XLI = XLAMO
    XNI = No
    ATIME = 0.0D0
    XN = No + DNDT
   ENDIF! Ires non-zero
  RETURN
  END! end dsinit
               SUBROUTINE DSPACE
 This Subroutine provides deep space contributions to mean elements for
  perturbing third body. these effects have been averaged over one
  revolution of the sun and moon. for earth resonance effects, the
  effects have been averaged over no revolutions of the satellite.
  (mean motion)
  author
           : david vallado
                                 719-573-2600 28 jun 2005
                                                                                  Formatted: German
                                                                                   (Germany)
  d2201, d2211, d3210, d3222, d4410, d4422, d5220, d5232, d5421, d5433
  dedt
  del1, del2, del3 -
  didt
  dmdt
  dnodt
  domdt
          - flag for resonance
                                0-none, 1-one day, 2-half day
  irez
   argpo
           - argument of perigee
   argpdot - argument of perigee dot (rate)
         - time
  t
  tc
```

```
gsto
        - gst
xfact
xlamo
no
        - mean motion
atime
        - eccentricity
em
ft
          - argument of perigee
argpm
         - inclination
inclm
xli
mm
         - mean anomaly
xni
        - mean motion
nodem
          - right ascension of ascending node
outputs
atime
em
        - eccentricity
          - argument of perigee
argpm
         - inclination
inclm
xli
         - mean anomaly
mm
xni
          - right ascension of ascending node
nodem
dndt
        - mean motion
nm
coupling
none
references :
hoots, roehrich, norad spacetrack report #3 1980
hoots, norad spacetrack report #6 1986
hoots, schumacher and glover 2004
vallado, crawford, hujsak, kelso 2006
SUBROUTINE DSPACE(IRez, D2201, D2211, D3210, D3222, D4410,
            D4422, D5220, D5232, D5421, D5433, Dedt,
&
&
            Del1, Del2, Del3, Didt, Dmdt, Dnodt,
            Domdt, Argpo, ArgpDot, T, TC, GSTo,
&
            Xfact, Xlamo, No
&
            Atime, Eccm, Argpm, Inclm, Xli, Mm,
&
            XNi , nodem, Dndt , XN )
&
                                                                                 Formatted: French
                                                                                  (France)
  IMPLICIT NONE
 INTEGER IRez
                                                                                 Formatted: German
                                                                                  (Germany)
```

```
Real*8 D2201, D2211, D3210, D3222, D4410, D4422, D5220,
  &
         D5232, D5421, D5433, Dedt, Del1, Del2, Del3,
         Didt, Dmdt, Dnodt, Argpo, ArgpDot, T,
  &
         TC , GSTo , Xfact , Xlamo , No , Atime , Eccm ,
  &
         Argpm, Inclm, Xli, Mm, Xni, nodem, Dndt,
  &
  &
  ------ Local Variables -----
   INTEGER iretn, iret
   REAL*8 Delt , Ft , theta , x2li , x2omi , xl , xldot ,
         xnddt, xndt, xomi
   REAL*8 G22 , G32 , G44 , G52 , G54 , Fasx2 ,
         Fasx4, Fasx6, RPtim, Step2, Stepn, Stepp
   COMMON /DebugHelp/ Help
   CHARACTER Help
   INCLUDE 'ASTMATH.CMN'
* ------ Constants -----
                                                                          Formatted: French
                                                                           (France)
   FASX2 = 0.13130908D0
   FASX4 = 2.8843198D0
   FASX6 = 0.37448087D0
   G22 = 5.7686396D0
                                                                          Formatted: German
                                                                           (Germany)
   G32 = 0.95240898D0
   G44 = 1.8014998D0
   G52 = 1.0508330D0
   G54 = 4.4108898D0
   RPTIM = 4.37526908801129966D-3
   STEPP = 720.0D0
   STEPN = -720.0D0
   STEP2 = 259200.0D0
* ------ CALCULATE DEEP SPACE RESONANCE EFFECTS ------
   DNDT = 0.0D0
   THETA = DMOD(GSTo + TC*RPTIM,TwoPi)
   Eccm = Eccm + DEDT*T
                                                                          Formatted: German
                                                                           (Germany)
   Inclm = Inclm + DIDT*T
   Argpm = Argpm + DOMDT*T
   nodem = nodem + DNODT*T
   Mm = Mm + DMDT*T
c sgp4fix for negative inclinations
c the following if statement should be commented out
    IF(Inclm .lt. 0.0D0) THEN
c
      Inclm = -Inclm
```

```
Argpm = Argpm-PI
       nodem = nodem + PI
                                                                              Formatted: German
                                                                               (Germany)
      ENDIF
c
c sgp4fix for propagator problems
c the following integration works for negative time steps and periods
c the specific changes are unknown because the original code was so convoluted
   sgp4fix take out atime = 0.0 and fix for faster operation
    Ft = 0.0D0
                 ! Just in case - should be set in loops if used.
    IF (IREZ .ne. 0) THEN
* ----- UPDATE RESONANCES: NUMERICAL (EULER-MACLAURIN)
INTEGRATION ---
* ------ EPOCH RESTART ------
    ! sgp4fix streamline check
    IF ((atime .eq. 0.0D0) .or. (t * atime .le. 0.0D0) .or.
       (dabs(t) .lt. dabs(atime)) ) THEN
       atime = 0.0D0
       xni = no
       xli = xlamo
      ENDIF
     ! sgp4fix move check outside loop
     IF (t.gt. 0.0D0) THEN
       delt = stepp
      else
       delt = stepn
      ENDIF
      iretn = 381! added for do loop
      iret = 0! added for loop
      DO WHILE (IRetn.eq.381)
* ----- DOT TERMS CALCULATED -----
* ----- NEAR - SYNCHRONOUS RESONANCE TERMS -----
      IF (IREZ .ne. 2) THEN
       XNDT = DEL1*DSIN(XLI-FASX2) +
             DEL2*DSIN(2.0D0*(XLI-FASX4)) +
                                                                              Formatted: German
                                                                              (Germany)
             DEL3*DSIN(3.0D0*(XLI-FASX6))
  &
        XLDOT = XNI + XFACT
        XNDDT = DEL1*DCOS(XLI-FASX2) +
  &
          2.0D0*DEL2*DCOS(2.0D0*(XLI-FASX4)) +
          3.0D0*DEL3*DCOS(3.0D0*(XLI-FASX6))
  &
        XNDDT = XNDDT*XLDOT
       ELSE
* ------ NEAR - HALF-DAY RESONANCE TERMS ------
```

```
XOMI = Argpo + ArgpDot*ATIME
              X2OMI= XOMI + XOMI
              X2LI = XLI + XLI
              XNDT = D2201*DSIN(X2OMI+XLI-G22) + D2211*DSIN(XLI-G22) +
                          D3210*DSIN( XOMI+XLI-G32) +
&
                                                                                                                                                                                                Formatted: German
                                                                                                                                                                                                 (Germany)
&
                          D3222*DSIN(-XOMI+XLI-G32) +
&
                         D4410*DSIN(X2OMI+X2LI-G44)+ D4422*DSIN(X2LI-G44)+
&
                         D5220*DSIN(XOMI+XLI-G52)+
                         D5232*DSIN(-XOMI+XLI-G52) +
&
&
                         D5421*DSIN( XOMI+X2LI-G54)+
                                                                                                                                                                                                Formatted: French
                                                                                                                                                                                                 (France)
&
                         D5433*DSIN(-XOMI+X2LI-G54)
              XLDOT = XNI + XFACT
             XNDDT = D2201*DCOS(X2OMI+XLI-G22) + D2211*DCOS(XLI-G22) + D2211*
                           D3210*DCOS(XOMI+XLI-G32)+
&
                                                                                                                                                                                                Formatted: German
                                                                                                                                                                                                 (Germany)
&
                           D3222*DCOS(-XOMI+XLI-G32) +
&
                           D5220*DCOS( XOMI+XLI-G52) +
&
                           D5232*DCOS(-XOMI+XLI-G52) +
                           2.0D0*(D4410*DCOS(X2OMI+X2LI-G44) +
&
                                                                                                                                                                                                Formatted: French
&
                           D4422*DCOS(X2LI-G44) +
&
                           D5421*DCOS( XOMI+X2LI-G54) +
                           D5433*DCOS(-XOMI+X2LI-G54))
&
              XNDDT = XNDDT*XLDOT
           ENDIF
                       ----- INTEGRATOR -----
           ! sgp4fix move end checks to end of routine
           IF (DABS(T-ATIME).ge.STEPP) THEN
                IRET = 0
                IRETN = 381
              ELSE
                FT = T-ATIME
                IRETN = 0
              ENDIF
           IF (IRETN.EQ.381) THEN
                XLI = XLI + XLDOT*DELT + XNDT*STEP2
                XNI = XNI + XNDT*DELT + XNDDT*STEP2
                ATIME = ATIME + DELT
              ENDIF
           ENDDO
         XN = XNI + XNDT*FT + XNDDT*FT*FT*0.5D0
         XL = XLI + XLDOT*FT + XNDT*FT*FT*0.5D0
         IF(IREZ .ne. 1) THEN
              Mm = XL-2.0D0*nodem+2.0D0*THETA
```

```
DNDT = XN-No
     ELSE
      Mm = XL-nodem-Argpm+THETA
      DNDT = XN-No
     ENDIF
    XN = No + DNDT
   ENDIF
 RETURN
 END ! end dspace
             SUBROUTINE JDay1
This subroutine finds the Julian date given the Year, Month, Day, and Time.
Author
          : David Vallado
                                  719-573-2600 1 Mar 2001
Inputs
           Description
                                Range / Units
                              1900 .. 2100
          - Year
 Year
 Mon
          - Month
                               1...12
                              1.. 28,29,30,31
 Day
         - Day
         - Universal Time Hour
 Hr
                                   0 \dots 23
         - Universal Time Min
                                   0..59
 Min
         - Universal Time Sec
                                   0.0D0 .. 59.999D0
 Sec
 WhichType - Julian .or. Gregorian calender 'J' .or. 'G'
Outputs
 JD
         - Julian Date
                               days from 4713 BC
Locals
        - Var to aid Gregorian dates
 В
Coupling
 None.
References:
 Vallado
            2007, 189, Alg 14, Ex 3-14
 SUBROUTINE JDAY1
                          (Year,Mon,Day,Hr,Min, Sec, JD)
  IMPLICIT NONE
  INTEGER Year, Mon, Day, Hr, Min
  REAL*8 Sec, JD
```

```
! ----- Implementation -----
  JD= 367.0D0 * Year
      - INT( (7* (Year+INT ( (Mon+9)/12.0) ) ) * 0.25D0 )
                                                                                  Formatted: French
                                                                                   (France)
      + INT( 275*Mon / 9.0 )
&
&
      + Day + 1721013.5D0
&
      + ((Sec/60.0D0 + Min) / 60.0D0 + Hr) / 24.0D0
      -0.5D0*DSIGN(1.0D0, 100.0D0*Year + Mon - 190002.5D0) + 0.5D0
 RETURN
 END! end jday1
             SUBROUTINE DAYS2MDHMS
This subroutine converts the day of the year, days, to the equivalent month
 day, hour, Minute and second.
Algorithm : Set up array for the Number of days per month
         Find Leap Year - be sure to account for the 400 years
         Loop through a Temp value for WHILE the value is .lt. the days
         Perform INTEGER conversions to the correct day and month
         Convert remainder into H M S using type conversions
Author
          : David Vallado
                                  719-573-2600 1 Mar 2001
Inputs
           Description
                                Range / Units
         - Year
                             +1900 .. 2100+
 Year
                                   0.0D0 .. 366.0D0
 Days
          - Julian Day of the year
OutPuts
 Mon
          - Month
                               1...12
                             1.. 28,29,30,31
         - Dav
 Day
 Hr
         - Hour
                             0 .. 23
         - Minute
                              0...59
 Min
         - Second
                              0.0D0 .. 59.999D0
 Sec
Locals
 DayofYr - Day of year
          - Temporary REAL*8 values
 IntTemp - Temporary INTEGER value
 LMonth[12] - INTEGER Array containing the Number of days per month
Coupling
```

```
None.
  SUBROUTINE DAYS2MDHMS (Year, Days, Mon, Day, Hr, Min, Sec)
   IMPLICIT NONE
   REAL*8 Days,Sec
   INTEGER Year, Mon, Day, Hr, Min
* ------ Locals -----
   INTEGER IntTemp,i,DayofYr, LMonth(12)
   REAL*8 Temp
   ! ----- Implementation -----
   ! ----- Set up array of days in month -----
   DO i = 1.12
     LMonth(i) = 31
    ENDDO
   LMonth(2) = 28
   LMonth(4) = 30
   LMonth(6) = 30
   LMonth(9) = 30
   LMonth(11) = 30
   DayofYr=IDINT(Days)
   ! ----- Find month and Day of month -----
   IF (MOD(Year,4).eq.0) THEN
     LMonth(2)=29
    ENDIF
   i=1
   IntTemp=0
   DO WHILE ( (DayofYr.gt.IntTemp + LMonth(i) ) .and. (i.lt.12))
     IntTemp = IntTemp + LMonth(i)
     i=i+1
    ENDDO
   Mon=i
   Day= DayofYr - IntTemp
   ! ----- Find hours Minutes and seconds -----
   Temp= (Days - DayofYr)*24.0D0
   Hr = IDINT(Temp)
   Temp = (Temp-Hr) * 60.0D0
                                                                            Formatted: French
                                                                             (France)
   Min = IDINT( Temp )
   Sec = (Temp-Min) * 60.0D0
  RETURN
  END! end days2mdhms
```

A.2 Other Codes

Attached on the accompanying CD, there are other codes in their entirety. These include:

- PREDICT (Feb. Version) with updated SGP4 propagator.
- MATLAB version of Vallado's SGP4 propagator.
- FORTRAN version of Vallado's SGP4 propagator.

Additionally, the analysis files are also provided on the CD.

APPENDIX B. Verification TLE File

# Verification test cases			
# # TEME example			
1 00005U 58002B 00179.78495062 .00000023 00000-0 28098-4 0 4753			
2 00005 34.2682 348.7242 1859667 331.7664 19.3264 10.82419157413667	0.00	4320.0	360.00
# ## fig show lyddane fix error with gsfc ver			
1 04632U 70093B	51040	4006.0	120.00
2 04632 11.4628 273.1101 1450506 207.6000 143.9350 1.20231981 44145	-5184.0	-4896.0	120.00
# DELTA 1 DEB # near earth normal drag equation # perigee = 377.26km, so moderate drag case			
1 06251U 62025E 06176.82412014 .00008885 00000-0 12808-3 0 3985			
2 06251 58.0579 54.0425 0030035 139.1568 221.1854 15.56387291 6774	0.0	2880.0	120.00
# MOLNIYA 2-14 # 12h resonant ecc in 0.65 to 0.7 range	0.0	2000.0	120.00
1 08195U 75081A 06176.33215444 .00000099 00000-0 11873-3 0 813			
2 08195 64.1586 279.0717 6877146 264.7651 20.2257 2.00491383225656	0.0	2880.0	120.00
# MOLNIYA 1-36 ## fig 12h resonant ecc in 0.7 to 0.715 range			
1 09880U 77021A 06176.56157475 .00000421 00000-0 10000-3 0 9814			
2 09880 64.5968 349.3786 7069051 270.0229 16.3320 2.00813614112380	0.0	2880.0	120.00
# SMS 1 AKM # show the integrator problem with gsfc ver			
1 09998U 74033F 05148.7941792800000112 00000-0 00000+0 0 4480	1.440.0	720.00	60.0
2 09998 9.4958 313.1750 0270971 327.5225 30.8097 1.16186785 45878 - # # Original STR#3 SDP4 test	1440.0	-720.00	60.0
# # Original STR#3 SDP4 test 1 11801U 80230.29629788 .01431103 00000-0 14311-1 13			
	0.0 14	440.0	360.00
# EUTELSAT 1-F1 (ECS1)## fig lyddane choice in GSFC at 2080 min	0.0 1	110.0 .	00.00
1 14128U 83058A 06176.0284489300000158 00000-0 10000-3 0 9627			
2 14128 11.4384 35.2134 0011562 26.4582 333.5652 0.98870114 46093	0.0	2880.0	120.00
# SL-6 R/B(2) # Deep space, perigee = $82.48 (< 98)$ for			
# # s4 > 20 mod			
1 16925U 86065D 06151.67415771 .02550794 -30915-6 18784-3 0 4486			
2 16925 62.0906 295.0239 5596327 245.1593 47.9690 4.88511875148616	0.0	1440.0	120.00
# SL-12 R/B # Shows Lyddane choice at 1860 and 4700 min			
1 20413U 83020D 05363.79166667 .00000000 00000-0 00000+0 0 7041	1440.0	4220.0	120.00
2 20413 12.3514 187.4253 7864447 196.3027 356.5478 0.24690082 7978 # MOLNIYA 1-83 # 12h resonant, ecc > 0.715 (negative BSTAR)	1440.0	4320.0	120.00
1 21897U 92011A 06176.0234124400001273 00000-0 -13525-3 0 3044			
2 21897 62.1749 198.0096 7421690 253.0462 20.1561 2.01269994104880	0.0	2880.0	120.00
# SL-6 R/B(2) # last tle given, decayed 2006-04-04, day 94	0.0	2000.0	120.00
1 22312U 93002D 06094.46235912 .99999999 81888-5 49949-3 0 3953			
2 22312 62.1486 77.4698 0308723 267.9229 88.7392 15.95744531 98783	54.2028	672 1440.	0 20.00
# SL-6 R/B(2) # 12h resonant ecc in the > 0.715 range			
1 22674U 93035D 06176.55909107 .00002121 00000-0 29868-3 0 6569			
2 22674 63.5035 354.4452 7541712 253.3264 18.7754 1.96679808 93877	0.0	2880.0	120.00
# ARIANE 44L+ R/B # Lyddane bug at <= 70 min for atan2(),			
# # no quadrant fix			
1 23177U 94040C 06175.45752052 .00000386 00000-0 76590-3 0 95 2 23177 7.0496 179.8238 7258491 296.0482 8.3061 2.25906668 97438	0.0	1440.0	120.00
# WIND # STR#3 Kepler failes past about 200 min	0.0	1440.0	120.00
1 23333U 94071A 94305.4999999900172956 26967-3 10000-3 0 15			
2 23333 28.7490 2.3720 9728298 30.4360 1.3500 0.07309491 70 0.	0 160	0.0 12	0.00
# ARIANE 42P+3 R/B ## fig Lyddane bug at > 280.5 min for AcTan()			
1 23599U 95029B 06171.76535463 .00085586 12891-6 12956-2 0 2905			
2 23599 6.9327 0.2849 5782022 274.4436 25.2425 4.47796565123555	0.0	720.0	20.00
# ITALSAT 2 # 24h resonant GEO, inclination > 3 deg			
1 24208U 96044A 06177.0406174000000094 00000-0 10000-3 0 1600			100.00
2 24208 3.8536 80.0121 0026640 311.0977 48.3000 1.00778054 36119	0.0	1440.0	120.00
# AMC-4 ## fig low incl, show incl shift with # gsfc version from 240 to 1440 min			
## gstc version from 240 to 1440 min 1 25954U 99060A 04039.6805728500000108 00000-0 00000-0 0 6847			
12575.657700011 01057.0005720500000100 00000-0 00000-0 0 00-7			

2 25954 0.0004 243.8136 0001765 15.5294 22.7134 1.00271289 15615 -1 # INTELSAT 902 # negative incl at 9313 min then # 270 deg Lyddane bug at 37606 min	440.0	1440.0	120.00
1 26900U 01039A 06106.74503247 .00000045 00000-0 10000-3 0 8290 2 26900 0.0164 266.5378 0003319 86.1794 182.2590 1.00273847 16981 9 COSMOS 1024 DEB # 12h resonant ecc in 0.5 to 0.65 range	9300.00	9400.00	60.00
1 26975U 78066F 06174.85818871 .00000620 00000-0 10000-3 0 6809 2 26975 68.4714 236.1303 5602877 123.7484 302.5767 2.05657553 67521 # CBERS 2 # Near Earth, ecc = 8.84E-5 (< 1.0e-4)	0.0	2880.0	120.00
# # drop certain normal drag terms 1 28057U 03049A 06177.78615833 .00000060 00000-0 35940-4 0 1836 2 28057 98.4283 247.6961 0000884 88.1964 271.9322 14.35478080140550 # NAVSTAR 53 (USA 175)# 12h non-resonant GPS (ecc < 0.5 ecc)	0.0	2880.0	120.00
1 28129U 03058A 06175.5707113600000104 00000-0 10000-3 0 459 2 28129 54.7298 324.8098 0048506 266.2640 93.1663 2.00562768 18443 # COSMOS 2405 # Near Earth, perigee = 127.20 (< 156) s4 mod	0.0	1440.0	120.00
# Real Earth, perigee = 127.20 (< 130) \$4 inod 1 28350U 04020A 06167.21788666 .16154492 76267-5 18678-3 0 8894 2 28350 64.9977 345.6130 0024870 260.7578 99.9590 16.47856722116490 # H-2 R/B # Deep space, perigee = 135.75 (<156) \$4 mod	0.0	2880.0	120.00
1 28623U 05006B 06177.81079184 .00637644 69054-6 96390-3 0 6000 2 28623 28.5200 114.9834 6249053 170.2550 212.8965 3.79477162 12753 # XM-3 # 24h resonant geo, incl < 3 deg goes	0.0	1440.0	120.00
# MINOTAUR R/B # Sub-orbital case - Decayed 2005-11-29	0.0	1440.0	120.00
# #(perigee = -51km), lost in 50 minutes 1 28872U 05037B 05333.02012661 .25992681 00000-0 24476-3 0 1534 2 28872 96.4736 157.9986 0303955 244.0492 110.6523 16.46015938 10708 # SL-14 DEB # Last stage of decay - lost in under 420 min	0.0	50.0	5.00
1 29141U 85108AA 06170.26783845 .99999999 00000-0 13519-0 0 718 2 29141 82.4288 273.4882 0015848 277.2124 83.9133 15.93343074 6828 # SL-12 DEB # Near Earth, perigee = 212.24 < 220	0.0	440.0	20.00
# # simplified drag eq 1 29238U 06022G 06177.28732010 .00766286 10823-4 13334-2 0 101 2 29238 51.5595 213.7903 0202579 95.2503 267.9010 15.73823839 1061 # # Original STR#3 SGP4 test	0.0	1440.0	120.00
1 88888U 80275.98708465 .00073094 13844-3 66816-4 0 87 2 88888 72.8435 115.9689 0086731 52.6988 110.5714 16.05824518 1058 #	0.0	1440.0	120.00

APPENDIX C – Verification Results Data

```
5 xx
             0.00000000 7022.46529266 -1400.08296755
                                                                                                                                                                                          0.03995155 1.893841015 6.405893759 4.534807250
       360 00000000 -7154 03120202 -3783 17682504 -3536 19412294 4 741887409 -4 151817765 -2 093935425
       1080.00000000 5568.53901181 4492.06992591
                                                                                                                                                                                       3863.87641983 -4.209106476 5.159719888 2.744852980
     1800.00000000 -9680.56121728 2802.47771354 124.10688038 -0.905874102 -4.659467970 -3.227347517
     2160.000000000
                                                               190.19796988 7746.96653614 5110.00675412 -6.112325142 1.527008184 -0.139152358
     2520.00000000 5579.55640116 -3995.61396789 -1518.82108966 4.767927483 5.123185301 4.276837355
     3240.00000000 -5429.79204164 7574.36493792 3747.39305236 -4.999442110 -1.800561422 -2.229392830
     3600.00000000 6759.04583722 2001.58198220 2783.55192533 -2.180993947 6.402085603 3.644723952
    3960 00000000 -3791 44531559 -5712 95617894 -4533 48630714 6 668817493 -2 516382327 -0 082384354
    4320.00000000 -9060.47373569 4658,70952502 813.68673153 -2.232832783 -4.110453490 -3.157345433
4632 xx
             0.00000000 2334.11450085 -41920.44035349
                                                                                                                                                                                           -0.03867437 2.826321032 -0.065091664 0.570936053
    -5064\ 00000000\ -32982\ 56870101\ -11125\ 54996609\ -6803\ 28472771\ 0\ 617446996\ -3\ 379240041\ 0\ 085954707
    -4944.0000000 -22097.68730513 -31583.13829284 -4836.34329328 2.230597499 -2.166594667 0.426443070
   -4896.00000000 \ \ -15129.94694545 \ \ -36907.74526221 \ \ \ -3487.56256701 \ \ 2.581167187 \ \ -1.524204737 \ \ 0.504805763
6251 xx
             0.00000000 3988.31022699 5498.96657235
                                                                                                                                                                                         0.90055879 -3.290032738 2.357652820 6.496623475
       120.00000000 -3935.69800083 409.10980837
                                                                                                                                                                                    5471.33577327 -3.374784183 -6.635211043 -1.942056221
       240.00000000 -1675.12766915 -5683.30432352 -3286.21510937 5.282496925 1.508674259 -5.354872978
       480.000000000 \quad -1115.07959514 \quad 4015.11691491 \quad 5326.99727718 \quad -5.524279443 \quad -4.765738774 \quad 2.402255961 \quad -4.765738774 \quad -4.76573774 \quad -4.76573774 \quad -4.7657774 \quad -4.7657774 \quad -4.7657774 \quad -4.765774 \quad -4.765774 \quad
       600.00000000 \quad -4329.10008198 \quad -5176.70287935 \quad 409.65313857 \quad 2.858408303 \quad -2.933091792 \quad -6.509690397 \quad -6.50969007 \quad -6.5096907 \quad -6.5096907 \quad -6.509607 \quad -6.509607 \quad -6.50907 \quad -6.509607 \quad -6.5097 \quad -6.509607 \quad -6.509607 \quad -6.509
       720.000000000 \quad 3692.60030028 \quad -976.24265255 \quad -5623.36447493 \quad 3.897257243 \quad 6.415554948 \quad 1.429112190 \quad -976.24265255 \quad -976.2426525 \quad -976.242
```

840.00000000 2301.83510037 5723.92394553 2814.61514580 -5.110924966 -0.764510559 5.662120145 960 00000000 -4990 91637950 -2303 42547880 3920 86335598 -0 993439372 -5 967458360 -4 759110856 $1080.00000000 \qquad 642.27769977 \quad -4332.89821901 \quad -5183.31523910 \quad 5.720542579 \quad 4.216573838 \quad -2.846576139 \quad -2.8467676139 \quad -2.8467676197761977019 \quad -2.84676777779 \quad -2.8467677779 \quad -2.846767779 \quad -2.846767779 \quad -2.846767779 \quad$ $1200.00000000 \quad 4719.78335752 \quad 4798.06938996 \quad -943.58851062 \quad -2.294860662 \quad 3.492499389 \quad 6.408334723 \quad -2.294860662 \quad -2.294860666 \quad -2.2948606666 \quad -2.294860666 \quad -2.2948606666 \quad -2.294860666 \quad -2.29486066666 \quad -2.294860666 \quad -2.2948606666 \quad -2.2948606666 \quad -2.29486066666 \quad -2.2948606666 \quad -2.29486066666 \quad -2.294860666$ $1320.00000000 \quad -3299.16993602 \quad 1576.83168320 \quad 5678.67840638 \quad -4.460347074 \quad -6.202025196 \quad -0.885874586 \quad -0.885874686 \quad -0.8858746 \quad -0.885846 \quad -0.8858746 \quad -0.885846 \quad -0.885846 \quad -0.885846 \quad -0.885866 \quad$ 1440.00000000 -2777.14682335 -5663.16031708 -2462.54889123 4.915493146 0.123328992 -5.896495091 1560.00000000 4992.31573893 1716.62356770 -4287.86065581 1.640717189 6.071570434 4.338797931 1680.00000000 -8.22384755 4662.21521668 4905.66411857 -5.891011274 -3.593173872 3.365100460 1800.00000000 -4966.20137963 -4379.59155037 1349.33347502 1.763172581 -3.981456387 -6.343279443 $1920.00000000 \quad 2954.49390331 \quad -2080.65984650 \quad -5754.75038057 \quad 4.895893306 \quad 5.858184322 \quad 0.375474825 \quad -695984650 \quad -69598460 \quad -69598400 \quad -695984000 \quad -69598400 \quad -695984000 \quad -695984000 \quad -695984000 \quad -695984000 \quad -695984000 \quad -695984000 \quad$ 2040.0000000 3363.28794321 5559.55841180 1956.05542266 -4.587378863 0.591943403 6.107838605 2160.00000000 -4856.66780070 -1107.03450192 4557.21258241 -2.304158557 -6.186437070 -3.956549542 $2280.00000000 \quad -497.84480071 \quad -4863.46005312 \quad -4700.81211217 \quad 5.960065407 \quad 2.996683369 \quad -3.767123329 \quad -4700.81211217 \quad -4863.46005312 \quad -4863.46005312$ 2400.00000000 5241.61936096 3910.75960683 -1857.93473952 -1.124834806 4.406213160 6.148161299 2520.00000000 -2451.38045953 2610.60463261 5729.79022069 -5.366560525 -5.500855666 0.187958716 2640.00000000 -3791.87520638 -5378.82851382 -1575.82737930 4.266273592 -1.199162551 -6.276154080 2760,00000000 4730,53958356 524.05006433 -4857.29369725 2.918056288 6.135412849 3.495115636 2880.00000000 1159.27802897 5056.60175495 4353.49418579 -5.968060341 -2.314790406 4.230722669

8195 xx

0.00000000 2349.89483350 -14785.93811562 0.02119378 2.721488096 -3.256811655 4.498416672 120.00000000 15223.91713658 -17852.95881713 25280.39558224 1.079041732 0.875187372 2.485682813 240.00000000 19752.78050009 -8600.07130962 37522.72921090 0.238105279 1.546110924 0.986410447

360.00000000 19089.29762968 3107.89495018 39958.14661370 -0.410308034 1.640332277 -0.306873818 480.00000000 13829.66070574 13977.39999817 32736.32082508 -1.065096849 1.279983299 -1.760166075 $600.000000000 \quad 3333.05838525 \quad 18395.31728674 \quad 12738.25031238 \quad -1.882432221 \quad -0.611623333 \quad -4.039586549 \quad -0.611623333 \quad -4.039586549 \quad -0.611623333 \quad -0.61162333 \quad -0.611623$ 720 00000000 2622 13222207 -15125 15464924 474 51048398 2 688287199 -3 078426664 4 494979530 840.00000000 15320.56770017 -17777.32564586 25539.53198382 1.064346229 0.892184771 2.459822414 $960.00000000 \quad 19769.70267785 \quad -8458.65104454 \quad 37624.20130236 \quad 0.229304396 \quad 1.550363884 \quad 0.966993056 \quad 0.229304396 \quad 0.22930496 \quad 0.229306 \quad 0.$ 1200.00000000 13729.19205837 14097.70014810 32547.52799890 -1.074511043 1.270505211 -1.785099927 $1320.000000000 \quad 3148.86165643 \quad 18323.19841703 \quad 12305.75195578 \quad -1.895271701 \quad -0.678343847 \quad -4.086577951 \quad -4.086777951 \quad -4.086577951 \quad -4.086777951 \quad -4.086777951 \quad -4.08677791 \quad -4.086777791 \quad -4.086777791 \quad -4.086777791$ 1440.00000000 2890.80638268 -15446.43952300 948.77010176 2.654407490 -2.909344895 4.486437362 1560 00000000 15415 98410712 -17699 90714437 25796 19644689 1.049818334 0.908822332 2.434107329 $1680.00000000 \quad 19786.00618538 \quad -8316.74570581 \quad 37723.74539119 \quad 0.220539813 \quad 1.554518900 \quad 0.947601047 \quad 0.9$ 2040.00000000 2963.26486560 18243.85063641 11868.25797486 -1.908015447 -0.747870342 -4.134004492 2160.00000000 3155.85126036 -15750.70393364 1422.32496953 2.620085624 -2.748990396 4.473527039 2280.00000000 15510.15191770 -17620.71002219 26050.43525345 1.035454678 0.925111006 2.408534465 2400.00000000 19801.67198812 -8174.33337167 37821.38577439 0.211812700 1.558576937 0.928231880 $2520.00000000 \quad 18965.46529379 \quad 3565.19666242 \quad 39847.97510998 - 0.433459945 \quad 1.637120585 - 0.364653213 \quad 1.637120585 - 0.36465213 \quad 1.637120585 - 0.36465213 \quad 1.637120585 - 0.364653213 \quad 1.637120585 - 0.3646520 - 0.3646520 \quad 1.63712058 - 0.3646520 - 0.364600 - 0.364600 - 0.364600 - 0.364600 - 0.364600 - 0.364600 - 0.364600 - 0.364600 - 0.364600 - 0.364600 - 0.364600 - 0.$ $2640.00000000 \quad 13525.88227400 \quad 14335.15978787 \quad 32162.13236536 \cdot 1.093537945 \quad 1.250868256 \cdot 1.835451681 \quad 1.250868256 \cdot 1.8354681 \quad 1.250868256 \cdot 1.8354681 \quad 1.250868256 \cdot 1.250868256 \quad 1.250868676 \quad 1.2508668676 \quad 1.2508668676 \quad 1.250866866 \quad 1.250$ 2760.00000000 2776.30574260 18156.98538451 11425.73046481 -1.920632199 -0.820370733 -4.181839232

9880 xx

 $0.00000000 \quad 13020.06750784 \quad \text{-}2449.07193500$ 1.15896030 4.247363935 1.597178501 4.956708611 240.00000000 11332.67806218 16517.99124008 38569.78482991 -1.400974747 0.710947006 0.923935636 360.00000000 328.74217398 19554.92047380 40558.26246145 -1.593281066 0.126772913 -0.359627307 480.00000000 -10684.90590680 18057.15728839 33158.75253886 -1.383205997 -0.582328999 -1.744412556 $600.00000000 - 17069.78000550 \quad 9944.86797897 \quad 13885.91649059 \quad 0.044133354 - 1.853448464 - 3.815303117 - 1.853448464 - 1.85344864 - 1.8534864 - 1.85344864 - 1.85344864 - 1.85344864 - 1.85344864 - 1.85344864 - 1.85344864 - 1.8534864 - 1.8534864 - 1.85344864 - 1.8534864 - 1.8534864 - 1.8534864 - 1.8534864 - 1.8534864 - 1.8534864 - 1.8534864 - 1.8534864 - 1.8534864 - 1.8534864 - 1.8534866 - 1.8534866 - 1.8534866 - 1.8534866 - 1.853486 - 1.853486 - 1.853486 - 1.853486 - 1.853486 - 1.853486 - 1.853486 - 1.853486 - 1.853486 - 1.853486 - 1.853486 - 1.85348 - 1.853486 - 1.8$ $720.00000000 \quad 13725.09398980 \quad -2180.70877090 \quad 863.29684523 \quad 3.878478111 \quad 1.656846496 \quad 4.944867241 \quad 4.9448672411 \quad 4.944867241 \quad 4.944$ 840.00000000 19089.63879226 9456.29670247 27026.79562883 -0.656614299 1.309112636 2.449371941 $960.00000000 \quad 11106.41248373 \quad 16627.60874079 \quad 38727.35140296 - 1.409722680 \quad 0.698582526 \quad 0.891383535 \quad 0.698582526 \quad 0.8913835 \quad 0.6985825 \quad 0.6985825$ 1080.00000000 72.40958621 19575.08054144 40492.12544001 -1.593394604 0.113655142 -0.390556063 1320.00000000 -17044.61207568 9635.48491849 13212.59462953 0.129244030 -1.903551430 -3.884569098 $1440.00000000 \quad 14369.90303735 \quad -1903.85601062 \quad 1722.15319852 \quad 3.543393116 \quad 1.701687176 \quad 4.913881358 \quad 1.901687176 \quad 4.913881358 \quad 1.901687176 \quad 4.913881358 \quad 1.901687176 \quad 1.90167176 \quad 1.90167176 \quad 1.90167176 \quad 1.90167176 \quad 1.90167176 \quad 1.90167176 \quad 1.90167176$ 1560,00000000 18983,96210441 9661,12233804 27448,99557732 -0.687189304 1,293808870 2,403630759 $1680.00000000 \quad 10878.79336704 \quad 16735.31433954 \quad 38879.23434264 \quad -1.418239666 \quad 0.686235750 \quad 0.858951848 \quad -1.418239666 \quad 0.686235750 \quad 0.85895184 \quad -1.418239666 \quad 0.686235750 \quad 0.85896184 \quad -1.418239666 \quad 0.866235750 \quad 0.85896184 \quad 0.85896184 \quad -1.418239666 \quad 0.866235750 \quad 0.85896184 \quad -1.418239666 \quad 0.866235750 \quad 0.85896184 \quad 0.8$ $1800.00000000 \quad -184.03743100 \quad 19593.09371709 \quad 40420.40606889 \quad -1.593348925 \quad 0.100448697 \quad -0.421571993 \quad -0.421571999 \quad -0.421571999 \quad -0.421571999 \quad -0.42157199 \quad -0.42157199 \quad -0.42157199 \quad -0.42157199 \quad -0.$ 1920.00000000 -11125.12138631 17870.19488928 32534.21521208 -1.359116236 -0.621413776 -1.821629856 2040.00000000 -17004.43272827 9316.53926351 12526.11883812 0.220330736 -1.955594322 -3.955058575 2160.00000000 14960.06492693 -1620.68430805 2574.96359381 3.238634028 1.734723385 4.868880331 2280.00000000 18873.46347257 9863.57004586 27863.46574735 -0.716736981 1.278632817 2.358448535 2400.00000000 10649.86857581 16841.14172669 39025.48035006 -1.426527152 0.673901057 0.826632332 $2520.00000000 \quad -440.53459323 \quad 19608.95524423 \quad 40343.10675451 \quad -1.593138597 \quad 0.087147884 \quad -0.452680559 \quad -0.087147884 \quad -0.45268059 \quad -0.087147884 \quad -0.08714884 \quad -0.0871488$ 2640.00000000 -11342.45028909 17771.44223942 32211.12535721 -1.346344015 -0.641464291 -1.860864234 2760.00000000 -16948.06005711 8987.64254880 11826.28284367 0.318007297 -2.009693492 -4.026726648 $2880.00000000 \quad 15500.53445068 \quad -1332.90981042 \quad 3419.72315308 \quad 2.960917974 \quad 1.758331634 \quad 4.813698638 \quad 1.960917974 \quad 1.758331634 \quad 1.960917974 \quad 1.96$

9998 xx

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        31076.77273609
        22063.44379776
        6325.93403705 - 1.794027976
        2.642072476
        0.083556127

        -900.0000000
        23341.26015320
        30460.88002531
        6342.91707895 - 2.469409743
        1.990861658 - 0.073612096

        -840.00000000
        13568.39733054
        36204.45930900
        5806.79548733 - 2.919354203
        1.178920217 - 0.221646814

        -720.00000000
        -8535.81598158
        38171.79073851
        3331.00311285 - 3.043839958 - 0.644462527 - 0.445808894
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14128 xx

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16925 xx

20413 xx

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21897 xx

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22312 xx

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23177 xx

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23333 xx

23599 xx

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280.00000000 884.59720467 -4465.74516163 4725.83632696 0.380656028 5.691554046 5.303910983
300.00000000
                                                                                        446.40767236 4086.66839620 5093.05596650 -0.982424447 6.072965199 -4.791630682
320.00000000 -752.24467495 5588.35473301 -3275.04092573 -0.661161370 -4.016290740 -6.676898026
340.00000000 -643.72872525 -2585.02528560 -5923.01306608 0.807922142 -7.171597814 3.041115058
360.00000000
                                                                                           584.40295819 -6202.35605817 1781.00536019 0.869250450 2.226927514 7.471676765
                                                                                         779.59211765 1100.73728301 6311.59529480 -0.599552305 7.721032522 -1.275153027
380 000000000
400 00000000 -403 03155588 6399 18000837 -364 12735875 -1 008861924 -0 516636615 -7 799812287
420.00000000 -852.93910071
                                                                                                                                                                                         192.65232023 -6322.47054784 0.396006194 -7.882964919 -0.289331517
```

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APPENDIX D. REFERENCES

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ⁱⁱ Hoots, Felix R., et al. "History of Analytical Orbit Modeling in the U.S. Space Surveillance System." Journal of Guidance Control, and Dynamics March-April 2004: 174-185.

iii Vallado, David, et al. 'Revisiting Spacetrack Report #3', AIAA 2006-6753,

iv Vallado, David, et al. 'Revisiting Spacetrack Report #3', AIAA 2006-6753

^v Abercromby, Kira. "Object 8832" E-Mail to N. Miura, 26 May 2009.

vi "Selected Orbital Data" 2009. http://www.heavens-above.com

vii http://en.wikipedia.org/wiki/WGS84

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